Columbia University
Department of Physics
QUALIFYING EXAMINATION
Monday, January 8, 2007
9:00 AM – 11:00 AM

Classical Physics
Section 1. Classical Mechanics

Two hours are permitted for the completion of this section of the examination. Choose 4 problems out of the 5 included in this section. (You will not earn extra credit by doing an additional problem). Apportion your time carefully.

Use separate answer booklet(s) for each question. Clearly mark on the answer booklet(s) which question you are answering (e.g., Section 1 (Classical Mechanics), Question 1; Section 1(Classical Mechanics) Question 3, etc.)

Do **NOT** write your name on your answer booklets. Instead clearly indicate your **Exam Letter Code**

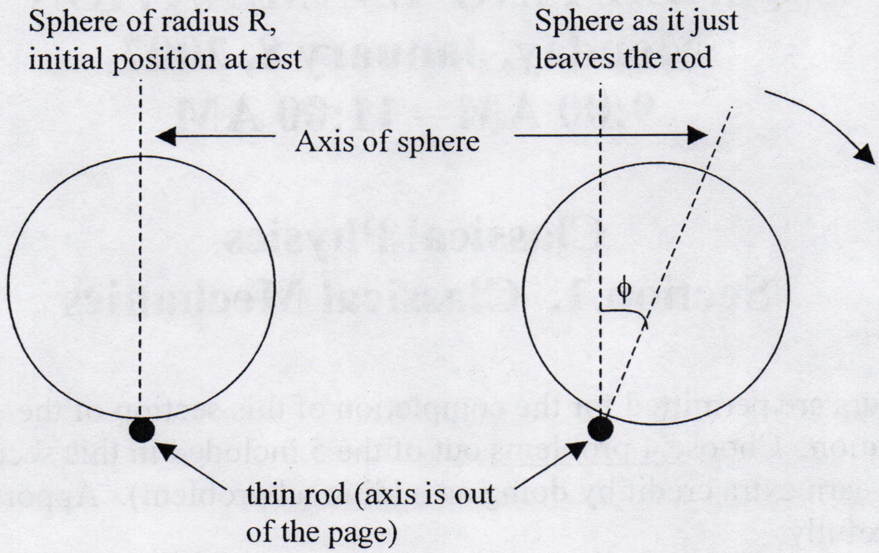
You may refer to the single handwritten note sheet on 8 ½ x 11" paper (double-sided) you have prepared on Classical Physics. The note sheet cannot leave the exam room once the exam has begun. This note sheet must be handed in at the end of today's exam. Please include your Exam Letter Code on your note sheet. No other extraneous papers or books are permitted.

Simple calculators are permitted. However, the use of calculators for storing and/or recovering formulae or constants is NOT permitted.

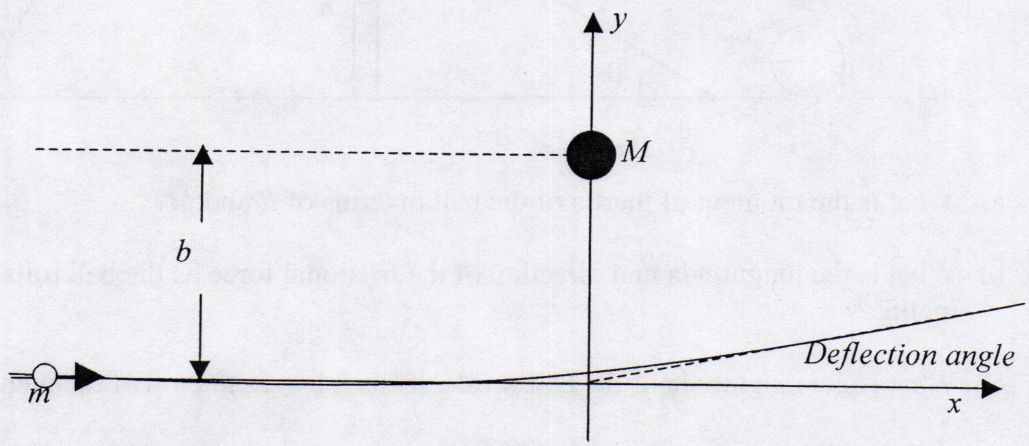
Questions should be directed to the proctor.

Good luck!!

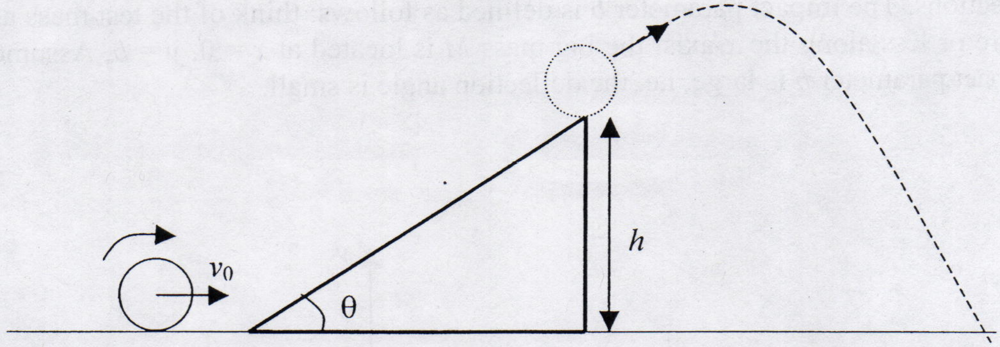
1. A solid sphere of radius R sits directly on top of a thin fixed rod as shown. The sphere (starting at rest) begins to roll about the rod without slipping until it finally falls off the rod. Use the method of Lagrange multipliers to find the angle, ϕ, at which the sphere leaves the rod.



