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Using option prices to estimate realignment probabilities in the European Monetary System: the case of sterling–mark

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This paper describes a procedure for estimating the market's perceived probability distribution of future exchange rates from the prices of risk reversals, strangles and other currency options, and uses the procedure to estimate the risk neutral *ex ante* probability of a realignment of the pound sterling. The procedure for estimating the realignment probabilities relies on the jump-diffusion model of exchange rate behavior and the resulting option pricing formula. By fitting this model to market option price data, I retrieve the unobserved parameters of the jump-diffusion process. I then use these parameter estimates to estimate the *ex ante* probability distribution of exchange rates and thus the realignment probabilities. (JEL F31, F33). Copyright © 1996 Elsevier Science Ltd

The use of derivatives prices to draw conclusions about future asset prices is common. The forward exchange rate can be interpreted as the risk-neutral first moment of the future spot rate. Analogously, implied volatilities calculated from currency options have been interpreted as the market estimate of the future second moment. In the same spirit, the prices of certain options can be interpreted as indicators of the market view on the kurtosis and skewness in the distribution of future exchange rates.

Options are frequently sold in combinations. Among the most common in over-the-counter currency option markets are *risk reversals* and *strangles*, both consisting of an out-of-the-money call and out-of-the-money put. The exercise

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price of the call component is higher than the forward exchange rate, and the exercise price of the put is lower. In a risk reversal, the dealer buys one of the options from the counterparty and sells the other option to the counterparty. As we shall see, the prices of risk reversals indicate market sentiment concerning the likelihood of exchange rate moves in a particular direction. In a strangle, the dealer sells both options to or buys both options from the counterparty. The prices of strangles indicate market sentiment concerning the likelihood of large moves in the exchange rate.

This paper describes a procedure for estimating the market's perceived probability distribution of future exchange rates from the prices of risk reversals, strangles, and at-the-money currency options and uses this procedure to estimate the risk-neutral probability of a realignment of the pound sterling in the European Monetary System (EMS). Bates (1991, 1996a,b) applies a jump-diffusion model of asset price behavior to the prices of exchange-traded options to extract the asset price's risk-neutral probability density function. The model described here is similar, but uses a simpler procedure made possible with over-the-counter currency option data to estimate the risk-neutral probability distribution of the pound sterling's exchange rate against the mark ('sterling–mark'). In contrast to previous studies of Exchange Rate Mechanism (ERM) exchange rates, it is possible with this procedure to identify both the risk-neutral probability and magnitude of a realignment.

I. The Crisis of the EMS[1]

The ERM, which began operations on March 12, 1979, consists of (i) a grid of bilateral central parities; (ii) rates for compulsory intervention, or fluctuation limits, set until August 2, 1993, at 2.25 percent or 6 percent above and below the parities; and (iii) the obligation of central banks on both sides of a currency pair to purchase or sell unlimited amounts of currency at the fluctuation limits. Bundesbank concerns about the potential for the ERM to undermine its control of the German money supply were addressed by a public commitment from the German government to shield it from a potential conflict between the intervention obligations and monetary stability either by means of a realignment or a temporary suspension of the intervention obligations. The unilateral Bundesbank reservation has remained in effect throughout the existence of the ERM.[2] Figures 1 and 2 summarize exchange and interest rate developments for the pound sterling in the ERM.

Until 1987, realignments in the ERM were frequent, and the system relied heavily on capital controls to counter selling pressures on weak currencies. From 1987 until 1992, there were no EMS realignments.[3] This period also witnessed a burst of political activity aimed at establishing a currency union, the European Monetary Union (EMU), by the end of the 1990s. The UK brought the pound into the ERM, with a ± 6 percent fluctuation band, on October 8, 1990. Several European countries outside the ERM pegged their currencies to the mark or to the European Currency Unit (ECU). Investors began to pour funds into assets denominated in non-mark European currencies in the conviction that further realignments were unlikely. Persistent but dimin-

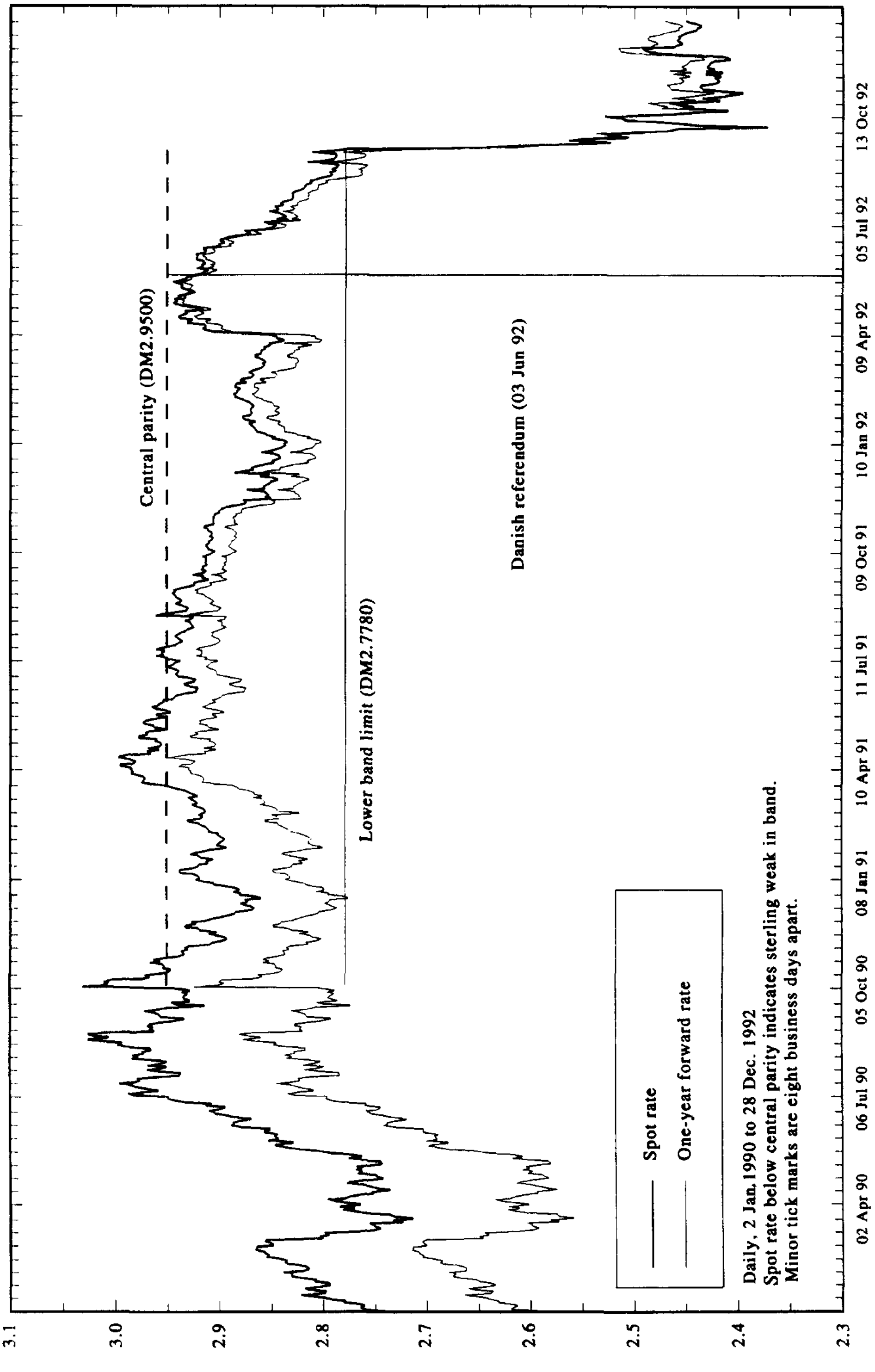
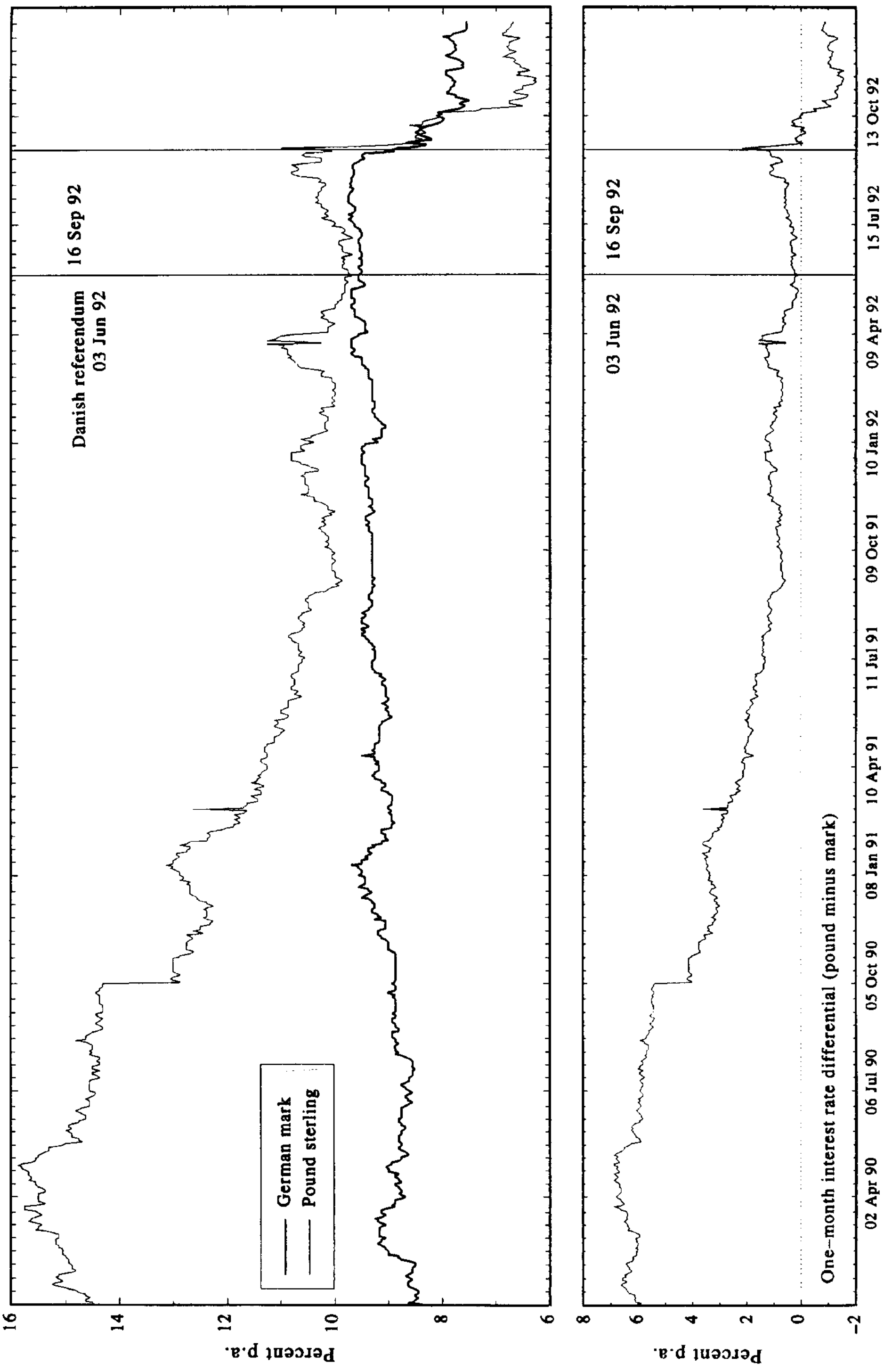


FIGURE 1. The sterling–deutsche mark exchange rate in the ERM.



Daily, 2 Jan. 1990 to 28 Dec. 1992 Minor tick marks are eight business days apart.

FIGURE 2. One-month Eurodeposit rates, German mark and pound sterling.

ishing interest rate differentials *vis-à-vis* mark-denominated assets were seen as more than adequate compensation for the dwindling exchange rate risk.

