Contextual Logic with Modalities for Time and Space

Haim Gaifman*

1. Introduction

Contextuality is trivially pervasive: all human experience takes place in endlessly changing environments and inexorably moving time frames. In order to have any meaning, the changing items must be placed within a more stable setting, a framework that is not subject to the same kind of contextual change. Total contextuality collapses into chaos, or becomes ineffable. While basic learning is highly contextual (one learns by example), *what* is learned transcends the examples used in the learning. Perhaps, in a similar manner, artistic expression transcends context by fully embracing it. In any case, a philosophical account of contextuality is itself stated in a more absolute mode, not necessarily a picture from an "eternal" view point, but at least one that avoids the contextuality which it describes.

In the case of language, contextuality means the dependence of the meaning of linguistic expressions ─ or, more concretely, the truth-values and denotations ─ on the context in which they are used. I shall be concerned with declarative sentences, used to make factual statements. Accordingly, contextuality is manifested in occurrences of the same sentence-type that have different truth-value in different contexts.† Often the difference is traceable to the effect of the context on smaller units, names or predicates. But this is often not the case. Philosophers have tended to view the situation through some preconceived theory of *propositions*. On that view, the semantics of the language and the context in which a sentence is used determine a proposition, the one expressed in that context by the sentence. From this view point the role of the context consists in its contribution to the proposition. My goal in this paper is to avoid any appeal to some preconceived theory, or picture of propositions. I shall use 'proposition' as a term of art, meaning loosely some sort of abstract object representing the truth-conditions of a sentence. I think that propositions should be tailored to reflect those aspects of usage that we find important or interesting. I also think that preconceptions regarding propositions may slant our views and blind us to significant aspects. And there is the danger of questions being begged and of controversial, if not dubious, metaphysics being smuggled in under the cover of linguistic theory.

---

* I would like to thank Dan Rothschild for useful discussion and comments and an anonymous referee for careful reading and stylistic suggestions.

† Usually, one speaks here of tokens and often of utterances. There are several reasons for preferring 'occurrences'. Tokens are concrete physical items, whereas occurrences are abstract elements and are more suitable in a general theoretical account; an account, moreover, which speaks of possible tokens rather than actual ones. I am assuming that in every context, sentence-tokens give rise to corresponding occurrences. On the other hand, a sign, 'X', can be used as an abbreviation for a compound sentence, '__ and …' (or any piece of text, for that matter), yet a token of 'X' does not "decompose" into tokens of '__' and '…'; and we do want to speak of the occurrences of the components that are introduced through the use of 'X'. Also some types of contextuality relate directly to types, rather than tokens, cf. section 2.

At the same time, my approach is not linguistically oriented;[‡] it ignores detailed aspects of particular natural languages that the linguist might focus on. The proposals outlined in this paper belong to the philosophy of language and my approach is best characterized as *logical*, being concerned with basic patterns of thought and reasoning.

One proposal amounts to enriching first order logic, by adding to it *context operators*, which act on sentences (more generally, well-formed formulas, and sometimes on smaller units) and which represent contexts of various types. The effect of such an operator is that of the context wherein the sentence occurs. Originally, I introduced a particular context operator in order to formalize semantic tolerance (the insensitivity of various predicates to sufficiently small changes in the argument) and to analyze and resolve the Sorites paradox. I have worked on that system on and off in the last 10 years.[§] In [Gaifman 2002] a system based on that operator was called CL (Contextual Logic); it was however a very particular case of this type of logic. In that work I also suggested a generalization to the treatment of contexts of other kinds. The present work pursues this idea, mostly for the cases of indexicals, demonstratives and proper names of natural language. I shall not consider here the operator used in the treatment of tolerance (which involves some theorems of technical interest) but focus on the generic form of such operators, pointing out their general usefulness. The notation can be easily extended to include the action of contextual operators on units smaller than sentences.

There are also innovations, as far as I know, in the treatment of temporal and of spatial modalities. The latter also leads to a new way of handling adverbial phrases. My presentation will not involve much technical detail, except for the last section, where the syntax and the semantic of a full fledged-system, comprising a variety of operators and modalities, are specified with formal rigor; there is also a deductive system and a statement (without proof) of a completeness result. Before proceeding to the substance of my proposals, some general remarks are due concerning the role of logic in analyzing phenomena often considered to be within the purview of pragmatics.

Symbolic logic is indispensable when it comes to revealing, in a precise systematic way, the truth conditions of various sentences in natural language, as well as basic reasoning patterns. These applications of logic involve *local regimentation*, that is, the recasting of pieces of text into formal, or semi-formal, sentences, written in some artificial notation. Such uses of the formalism are, as a rule, local and schematic; the sentences are restructured according to some formal scheme. The schemes derives from a logical calculus, such as first-order logic, or modal logic, or any of the numerous artificial languages that have been developed in the last fifty years. These languages can be given formal semantics: interpretations, in the form of models that are defined in a mathematical framework, usually set theory or a fragment thereof. There have been in the twentieth century grand projects (initiated by Russell and continued by Carnap) aiming at

---

[‡] As exemplified in [Kamp and Weyle, 1993]. I admit, though, that the borderline between logical and linguistic can be fuzzy.

[§] Versions of this logic were presented in various meetings and workshops from 1996 on. The system in its later simplified form was described in an invited talk at the 2001 annual meeting of the Association of Symbolic Logic and a short sketch is given in an abstract, in the Bulletin of Symbolic Logic [Gaifman 2001]. The full description can be found in a preparatory paper posted on my website, cf. [Gaifman 2002]. That paper contains additional material about vagueness, which is not relevant for the present subject.

expressing the totality of human cognition, as it embraces the facts of the world, within one comprehensive formal language; or at least indicating how, in principle, this can be done. Montague Grammar is another kind of a grand project, which is linguistically, rather than metaphysically or epistemologically oriented, and which is tailored to the grammar of English. It is not my intention to evaluate projects of this kind. I mention them as a contrast to the top-down approach, which is adopted here. That approach is more in the spirit of Quine's "shallow analysis":