2. This is a problem where gravity is the only force present, and the particle motion is non-relativistic. Calculate the deflection angle of a test mass m which approaches a big mass M ($m << M$), with an impact parameter of b and a velocity of v_0 in the far past. In other words, the test mass will move in a direction in the far future that is different from the incoming direction in the far past. Compute the difference in angle between these two directions. The impact parameter b is defined as follows: think of the test mass as moving more or less along the x-axis, the big mass M is located at $x = 0$, $y = b$. Assume that the impact parameter b is large, i.e. the deflection angle is small.



3. A uniform spherical ball of radius R, and mass M, rolls (without slipping) up an incline of height h, and angle θ. The ball has an initial velocity of v_0 at the bottom of the incline. The velocity v_0 is sufficiently large that the ball projects off the top of the incline and hits the ground a distance from the end of the incline.



a) What is the moment of inertia of the ball in terms of R and M?

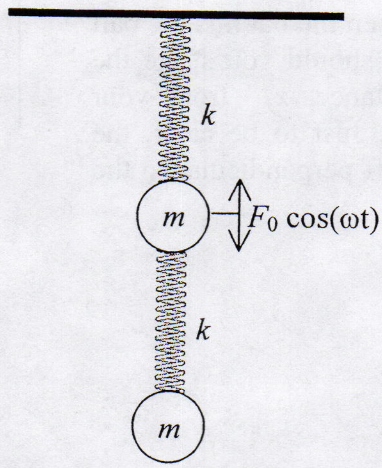
b) What is the magnitude and direction of the frictional force as the ball rolls up the incline?

c) What is the magnitude of the ball's velocity as it leaves the top of the incline?

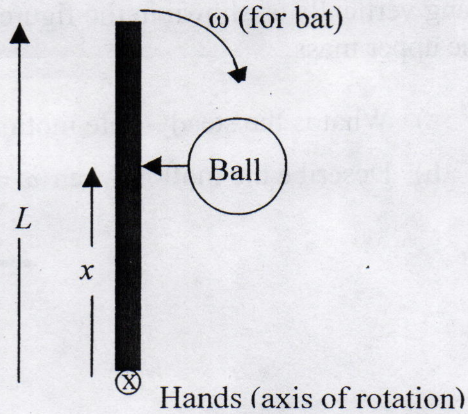
4. Two massless springs with spring constant k are connected to two masses, m, that hang vertically as shown in the figure. A vertical harmonic force, $F_0 \cos(\omega t)$, is applied to the upper mass.

a) What is the steady-state motion for each mass?

b) Describe the motion when $\omega = \omega_0 = \sqrt{k/m}$.



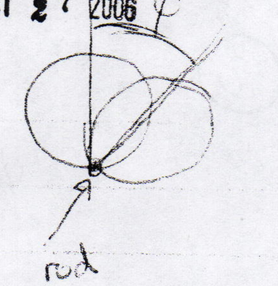
5. Imagine you strike a thrown ball with a bat. Consider the bat to be a uniform rod of length L, which you hold at one end with your hands. The way you strike the ball is by swinging (rotating) the bat with your hands, but not displacing your hands at all (i.e. your hands are the axis of rotation for the bat). Suppose you would like to minimize the force on your hands when the bat hits the ball – where along the bat should you strike the ball? (specify the distance, x, from your hands). Ignore gravity. Just to be clear, the ball strikes and goes off perpendicular to the long axis of the bat.



Sec 1 # 1

MILLER

OCT 27 2006 ϕ



Mechanics Quals Problem Solution - Miller

(1)

Kinetic Energy $T = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$

$$I = I_{cm} + Mr^2$$

parallel axis theorem

$$= \frac{2}{5}MR^2 + Mr^2$$

r = distance from axis to center of sphere

$$T = \frac{1}{2}m\dot{\phi}^2\left[\frac{2}{5}R^2 + r^2\right] + \frac{1}{2}m\dot{r}^2$$

$$U = -mgr\cos\phi$$

$$\mathcal{L} = \frac{1}{2}m\left[\frac{3}{5}R^2 + r^2\right]\dot{\phi}^2 + \frac{1}{2}mr^2 - mgr\cos\phi$$

5pts

$$\frac{\partial \mathcal{L}}{\partial r} - \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{r}}\right) + \lambda = 0$$

$$mr\dot{\phi}^2 - mg\cos\phi - m\ddot{r} + \lambda = 0$$

$$\ddot{r} - r\dot{\phi}^2 + g\cos\phi = \lambda$$

Constraint $r = R$

$$-R\phi^2 + g\cos\phi = \lambda$$

Want $\angle$ at which $\lambda = 0$

$$\dot{\phi}^2 = \frac{g\cos\phi}{R}$$

5pts

Use conservation of energy to get rid of $\dot{\phi}^2$

$$MgR = \frac{1}{2}M\left[\frac{2}{5}R^2 + R^2\right]\dot{\phi}^2 + \cancel{\frac{1}{2}m\dot{r}^2} + mgR\cos\phi$$

$$MgR = \frac{7}{10}mR^2\dot{\phi}^2 + MgR\cos\phi$$

OCT 27 2006

②

$$\dot\phi^2 = MgR(1-\cos\phi)\left(\frac{10}{7mR^2}\right)$$

$$= \frac{10}{7}\frac{g}{R}(1-\cos\phi)$$

$$\frac{10}{7}\frac{g}{R}(1-\cos\phi) = \frac{g}{R}\cos\phi$$

$$\frac{10}{7}\frac{g}{R} = \frac{10}{7}\frac{g}{R}\cos\phi + \frac{g}{R}\cos\phi$$

$$= \frac{17}{7}\frac{g}{R}\cos\phi$$

$\cos\phi = \frac{10}{17}$ $\quad\quad$ $\phi = 54°$ $\quad\quad$ 5pts

Hui Sec 1 #2

To the Quals Committee,
Below please find two problems: one mechanics and one general.
Lam