In 1991 and early 1992, 'convergence trading' was undermined by economic and political shocks. German unification was financed by borrowing from the public. The Bundesbank tightened monetary policy to thwart the perceived threat to monetary stability: two-thirds of the DM 108 billion swing in the 1991 current account was financed by short-term funds attracted to mark deposits by rising interest rates. The difficult Maastricht summit in December 1991 and the Danish public's rejection of the draft Treaty on European Union on June 2, 1992, highlighted the obstacles to EMU. Interest rate differentials *vis-à-vis* the mark had diminished, so the buffer which might have absorbed part of the Bundesbank's interest rate hikes was thinner, and the return for bearing realignment risk had fallen. By late August 1992, positions in non-mark European assets were being liquidated and ERM central banks were intervening in currency markets on a large scale.[4]

The lira was devalued by 7 percent on September 13 and the Bundesbank announced a 25 basis point reduction in the Lombard rate on Monday, September 14, briefly calming trading in the exchange markets. By the next day, sterling and the lira came under renewed pressure. That night, Bundesbank President Schlesinger was reported to have told an interviewer he favored a sterling devaluation.[5] On September 16, the Bank of England responded to the unprecedented pressure on sterling by raising its minimum lending rate, first from 10 to 12 percent and three hours later to 15 percent. That evening, the UK suspended its participation in the ERM and rescinded the second interest rate increase.

Italy also withdrew from the ERM and several currencies were devalued once or several times over the next eight months. By mid-1993, calm appeared to have returned. Abruptly, in July 1993, provoked by an aggressive attempt by the Banque de France to cut interest rates to sub-German levels, the crisis flared again. On August 1–2, 1993, the fluctuation margins for all currency pairs other than mark–Dutch guilder were widened to ± 15 percent, but the parities were left unchanged.

II. Measuring the credibility of ERM parities

Empirical assessments of the credibility of ERM parities have relied heavily on interest rate differentials and the open interest parity hypothesis, which sets expected appreciation $E[s_{t+n} - s_t|\Theta_t]$, where s_t is the logarithm of the exchange rate (German marks per foreign currency unit), equal to the forward premium of the foreign currency *vis-à-vis* the mark. Forecasts of exchange rate changes in n years can be written as a probability-weighted average of forecasts conditional on current information and the occurrence or non-occurrence of a realignment:

$$\langle 1 \rangle \qquad E[s_{t+n} - s_t|\Theta_t] = \pi_t^n E[s_{t+n} - s_t|\Theta_t, \text{realignment}] + (1 - \pi_t^n) E[s_{t+n} - s_t|\Theta_t, \text{no realignment}],$$

where π_t^n is the subjective probability of a realignment during $(t, t+n)$ and Θ_t is time-t information.

In this framework, Collins (1985, 1992) treats $E[s_{t+n} - s_t|\Theta_t$, no realignment], the anticipated appreciation in the absence of realignment, as an iid normal variate. The probability π_t^n is assumed to be determined by the foreign exchange reserves of the non-German central bank and the domestic interest rate. More recent work has used target zone models incorporating realignment risk. These interpret $E[s_{t+n} - s_t|\Theta_t$, no realignment] as the exchange rate movement within the fluctuation limits. This component of expected exchange rate changes is a mean-reverting function i.a. of the exchange rate's position in the band and can be large relative to the interest rate differential.[6]

Recent work has attempted to use option price data to study target zone exchange rates. In the Dumas *et al.* (1993) model of option values in a credible target zone, the volatility of the exchange rate is greatest at the center of the band, where it has the most room to wander towards the fluctuation limits, and lowest at the limits. Campa and Chang (1996) test this relationship using sterling–mark options and find evidence against credibility of the target zone. They also use model- and preference-free restrictions on option prices to estimate lower bounds for $\pi_t^n E[s_{t+n} - s_t|\Theta_t$, realignment].

This framework, closely related to the 'peso problem' approach to the forward exchange rate prediction bias, assumes that only zero or one realignment can occur between times t and $t+n$, corresponding to the infrequent changes in ERM parities prior to September 1992. The withdrawal of sterling and the lira from the ERM, the repeated devaluations of the peseta and escudo, and the widening of the bands do not fit perfectly into a two-state model. The realignment model is nonetheless a viable approximation, since these discrete events resulted in immediate, large changes in exchange rates.

III. The stochastic behavior of nominal exchange rates

The Black–Scholes model, the benchmark model for pricing and managing the risks of options, rests on the assumption that nominal exchange rate returns follow a random walk. To extract information about realignment expectations from option prices, I use an alternative model of option values based on the jump-diffusion model of exchange rate changes. To motivate the model, I sketch the present state of knowledge about the statistical properties of exchange rates.[7]

Floating exchange rates, it is generally agreed, are unit root processes, and daily log price relatives or nominal returns are stationary and serially uncorrelated. Beyond that the results are less conclusive, in part because of the wide range of hypotheses about the process the returns follow. The hypothesis that they are normal iid is widely rejected. The distribution of nominal returns is kurtotic or 'fat-tailed', that is, large values occur too often to be consistent with normality. The distribution appears to be skewed, so positive and negative returns of a given size are not equally likely. Finally, the distribution appears to be time-varying. The lack of autocorrelation in nominal returns indicates that the variance rather than mean varies; the volatility of returns 'clusters', with large absolute values of nominal returns often followed by large values. Thus, returns are not both iid and normally distributed, suggesting two approaches: non-normal distributions and time-varying distribution parameters.

There is strong evidence that flexible exchange rate returns follow jump-diffusions, that is, a sum of iid normal and Poisson-distributed jump components, which can account for both the kurtosis and the skewness in nominal returns.[8] If jumps in either direction are equally likely, then kurtosis, but no skewness will be apparent. If jumps in one direction are larger or more frequent, the distribution will also be skewed. Time-varying parameters can be represented by autoregressive conditional heteroscedasticity (ARCH) models, which can account for kurtosis as well as for the time variation of volatility.[9]

ERM currencies should display rather different stochastic properties from flexible exchange rates. Models of credible target zones as well as target zones with realignment risk imply that the exchange rate is mean-reverting, tending to return to the central parity. However, the implied distribution of nominal exchange rates as well as returns depends crucially on how the monetary authorities are assumed to maintain the target zone and on the way realignment risk is modeled.[10] Nieuwland *et al.* (1991, 1994) find little evidence of mean reversion but strong evidence of jumps in ERM exchange rates. However, they also find that allowing for kurtosis diminishes the presence of jumps. Ball and Roma (1993) find that processes incorporating both jumps and mean reversion fit ERM currencies well.

IV. Over-the-counter currency option price conventions

The data used in this study are drawn from the over-the-counter markets in which most currency option dealing takes place. These markets use conventions based on the Black–Scholes model to express the terms and prices of currency options.[11] Although option dealers are well aware that exchange rate behavior does not conform precisely to the model, they use it as a benchmark for valuation, and draw their terminology and metrics from it. The model's key assumption is that the logarithm of the forward exchange rate follows geometric Brownian motion, resulting in the risk-neutral process for the spot rate

$$\langle 2 \rangle \qquad S_T = S_0 + (r - r^*)\int_0^T S_t\,dt + \sigma\int_0^T S_t\,dW_t,$$

where W_t denotes a standard Brownian motion, S_t the level of the exchange rate, σ the variance rate or volatility, and r (r^*) the domestic (foreign) risk-free interest rate; σ, r and r^* are assumed constant. The model results in the Black–Scholes formulas for the values of European currency options. Let us for concreteness take sterling–mark (marks per pound) as the underlying currency. Then r and r^* are the German mark and pound sterling risk-free rates, the value of a sterling call denominated in marks is

$$\langle 3 \rangle \qquad v(S_t, \tau; X, \sigma, r, r^*) = S_t e^{-r^*\tau}\Phi(d + \sigma\sqrt{\tau}) - Xe^{-r\tau}\Phi(d),$$

and the value of a put is

$$\langle 4 \rangle \qquad w(S_t, \tau; X, \sigma, r, r^*) = Xe^{-r\tau}\Phi(-d) - S_t e^{-r^*\tau}\Phi(-d - \sigma\sqrt{\tau}),$$

where $\tau \equiv T - t$ is expressed in years, $\Phi(\cdot)$ represents the standard cumulative normal distribution, and

$$\langle 5 \rangle \qquad d = \frac{\ln\left(\frac{S_t}{X}\right) + \left(r - r^* - \frac{\sigma^2}{2}\right)\tau}{\sigma\sqrt{\tau}}.$$

Exchange-traded currency options are priced in currency units.[12] Alternatively, one can replace the model value on the left-hand side of equations ⟨3⟩ or ⟨4⟩ with an observed option price v_t or w_t and extract volatility as an implicit function of v_t or w_t, S_t, τ, X, r and r^*. In this context, σ is called the *Black–Scholes implied volatility*. The Black–Scholes values increase monotonically in σ, so the implied volatility is a unique inverse function of $v(S_t, \tau; X, \sigma, r, r^*)$ or $w(S_t, \tau; X, \sigma, r, r^*)$.

In over-the-counter currency option markets, dealers quote implied volatilities or 'vols' rather than option prices denominated in currency units.[13] Exercise prices of over-the-counter currency options are generally set equal to the forward exchange rate of the same maturity as the option, in which case the option is called *at-the-money forward*. A dealer asked to quote a one-month call option on sterling–mark (the value of the pound in German marks) might say 'one-month at-the-money forward calls are 5 at 5.5', meaning that he buys a one-month at-the-money sterling–mark call option with an exercise price equal to the current forward exchange rate for 5 volatility points (5 vols) and sells them for 5.5. When a deal is struck, the agreed price is translated from vols to currency units via the Black–Scholes formulas.[14]

The over-the-counter option markets also have a special metric for expressing the moneyness of options, that is the degree to which they are in- or out-of-the-money: the option *delta*, or the derivative of the Black–Scholes option value with respect to the spot rate. The delta of a currency call is

$$\langle 6 \rangle \qquad \delta_v(S_t, \tau; X, \sigma, r, r^*) \equiv \frac{\partial v(\cdot;\cdot)}{\partial S_t} = e^{-r^*\tau}\Phi(d + \sigma\sqrt{\tau}),$$

and that of currency put is

$$\langle 7 \rangle \qquad \delta_w(S_t, \tau; X, \sigma, r, r^*) \equiv \frac{\partial w(\cdot;\cdot)}{\partial S_t} = 1 - \delta_v(S_t, \tau; X, \sigma, r, r^*).$$

The delta of an at-the-money forward option is approximately 50 percent. In interbank dealing, exercise prices are often set to an exchange rate such that delta is equal to 25 or 30 percent. If a counterparty buys, say, a 25-delta sterling–mark call, the exercise price is calculated by setting the left-hand side of equation ⟨6⟩ equal to 0.25 and solving for X.[15]

A property of delta which makes it a convenient metric for moneyness is the following. Consider a 25-delta call and a 25-delta put with the same maturity and the same implied volatility. The exercise prices of the two options are then an equal percentage distance from the current forward exchange rate. Denoting the forward exchange rate by $F_{t,t+n}$ and the exercise prices of the 25-delta call and put by $X^{25\delta}$ and $X^{75\delta}$, we can express this property as $X^{75\delta}/F_{t,t+n} = F_{t,t+n}/X^{25\delta}$.