> A *maxim of shallow analysis* prevails: *expose no more logical structure than seems useful* for the deduction or other inquiry at hand. In the immortal words of Adolf Meyer, where it doesn't itch don't scratch. [Quine 1960, p. 160; italics in the original]

For example, the statement from *The Twelfth Night*,

(1) Some are born great, some achieve greatness, and some have greatness thrust upon them,

can be analyzed as:

(1*) $A \wedge B \wedge C$

where, for convenience, we ignore grouping in the repeated conjunction. We can go further down:

(1**) $\exists\, x\, [P(x) \wedge BG(x)] \wedge \exists\, x\, [P(x) \wedge AG(x)] \wedge \exists\, x\, [P(x) \wedge GT(x)]$,

where '$P(x)$', '$BG(x)$', '$AG(x)$', '$GT(x)$' are read, respectively, as 'x is a person', 'x was born great', 'x achieved greatness', 'greatness was thrust upon x'. Or, if you think that 'some' means more than one, replace '$\exists x$' by '$\exists^{>1} x$', where '$\exists^{>1} x\ \alpha(x)$' is a shorthand for '$\exists x_1, x_2\, (\alpha(x_1) \wedge \alpha(x_2) \wedge x_1 \neq x_2)$'. But that is as far as we can go without undertaking an in-depth analysis of these predicates, and there is no point in attempting it, unless we have a particular goal and the payoff is worth it. We can, if we wish, imagine a "model" for the formal language of (1**), whose universe is some class of objects (including all humans), with the predicates interpreted as the subclass of persons, the subclass of objects that were born great (perhaps also animals?), the subclass of those that achieved greatness and the subclass of those that have greatness thrust upon them. But surely it is only a heuristic pretense to imagine that we have defined thereby a model of a formal language; do we have a clear conception of any of these classes? what exactly counts as having greatness thrust upon one? Other examples may be more amenable to formal modeling, but cases that lead to well defined models have to be handcrafted, as one does in constructing word problems for a logic course, and the models are "small", designed specifically for the problem at hand. Only in mathematics do we get true models for full-fledged formal languages. Formal systems can reveal essential patterns of thought and reasoning, but this does not mean that the world is some enormously complicated model for some enormously complicated formal language.

Regimentation obviously involves considerable simplification; nuance and ambiguity are trimmed off. Quine considers this a great virtue: "Quantification cuts across the vernacular use of 'all,' 'every,' 'any' and also 'some,' 'a certain' etc. ... in such a fashion as to clear away the baffling tangle of ambiguities and obscurities ... The device of quantification subjects this level of discourse for the first time, to a clear and general algorithm." [**] The "baffling tangle" may contain however elements that can be systematically explored and formally expressed. For example, first-order logic captures the distributive uses of 'and' and 'all', but not the collective ones. It provides adequate paraphrases of (2), but not (3); of (4), but not (5):[††]

(2) Alice and Beth and Carl and Dan enrolled in the course.
(3) Alice and Beth and Carl and Dan discussed it and decided to drop the project.

(4) All the pieces on the table are red.
(5) All the pieces on the table fit together.

Now collective uses can be formalized, in more than one way. Let us add a variable-binding "collecting operator" say $\mathcal{C}$, such that $\mathcal{C}x\alpha(x)$, is the collection of the objects satisfying $\alpha(x)$: some entity that — in the case of people — can engage in discussion and decision making, and — in the case of physical objects — can consist of pieces that fit together. (For (3) we can use $\mathcal{C}x(x\text{=Alice} \vee x\text{= Beth} \vee x\text{=Carl} \vee x\text{= Dan})$) and for (5) — $\mathcal{C}x$(x is a piece on the table)). Collections are rudimentary sets (not mereological fusions); they can appear under predicates and function-symbols, yet our setup need not have quantifiable variables ranging over collections, neither do we need higher set-theoretic entities (collections of collections). These additional items are optional. Collections are the obvious entities by which we can regiment certain portions of natural language. This being an illustrative example, I shall not specify further details. Admittedly, the distributive use is logically more basic, but collective use can and should be formalized if a deeper analysis of reasoning patterns is required. Whether the system that incorporates $\mathcal{C}$ qualifies as "pure logic", depends on further unspecified features;[‡‡] the question is legitimate, but it can be raised only if we have a clearer view of what qualifies as "pure logic".[§§]

2. Three Kinds of Context Dependence

Since contextuality is conveniently described as the dependence of meaning on occasions of use, and the most obvious use of language is in speaking, contextuality is frequently associated with communication by of speech. Illustrative scenarios are conveniently described in terms of a speaker addressing some audience. Yet, important types of context-dependence do not involve communication in any substantial way. In these cases the role of the audience is only to provide a natural occasion for producing a token in

[**] [Quine 1951] p. 70.
[††] (4) and (5), are essentially from [Vendler 1962] , where they are used to make a similar point.
[‡‡] It depends, among other things, on the availability of quantifiers over collections, but more than that — on the semantics of these quantifiers. In general, the enrichment with $\mathcal{C}$ falls short of second order logic.
[§§] What should count as "pure logic" is controversial and not my concern in this paper. Arguably some very basic patterns of reasoning, for example, in arithmetic, are not necessarily part of pure logic. Neither can facts pertaining to usage in natural language decide such questions.

given circumstances. As far as the analysis is concerned the audience can be ignored. Such cases are better classified as *one-person* contextuality. As the examples below will indicate this is not a grammatically matter, but rather a philosophical, or logical, one. The second kind is *communicative contextuality*, where the audience plays a true role. The third kind is *contextuality of text*, or *textual contextuality*; it is exemplified in reports, written or recorded,[***] where the meaning of a component shifts according to a larger piece of text in which it is embedded. This is a rough non-exhaustive classification. Needless to say, there are borderline cases and overlaps, and parallels. But it will serve as a rough preliminary map.