Mechanics -
Problem:

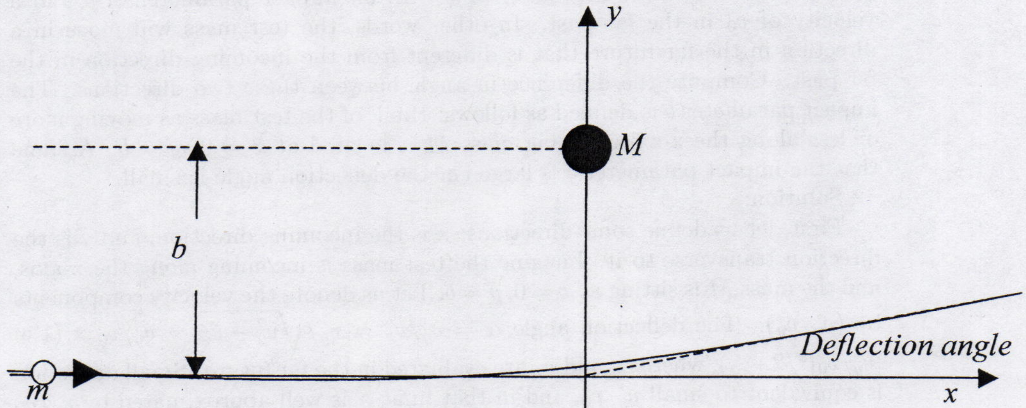
This is a problem where gravity is the only force present, and the particle motion is non-relativistic. Calculate the deflection angle of a test mass m which approaches a big mass M ($m \ll M$), with an impact parameter of b and a velocity of v_0 in the far past. In other words, the test mass will move in a direction in the far future that is different from the incoming direction in the far past. Compute the difference in angle between these two directions. The impact parameter b is defined as follows: think of the test mass as moving more or less along the x-axis, the big mass M is located at $x = 0$, $y = b$. Assume that the impact parameter b is large i.e. the deflection angle is small.

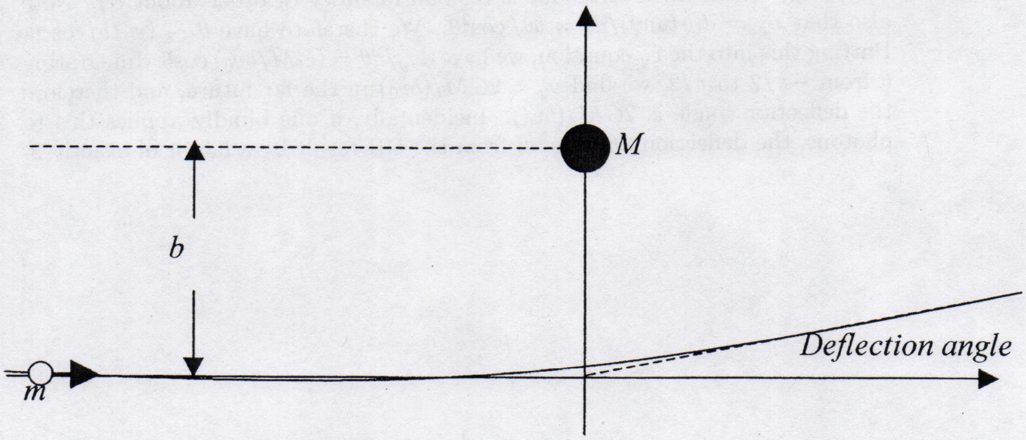
Solution:

First, let us define some directions: x is the incoming direction, and y is the direction transverse to it. Imagine the test mass is incoming along the x-axis, and the mass M is sitting at $x = 0, y = b$. Let us denote the velocity components by (v_x, v_y). The deflection angle $\alpha = v_y/v_x = v_y/\sqrt{v_0^2 - v_y^2} \sim v_y/v_0 \times (1 + [v_y/v_0]^2/2 + ...)$, where v_y and v_x are evaluated in the far future. Small deflection is equivalent to small v_y/v_0, and in that limit α is well approximated by v_y/v_0 (the correction is order α^3). Our task is therefore to find v_y in the far future. Let us define an angle θ such that $b\tan\theta$ equals the x component of the separation between the test mass and M, and $b/\cos\theta$ equals the actual separation. (In the limit of small deflection, the y component of the separation between test mass and M is always going to be roughly b.) The equation of motion in the y direction is $\dot{v}_y = \cos\theta GM/(b/\cos\theta)^2$. Note that $v_x \sim v_0$ (this is actually not so obvious, but can be seen by noting that $v_x \sim v(1 - [v_y/v]^2/2 + ...)$, and $v_y/v \ll 1$, and v and v_0 should differ from each other by no more than $(v - v_0)/v_0 < GM/(bv_0^2)$ which as it turns out is a small quantity of order about α). Note also that $v_x = d(b\tan\theta)/dt = b\dot{\theta}/\cos^2\theta$. We therefore have $\dot{\theta} \sim (v_0/b)\cos^2\theta$. Putting this into the $\dot{v}_y$ equation, we have $dv_y/d\theta = (GM/bv_0)\cos\theta$. Integrating θ from $-\pi/2$ to $\pi/2$, we find $v_y = 2GM/(bv_0)$ in the far future, and therefore the deflection angle is $2GM/(bv_0^2)$. Incidentally, if one blindly applies this to photons, the deflection angle is off from the GR result by a factor of exactly 2.