V. Kurtosis, skewness and option prices

The Black–Scholes model implies that all options on the same currency have identical implied volatilites, regardless of time to maturity and moneyness. However, out-of-the money options often have different implied volatilities from at-the-money options. This phenomenon is evidence that market participants view exchange rates as kurtotic. Out-of-the money call options often have different implied volatilities from equally out-of-the money puts, indicating that the market perceives directional bias in exchange rates. The market refers to these phenomena as the *volatility smile*, because of the characteristic shape of the plot of implied volatilities of options of a given maturity against delta.[16]

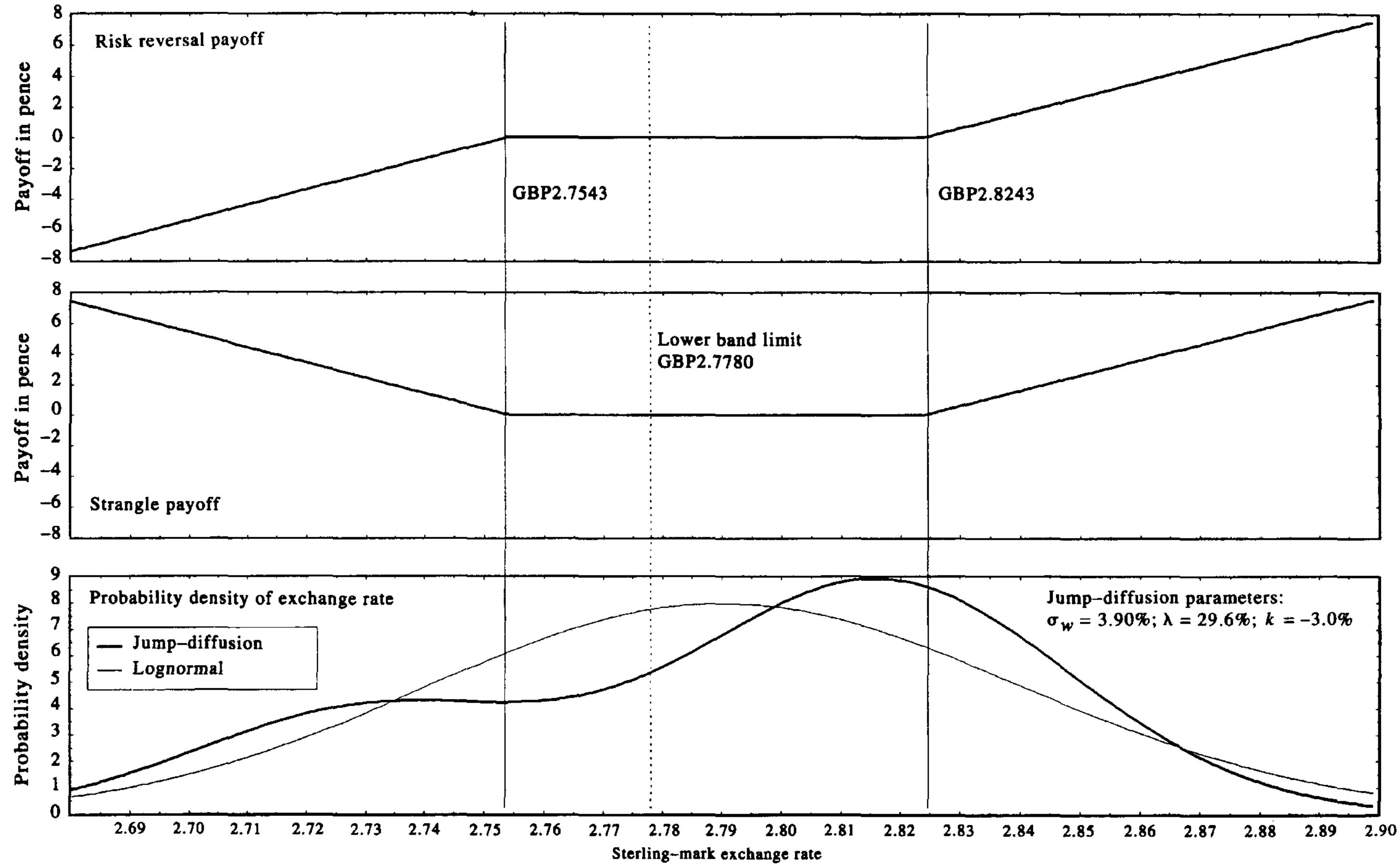
Two instruments actively traded in over-the-counter currency option markets, risk reversals and strangles, summarize the shape of the prevailing volatility smile. A strangle consists of an out-of-the-money put and call with the same delta, usually 25 or 30 percent; the dealer sells or buys both from the counterparty. Dealers usually quote strangle prices by stating the average implied volatility at which they buy or sell the out-of-the-money options and record strangle prices as the spread of the strangle volatility over the at-the-money forward volatility; this measure indicates the degree of curvature of the volatility smile.

A risk reversal also consists of an out-of-the-money put and call. In a risk reversal, the dealer exchanges one of the options for the other with the counterparty. Because the put and the call generally have different implied volatilities, the dealer pays or receives a premium for exchanging the options. The premium is expressed as the implied volatility spread at which he exchanges a 25-delta call for a 25-delta put and indicates the skewness of the volatility smile. For example, if sterling–mark is expected to fall sharply (sterling depreciation), an option dealer might quote sterling–mark risk reversals as follows: 'one-month 25-delta risk reversals are 0.7 at 1.3 mark calls over', meaning he stands ready to pay a net premium of 0.7 vols to buy a 25-delta mark call (sterling put) and sell a 25-delta mark put against the pound, and charges a net premium of 1.3 vols to sell a 25-delta mark call and buy a 25-delta mark put.[17]

The midpoint of the time t strangle price can be expressed as $str_t = 0.5(\sigma_t^{75\delta} + \sigma_t^{25\delta})\text{-}atm_t$ and the risk reversal price as $rr_t = \sigma_t^{25\delta} - \sigma_t^{75\delta}$, where str_t, rr_t, and atm_t denote the strangle price, risk reversal price, and at-the-money volatility, and $\sigma_t^{25\delta}$ and $\sigma_t^{75\delta}$ refer to the implied volatilities of the 25 delta call and the 25 delta put. Using these definitions, the market quotes for the strangle price, risk reversal price, and at-the-money volatility can be solved for $\sigma_t^{25\delta}$ and $\sigma_t^{75\delta}$:[18]

$$\begin{aligned} \sigma_t^{25\delta} &= atm_t + str_t + 0.5rr_t \\ \sigma_t^{75\delta} &= atm_t + str_t - 0.5rr_t. \end{aligned} \quad \langle 8 \rangle$$

Figure 3 attempts to convey some intuition for the relationship between risk reversal and strangle prices and the risk-neutral distribution of the future exchange rate. The first two panels display the terminal payoffs of one-month 25-delta sterling–mark risk reversals and strangles on August 31, 1992. The bottom panel displays two density functions. If the Black–Scholes model were



Sterling–mark, 31 Aug. 1992: spot GBP2.7922;

ATM option price 6.20%; risk reversal price −1.00%; strangle price 0.25%.

FIGURE 3. Option prices and the probability density function.

correct, the n-year forward exchange premium and $\sqrt{n}$ times the market implied volatility could be interpreted as the risk-neutral first and second moments of $(S_{t+n} - S_t)/S_t$. The thin line in Figure 3 is the lognormal density based on the August 31 sterling–mark one-month forward rate (DM 2.7913) and at-the-money forward volatility (6.2 percent). The second density, estimated with the jump-diffusion model described in the next section, is consistent with the out-of-the-money volatilities as well as the at-the-money volatility.

On August 31, the exercise price of the sterling put component of the 25-delta risk reversal and strangle was DM 2.7543 and that of the sterling call DM 2.8243. The risk reversal price, -1.00 vols, indicates that the market considered the expected value of exchange rate realizations on September 31 below DM 2.7543 to be greater than that of realizations in excess of DM 2.8243. The strangle price, 0.25 vols, indicates that the market considered the expected value of exchange rate realizations outside the range {2.7543, 2.8243} to be greater than consistent with a lognormal distribution.

Correspondingly, the density based on the jump-diffusion model has fatter tails than the lognormal, and the left tail is fatter than the right, particularly for the extreme exchange rates which make the greatest contribution to the values of the 25-delta put and call. The risk reversal price would be zero if the market believed that the tails of the density function of percent changes in the forward rate were symmetrical, that is, increases or decreases in the forward rate of a given magnitude were equally likely.[19]

VI. Currency option prices in the presence of realignment risk

VI.A. Option pricing formulas for jump-diffusions

The evidence from option prices on skewness and kurtosis in expected future asset prices suggests that one might improve the Black–Scholes estimate of the perceived probability distribution using simultaneous observations of option prices with different strike prices. Breeden and Litzenberger (1978) pointed out that the second derivative of European call option prices with respect to the exercise price is the risk-neutral probability density of the time T asset price. This result has motivated attempts to numerically reconstruct a distribution consistent with observed option prices.[20]

Skewness and kurtosis also suggest alternative option pricing models based on alternative distributions to the lognormal iid. Bates (1991, 1996a) fits option prices with varying exercise prices to a jump-diffusion option valuation formula to estimate the parameters of the jump-diffusion.

Following Bates, I explain the option skewness and kurtosis using an asymmetric jump-diffusion model of the stochastic process for the exchange rate, the risk-neutral process for which can be written

$$\langle 9 \rangle \qquad S_T = S_0 + \int_0^T (r - r^* - \lambda E[k])S_t dt + \int_0^T \sigma_w S_t dW_t + \int_0^T S_t k dq_{t,T},$$

where σ_w denotes the diffusion volatility of the exchange rate, $q_{t,T}$ is a Poisson counter over the interval (t,T) with average rate of occurrence of jumps λ, and

k is the possibly random jump size. The option valuation formulas for this model are derived by Merton (1976) and Bates (1988, 1991).[21] The parameters are those of the risk-neutral distribution and are not in general equal to the true parameters.[22]

For the mark cross rates in the ERM, it seems more appropriate to employ a simplified version of the jump-diffusion model, in which k is non-stochastic and there is either zero or one jump in the exchange rate over the life of the option. Ball and Torous (1983, 1985) refer to this as the Bernoulli distribution version of the model. The formula for a call is

⟨10⟩

$$c(S_t,\tau;X,\sigma_w,r,r^*,\lambda,k)$$
$$= (1-\lambda\tau)\left[\frac{S_t e^{-r^*\tau}}{1+\lambda k\tau}\Phi\left(d_0+\sigma_w\sqrt{\tau}\right) - Xe^{-r\tau}\Phi(d_0)\right]$$
$$+\lambda\tau\left[\frac{S_t e^{-r^*\tau}}{1+\lambda k\tau}(1+k)\Phi\left(d_1+\sigma_w\sqrt{\tau}\right) - Xe^{-r\tau}\Phi(d_1)\right]$$
$$= (1-\lambda\tau)v\left(\frac{S_t}{1+\lambda k\tau},\tau;X,\sigma_w,r,r^*\right) + \lambda\tau v\left[\frac{S_t}{1+\lambda k\tau}(1+k),\tau;X,\sigma_w,r,r^*\right],$$

and that of a put is

⟨11⟩

$$p(S_t,\tau;X,\sigma_w,r,r^*,\lambda,k)$$
$$= (1-\lambda\tau)\left[Xe^{-r\tau}\Phi(-d_0) - \frac{S_t e^{-r^*\tau}}{1+\lambda k\tau}\Phi\left(-d_0-\sigma_w\sqrt{\tau}\right)\right]$$
$$+\lambda\tau\left[Xe^{-r\tau}(1+k)\Phi(-d_1) - \frac{S_t e^{-r^*\tau}}{1+\lambda k\tau}\Phi\left(-d_1-\sigma_w\sqrt{\tau}\right)\right]$$
$$= (1-\lambda\tau)w\left(\frac{S_t}{1+\lambda k\tau},\tau;X,\sigma_w,r,r^*\right) + \lambda\tau w\left[\frac{S_t}{1+\lambda k\tau}(1+k),\tau;X,\sigma_w,r,r^*\right],$$

where

⟨12⟩
$$d_0 = \frac{\ln(S_t/X) - \ln(1+\lambda k\tau) + \left(r - r^* - \sigma_w^2/2\right)\tau}{\sigma_w\sqrt{\tau}}$$

and

⟨13⟩
$$d_1 = \frac{\ln(S_t/X) - \ln(1+\lambda k\tau) + \ln(1+k) + \left(r - r^* - \sigma_w^2/2\right)\tau}{\sigma_w\sqrt{\tau}}.$$

Each formula is an average of the Black–Scholes option value given a jump, weighted by the probability of a jump, and the Black–Scholes value absent a jump, weighted by the probability of no jump.