The reader will notice that I am not using the well known traditional semantic/pragmatic distinction, a theme that has been considerably debated in recent years.[†††] The approach I am suggesting does not presuppose this or that position in the debates. The semantic/pragmatic border has been a moving line and I suspect that it is crucially affected by the kind of account we can give. We tend to classify under 'semantics' features for which we have a systematic, sufficiently precise and sufficiently general account; features for which such an account is unavailable are put under 'pragmatics'. Now we can introduce context operators, without deciding whether they come under the one or the other. Often there is little that can be further analyzed; we can tell a story explaining the context and its effect, but the formal representation remains essentially a schematic symbol, like the letters in (1*). Communicative contextuality is by and large of this kind. The functioning of indexicals, what Kaplan [1989] calls *character*, is usually considered now a semantic matter, though previously they were not so considered.[‡‡‡] Unsurprisingly, context operators representing indexicals are subject to systematic relatively simple rules. The rules apply no matter whether we consider them semantic or pragmatic.

One-person contextuality includes all context dependencies that are due to standard indexicals, words such as 'I', 'here', 'now'. Temporal adverbs ('today', 'next year', 'in the future', etc.) also involve this kind of context-dependence. The same goes for demonstratives ('this', 'that', and also 'you', 'he' and 'she'). For convenience, I include the latter under 'indexicals'. A person can tell someone, 'Today is a very nice day', or say it to oneself, or write it in a diary, or think it. In the first case, a speech act is performed and there is an audience; the role of 'today' however is the same: to fix the reference to the day in which the sentence is uttered, or is being written (or is being thought). The speech act serves only as means for creating a token. But when an utterance is interpreted differently than the speaker intended ─ say when a demonstration (pointing gesture) that accompanies a demonstrative is misconstrued ─ the audience enters the picture, because we should give some account of what the utterance says to the hearer. In the same way the audience enters the picture when the speaker who has uttered 'I' is unknown and unseen.

[***] A report can be in first or third person. If read aloud, the person who does the reading serves merely as a device and, in this role, does not enter the meaning analysis. But a report, written or recorded in some way, which commits its author, amounts to an assertion by the author, no matter who reads it aloud.

[†††] Cf. Bach [1999] for a useful overview of the history of the distinction and some of the debates.

[‡‡‡] Bach [1999], pp. 71,72.

There are other cases, which appear to involve communication but are in fact of the single-person kind. Soames [2002, p. 78] considers a man, sitting at a counter of a coffee shop who, when asked by the waitress what he wants, replies, "I would like some coffee please." In that context 'coffee' means a cup of freshly brewed coffee (which the waitress brings him), not a gallon of coffee, nor any quantity in the form of beans or powder. The communication with the waitress is part of the context in the same way that the coffee shop and the sitting at the counter are; it does not play a role *qua* communication. In Soames' scenario the waitress interprets the request correctly; this serves only to highlight the fact that the usage is common. Had she misunderstood it (say, she is unaccustomed to the polite formal style), the occurrence of 'coffee', in the man's request, would have had the same meaning. This is also its meaning in "I would like to get some coffee here," said by a person (to a friend, or to oneself) when passing a coffee bar. The contextual dependency carries over to reports — e.g., "John noticed a coffee bar and decided he needed some coffee." It is a parallel case of the third kind, contextuality of text. But 'coffee' has a different meaning when a person, pausing before a grocery store, says, "I need to get some coffee", i.e., a certain quantity of ground or unground coffee beans. This, as well, carries over to the third kind.

Communicative contextuality is exemplified in those cases where the meaning is adjusted so that it enables, or accords with, successful communication. Suppose that, speaking with Carl, Bess says:

(6) I called Jane Smith and she is doing well,

where the rest of the context is this: Bess and Carl have a common friend, named Jane Smith, who — it is common knowledge among them — has been recently ill; no other person by that name has figured recently in their discussions. Let C_1 be this context and let *a* be the common friend. Then (6), in context C_1, says:

(6a) Bess phoned *a*, and *a* is recovering from her recent illness.

Suppose Bess has another friend with the name Jane Smith. *That* friend, call her *b*, is also known to Dan, who knows no other person with this name. Bess and Dan discussed recently *b*'s moving to a new challenging job. Let C_2 be the context of Bess uttering (6) when speaking to Dan, a day after that discussion. In context C_2, (6) says:

(6b) Bess phoned *b*, and *b* is succeeding in her new job.

In both cases there is also the implicature that the information was given to Bess in the conversation initiated by the call. But we need not go into this.

Now suppose that Bess utters (6) when conversing with Dan, but intending to refer to *a* (she met Dan while thinking of *a*, having forgotten for a moment that Dan did not know *a*). What does (6) say in *that* context? This is a bad question; (6) says different things to different people.

Often one finds in the literature statements to the effect that the reference is determined by the speaker's intention. This is misleading at best. Perhaps it is due to the term ‘speaker’s reference’ coined by Kripke [1977] in discussing Donnellan’s paper [1966]. Donnellan has shown, by convincing examples, that definite descriptions can be used *referentially*, that is, to refer to particular objects even when the objects do not satisfy the description. In one case an assertion of “Smith’s murderer must be insane” is taken to refer to Jones, a known person who is on trial for Smith’s murder and who is believed by the speaker to be guilty; when later it turns out that Jones was not the murderer, people still consider Jones the subject of the assertion, given that it was motivated by Jones’ behavior during the trial. Then there is the question someone asks in a party, “Who is the man drinking a martini?” where the asker intends a particular man drinking from a martini glass; it turns out that the man was drinking water, yet the reference to that particular man succeeds. These cases fall squarely within context-dependency of the second kind. But Donnellan seems to claim much more: a speaker may use a name to refer to a particular object he, or she, has in mind, so that only the speaker can determine the intended object and judge the success of this act.[§§§] I doubt that philosophy of language should be tailored to accommodate that metaphysical view.[****] Donnellan’s examples are convincing because, in his scenarios, there are indicators that make the speaker’s intentions pretty obvious. Change them slightly so that communication is lost (this is easily done), and it becomes far from clear what, if anything, should count as *the* reference.[††††] “Determined by the speaker’s intention” is perhaps better read as: “determined by what can be plausibly inferred by a competent audience about the speaker’s intention, in the given circumstance.” In cases of miscommunication the speaker fails to transmit to the audience what he or she intended. There is a host of possible failures, full and partial, and it is futile to try to find here a systematic account. The principle that should hold in any case is that the speaker cannot be, like Humpty Dumpty in *Alice through the looking glass*, a dictator of meaning ─ neither in semantics, nor in the rules of conversation.[‡‡‡‡] In both we have public norms, albeit different ones, to which the speaker is beholden no less than the audience.