Quals 2007
Section 1, Mechanics, Question 2 by Lam Hui

Figure for Lam Hui Mechanics problem (edited by Mike Tuts)





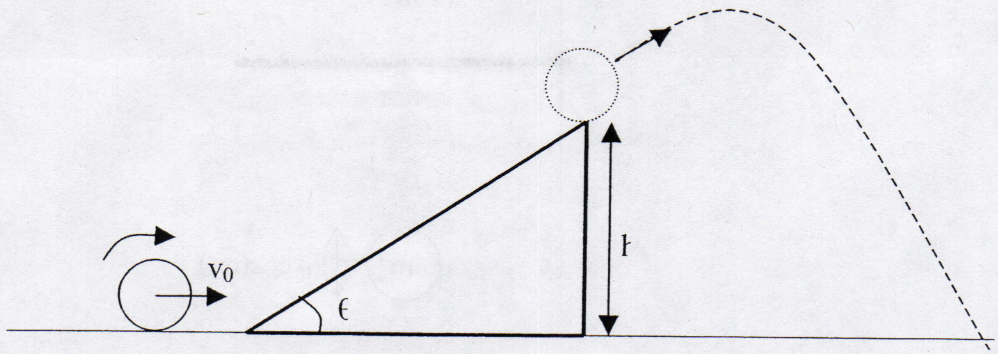
sec 1 # 3

NOV 2 ? 2006

Quals Problem 1 M. Shaevitz

Mechanics Fall, 2006

A uniform spherical ball of radius, R, and mass, M, rolls (without slipping) up an incline of height, h, and angle, θ. The ball has an initial velocity of v_0 at the bottom of the incline. The velocity v_0 is sufficiently large that the ball projects off the top of the incline and hits the ground a distance from the end of the incline.



a) What is the moment of inertia of the ball in terms of R and M?

b) What is the magnitude and direction of the frictional force as the ball rolls up the incline?

c) What is the magnitude of the ball's velocity as it leaves the top of the incline?

Solution:

a)

$$I_{sphere} = \frac{M}{\frac{4}{3}\pi R^3}\int_0^R \pi\left(R^2 - y^2\right)^2 dy = \frac{2}{5}MR^2$$

b)

Assuming that f point up the incline

$-mg\sin\theta + f = ma$

$-fR = I\alpha = Ia/R$

$$f = \left(\frac{1}{1+\frac{mR^2}{I}}\right)mg\sin\theta = \frac{2}{7}mg\sin\theta$$

Since f comes out positive with our assumption, the direction is up the incline

c)

Energy conservation:

$$E_i = \frac{1}{2}mv_0^2 + \frac{1}{2}I\left(\frac{v_0}{R}\right)^2 = E_f = \frac{1}{2}mv^2 + \frac{1}{2}I\left(\frac{v}{R}\right)^2 + mgh$$

$$v^2 = v_0^2 - \frac{5}{7}gh$$

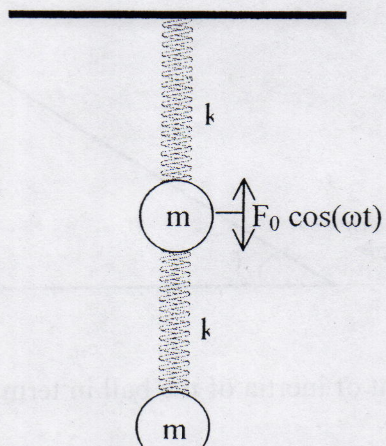
Sec 1 # 4

2007 Quals Problem 3 M. Shaevitz
Mechanics Fall, 2006

Two massless springs with spring constant k are connected to two masses of mass m that hang vertically as shown in the figure. A vertical harmonic force, $F_0 \cos(\omega t)$, is applied to the upper mass.

a) What is the steady-state motion for each mass?

b) Describe the motion when $\omega = \omega_0 = \sqrt{k/m}$.



Solution:

a) The equations of motion can be found from Newton's laws or the Lagrangian

$$m\ddot{x}_1 + kx_1 - k(x_2 - x_1) = F_0 \cos(\omega t)$$

$$m\ddot{x}_2 + k(x_2 - x_1) = 0$$

where $x_{1(2)} =$ position of the upper (lower) mass relative the equilibrium point

For steady state motion, $x_1 = A\cos(\omega t + \phi_1)$ and $x_2 = B\cos(\omega t + \phi_2)$. Since there is no damping term, the motion will be in phase and $\phi_1 = \phi_2 = 0$. Substituting into the equations of motion gives

$$-A\omega^2 m + 2kA - kB = F_0$$

$$-B\omega^2 m - kA + kB = 0$$

Solving these using $\omega_0 = \sqrt{k/m}$ gives

$$A = \frac{F_0/m\left(\omega_0^2 - \omega^2\right)}{\left(2\omega_0^2 - \omega^2\right)\left(\omega_0^2 - \omega^2\right) - \omega_0^4} \quad \text{and} \quad B = \frac{F_0/m\,\omega_0^2}{\left(2\omega_0^2 - \omega^2\right)\left(\omega_0^2 - \omega^2\right) - \omega_0^4}$$

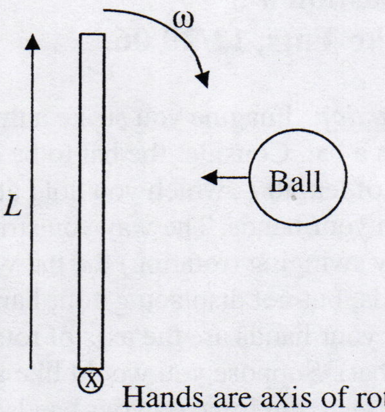
b) When $\omega = \omega_0 = \sqrt{k/m}$, A = 0 and B = F_0/k. The upper mass stays at rest and the lower mass oscillates with a amplitude of F_0/k. This happens since the external force cancels the lower spring force on mass m_1.