The spot exchange rate is divided by the expected value of a jump $(1+\lambda k\tau)$ in the formulas. Intuitively, sterling–mark must already have appreciated by, say, 5 percent, to reflect a jump with an expected value of 5 percent. Otherwise, the weighted average of the zero-jump and one-jump future spot rates would not equal the current forward rate and the option price would not be risk-neutral. The $(1+\lambda k\tau)^{-1}$ term implies that if there is no jump, the exchange rate will depreciate by $(1+\lambda k\tau)$; if there is a jump, the exchange rate will appreciate by $(1+k)(1+\lambda k\tau)^{-1}$. In either case, one also adds the forward points $S_t[e^{(r-r^*)\tau}-1]$ to arrive at the risk-neutral first moment of the distribution.

The jump-diffusion model captures the widespread market view that the risk for sterling was that a realignment would bring about a single sharp change in the currency's value, but that the diffusion volatility σ_w would remain constant, implying no change in the band width. The model does not capture the sense among at least some market participants by September 1992—and the actual outcome—of a suspension of participation in the target zone or no realignment plus widening of the band.

VI.B. Normalized Black–Scholes and jump-diffusion option price formulas

The conventions of the over-the-counter currency option markets permit a simplification of the valuation formulas used in estimating the jump-diffusion parameters from Black–Scholes prices. Data on the spot exchange rate, the foreign and domestic interest rates, and the exercise prices of the options are not needed.[23] The data pertain to one-month options, so τ is set to 1/12 throughout. Dividing equation ⟨3⟩ through by $X_t e^{-r\tau}$ yields the simplified Black–Scholes formula for the value of a call

$$\langle 14\rangle \qquad v(R_t,\sigma)\equiv e^r v\left(R_t,1;\frac{1}{12},\sigma,0,0\right)=R_t\Phi(d+\sigma)-\Phi(d),$$

where $R_t \equiv F_{t,t+1/12}/X$, $F_{t,t+1/12}=S_t e^{(r-r^*)/12}$, and

$$\langle 15\rangle \qquad d=\frac{\ln(R_t)}{\sigma}-\frac{\sigma}{2}.$$

The simplified Black–Scholes formula for a currency put is

$$\langle 16\rangle \qquad w(R_t,\sigma)\equiv e^r w\left(R_t,1;\frac{1}{12},\sigma,0,0\right)=\Phi(-d)-R_t\Phi(-d-\sigma).$$

The jump-diffusion formulas can be similarly normalized. The formula for a call is

$$\begin{aligned}\langle 17\rangle \qquad c(R_t,\sigma_w,\lambda,k) &\equiv e^r c\left(R_t,1;\frac{1}{12},\sigma_w,0,0,\lambda,k\right)\\ &= e^r\left[(1-\lambda)v\left(\frac{R_t}{1+\lambda k},1;\frac{1}{12},\sigma_w,0,0\right)\right.\\ &\quad\left.+\lambda v\left(\frac{R_t}{1+\lambda k}(1+k),1;\frac{1}{12},\sigma_w,0,0\right)\right]\\ &=(1-\lambda)v\left(\frac{R_t}{1+\lambda k},\sigma_w\right)+\lambda v\left[\frac{R_t}{1+\lambda k}(1+k),\sigma_w\right],\end{aligned}$$

and for a put

$$\langle 18\rangle \quad p(R_t,\sigma_w,\lambda,k) \equiv e^r p\left(R_t,1;\frac{1}{12},\sigma_w,0,0,\lambda,k\right)$$
$$= (1-\lambda)w\left(\frac{R_t}{1+\lambda k},\sigma_w\right) + \lambda w\left[\frac{R_t}{1+\lambda k}(1+k),\sigma_w\right].$$

The normalized prices are future values rather than being discounted back to the present, and are expressed as a fraction of the exercise price of the option.

VII. Estimates of realignment probabilities using option prices

VII.A. Data

While options on currencies and currency futures are traded on several exchanges, liquidity in currency option trading is centered in the over-the-counter markets. Mostly American options are traded on the exchanges, while primarily European options, which are simpler to evaluate, are traded over-the-counter. Exchange-traded options mature on fixed dates, so that prices on successive days pertain to options of decreasing maturity. In over-the-counter markets, a fresh option for standard maturities can be purchased daily, so a series of prices of options of like maturity can be constructed.

Trading in dollar options is particularly liquid, but much trading also takes place in options on German marks against other European currencies and the yen. In 1992, the markets in European cross-rate options were less developed than today; sterling–mark options were dominant, accounting for over 80 percent of European cross-rate option transactions.[24]

Price data for out-of-the-money over-the-counter European cross-rate options are difficult to obtain for the narrow-band ERM era. One major dealer has recorded prices of sterling–mark one-month at-the-money forward options, and one-month 25-delta risk reversals and strangles data since March 31, 1992. The data were entered daily at noon London time by the option traders.[25]

Liquidity in major European cross-rate options such as sterling–mark and mark–French franc was generally good in 1992 in the sense that, at least for such standardized products as at-the-money forward options, straddles, risk reversals and strangles, there were at most times prices in the interbank market which dealers could easily discover and at which they could adjust their positions without significantly moving prices. The growing crisis in the ERM strained option market liquidity in some of the smaller currency segments by the end of August. Liquidity held up longer for mark–French franc and particularly for sterling–mark. However, by September 15, and certainly on September 16, 1992, the prices recorded by traders may not have reflected actual transactions, which had become infrequent. Sterling–mark at-the-money implied volatilities are displayed in Figure 4.

VII.B. Estimation procedure

The task is to estimate the risk-neutral parameters σ_w, λ and k in the

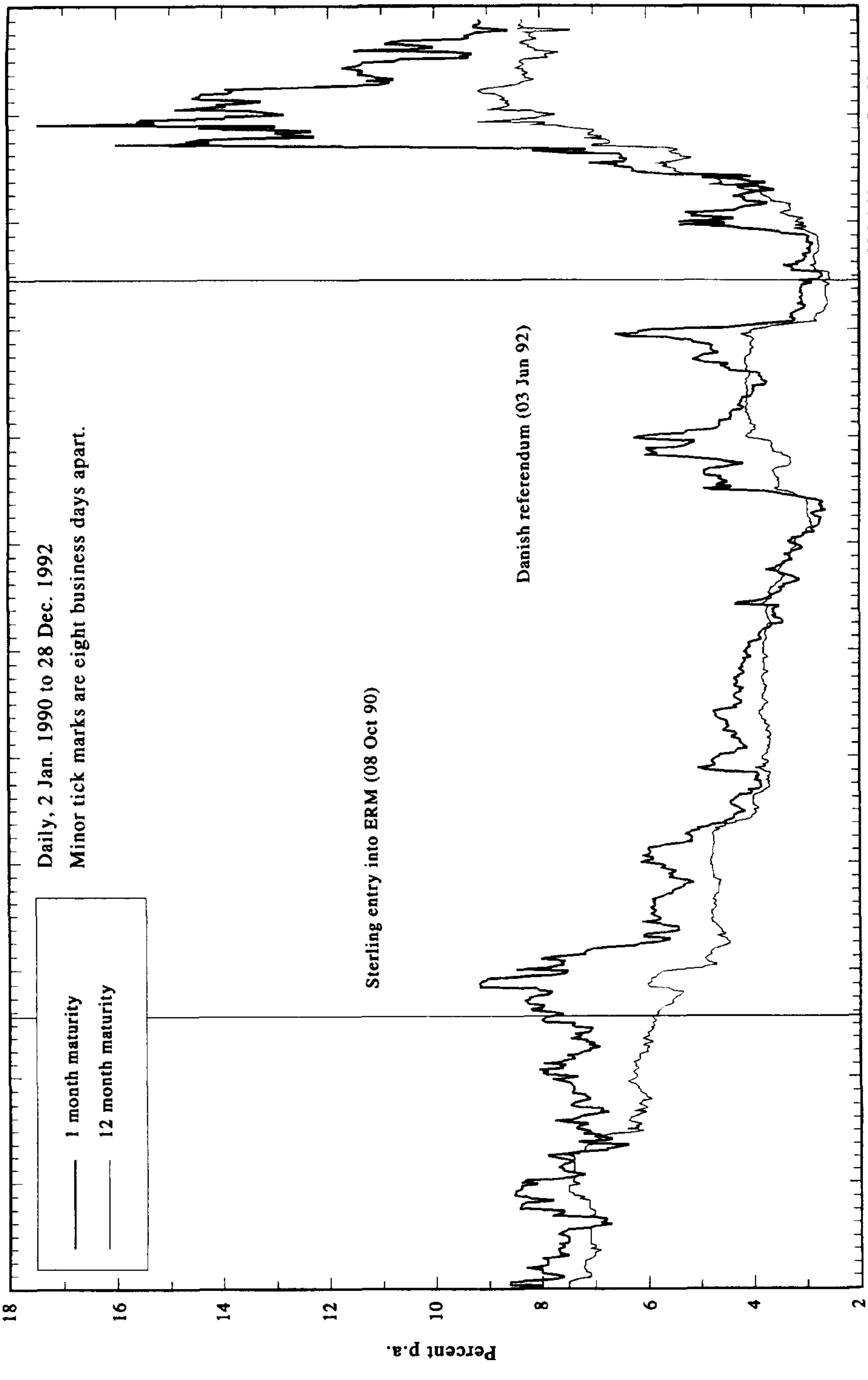


FIGURE 4. Sterling–mark at-the-money implied volatility.

jump-diffusion option valuation formulas ⟨17⟩ and ⟨18⟩. The data are the prices, in volatilities, of at-the-money forward one-month options, one-month 25-delta risk reversals, and one-month 25-delta strangles.

It is worth reviewing several implicit assumptions made here. The jump-diffusion model postulates that the parameters σ_w, λ and k are constants. However, extracting daily parameter estimates implicitly assumes that the market views the parameters as constant over the one-month maturity of the options, but revises its estimates of the parameters each day.[26] It is also assumed that one-month implied volatilities, although quoted at an annual rate, are in fact monthly volatilities scaled up by $\sqrt{12}$, so rescaling them to a monthly basis in the formulas imposes no additional structure.

The estimation procedure is as follows. I first calculate the option prices from the at-the-money volatilities and risk reversal and strangle prices expressed in vols. To do so:

- I find $\sigma_t^{25\delta}$ and $\sigma_t^{75\delta}$, the implied volatilities of 25- and 75-delta calls, from the at-the-money volatilities and risk reversal and strangle prices via equation ⟨8⟩, and set $\sigma_t^{50\delta} = atm_t$.[27]
- Next, I use these volatilities and equation ⟨6⟩ to solve for the forward rate/exercise price ratio at which the call option delta is 25, 50, or 75 percent.
- Using the implied volatility and the forward rate/exercise price ratio, I transform the volatilities into option prices using the normalized Black–Scholes formulas.