Finally, some observations about the third kind. Many contextual dependencies of the other kinds have parallels here. We have seen such an example above: the context dependency of ‘coffee’, occurring in speech acts, corresponds to a textual context dependency of the word as it occurs in reports. This holds quite broadly but not always. Dependencies of the communicative kind need not have textual parallels.

[§§§] “I can be referring to a particular man when I use the description "the man drinking a martini," even though the people to whom I speak fail to pick out the right person or any person at all. Nor, as we have stressed, do I fail to refer when nothing fits the description. But perhaps I fail to refer in some extreme circumstances, when there is nothing that *I* am willing to pick out as that to which I referred.” [Donnellan 1966] p. 362

[****] It echoes Russell’s metaphysic of the period 1913 ─ 1920. Russell was of course up front about his motivation and about his project in metaphysics and epistemology. His linguistic theory derived from it.

[††††] Getting right the intended referential use can depend on very small details. It makes a difference whether the question about “the man drinking a martini” is asked while looking at a group of people, or in a separate room where the man is not in sight. Donnellan’s scenarios are optimal in that they provide the audience with all the pointers. Suppose the speaker knows nothing about martini, but for some reason thinks that it is a dark brown drink; by ‘the man drinking a martini’ he intends to refer to someone drinking coca cola. Ironically, he would succeed in getting the reference across, if his audience shared his misconception. If the audience is more knowledgeable, then the question, what *is*, in that context, *the* reference of the description, has no answer.

[‡‡‡‡] “When *I* use a word,” Humpty Dumpty said, in rather a scornful tone, “it means just what I choose it to mean ─ neither more nor less.”

'I' in written reports retains its function of referring to the originator of the sentence, i.e., the writer (except in direct discourse, e.g., "I am tired, thought John"). But other indexicals can behave differently. *Now* and *here* may refer to the time and place of the writing; they may also refer to the time and place of the story, and they can move with it:

(9) I entered the room. *Here* it was totally dark and I could see nothing. I waited some time… and *now* I could see … etc. etc.

Demonstratives, including pronouns, become anaphors, except for 'you' which can address the reader, or listener, but which can also function as a proxy for the indefinite 'one' (any person).

Since dependencies of the third kind do not involve speech acts, they lead naturally to the study of linguistic structures by themselves. Some questions about compositional semantics arise here, where context operators can do some good work, but they are not of the kind discussed in this paper, except for short observations towards the end of section 3, before the subsection on direct reference.

Another dimension that can serve for organizing contexts is the extent to which the context is necessary for determining a proposition ─ a subject of considerable debate in the last ten years.[§§§§] Since the underlying notion of proposition is not clear, some of the arguments are vague. For the sake of precision let us assume that to determine the proposition, which is expressed by a sentence-occurrence, is to determine the truth conditions of the occurrence. On Grice's theory, sometimes called minimalism, a proposition is determined, once the references of the relevant standard indexicals and demonstratives, and their derivatives ('tomorrow', 'in the future', 'next room', etc.) are determined. This proposition is what the sentence "literally" says. What is communicated can be a different proposition, obtained from the first (and the relevant context), through various kinds of implicature. At the other extreme, Recanati [2004] argued for a view, termed "Contextualism", according to which a sentence-type in natural language does not, as a rule, determine a proposition, but provides only a sort of scheme; in order to get a proposition, context is needed on an extensive scale ─ much beyond the fixing of standard indexicals. Views vary according to the weight assigned to context vis-à-vis the semantics. What I am proposing does not hinge on adopting this or that position, so I will not enter into the issues.

Specificating Contexts

There are cases in which the determination of certain implicit parameters, provides additional information that strengthens an existing proposition. The clearest case is that of fixing the location of an event. (10) below expresses a proposition (ignore, for the sake of illustration, any ambiguity one might find in "rode her bike"); (11) expresses a different, stronger proposition.

(10) Ann rode her bike before May 25 2007.[*****]
(11) Ann rode her bike before May 25 2007 in Central Park, Manhattan.

[§§§§] For an earlier, short and neat discussion of contextuality see Neil Tennant [1981]..
[*****] The past tense is required in (22) by the grammar. A formal version should be construed tenselessly.

Note that (10) is equivalent to the existential statement:

(10*) There is some place in which Ann rode her bike before May 25 2007.

The proposition expressed by (10) is made more specific in (11) through additional information. I shall call a context that provides such information a *specificating context*. Those who incline to give the semantics a greater weight also tend to regard many contexts as specificating contexts. Soames [2002] treats "I would like some coffee, please" as a case in which the instantiation of the indexical 'I' yields a minimal proposition; it is the common denominator of all possible specifications ─ one of which is the scenario in the coffee shop. This amounts to treating the minimal proposition as a disjunction of all its specifications. The problem here is that the disjunction is extremely vague, and may be considered an open texture. I find it very difficult to survey what might count as "bringing some coffee"; will a sealed canister filled with vaporized coffee, or ground coffee mixed with earth, do?[†††††] The point is that we do not know, until the case arrives, and *then* we will make some decision. This is directly related to the issue of open texture, raised by Wittgenstein and elaborated by Waissman. Spatial locations, on the other hand, can be quantified over, and they do not give rise to this type of difficulty.

3. Context Operators

Context operators are suggested here, first as a notational device, useful for carrying out logical analysis and regimentation at the semi-formal level. There is also a formal system (syntax, semantics, and deductive system) which incorporates the more obvious cases of context operators, and whic is presented in the last section.

A context operator has a generic form [C], where C represents a context; it is appended to a well formed formula (henceforth wff), α, the result being another wff::

(11) $[C]\alpha$ (read: α *in the context* C)

$[C]\alpha$ states what is stated by α in the context C. Usually, we have rules that imply biconditionals of the form:

(12) $[C]\alpha \leftrightarrow \alpha'$

where α' is a wff in which items that depend on the context C have been changed to context-independent ones. The formalism should have means for writing down α'. If C covers all the context-dependent aspects of α, then α' is context-independent. There are also cases where C adds specifications to a wff that is already context independent. Wffs of the form $[C]\alpha$ are subject to the recursive rules that generate wffs. They can be negated, combined by sentential connectives, quantified, etc. We have for example the conjunction:

[uttered by *a*](I am happy) $\wedge$ [uttered by *b*](I am miserable)

[†††††] A similar point, using a steak-and-potatoes scenario, is made by Searle [1992], p. 180.