sec 1 #5

NOV 22 2006

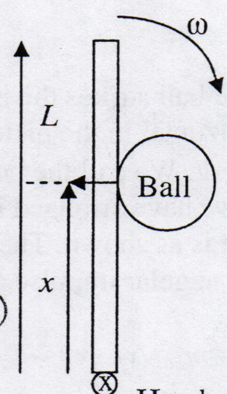
Quals Question – Mechanics
Mike Tuts, 11/22/06

Question: Imagine you strike a thrown ball with a bat. Consider the bat to be a uniform rod of length L, which you hold at one end with your hands. The way you strike the ball is by swinging (rotating) the bat with your hands, but not displacing your hands at all (i.e. your hands are the axis of rotation for the bat). Suppose you would like to minimize the force on your hands when the bat hits the ball – where along the bat should you strike the ball? (specify the distance from your hands). Ignore gravity. Just to be clear, imagine the ball strikes when the bat is in the position shown in the diagram – in that case you want to minimize the "horizontal" force on your hands.



Hands are axis of rotation

Answer: Let's call the place where the ball strikes the bat x, and the contact force F (horizontal in the picture) which is present for a short time Δt. We call the horizontal force on your hands F_1 – we have arranged it so that the ball strikes when the bat is as shown. Then the equations for the impulse and angular impulse are:



Hands are axis of rotation

$-(F+F_1)\,\Delta t = m_{bat}\,(v_f - v_i)$ $\longrightarrow$ $\dfrac{v_f - v_i}{\Delta t} = \dfrac{-(F+F_1)}{m_b}$

And

$-F\,x\,\Delta t = I\,(\omega_f - \omega_i)$, where $I = 1/3\ m_{bat}\,L^2$, and $(L/2)\,\omega = v$ $\qquad -Fx\Delta t = \dfrac{m_b}{3}L^2\left(\dfrac{2v_f}{L} - \dfrac{2v}{L}\right.$

Solving for $F_1 = (3x/2L - 1)\,F$,

So we find that the minimum value for F_1 occurs when $x = 2/3\ L$ $\qquad -Fx\Delta t = \dfrac{2}{3}m_b L(v_f - V$

$$-Fx\frac{3}{2m_bL} = \frac{v_f - v_i}{\Delta t}$$

$$-F\Delta t = m_b(v_f - v_i)$$

$$-F\Delta t = \frac{m_b L^2}{3x}(\omega_f - \omega_i)$$

$$= \frac{m_b L^2}{3x}\left(\frac{2v_f}{L} - \frac{2v_i}{L}\right)$$

$$= \frac{m_b 2L}{3x}(v_f - v_i)$$

$$-\frac{Fx3}{2m_bL} = -\frac{(F+F_1)}{m_b}$$

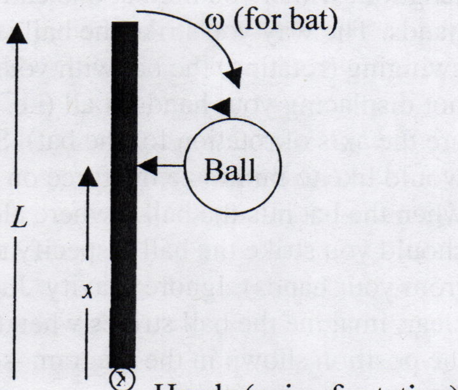
$$F_1 = F - \frac{Fx3}{2L}$$

$$F_1 = F\left(\frac{3x}{2L} - 1\right) \Rightarrow F_1^{min} \Rightarrow \frac{3x}{2L} = 1$$

$$x = \frac{2}{3}L$$

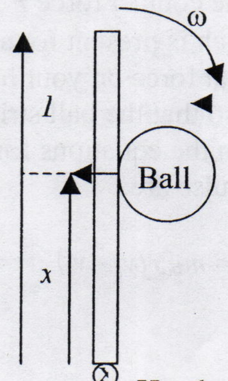
Quals 2007
Section 1, Mechanics
Question # 5
Mike Tuts, 12/20/06

Question: Imagine you strike a thrown ball with a bat. Consider the bat to be a uniform rod of length L, which you hold at one end with your hands. The way you strike the ball is by swinging (rotating) the bat with your hands, but not displacing your hands at all (i.e. your hands are the axis of rotation for the bat). Suppose you would like to minimize the force on your hands when the bat hits the ball – where along the bat should you strike the ball? (specify the distance, x, from your hands). Ignore gravity. Just to be clear, the ball strikes and goes off perpendicular to the bat.



Hands (axis of rotation)

Answer: The ball strikes the bat at x, with contact force F (horizontal in the picture) which is present for a short time Δt. We call the horizontal force on your hands F_1 – we have arranged it so that the ball strikes when the bat is as shown. Then the equations for the impulse and angular impulse are:



Hands are axis of rotation

$-(F+F_1)\,\Delta t = m_{bat}\,(v_f - v_i)$

And

$-F\,x\,\Delta t = I\,(\omega_f - \omega_i)$, where $I = 1/3\ m_{bat}\,L^2$, and $(L/2)\,\omega = v$

Solving for $F_1 = (3x/2L - 1)\,F$,

So we find that the minimum value for F_1 occurs when $x = 2/3\ L$

Columbia University
Department of Physics
QUALIFYING EXAMINATION
Monday, January 8, 2007
11:10 AM – 1:10 PM

Classical Physics
Section 2. Electricity, Magnetism & Electrodynamics

Two hours are permitted for the completion of this section of the examination. Choose 4 problems out of the 5 included in this section. (You will not earn extra credit by doing an additional problem). Apportion your time carefully.

Use separate answer booklet(s) for each question. Clearly mark on the answer booklet(s) which question you are answering (e.g., Section 2 (Electricity etc.), Question 2; Section 2(Electricity etc.) Question 4, etc.)