Next, I estimate the parameters by finding $\underset{\{\lambda_t, k_t, \sigma_{w,t}\}}{\operatorname{argmin}} \sum_{i=1}^{3} (u_t^i)^2$, with the u_t^i defined by

$$\begin{aligned} v\left(R_t^{25\delta}, \sigma_t^{25\delta}\right) &= c\left(R_t^{25\delta}, \sigma_{w,t}, \lambda_t, k_t\right) + u_t^1 \\ v\left(R_t^{50\delta}, \sigma_t^{50}\right) &= c\left(R_t^{50\delta}, \sigma_{w,t}, \lambda_t, k_t\right) + u_t^2 \qquad \langle 19\rangle \\ w\left(R_t^{75\delta}, \sigma_t^{75}\right) &= p\left(R_t^{75\delta}, \sigma_{w,t}, \lambda_t, k_t\right) + u_t^3, \end{aligned}$$

where $R_t^{25\delta}$, $R_t^{50\delta}$ and $R_t^{75\delta}$ refer to the forward rate/exercise price ratio of the 25-delta call, the 50-delta call, and the 25-delta put.[28]

It is difficult to estimate all three parameters simultaneously. If $\sigma_{w,t}$ in the jump-diffusion formula is close to the Black–Scholes implied volatility, λ_t and k_t become small and difficult to distinguish, so $\sigma_{w,t}$ needs to be held constant while searching over values of λ_t and k_t. I therefore employed a two-step procedure, holding $\sigma_{w,t}$ constant while estimating λ_t and k_t, repeating the procedure for a grid of $\sigma_{w,t}$ values, and selecting the estimate which minimized $\sum_{i=1}^{3} (u_t^i)^2$.[29]

It is difficult to distinguish sharply between the contribution of λ and k to the observed volatility, since they generally appear paired as λk in equations

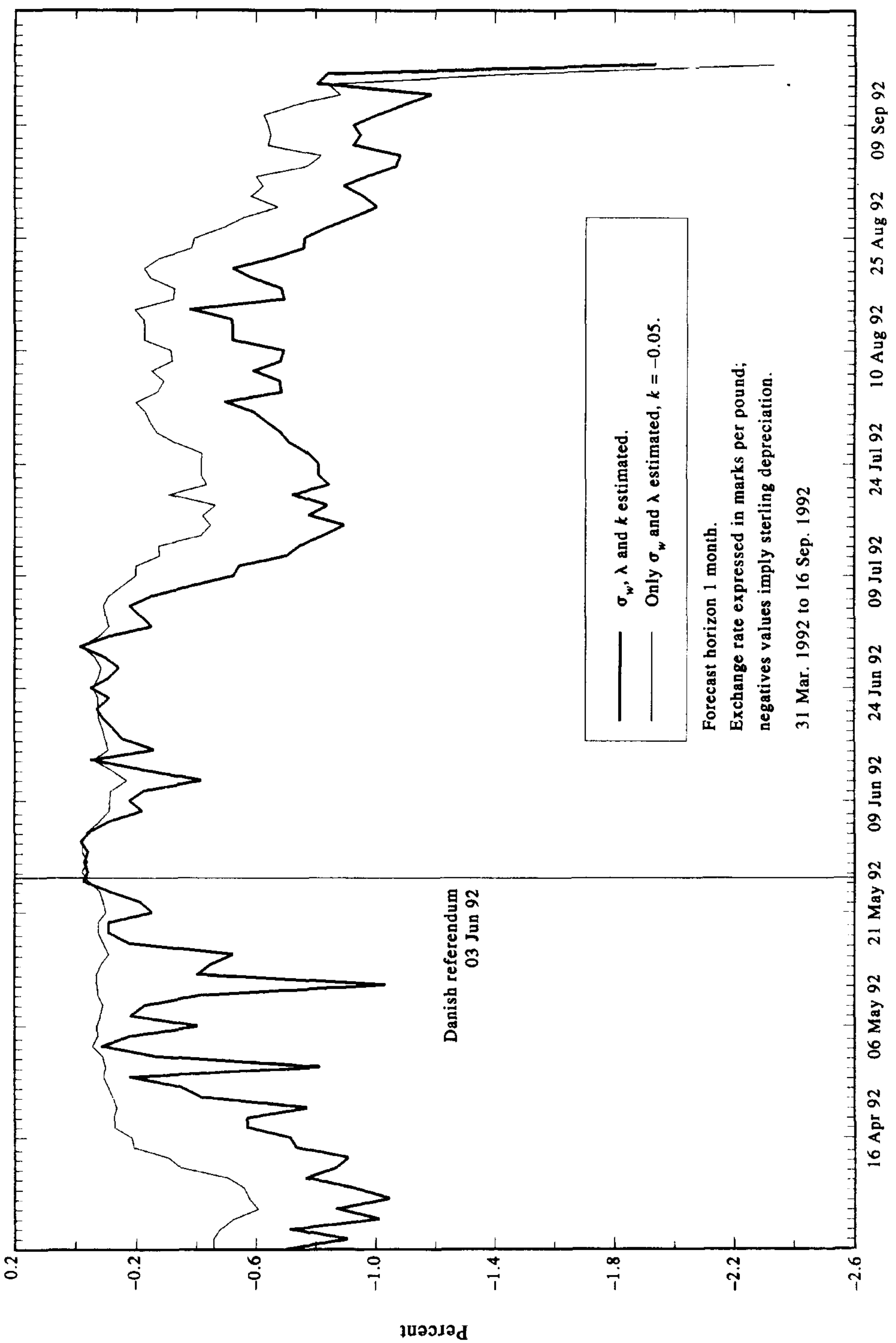


FIGURE 5. Expected value of jump in sterling–mark exchange rate.

⟨17⟩ and ⟨18⟩. As a check of robustness, I estimated both λ_t and k_t jointly, and λ_t alone, with k_t set to a constant value of -0.05. This is somewhat below the general range of estimated k_t values and well below the depreciation in the month after the suspension of participation in the ERM, but permits a comparison with Rose's (1993) target zone model estimates of the realignment probability.[30] The results are displayed in Tables 1 and 2. Figure 5 displays estimates of $\lambda_t k_t$, the expected value of a jump over the next month. The sterling–mark exchange rate is expressed in marks per pound, so a sudden depreciation of the pound means a negative jump in the rate.[31]

The estimates of $\lambda_t k_t$ are plausible, near zero before the onset of the crisis but falling modestly after the Danish referendum and dramatically from the end of August 1992. The estimates of k_t are about 3 to 4 percent on the days of acute selling pressure, quite different from the *ex post* value of about 0.125, suggesting the market was surprised by the extent, if not the timing, of sterling's depreciation.

As an indicator of the accuracy of the estimated parameters, Figure 6 compares the actual with the fitted option values.[32] The fit is generally quite good, with the out-of-the-money sterling calls somewhat, but not dramatically, underpriced. The fit is poor on the last day of sterling's participation in the ERM, a day on which, as noted above, trading in sterling–mark options had fallen off considerably and the recorded prices may be inaccurate. The poor fit on that day may also reflect model misspecification.

VII.C. The implied distribution of future exchange rates and realignment probabilities

If the exchange rate follows the geometric Brownian process represented in equation ⟨2⟩, as assumed by the Black–Scholes model, the risk-neutral distribution of S_T is lognormal. Substituting $F_{t,t+\tau} = S_t e^{(r-r^*)\tau}$ for notational convenience, we can write its cumulative probability distribution function as

⟨20⟩
$$\text{prob}\,\{S_T \le X\} = \Phi\left[\frac{\ln\left(\dfrac{X}{F_{r,t+\tau}}\right) + \dfrac{\sigma^2}{2}\tau}{\sigma\sqrt{\tau}}\right].$$

If S_t follows the jump-diffusion process represented in equation ⟨9⟩, the risk-neutral distribution of S_T is

⟨21⟩
$$\text{prob}\,\{S_T \le X\} = \sum_{n=0}^{\infty} \frac{e^{-\lambda\tau}(\lambda\tau)^n}{n!}\Phi\left[\frac{\ln\left(\dfrac{X}{F_{t,t+\tau}}\right) + \left(\lambda k + \dfrac{\sigma_w^2}{2}\right)\tau - n\ln(1+k)}{\sigma_w\sqrt{\tau}}\right].$$

In the Bernoulli distribution model, the Poisson counter $q_{t,T}$ is zero with probability $(1-\lambda_t)$ and unity with probability λ_t, and equation ⟨21⟩ describes a mixture of two lognormals with distribution function

⟨22⟩

$$\text{prob}\{S_T \leq X\} = (1-\lambda)\Phi\left[\frac{\ln\left(\frac{X}{F_{t,t+1}}\right) + \ln(1+\lambda k) + \frac{\sigma_w^2}{2}}{\sigma_w}\right]$$

$$+\lambda\Phi\left[\frac{\ln\left(\frac{X}{F_{t,t+1}}\right) + \ln(1+\lambda k) - \ln(1+k) + \frac{\sigma_w^2}{2}}{\sigma_w}\right].$$

The probability density function of sterling–mark corresponding to the distribution in equation ⟨22⟩ is plotted in Figure 7. It is calculated for each day by substituting for λ_t, k_t, $\sigma_{w,t}$, and $F_{t,t+\tau}$ their estimated or observed values. The distribution is tight and centered at high values of $F_{t,t+\tau}$ in spring of 1992. As sterling weakens and the credibility of the target zone dissolves, the distribution becomes more dispersed and its center moves lower. For days on which the market price of protection against realignment risk was high, the distribution is bimodal.

Another perspective on the estimation procedure can be gained by returning for a moment to the volatility smile. Figure 8 displays the sterling–mark volatility smile on August 31, 1992, implied by the jump-diffusion model, together with a volatility smile drawn from the same data by interpolation and the volatility smile implied by the Black–Scholes model—a constant equal to the at-the-money volatility.

The interpolated volatility smile does not incorporate a distributional hypothesis regarding the exchange rate. Similar interpolated volatility smiles can be drawn for flexible exchange rates such as dollar–mark and dollar–yen which have active over-the-counter option markets, and for which strangle and risk reversal prices are easily obtained. The interpolated volatility smile can in turn be used to extract numerically an associated risk-neutral probability distribution.[33] The volatility smile based on the jump-diffusion model incorporates the distributional hypothesis that sterling–mark follows a jump-diffusion. Its shape corresponds to the bimodal probability density function implied by the estimated parameters.

The risk-neutral realignment probability is the likelihood that $S_T \leq S_*$, where S_* denotes the lower fluctuation limit, and is calculated by substituting S_* for X in equation ⟨22⟩: $\pi_t^{1/12} \equiv \pi_t = \text{prob}\{S_T \leq S_*\}$. Figure 9 displays π_t with λ_t, $\sigma_{w,t}$ and k_t estimated and with λ_t and $\sigma_{w,t}$ estimated and k_t set to -0.05, $\forall t$. The series based on option prices are compared with one based on the target zone (Svensson–Rose) model.[34]

The estimated realignment probabilities conform closely to narratives of the ERM's unraveling. The probabilities were zero in the spring of 1992 and rose sharply in the second half of August, peaking on September 16. Expectations

TABLE 1. Parameter estimates for the jump-diffusion model; λ_t, k_t and $\sigma_{w,t}$ estimated

Date	λ_t	k_t	$\sigma_{w,t}$	Date	λ_t	k_t	$\sigma_{w,t}$	Date	λ_t	k_t	$\sigma_{w,t}$
31 Mar 92	0.3021	−0.0233	0.0375	29 May 92	0.0438	−0.0096	0.0275	24 Jul 92	0.3537	−0.0219	0.0345
01 Apr 92	0.3836	−0.0236	0.0350	01 Jun 92	0.0054	−0.0313	0.0275	27 Jul 92	0.3538	−0.0203	0.0300
02 Apr 92	0.2987	−0.0239	0.0395	02 Jun 92	0.0237	−0.0209	0.0275	28 Jul 92	0.3734	−0.0184	0.0280
03 Apr 92	0.3906	−0.0259	0.0400	03 Jun 92	0.0535	−0.0214	0.0275	29 Jul 92	0.3613	−0.0177	0.0275
06 Apr 92	0.3136	−0.0277	0.0495	04 Jun 92	0.1433	−0.0154	0.0275	30 Jul 92	0.3463	−0.0172	0.0275
07 Apr 92	0.3749	−0.0279	0.0425	05 Jun 92	0.0880	−0.0200	0.0275	31 Jul 92	0.3062	−0.0162	0.0275
08 Apr 92	0.3463	−0.0267	0.0435	09 Jun 92	0.1426	−0.0159	0.0275	03 Aug 92	0.3734	−0.0184	0.0280
09 Apr 92	0.3184	−0.0241	0.0440	10 Jun 92	0.2792	−0.0149	0.0275	04 Aug 92	0.3504	−0.0194	0.0280
10 Apr 92	0.4305	−0.0201	0.0305	11 Jun 92	0.1426	−0.0159	0.0275	05 Aug 92	0.3226	−0.0183	0.0275
13 Apr 92	0.5007	−0.0181	0.0310	12 Jun 92	0.0082	−0.0598	0.0275	06 Aug 92	0.3322	−0.0206	0.0285
14 Apr 92	0.4998	−0.0147	0.0275	15 Jun 92	0.2004	−0.0129	0.0275	07 Aug 92	0.3484	−0.0199	0.0295
15 Apr 92	0.4996	−0.0144	0.0275	16 Jun 92	0.0856	−0.0186	0.0275	10 Aug 92	0.2947	−0.0177	0.0275
16 Apr 92	0.4970	−0.0115	0.0275	17 Jun 92	0.0587	−0.0218	0.0275	11 Aug 92	0.2947	−0.0177	0.0275
21 Apr 92	0.4970	−0.0115	0.0275	18 Jun 92	0.0340	−0.0276	0.0275	12 Aug 92	0.2947	−0.0177	0.0275
22 Apr 92	0.6147	−0.0125	0.0275	19 Jun 92	0.0127	−0.0567	0.0275	13 Aug 92	0.2058	−0.0185	0.0275
23 Apr 92	0.3502	−0.0119	0.0275	22 Jun 92	0.0538	−0.0204	0.0275	14 Aug 92	0.3477	−0.0201	0.0300
24 Apr 92	0.3058	−0.0113	0.0275	23 Jun 92	0.0082	−0.0598	0.0275	17 Aug 92	0.3314	−0.0208	0.0290
27 Apr 92	0.1176	−0.0152	0.0275	24 Jun 92	0.0538	−0.0204	0.0275	18 Aug 92	0.3226	−0.0183	0.0275