Context operators can be iterated; e.g., [uttered by *a*][uttered by *b*]α; this particular case will turn out to be equivalent to [uttered by *b*]α, since all occurrences of the indexical 'I' are captured by [uttered by *b*].

Let me recall (6) of section 2, which will serve throughout as an illustration in the discussion.

(6) I called Jane Smith and she is doing well.

The formal system contains the indexicals 'I' and 'now', as well English names, such as 'Jane Smith' and 'Bess'. All of these have the status of names whose references are determined by context; in particular, 'now' is construed as a name referring to the time of the utterance. Henceforth such names will be called *contextual names*. The formal system contains also *permanent names* whose references are context independent.

Consider an utterance of (6) made by Bess who is speaking to Carl. Let us use '*a*' as a permanent name for the Jane Smith in question. The context operator that fixes the reference of 'Jane Smith' is:

$$[\ \_\_\ ,\ \text{Jane Smith} / \underline{a}\ ]$$

where '__' is reserved for any optional added information. It may contain the relevant details of the scenario, or some agreed mark, e.g., *common-friend*. (The context operator for 'Aristotle' may include *ancient-philosopher* indicating that he is not Aristotle Onassis). It can also be left empty. The axioms of the formalism imply:

(13) $[\text{Jane Smith} / \underline{a}]\ \alpha(\ \ldots\ \text{Jane Smith}\ \ldots) \leftrightarrow \alpha(\ldots\ \underline{a}\ \ldots)$

The indexical 'she' can be included as well. For the sake of simplicity, let us assume that at some pre-processing of (6), the anaphoric 'she' is resolved and replaced by 'Jane Smith'. Then, letting 'JS' stand for 'Jane Smith', the formalism yields:

(14) $[\text{JS}/\underline{a}]$ (I called JS and JS is doing well) ↔ I called $\underline{a}$ and $\underline{a}$ is doing well

The indexical 'I', henceforth written as *I*, is treated by applying an *I*-operator:

$$[I/\underline{c}] \qquad \text{(read: the speaker } = \underline{c}\text{)}$$

In our case, $\underline{c}$ is a permanent name of Bess. We can apply the two operators successively; the rules will imply:

(15) $[I/\underline{c}]\ [\text{JS}/\underline{a}]$ (*I* called JS and JS is doing well) ↔ $\underline{c}$ called $\underline{a} \wedge \underline{a}$ is doing well

The two operators can be combined into a single one, $[I/\underline{c}\ ;\ \text{JS}/\underline{a}]$, via the notation:

$$[C\ ;\ D]\ \alpha\ =_{\text{Df}}\ [C][D]\ \alpha$$

This generalizes to any sequence of operators. Two operators are said *commute*, if the following scheme is derivable:

(com) $[C]\,[D]\,\alpha \leftrightarrow [D]\,[C]\,\alpha$

It is obvious that in our example the operators should commute. Note that two operators of the same type, e.g., $[I/\underline{c}]$, $[I/\underline{c'}]$, do not, as a rule, commute.

The operator $[I\,/\underline{c}]$ represents the context where the speaker is *c*. No other data are needed. On the other hand, $[\text{JS}/\underline{a}]$ represents an assignment of a value, determined by pragmatic considerations that involve a wider context. This is indicated by the fact that JS is an English name; we may also use here the additional-information slot. The additional details of our story, given as a context C_1, have also the effect of reading ‘does well’ as ‘is recovering from her recent illness’. This is expressible in the form:

$$(15^{+})\ [I/\underline{c}]\ [\text{JS}/\underline{a}]\ [C_1]\ (I \text{ called JS and JS is doing well}) \leftrightarrow$$
$$\underline{c} \text{ called } \underline{a} \wedge \underline{a} \text{ is recovering from her recent illness}$$

The treatment of the demonstratives *this* and *that* (which are not present in this example) is analogous to the treatment of *I*, using operators such as [*this/q*]. The optional slot may contain any items regarding the demonstration.

Temporal Contexts and Temporal Modalities

Continuing with the same example, let us go on to handle the temporal aspects. For the sake of simplicity I shall omit C_1 and skip over (15^{+}). Rewrite the right-hand side of (15) so that the implicit indexical *now* is made explicit:

$$(16)\quad \text{before } now\ \underline{c} \text{ called } \underline{a} \wedge now\ \underline{a} \text{ does well}$$

The *now*-operator has the form:

$$[now/\underline{d}] \qquad (\text{read: time of the utterance} = \underline{d}).$$

Here *d* is a permanent name of a time point. By the rules of the system we get:

$$(17)\ [now/\underline{d}]\ (\text{before } now\ \underline{c} \text{ called } \underline{a} \wedge now\ \underline{a} \text{ does well}) \leftrightarrow$$
$$\text{before } \underline{d},\ \underline{c} \text{ called } \underline{a} \wedge \text{at time } \underline{d},\ \underline{a} \text{ does well.}$$

Combining all the operators into one, by means of the notation, we have:

$$[now/\underline{d}\ ;\ I/\underline{c}\ ;\ \text{JS}/\underline{a}]\ [\ (I \text{ called JS and JS is doing well}) \leftrightarrow$$
$$\text{before } \underline{d},\ \underline{c} \text{ called } \underline{a} \wedge \text{at time } \underline{d},\ \underline{a} \text{ does well}$$

It is easily seen that the operators should commute.

‘At time *d*’ and ‘before *d*’ are handled by incorporating into the setup means for handling time. Once the contextual machinery has done its job by fixing the reference of *now*, temporal logic (known also as tense logic) takes over. Any of the well-known versions of temporal logic can be used.[‡‡‡‡‡] I shall suggest below some new versions, arising out of

[‡‡‡‡‡] Temporal logic was introduced under the name Tense Logic, in the late fifties and the early sixties by Prior, whose motivation derived from metaphysical views on logic and time. It was investigated by

considerations relating to the present setup. The reader can easily see how the customary versions can be used, instead.