Do **NOT** write your name on your answer booklets. Instead clearly indicate your **Exam Letter Code**

You may refer to the single handwritten note sheet on 8 ½ x 11" paper (double-sided) you have prepared on Classical Physics. The note sheet cannot leave the exam room once the exam has begun. This note sheet must be handed in at the end of today's exam. Please include your Exam Letter Code on your note sheet. No other extraneous papers or books are permitted.

Simple calculators are permitted. However, the use of calculators for storing and/or recovering formulae or constants is NOT permitted.

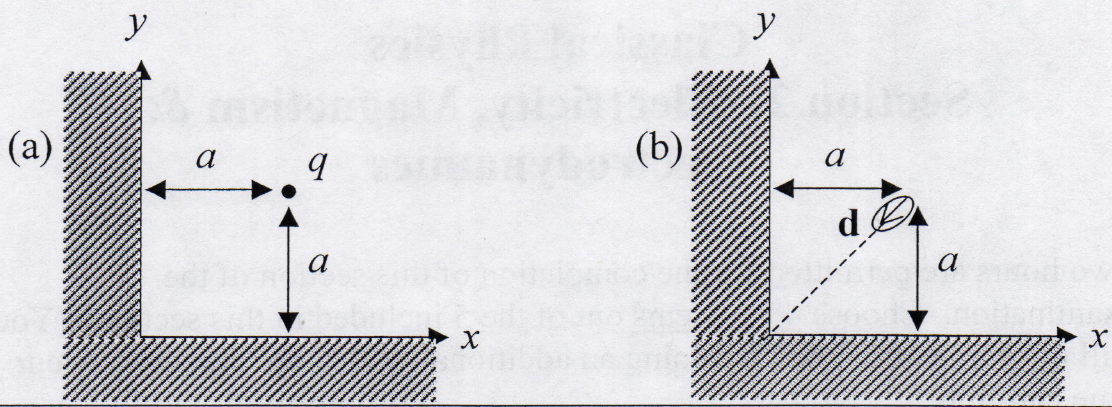
Questions should be directed to the proctor.

Good luck!!

1. (a) A point charge, q, is placed in a region bounded by two perpendicular conducting planes that are held at zero potential (see figure a). The charge is at equal distance a from the two planes. Find the force acting on the charge.
(b) Replace the charge q in part (a) with an electric dipole **d**, as indicated in figure b. The dipole moment **d** lies in the (x, y) plane and points to the intersection of the two conducting planes. Find the force acting on the electric dipole.

Hints:

- use the results obtained in part (a)
- Assume that the charge separation in the electric dipole is $h << a$.



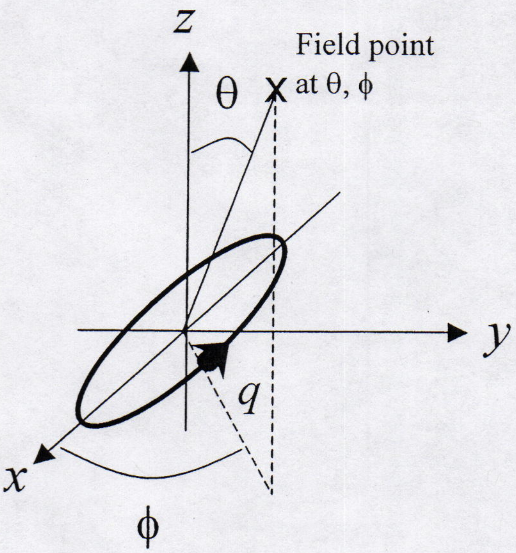
2. A particle of charge q moves in the x–y plane along an elliptical orbit, as shown in the diagram. The particle's position vector $\vec{s}(t)$ depends on time t as:

$$\vec{s}(t) = \hat{x}\, R_1 \cos(\omega t) + \hat{y}\, R_2 \sin(\omega t)$$

where ω, R_1, and R_2 are given positive constants, and $\hat{x}$ and $\hat{y}$ are unit vectors. Assume $R_1 > R_2$. Also, assume that the particle is in <u>non-relativistic</u> motion $(v << c)$.

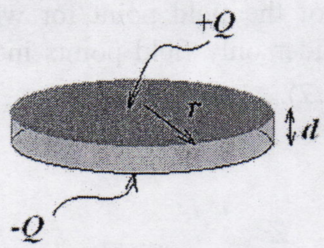
(a) Find the time-averaged power radiated (emitted) per unit solid angle $\left(\dfrac{dP}{d\Omega}\right)_{avg}$ in the direction specified by the polar and azimuthal angles θ and ϕ.

(b) Find the polar angle θ of the field point for which the detected radiation is circularly polarized. Consider only field points in the first quadrant of the x–z plane ($\phi = 0$ and $0 < \theta < \pi/2$).



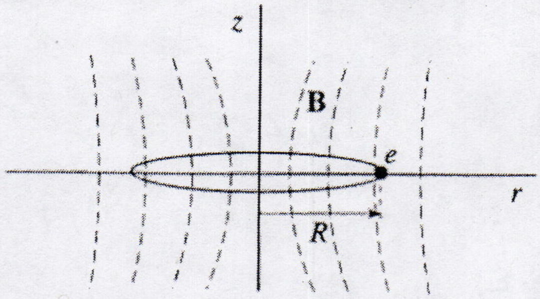
3. Two conducting disks of radius r are separated by a distance d, as shown in the figure. The region between the disks is filled with a conducting medium with conductivity σ. At $t = 0$ the top and bottom disks carry charge $\pm Q_0$ respectively. Assume $d << r$ and neglect the effects of fringing fields.