28 Apr 92	0.7195	−0.0113	0.0275	25 Jun 92	0.0847	−0.0170	0.0275	19 Aug 92	0.2947	−0.0177	0.0275
29 Apr 92	0.2399	−0.0111	0.0275	26 Jun 92	0.0494	−0.0195	0.0275	20 Aug 92	0.3447	−0.0190	0.0275
30 Apr 92	0.0463	−0.0182	0.0275	29 Jun 92	0.0027	−0.0427	0.0275	21 Aug 92	0.3563	−0.0213	0.0325
04 May 92	0.1471	−0.0119	0.0275	30 Jun 92	0.0538	−0.0204	0.0275	24 Aug 92	0.3557	−0.0215	0.0330
05 May 92	0.4905	−0.0082	0.0275	01 Jul 92	0.1825	−0.0138	0.0275	25 Aug 92	0.3429	−0.0244	0.0375
06 May 92	0.1386	−0.0132	0.0275	02 Jul 92	0.1508	−0.0143	0.0275	26 Aug 92	0.3463	−0.0267	0.0435
07 May 92	0.1886	−0.0122	0.0275	03 Jul 92	0.1176	−0.0152	0.0275	27 Aug 92	0.3346	−0.0301	0.0495
08 May 92	0.4873	−0.0085	0.0275	06 Jul 92	0.1825	−0.0138	0.0275	28 Aug 92	0.3497	−0.0274	0.0450
11 May 92	0.8490	−0.0122	0.0275	07 Jul 92	0.2632	−0.0145	0.0275	31 Aug 92	0.2955	−0.0302	0.0390
12 May 92	0.4905	−0.0082	0.0275	08 Jul 92	0.3335	−0.0158	0.0275	01 Sep 92	0.3475	−0.0280	0.0470
13 May 92	0.4907	−0.0091	0.0275	09 Jul 92	0.3457	−0.0158	0.0275	02 Sep92	0.3284	−0.0326	0.0525
14 May 92	0.4955	−0.0106	0.0275	10 Jul 92	0.3865	−0.0182	0.0290	03 Sep 92	0.3262	−0.0332	0.0545
15 May 92	0.1176	−0.0152	0.0275	13 Jul 92	0.4043	−0.0185	0.0275	04 Sep 92	0.3093	−0.0300	0.0435
18 May 92	0.0538	−0.0204	0.0275	14 Jul 92	0.3954	−0.0209	0.0355	07 Sep 92	0.3136	−0.0304	0.0440
19 May 92	0.0538	−0.0204	0.0275	15 Jul 92	0.4018	−0.0223	0.0385	08 Sep 92	0.3093	−0.0300	0.0435
20 May 92	0.2025	−0.0125	0.0275	16 Jul 92	0.3537	−0.0219	0.0345	09 Sep 92	0.3509	−0.0287	0.0485
21 May 92	0.1656	−0.0129	0.0275	17 Jul 92	0.3618	−0.0232	0.0370	10 Sep 92	0.3544	−0.0309	0.0545
22 May 92	0.0535	−0.0214	0.0275	20 Jul 92	0.3774	−0.0191	0.0295	11 Sep 92	0.3574	−0.0332	0.0610
25 May 92	0.0065	−0.0440	0.0275	21 Jul 92	0.3859	−0.0219	0.0360	14 Sep 92	0.2395	−0.0337	0.0610
26 May 92	0.0438	−0.0096	0.0275	22 Jul 92	0.3819	−0.0213	0.0350	15 Sep 92	0.1996	−0.0420	0.0710
27 May 92	0.0096	−0.0322	0.0275	23 Jul 92	0.3819	−0.0213	0.0350	16 Sep 92	0.2537	−0.0762	0.0710

TABLE 2. Parameter estimates for the jump-diffusion model; λ_t and $\sigma_{w,t}$ estimated, k_t set to -0.05

Date	λ_t	$\sigma_{w,t}$	Date	λ_t	$\sigma_{w,t}$	Date	λ_t	$\sigma_{w,t}$
31 May 92	0.0920	0.0280	29 May 92	0.0040	0.0275	24 Jul 92	0.0839	0.0275
01 Apr 92	0.0917	0.0295	01 Jun 92	0.0046	0.0275	27 Jul 92	0.0844	0.0275
02 Apr 92	0.0960	0.0295	02 Jun 92	0.0084	0.0275	28 Jul 92	0.0669	0.0275
03 Apr 92	0.1046	0.0345	03 Jun 92	0.0162	0.0275	29 Jul 92	0.0548	0.0275
06 Apr 92	0.1215	0.0390	04 Jun 92	0.0224	0.0275	30 Jul 92	0.0499	0.0275
07 Apr 92	0.1158	0.0370	05 Jun 92	0.0228	0.0275	31 Jul 92	0.0466	0.0275
08 Apr 92	0.1122	0.0355	09 Jun 92	0.0233	0.0275	03 Aug 92	0.0401	0.0275
09 Apr 92	0.1010	0.0335	10 Jun 92	0.0334	0.0275	04 Aug 92	0.0548	0.0275
10 Apr 92	0.0702	0.0275	11 Jun 92	0.0233	0.0275	05 Aug 92	0.0586	0.0275
13 Apr 92	0.0615	0.0275	12 Jun 92	0.0115	0.0275	06 Aug 92	0.0504	0.0275
14 Apr 92	0.0387	0.0275	15 Jun 92	0.0211	0.0275	07 Aug 92	0.0640	0.0275
15 Apr 92	0.0370	0.0275	16 Jun 92	0.0199	0.0275	10 Aug 92	0.0636	0.0275
16 Apr 92	0.0255	0.0275	17 Jun 92	0.0182	0.0275	11 Aug 92	0.0455	0.0275
21 Apr 92	0.0255	0.0275	18 Jun 92	0.0154	0.0275	12 Aug 92	0.0455	0.0275
22 Apr 92	0.0274	0.0275	19 Jun 92	0.0146	0.0275	13 Aug 92	0.0455	0.0275
23 Apr 92	0.0250	0.0275	22 Jun 92	0.0151	0.0275	14 Aug 92	0.0394	0.0275
24 Apr 92	0.0216	0.0275	23 Jun 92	0.0115	0.0275	17 Aug 92	0.0652	0.0275
27 Apr 92	0.0187	0.0275	24 Jun 92	0.0151	0.0275	18 Aug 92	0.0657	0.0275
28 Apr 92	0.0198	0.0275	25 Jun 92	0.0169	0.0275	19 Aug 92	0.0504	0.0275
29 Apr 92	0.0181	0.0275	26 Jun 92	0.0131	0.0275	20 Aug 92	0.0455	0.0275
30 Apr 92	0.0110	0.0275	29 Jun 92	0.0037	0.0275	21 Aug 92	0.0553	0.0275
04 May 92	0.0146	0.0275	30 Jun 92	0.0151	0.0275	24 Aug 92	0.0773	0.0275
05 May 92	0.0141	0.0275	01 Jul 92	0.0221	0.0275	25 Aug 92	0.0791	0.0275
06 May 92	0.0167	0.0275	02 Jul 92	0.0204	0.0275	26 Aug 92	0.0977	0.0300
07 May 92	0.0184	0.0275	03 Jul 92	0.0187	0.0275	27 Aug 92	0.1122	0.0355
08 May 92	0.0151	0.0275	06 Jul 92	0.0221	0.0275	28 Aug 92	0.1347	0.0410
11 May 92	0.0146	0.0275	07 Jul 92	0.0308	0.0275	31 Aug 92	0.1169	0.0370
12 May 92	0.0141	0.0275	08 Jul 92	0.0399	0.0275	01 Sep 92	0.1251	0.0320
13 May 92	0.0169	0.0275	09 Jul 92	0.0407	0.0275	02 Sep 92	0.1203	0.0390
14 May 92	0.0221	0.0275	10 Jul 92	0.0562	0.0275	03 Sep 92	0.1547	0.0435
15 May 92	0.0187	0.0275	13 Jul 92	0.0554	0.0275	04 Sep 92	0.1633	0.0445
18 May 92	0.0151	0.0275	14 Jul 92	0.0836	0.0275	07 Sep 92	0.1281	0.0355
19 May 92	0.0151	0.0275	15 Jul 92	0.0898	0.0310	08 Sep 92	0.1299	0.0365
20 May 92	0.0201	0.0275	16 Jul 92	0.0844	0.0275	09 Sep 92	0.1281	0.0355
21 May 92	0.0184	0.0275	17 Jul 92	0.0926	0.0295	10 Sep 92	0.1253	0.0405
22 May 92	0.0162	0.0275	20 Jul 92	0.0614	0.0275	11 Sep 92	0.1458	0.0455
25 May 92	0.0089	0.0275	21 Jul 92	0.0869	0.0285	14 Sep 92	0.1759	0.0500
26 May 92	0.0040	0.0275	22 Jul 92	0.0839	0.0275	15 Sep 92	0.1677	0.0450
27 May 92	0.0076	0.0275	23 Jul 92	0.0858	0.0275	16 Sep 92	0.2861	0.0435

were unstable during the ERM crisis: in a few days in late August, the market price of protection against realignment risk surged dramatically, supporting Eichengreen and Wyplosz's (1993) and Obstfeld's (1994) interpretation of the

