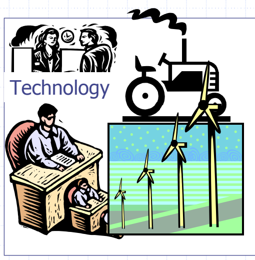
The Production Function

Intermediate Microeconomics

What do firms do?

Inputs (Capital, Labor, Raw materials, etc.) → Technology → Output(s) (Good or Service)



As a "first approximation," we treat production as a transformation of inputs into output.

Inputs (Capital, Labor, Raw materials, etc.) → **Black Box** → Output(s) (Good or Service)

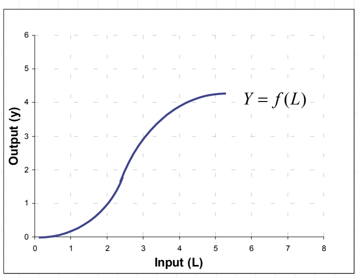
It tends to exhibit certain characteristics

- Diminishing marginal rate of substitution
- Diminishing marginal product
- These features can be conveniently summarized by a production function.

Production Functions
(Hypothetical example: wheat farm)

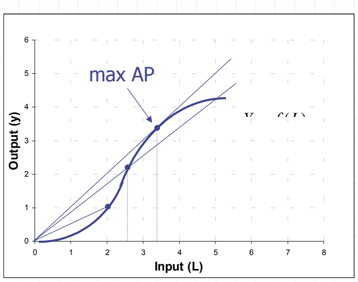
No. laborers	Acres of land	Bushels of wheat (000s)	Marginal product	Average product
0	1	0		
1	1	0.2		
2	1	1.0		
3	1	3.0		
4	1	3.9		
5	1	4.2		
6	1	4.2		

Production Function



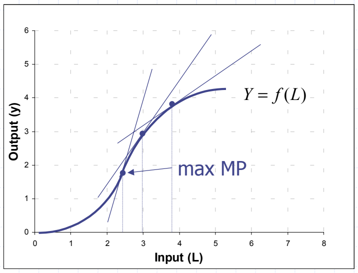
Economists prefer a graphical or functional representation of production over the tabular one.

Production Function
(Graphical representation of average product)



Economists prefer a graphical or functional representation of production over the tabular one.

Production Function
(Graphical representation of marginal product)



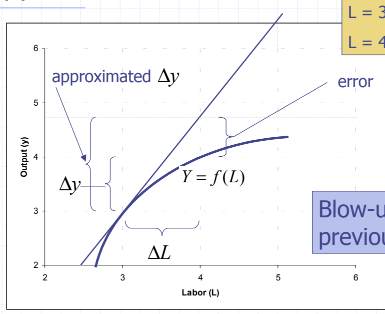
Economists prefer a graphical or functional representation of production over the tabular one.

How accurate is a linear approximation?

Suppose this data has been collected from actual experience:

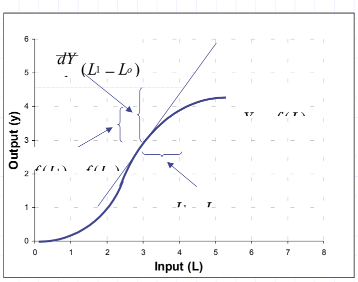
L = 3, y = 3

L = 4, y = 4



Blow-up of previous diagram

The Derivative & the Production Function
(Graphical representation)



The derivative (dY/dL) is

- the slope of the production function
- The marginal product of labor
- … actually it is a linear approximation, but because it is so easy to use, we "define" the MP_L as dY/dL

Differentiation = Linear Approximation

- The linear approximation of a movement along Y = f(L) is:

$$f(L_1) - f(L_o) \cong \frac{dY}{dL}(L_1 - L_o)$$

- Where the derivative of Y with respect to L is defined as:

$$\frac{dY}{dL} \equiv \left.\frac{f(L_1) - f(L_o)}{L_1 - L_o}\right|_{(L_1 - L_o)\to 0}$$

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Differential Calculus:
Seven Fundamental Rules of Differentiation

For any differentiable function, Y = f(x), there exists another function f′(x) = dY/dx that gives the derivative (slope) of the function, f(x).