As is well known, time-dependence can be handled by using predicates that can take names of time points as arguments. We can then instantiate the time indexicals as needed. For example, 'now *a* does well' is formalized as DoesWell(*a*, *now*) and the context operator substitutes a permanent name for *now*. This requires that the usual arity of the predicate be increased by 1. We should also need variables ranging over, time points, in order to express claims about the future or the past. The device of increased arity has the disadvantage of introducing temporality at the basic level of predicates. For many purposes, this is not necessary and even undesirable. A common pattern of natural language (and of our thinking) is the application of temporal modalities globally, to a big chunk of text as a whole, e.g.; "Presently life conditions are thus… But in the future, ..." A time, *t*, can serve as a global evaluation point: *the world at time t*. We might think of *t* as a marker of a possible world. In propositional temporal logic the basic units are time-dependent propositions, represented by single letters.[§§§§§] The use of temporal modalities in computer science is motivated by the computational possibilities of handling time by means of temporal modalities, which avoids the complexities of quantificational logic.

To accommodate the possible-world aspect of time, let us include temporal *at*-operators, (which are *not* context operators) of the form $\mathcal{A}(\tau)$, where τ is a temporal term: either a permanent name of a time point, or the indexical *now*. For any wff α we get a wff:

$$\mathcal{A}(\tau)\,\alpha \qquad \text{(read: at time } \tau,\ \alpha;\ \text{i.e., } \alpha \text{ holds at time } \tau)$$

In order to represent future and past, we add the temporal modalities:

$\mathcal{F}(\tau)\,\alpha$ (some time after τ, α) $\qquad$ $\mathcal{P}(\tau)\,\alpha$ (some time before τ, α) [******]

They correspond to the modal operators, *F*, *P* of the standard version of temporal logic;[††††††] but they are relativized to constant time points, which, as will become clear, makes a very big difference. We can define the duals, as usual:

---

philosophers and logician, mostly during and after the late sixties. The subject has been given a big technical push by computer scientists, following a program launched by Pnueli in 1977, whose aim was to use certain versions of temporal logic in order to specify and to check the correctness of program behavior cf. [Pnueli 77].

[§§§§§] Note also that the time-dependence of a predicate may depend on occupants of the argument places. Compare "Mary's brother is heavier than her", in which the time-dependence is obvious, with "Gold is heavier than silver", or "Neutrons are heavier than protons," where arguably the addition of a time-coordinate is inappropriate. Conceivably, one might want to use here predicates of variable polyadicity — an option to be later discussed lwith regard to spatial modality, where it seems more appropriate.

[******] $\mathcal{F}(\tau)$ and $\mathcal{P}(\tau)$ are interpreted as non-reflexive future and past, they exclude the present. The reflexive versions, say $\mathcal{F}^{\geq}(\tau)$ and $\mathcal{P}^{\leq}(\tau)$ can be defined as: : $\mathcal{F}^{\geq}(\tau)\,\alpha \Leftrightarrow_{\text{Df}} \mathcal{F}(\tau)\,\alpha \vee \mathcal{A}(\tau)\alpha$, and similarly for
$\mathcal{P}^{\leq}(\tau)\,\alpha$.

[††††††] For basic definitions and an historical overview see [Burgess 1984]. The subject has been considerably developed by computer scientists, in versions that are tailored to the needs of reasoning about

$\mathcal{G}(\tau)\, \alpha =_{Df}\, \neg F(\tau) \neg \alpha$ $\qquad\qquad$ $\mathcal{H}(\tau)\, \alpha =_{Df}\, \neg P(\tau) \neg \alpha$

Going on with our original example, we rewrite the right-hand side of (17) (‘before d, c called a ∧ at time d , a does well’) as:

(18) $\mathcal{P}$ (d) (c called a) ∧ $\mathcal{A}$(d) (a does well)

If desired, we can also include function symbols for forward and backward discrete time shifts say next( ) and prev( ). Any time unit can serve as a base; e.g., if τ denotes *t*, then next(τ) denotes $t + 1$ day, next (next(τ)) denotes $t + 2$ days, etc, prev(τ) denotes $t - 1$ day, etc. This type of machinery provides formal counterparts of ‘tomorrow’, ‘yesterday’, ‘next year’, etc. Finally, we can also include a binary relation for temporal ordering: $\tau < \tau'$ says that the time denoted by τ precedes the time denoted by τ'. Call this version of temporal logic version 0. The distinctive aspect of version 0 is the absence of quantifiable variables ranging over time points.

Note that we can express in version 0 ‘sometime’ and ‘always’:

$Sometime(\alpha) \Leftrightarrow_{Df} \mathcal{P}(\tau)\, \alpha \vee \mathcal{A}(\tau)\, \alpha \vee \mathcal{F}(\tau)(\alpha)$ $\qquad$ $Always(\alpha) \Leftrightarrow_{Df} \neg\, Sometime\, (\neg \alpha)$

The axioms will imply that different choices of τ yield equivalent wffs on the right-hand side.

In version 1 we allow also *at*-operators of the form $\mathcal{A}(u)$, where ‘*u*’ is an individual variable, ranging over time points. Modal operators for future and past are of the forms $\exists u > \tau \mathcal{A}(u)$, $\exists u < \tau\, \mathcal{A}(u)$, which can be applied to any wff; τ can be a variable or a term containing one. In version 1, temporal quantifiers and terms can appear only as parts of such operators. Thus, for a wff α, we get the wff

$$\exists\, u > v \mathcal{A}(u)\, \alpha$$

in which *u* is bound and *v* is free. Using negations, the corresponding universal modalities $\forall u > \tau\, \mathcal{A}(u)$ and $\forall\, u < \tau\, \mathcal{A}(u)$ are expressible in the usual way; e.g., $\forall u > \tau\, \mathcal{A}(u)\, \alpha$ is written as $\neg \exists\, u > v \mathcal{A}(u) \neg \alpha$. For example, the following wff

(19) $\qquad$ $\forall u > now\ \exists\, v > u\ \mathcal{A}(v)$ (It rains in New York)

says that for every time in the future there will be a later time at which it will rain in New York. It corresponds to *GF*(It rains in New York) of traditional temporal logic. It is not difficult to see that version 1 has at least the same expressive power with regard to temporal modalities as the traditional system. Vice versa, if we omit individual names for

program execution, see [Emerson 1990] for a comprehensive overview. These versions usually use reflexive temporal ordering and future modalities only.

time points and interpret *now* on all its occurrences as referring to the same time, then the quantifiers of version 1 provide essentially the same expressive power as the traditional *F*, *G*, *P*, *H*. Version 1 has more expressive power than the traditional system, due to the presence of individual names for times and the possibility of instantiating different occurrences of *now* to different times.