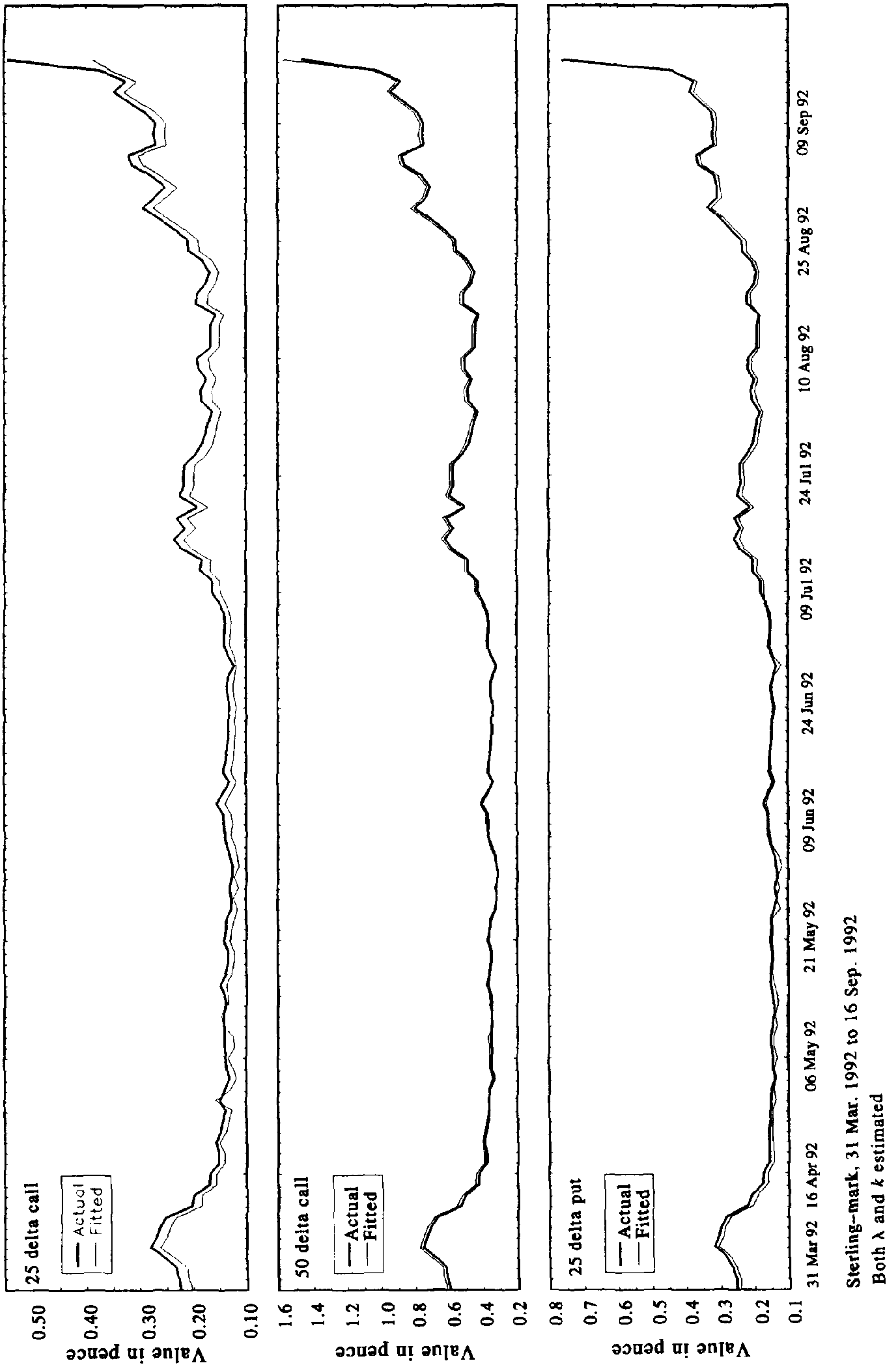


FIGURE 6. Actual and fitted option values.

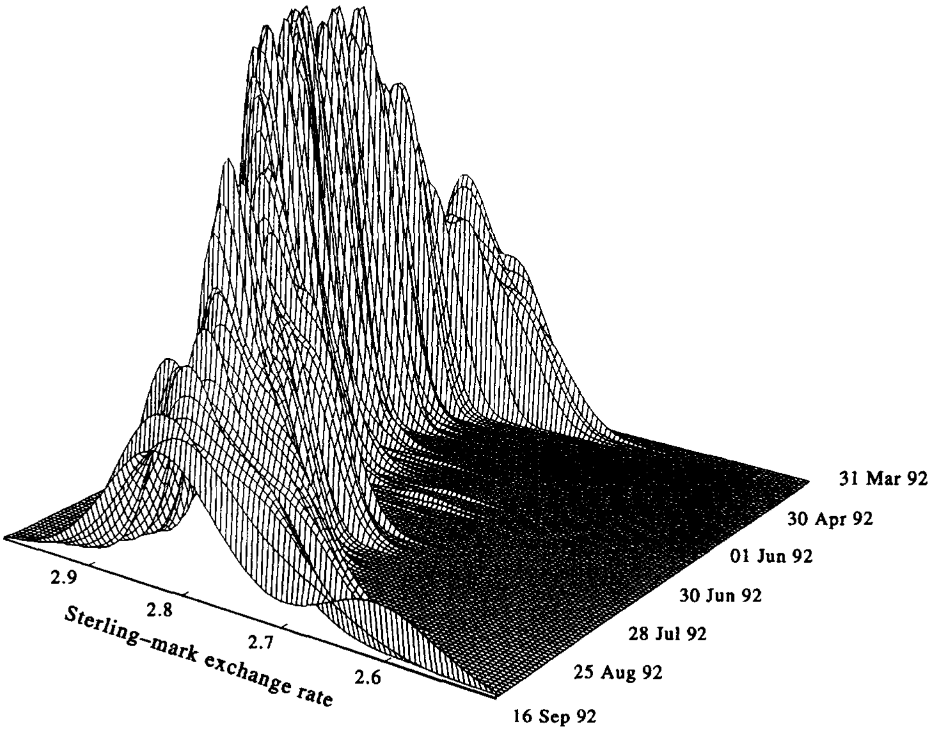


FIGURE 7. Probability density function of the sterling–mark exchange rate.

ERM crisis as a shift from one self-fulfilling set of expectations to another. Sterling's two brief respites from pressure, following the UK's announcement of plans to borrow ECU 10 billion to defend sterling on September 3, and following the lira devaluation on September 13, are also clearly reflected.[35]

The target zone model estimates tell much the same story. However, the target zone model estimates are negative, i.e. show a positive probability of a devaluation of the mark, from mid-April to mid-July 1992, while the estimates based on option prices are bounded below by zero. During the acute phase of the crisis, the target zone model estimates are implausibly low, ranging from 5 to 20 percent. The estimates for September 14, a day of relief for sterling, are a case in point: all three estimates fall on that day, but those based on the target zone model fall almost to zero.[36]

The target zone model estimates are low because they are based on interest rates and on the position of the exchange rate in the fluctuation margins. In the short run, permitting money market rates to rise is only one possible central bank response to exchange rate pressures. Alternative and complementary responses include intervention in exchange markets, public declarations of resolve, and formal and informal attempts to curb speculative techniques. These central bank responses may transitorily keep interest rates of currencies under selling pressure from rising as the perception of realignment risk mounts, but will not prevent option prices from adjusting to reflect those risks.

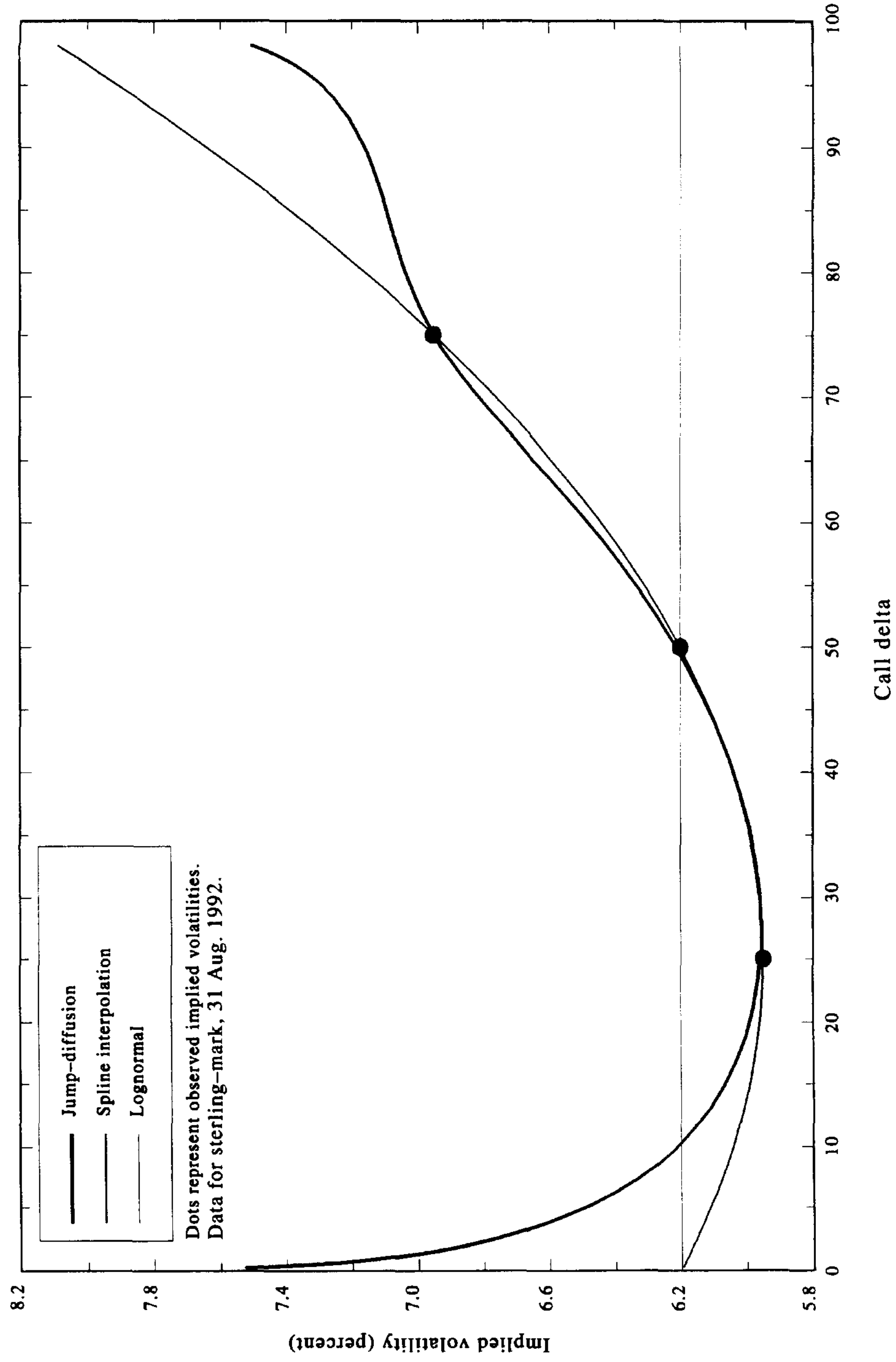


FIGURE 8. Volatility smiles under different models.