How can one find the derivative (slope) of f(x)? There are seven fundamental rules:

1. The constant rule
2. The power rule
3. The negative power rule
4. The sum-and-difference rule
5. The product rule
6. The quotient rule
7. The chain rule

See Handout: you must memorize these rules!!!

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Partial Differentiation: Linear Approximation, all else held constant

- The linear approximation of a movement along Y = f(L,K), letting L vary and K remain constant is:

$$f(L_1, K_o) - f(L_o, K_o) \cong \frac{\partial Y}{\partial L}(L_1 - L_o)$$

- Where the partial derivative of Y with respect to L is:

$$\frac{\partial Y}{\partial L} = \left.\frac{Y_1 - Y_o}{L_1 - L_o}\right|_{dL\to 0 \text{ and } dK = 0}$$

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The Cobb-Douglas Production Function

(The simplest commonly used explicit function for estimating production)

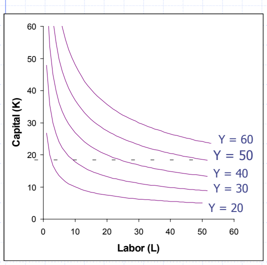
- Output as a function of two inputs, labor (L) and capital (K):

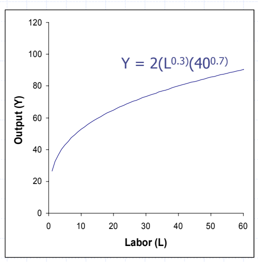
$$Y = f(L, K) = AL^{\alpha}K^{\beta}$$

  - where $\alpha > 0$ and $\beta > 0$ are technical parameters and $A > 0$ captures any influences that might shift the entire production function up or down, such as technical or institutional changes.
- Marginal products of labor and capital:
  - $MP_L = \dfrac{\partial Y}{\partial L} = \alpha AL^{\alpha-1}K^{\beta}$
  - $MP_K = \dfrac{\partial Y}{\partial K} = \beta AL^{\alpha}K^{\beta-1}$

What does the Cobb-Douglas Function look like?

Example: $Y = 2L^{0.3}K^{0.7}$





Properties of the functional form

- Do Cobb-Douglas production functions exhibit the properties we expect of a production function?
- What properties?
  - Diminishing marginal product
  - Standard-shaped isoquants with diminishing MRS
  - Returns to scale

Estimated Cobb-Douglas Production Functions for Several Canadian Industries

(Baldwin and Gorecki, 1986)

Industry	α	β	$\alpha + \beta$
Thread mill	0.64	0.18	0.82
Knitted fabrics	0.55	0.36	0.90
Lime manufacturers	0.60	0.25	0.84
Shoe factories	0.82	0.18	1.00
Hosiery mills	0.55	0.46	1.01
Jewelry and silverware	0.60	0.41	1.01
Concrete blocks and bricks	0.93	0.40	1.33
Paint	0.71	0.61	1.32
Orthopedic & surgical applicances	0.30	0.99	1.30

Some Uses of the Cobb-Douglas Production Function

- Estimating returns to scale in different industries.
- Accounting for sources of growth.
- Estimating productivity growth (or decline).

Summary

- What do production functions represent?
- Graphical and functional representations of average and marginal product
- Linear approximations of marginal production
  - how accurate?
  - how easy!
- Cobb-Douglas Production functions

For next lecture, you must be able to identify MP and AP from slopes of tangents and rays as if by "second nature." If not, you won't be able to keep up.

Innovation and Productivity Growth

Definition:

- An **innovation** is a modification in the production process through the adoption of a new method.
  - where "method" is interpreted broadly to include new techniques or technologies, new organizational arrangements, or new institutions.
- How does one account for innovations in the production processes?
  - in the Cobb-Douglas production function?
  - In a discrete process model …
  - In general, …