(a) What is the resistance of the material between the two disks?
(b) What is the capacitance of the two parallel disks?
(c) What is the charge on each disk as a function of time for $t > 0$?
(d) What is the magnetic field everywhere as a function of time for $t > 0$?



4. Consider the motion of electrons in an axially symmetric magnetic field. Suppose that at $z = 0$ (the "median plane") the radial component of the magnetic field is zero, so that in cylindrical coordinates, (r, z), $B(z=0) = B(r)\hat{z}$. Electrons at $z = 0$ then follow a circular path of radius R (see figure).

(a) What is the relationship between the electron momentum $\boldsymbol{p}$ and the orbit radius R?

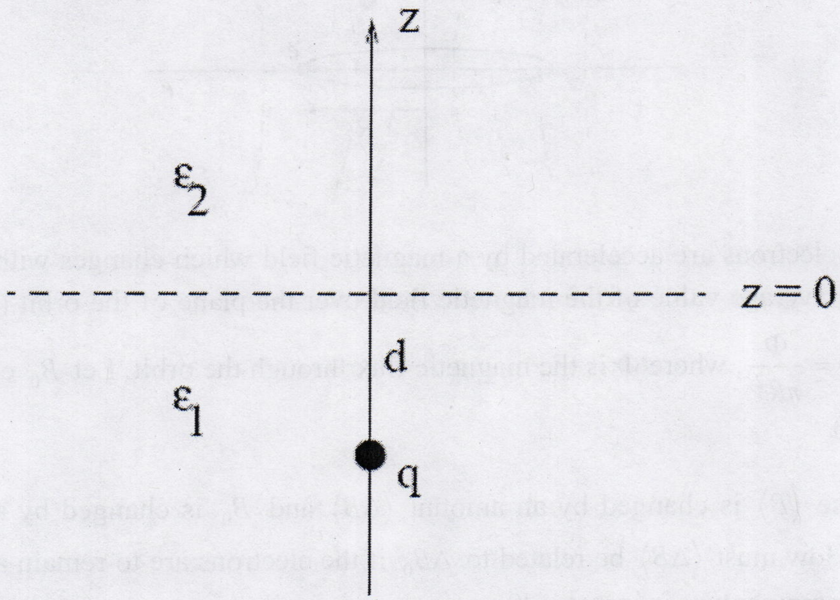


In a betatron, electrons are accelerated by a magnetic field which changes with time. Let $\langle B \rangle$ equal the average value of the magnetic field over the plane of the orbit (within the orbit), i.e. $\langle B \rangle = \frac{\Phi}{\pi R^2}$, where Φ is the magnetic flux through the orbit. Let B_0 equal $B(r = R, z = 0)$

(b) Suppose $\langle B \rangle$ is changed by an amount $\langle \Delta B \rangle$ and B_0 is changed by an amount ΔB_0. How must $\langle \Delta B \rangle$ be related to ΔB_0 if the electrons are to remain at radius R as their momentum is increased?

5. A point charge, q, is embedded in a semi-infinite dielectric of permittivity ε_1 a distance d away from a planar interface (at $z = 0$) which separates the first medium from another semi-infinite dielectric of permittivity ε_2, as shown in the figure.

(a) Obtain the electric potential for both for $z > 0$ and for $z < 0$.
(b) What is the surface charge density at the interface between the two dielectrics?



Sec 2 #1

Problem 1.

(a) Charge q is placed in region D bounded by two perpendicular conducting planes (held at zero potential). The charge is at equal distance a from the two planes. Find the force acting on the charge.

(b) Same problem with electric dipole $\mathbf{d}$ instead of charge q. The dipole moment $\mathbf{d}$ points to the line of intersection of the two conducting planes and is perpendicular to this line.

Solution:

(a) Using the method of images, one finds that the potential ϕ in region D is reproduced if one adds 3 charges behind the conducting planes: two charges $q_1 = q_2 = -q$ are symmetric to q with respect to the conducting planes, and the third charge $q_3 = q$ is symmetric to q_1 and q_2 about the same planes. Then $\phi = 0$ at both conducting planes as needed.

Now calculate the net electric field created by the three image charges at the location of the original charge,

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3 = \frac{(2\sqrt{2}-1)q}{8a^2}\,\mathbf{e},$$

where $\mathbf{e}$ is the unit vector directed from the original charge toward the intersection line of the conducting planes (and perpendicular to this line). The force acting on the charge is $\mathbf{E}q$.

Note that $\mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3$ may also be found anywhere on the line along $\mathbf{e}$. At a distance l from the intersection of the conducting planes, one gets

$$\mathbf{E}(l, l_0) = \left[\frac{2ql}{(l^2+l_0^2)^{3/2}} - \frac{q}{(l+l_0)^2}\right]\mathbf{e},$$

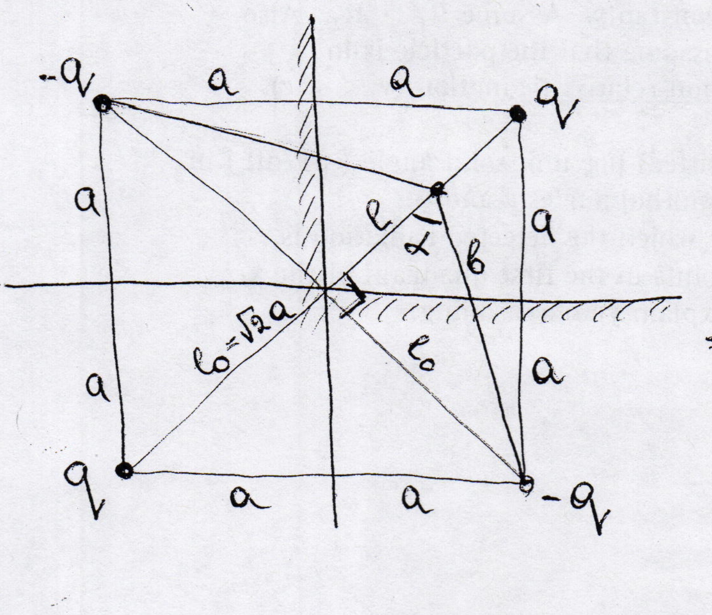
where $l_0 = \sqrt{2}a$ is the location of charge q on the line. This result can be used to solve part (b).