It can be shown that *GF*(It rains in New York) is not expressible by a wff of version 0. As is well known, the iterated modalities of traditional temporal logic do not correspond to natural-language constructions. Even the simple last example calls for tracking in English the quantificational structure, with explicit reference to time points ('it will always rain in New York' is too indefinite to be considered a faithful rendering). Version 0 accords more with what can be accomplished in English by tenses, though some constructions are beyond it, e.g., "Only once has such a storm occurred." Version 0 has also a nice formal feature: a sound and complete deductive system, where time points are modeled by the standard integers and the non-temporal part consists of any first-order language. This is impossible for traditional temporal logic.

Finally, consider version 2, a still stronger version that has relation symbols for equality and temporal order, time-point names and quantifiable variables ranging over time points. The temporal relations and terms can also appear in wffs, besides their appearance as parts of the modal operators. But there are no predicates under which both temporal and non-temporal terms can appear.

The interpretation of the temporal component of the system depends on the modeling of time. There are quite a few choices, which have been thoroughly investigated in the context of traditional temporal logic. The two most common are discrete time points and the points of some dense linearly ordered set (usually the real line). Furthermore, in computer science, branching-time models are used in the analysis of non-deterministic processes. In artificial intelligence, use is made of models of time-intervals. Indeed, *now* can be interpreted as referring to some interval whose length — depending on context — can be less than a minute ("The time is now one o'clock and six minutes"), or an unspecified number of years ("People live now longer than fifty years ago"). We can set up a system that provides for explicit references to time intervals. The obvious advantages come at a high cost of increased complexity.

The system is smoother if the reference of *now* is in all cases a sharp time point. The intervals can be introduced through the truth-conditions of the predicates (or verbs). For example, the reference of *now* in: "Ann is now finishing her breakfast" is the (sharp) time of the utterance. Let this be *d*; taken in context, the statement is:

[*now/d*] Ann is *now* finishing her breakfast

which is equivalent to:

(20) At time <u>*d*</u> Ann is finishing eating her breakfast,

The truth-conditions of (20), i.e., of "finishing eating," introduce a certain interval containing the point *d*; namely, *d* is inside and near the end of the time interval during which Ann eats breakfast. ('Near the end' is of course vague, but so is (20). It goes without saying that the formalism is not intended to handle also vagueness.)

'Today', 'tomorrow', 'this year', etc., which refer to time intervals, can be handled similarly. Thus, "today it is raining", uttered at time *d*, is analyzed as:

(21) [*now*/*d̲*] (In the day of *now* it is raining),

And this in turn is rewritten as:

(21') $\mathcal{A}$(*d̲*) RainyDay.

Here 'RainyDay' is a propositional constant and truth condition of (21') is that *d* belongs to a day in which it is raining.

Spatial Contexts, Spatial Modalities, Adverbs

The indexical *here* refers to some small spatial region surrounding the speaker (at the time of the utterance), or one that is determined by it. ('Here' can be also used as a demonstrative, but then it should be classified as a variant of 'this'). The instantiation of *here* to a spatial location is determined by context. The system therefore includes the indexical *here*, individual names for locations, and context *here*-operators of the form [*here*/*d̲*], where *d̲* is permanent name of a location.

As in the case of temporal names, there is a question, , whether to treat location names as terms that appear under predicates. It also arises when we specify additional parameters relating to a complete proposition. The sentence below requires specification of time but not of location.

(22) Ann rode her bicycle.

Yet we need in the formal language a device for adding spatial information to the information given in (22). The strategy of adding "location-coordinates" to predicates that are used for representing verbs is available, but not recommended. For one thing, indefinitely many adverbial phrases, which provide additional information, can be added; are we going to include an additional coordinate for each? This problem was used by Davidson as an argument in favor of including events, as object subject to quantification. Under his proposal, 'Ann rode her bike' is to be rephrased as a statement of the form:

There was an event *x*, such that:
(*x* is a bike-riding event ∧ rider in *x* = Ann ∧ bike in *x* = Ann's bike)

The location can be given by an additional conjunct under the existential quantifier, using a binary predicate that denotes the relation of an event to its location. There are probably good direct reasons for admitting quantification over events, but the attempt to derive ontological conclusions from the technical needs of formalization is often misguided. It is certainly misguided here, where alternative formal solutions are available. We can represent action verbs by using predicates with variable polyadicity; i.e., the number of argument can vary, allowing us to add in each case as many coordinates as needed.

Variable polyadicity has been incorporated into some artificial languages used in the context of computer science.

I think that locations are better handled in a manner analogous to times, by means of a (non-contextual) operator for *spatial modality*. It has the form:

$\mathcal{L}(\sigma)\ \alpha$ (read: in location σ, α)

Here σ is any term that denotes locations, including location-variables. The reasons for the spatial modality operator are similar to those for the temporal one. A location-assignment can be applied to a whole story; we can start by saying that it takes place in New York and it will follow that the described people, streets, events, the seasons, the weather ─ everything is in New York. For obvious topological reasons, there are no useful spatial analogues of time-slices. Nonetheless, spatial regions can play the role of "possible worlds", especially when speaking of other planets, other galaxies, or what not.