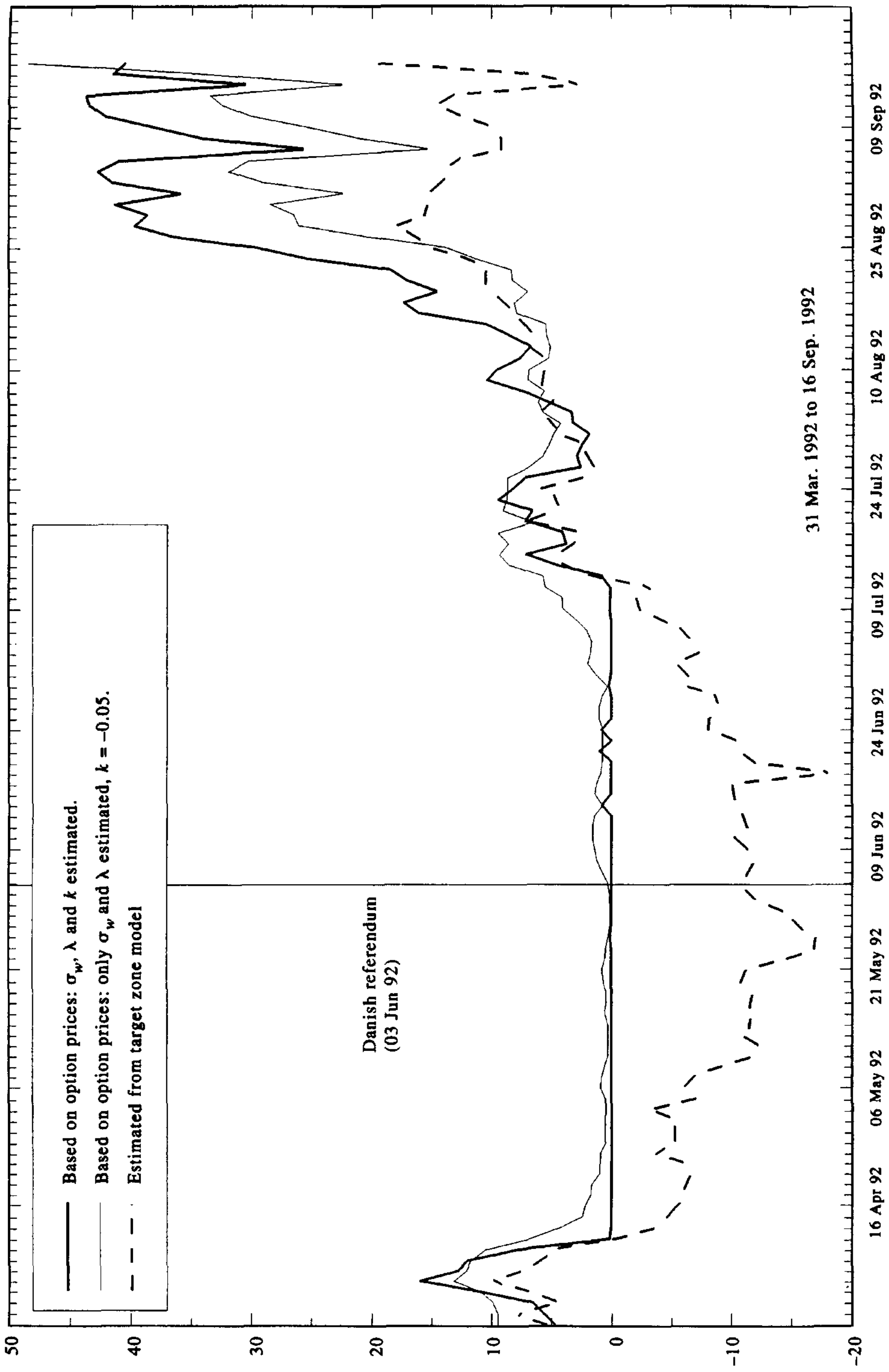


FIGURE 9. Probability of realignment of pound sterling in next month.

VIII. Conclusions

This paper argues that option prices are a useful indicator of the risk-neutral probability distribution of future exchange rates and presents an application to the ERM crisis of 1992. The resulting estimates of the probability of realignment differ considerably from previous estimates based on interest rate differentials and sterling's position in the ERM bands. Although we can only estimate risk-neutral parameters and realignment probabilities using this method (i.e. not true subjective probabilities), the information revealed is potentially quite useful. A central bank, for example, could safely interpret implied realignment probabilities as reflecting the market price of protection against realignment risk.

The results have implications for the defense of target zones against speculative attack. It has been recommended that central banks counter foreign exchange crises by raising interest rates early and gradually. Failing to ward off the crisis at the start may make realignment unavoidable if foreign exchange reserves are finite, since raising interest rates sharply once the crisis is well under way may be counterproductive.[37] However, the initial stages of a crisis are difficult to recognize. As we have seen, interest rate differentials appear to react slowly and incompletely to speculative pressure. Reserves may fluctuate widely for reasons unrelated to the credibility of the peg. Option prices offer central banks an additional indicator of these pressures.

Notes

1. The history and institutions of the ERM are surveyed in Giavazzi and Giovannini (1989), Ungerer *et al.* (1990), and Gros and Thygesen (1992).
2. See Emminger (1986, pp. 361f.).
3. The January 7, 1990, realignment of the Italian lira was a technical step to ease entry into the narrow ±2.25 fluctuation bands.
4. The crisis of the ERM through early 1993 is reviewed by Eichengreen and Wyplosz (1993), Group of Ten (1993), and Goldstein *et al.* (1993).
5. Peter Marsh and Quentin Peel, 'Bundesbank chief in row over ERM alignment,' *Financial Times*, September 16, 1992, p. 16.
6. Krugman's (1991) early model was of a credible target zone maintained by infinitesimally small interventions at the band limits. Bertola and Svensson (1993) present a model of a target zone with a Poisson-distributed realignment probability. Rose and Svensson (1991) and Rose (1993) estimate the probability of realignment for the French franc and pound sterling assuming a fixed realignment size. Svensson (1993b) surveys models and empirical research on target zones.
7. See Boothe and Glassman (1987), Hsieh (1988), Baillie and McMahon (1989), and de Vries (1994) for surveys.
8. See Akgiray and Booth (1988), Tucker and Pond (1988), and Jorion (1988).
9. See Hsieh (1988, 1989) and Baillie and Bollerslev (1989) for applications to currencies.
10. Chen and Giovannini (1992) discuss these issues and estimate the distributions of EMS exchange rates.
11. The original exposition of the Black–Scholes model is Black and Scholes (1973). An identical model was developed independently by Merton (1976). The application of the

model to foreign currency options is also called the Garman–Kohlhagen model, after its publication by Garman and Kohlhagen (1983). Merton (1982) provides an introduction to the two stochastic processes, geometric Brownian motion and jump-diffusion, on which this paper focuses.

12. A domestic currency call is also a foreign currency put. If I write a sterling call option stipulating that I agree to sell one pound in exchange for DM 3.00 in one month, I have also written a mark put, agreeing to buy DM 3.00 against the pound at a price of £0.33 per mark.
13. 'Vol' refers to both implied volatility and its unit of measure (percent per annum).
14. For example, on August 31, 1992, the sterling–mark spot rate was DM 2.7922, the at-the-money forward volatility was 6.2 percent, one-month forward sterling–mark traded at a discount of 9 points (DM 0.0009), and the one-month German mark Eurodeposit rate was 9.75 percent. A one-month at-the-money forward call option on one pound would have cost 1.98 pfennig, or 0.7 percent of the underlying value.
15. Continuing the example of Note 14, the implied volatility of a 25-delta sterling–mark call was 6.95 on August 31, 1992, so its exercise price was DM 2.8243, or 1.1 percent higher than the spot rate. The delta of an at-the-money forward option on August 31, 1992 would have been 49.93 percent. A sterling call with an exercise price of DM 2.7918, slightly higher than the forward exchange rate DM 2.7913, would have a delta of exactly 50 percent.
16. Cookson (1993, pp. 24ff.) and Murphy (1994) discuss the option smile and skewness from a market viewpoint. Shastri and Wethyavivorn (1987) discuss the implied volatility patterns associated with alternative stochastic processes for the exchange rate.
17. The risk reversal price is similar to Bates' (1991, forthcoming) x-percent skewness premium. A positive (negative) risk reversal price corresponds to a value of the skewness premium greater (less) than x percent.
18. Continuing the example of Notes 14 and 15, the risk reversal price on August 31, 1992 was -0.1 vols and the strangle price was 0.25 vols. The implied volatility of the 25-delta put, 6.95 percent, was thus significantly different from that of 25-delta calls (5.95 percent). The average volatility of the out-of-the-money options was 6.45 percent, higher than the at-the-money volatility (6.20 percent).
19. Asymmetry in the tails is not, however, a sufficient condition for a non-zero risk reversal price. The expected value of exchange rate changes beyond the 25-delta points might be equal, even if the tails are not mirror images of one another.
20. Shimko (1993), Derman and Kani (1994), Dupire (1994), Rubinstein (1994), and Malz (1996) present numerical techniques for recovering distributions from options prices.
21. See also Ball and Torous (1983, 1985) and Jarrow and Rudd (1983, pp. 164ff.). An important issue in deriving the option value is that the risk to a seller of options of an increase in the option price following a jump in the asset price cannot be managed by a continuous-adjustment hedging stragegy. The option might jump further in-the-money, in which case the writer will be underhedged. If he attemps to hedge in advance of jumps, he will be overhedged unless a jump occurs. Therefore, in contrast to the Black–Scholes model, the jump-diffusion model does not permit risk-neutral pricing techniques without additional assumptions.
22. The risk-neutral parameter λ might be greater than the true parameter if the jump size k is negative, and agents are risk-averse and willing to pay a premium for, say, a sterling–mark put to protect themselves against a relignment. The risk-neutral λ might be smaller, however, if agents can hedge partially by selling currency to the central bank, which then takes on part of the currency exposure.
23. The dimension reduction is similar to that of Merton (1973, p. 166). The volatilities are converted from the standard annual basis on which they are quoted to a monthly basis by dividing by $\sqrt{12}$. To avoid clutter, I do not incorporate this change of units in the notation, but take it into account in estimation. The jump parameter λ, however, is a monthly rate.

24. According to Bank for International Settlements (1993), daily turnover in foreign exchange options averaged US$37.7 billion equivalent in April 1992, of which US$31.0 billion took place over-the-counter. Daily turnover in over-the-counter sterling–mark options was US$2.2 billion equivalent, out of a total of US$2.5 billion in options on the mark against other EMS currencies.
25. Data on exchange-traded European cross-rate options are unavailable, as the first such option was the Philadelphia Stock Exchange's sterling–mark contract, introduced on September 25, 1992. Data on at-the-money forward implied volatilities are more widely available than out-of-the-money implied volatilities, so it was possible to check that part of the data against three other commercial and investment bank sources. Except for September 15–16, 1992, the differences among sources were nil or extremely small. On September 15, differences were 1 or 2 vols, and on September 16, 3 or 4 vols. Part, at least, of the differences may be attributable to the time of day at which option prices were recorded.
26. This, assumption, although logically inconsistent, is standard practice in the literature on option pricing models and their empirical tests.
27. As noted, the delta of an at-the-money forward option is not exactly 50 percent, introducing an insignificant error into the calculation of $R_t^{50\delta}$.
28. This step is similar to the technique outlined in Manaster and Rendleman (1982). I carried out the estimation procedure using TSP 4.3. I spot checked some calculations using Mathcad 6.0 and obtained numerically identical results.
29. The grid begins with 0.0270, the lowest recorded at-the-money forward volatility, and increasing in increments of 0.0005. The termination criterion was $\Sigma_{i=1}^{3}\ (u_t^i)^2 \leq 0.001$. The average value of the left hand side variables in equation ⟨19⟩ is approximately 0.3.
30. The pound closed at DM 2.4301 on October 5, 1992, some 12.5 percent below its floor of DM 2.7780.
31. Two other sets of estimates are not reported here. In one, $\sigma_{w,t}$ was set to a constant value of 0.0275, approximately equal to the lowest value of atm_t observed in the sample. In another, the constant value of $\sigma_{w,t}$ was permitted to double in the event of a realignment. In the first set of estimates, the implicit values of λ_t were somewhat higher than with a varying $\sigma_{w,t}$, while in the second set, λ_t was somewhat lower. In both cases, the implied realignment probabilities did not change by much.
32. As Bates (1995, p. 32) points out: 'A major problem with implicit parameter estimation is that we have no associated statistical theory.'
33. The interpolation method is described in detail, along with a procedure for extracting the risk-neutral probability distribution for flexible exchange rates, in Malz (1996).
34. The Svensson–Rose series is based on a regression of one-month changes in sterling–mark's position in the fluctuation margins $12^*(x_{t+1/12} - x_t)$ on its logarithmic position in the band (x_t), and the sterling (r_t^*) and mark (r_t) one-month Eurodeposit rates at the beginning of the month (see Rose, 1993). The results of the regression are:

$$x_{t+1/12} = -1.61 - 2.86\ x_t + 4.80\ r_t^* + 10.86 r_t + v_t.$$

$$(-1.65) \quad (3.25) \quad (1.57)$$

The regression is based on daily data, introducing the familiar overlapping observation problem. The t-statistics in parentheses are consistent.

35. See, for example, Goldstein *et al.* (1993) and Eichengreen and Wyplosz (1993).
36. Estimates of the realignment probability based on open interest parity and a 5 percent realignment size show the one-month probability of realignment remaining below 5 percent through September 16.
37. Market participants may interpret 'shock' interest rates as a signal of imminent devaluation, especially for countries such as the UK in which 'innocent bystanders' or financial stability are seen as particularly exposed to collateral damage from high interest rates. Market strategies such as buying mark call options or synthetically creating such options by delta hedging can induce automatic sales of a currency under attack in response to increases in interest rates (see Garber and Spencer, 1995).

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