(b) The dipole consists of two charges $\pm q$ on the line $\mathbf{e}$, which are separated by a small distance $h = d/q$. Charge $+q$ is located at l_0 and charge $-q$ at $l_0 + h$. Each of $\pm q$ has its own three images $\pm q_1, \pm q_2, \pm q_3$, which create the electric field $\mathbf{E}(l, l_0)$ and $-\mathbf{E}(l, l_0 + h)$, respectively. The net field of all these image charges for the dipole is

$$\mathbf{E}_d(l, l_0) = -\frac{\partial \mathbf{E}(l, l_0)}{\partial l_0}\,h.$$

The force acting on the dipole is $(\mathbf{d}\cdot\nabla)\mathbf{E}_d = -d\,\partial\mathbf{E}_d/\partial l$, so

$$\mathbf{F} = d\left.\frac{\partial^2 \mathbf{E}(l, l_0)}{\partial l\,\partial l_0}\right|_{l=l_0} = \frac{3d^2}{32a^4}(3\sqrt{2}-1)\,\mathbf{e}.$$



$$\vec{E}_1 + \vec{E}_2 = 2\frac{q}{b^2}\cos\alpha\,\vec{e}, \quad \vec{E}_3 = \frac{q(-\vec{e})}{(l_0+l)^2}$$

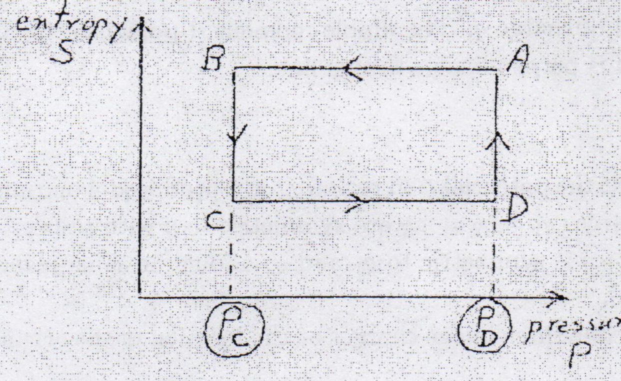
$$b^2 = l^2 + l_0^2, \quad \cos\alpha = \frac{l}{b}$$

$$\Rightarrow \vec{E}_1 + \vec{E}_2 + \vec{E}_3 = \left[\frac{2ql}{(l^2+l_0^2)^{3/2}} - \frac{q}{(l+l_0)^2}\right]\vec{e}$$

[illegible]

(1) Thermodynamics (general exam)

(30 points)



The reversible Brayton engine cycle A → B → C → D → A is shown in the entropy-pressure (S-P) diagram. During step D → A, the gas is heated at constant pressure with the absorption of an amount of heat Q_{DA}. During step B → C, the gas is cooled at constant pressure with the expulsion of an amount of heat Q_{BC}.

Let W_{ABCD} denote the net work done by the gas during a complete cycle. The efficiency of the engine is defined to be the ratio W_{ABCD}/Q_{DA} (because the expelled heat Q_{BC} is considered to be wasted). Consider an ideal gas as the working substance, so that the heat capacities C_P and C_V are temperature independent.

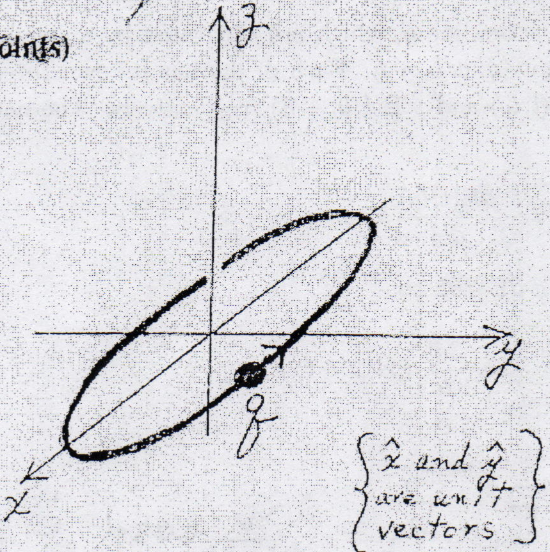
(a) Show this cycle in a pressure-volume (P-V) diagram.

(b) Calculate the efficiency of the engine in terms of C_P, C_V, and the pressure ratio P_C/P_D.

[illegible]

(2) Electromagnetism (classical exam)

(30 points)



A particle of charge q moves in the x-y plane along an elliptical orbit, as shown in the diagram. The particle's position vector $\vec{s}$ depends on time t:

$$\vec{s}(t) = \hat{x} R_1 \cos(\omega t) + \hat{y} R_2 \sin(\omega t)$$

where ω, R_1, and R_2 are given positive constants. Assume $R_1 > R_2$. Also, assume that the particle is in <u>non-relativistic</u> motion ($v << c$).

(a) Find the time-averaged power radiated (emitted) per unit solid angle ($\overbrace{dP/d\Omega}^{avg}$) in the direction specified by the polar and azimuthal angles θ and ϕ.

(b) Find the polar angle θ of the field point for which the detected radiation is circularly polarized. Consider only field points in the first quadrant of the x-z plane ($\phi = 0$ and $0 < \theta < \pi/2$). Briefly explain your reasoning.