Obviously, $\mathcal{L}(\sigma)\alpha \rightarrow \alpha$ is valid. The existential and universal spatial modalities are obtained by quantifying over locations:

$\exists x\ (\mathcal{L}(x)\ \alpha)$ (somewhere, α) $\forall x\ (\mathcal{L}(x)\ \alpha)$ (everywhere, α)

Now it is obvious that adverbs can be treated in similar way. Consider an adverb-operator Adv(η), where $\ulcorner\eta\urcorner$, which can be also a non-atomic structured item, signifies a particular adverb, or adverbial phrase. If α is a wff that asserts the carrying out of an action, then

Adv(η) α (read: α, in the manner η)

asserts the carrying out of the action in the manner described by $\ulcorner\eta\urcorner$. The following will be an axiom or derived rule: Adv(η) $\alpha\rightarrow\alpha$. Since Adv(η) α is itself a wff, we can apply such an operator again, thus piling up the adverbs:

Adv(η_2) Adv(η_1), Adv(η_3) Adv(η_2) Adv(η_1) α, … etc.

This formalization make for wider possibilities of applying adverbial phrases than the device of variable polyadicity. When the semantics is set correctly, the addition of an operator amounts to a conjunction: Adv(η_2) Adv(η_1) $\alpha \leftrightarrow$ Adv(η_2)$\alpha \wedge$ Adv(η_1)α is valid.

As in the case of times, the possible references of *here* can vary from a small region surrounding the speaker ("I am here"), to rooms ("The other room was noisy, but here it is quiet"), to cities ("I was told that she took recently a flight to New York, so she must be here"), to geographical hemispheres ("In the northern hemisphere it is winter, but here it is summer"). We can use for locations the strategy employed for time points. Namely, the possible references of *here* are space-points which can be used as reference points for larger spatial regions; the latter are determined by the semantics of the predicates. Say, the reference of *here* is the speaker's bodily center of gravity, then 'it is raining here' is true if the speaker's gravity center is included in some vaguely determined region associated with 'it rains'.

For obvious topological reasons locations are more complicated and fuzzier than time intervals; we have no natural periodic phenomena ─ not to speak of clocks ─ to set up a location scale. Occasionally, physical or legal features (the boundaries of a room, the official city limits) can serve for more precise determinations. Due to these reasons, the alternative of having terms that refer to spatial regions of various size is more plausible for space than for time. In our formal system *here* will refer to spatial points, but the spatial component of the language may contain terms that refer to regions, which can figure in the spatial modality $\mathcal{L}(\sigma)$.

Finally, let us add a specificating-context operator, which represents any context that indicates that what is described takes place in σ; here σ is a permanent term denoting a location (a spatial point or region). I shall use $[\ell{:}\sigma]$ for this operator. Note that the specification can be done explicitly by using $\mathcal{L}(\sigma)$. Indeed, $[\ell{:}\sigma]\alpha \leftrightarrow \mathcal{L}(\sigma)\alpha$ is valid in our system. The difference between the two is that $[\ell{:}\sigma]\alpha$ says that the context indicates that the location of what takes place is σ, whereas $\mathcal{L}(\sigma)\alpha$ says explicitly: such and such takes place in location σ.

This concludes the list of operators and modalities that will figure in the formal system of the last section.

Some uses of the formal system and the notation

It might appear that the introduction of operators of the forms [*I/a*], [JS/*a*], [*now/*τ], etc., is a mere trick: one displays in the language itself a substitution mechanism that one often employs on the meta-level. Yet, making things explicit at the level of the object language can be significant. In traditional temporal logic a sentence is evaluated at a (varying) single time point, which is hidden outside the language. Arthur Prior, who launched this logic, was motivated by the metaphysical picture of truth changing with time. The change should not be visible inside the language. Of course, the theorist who describes the semantics has a bird's-eye view of time; so the metaphysical picture is after all breached. In the traditional systems, (23), where $date_1$ and $date_2$ are different dates, is inexpressible, as are numerous other examples.

(23) [uttered on $date_1$](it is raining in NYC) ∧ [uttered on $date_2$](it is not raining in NYC)

In the system proposed here, (23) is expressible in the form:

(23') [$now/time_1$]$\mathcal{L}$(NYC) (Raining(*now*)) ∧ [$now/time_2$]$\mathcal{L}$(NYC) (Raining(*now*))

The temporal logics used in computer science share the single-time-point evaluation, but for a very different, practical reason. These logics are used in specifying and verifying program executions; simplicity and economy are therefore of paramount importance. If a limited system can suffice for our needs, so much the better. The same is true of dynamic logic, where wffs are evaluated at points that represent stages of program executions.

The system LD, in [Kaplan 1989], is a different matter. It is a complicated, extremely rich setup, representing a broad philosophical conception. The language contains *I*, *here*, various temporal operators and an apparatus of modal logic. It contains also other operators, into which I need not go here. Contexts appear only as evaluation points in the truth-definition. A model includes, among other things, a set of possible worlds and a set of contexts; each context consists of an agent (the speaker, or writer), a time point, a place (corresponding to my *location*), and an actual world, chosen from the set of possible worlds. Sentences are evaluated at a single context, which gives the values of *I*, *here*, and *now*, and also determines the actual world. In addition they are also evaluated at non-contextual possible worlds (for the purposes of metaphysical modality) and at non-contextual time points (for the purpose of temporal modality). Yet (23) is not expressible in LD.[‡‡‡‡‡‡] Consider also the treatment of *I*. At each context all occurrences of *I* are instantiated to the same speaker; conjunctions of the following form, where *a* and *b* are different persons, cannot be rendered in LD.

(24) [uttered by *a*] (I am happy) $\wedge$ [uttered by *b*] (I am miserable),

Now LD has individual names of people. Hence it can express (24) in the form: *a* is tall $\wedge$ *b* is tall. (LD can be extended so that (23) is similarly expressed). This means that *I* is treated outside LD, by substituting individual constants for its occurrences and only then writing the sentence in LD. The same goes for other indexicals. Contexts involving more than one speaker are essentially beyond LD, because they call for a machinery that indicates who says what. And if you think about it you will see that such a machinery amounts to introducing contexts at the level of syntax.

The operators [*n/p*] describe formally a trivial substitution mechanism and triviality that yields good insights is a virtue. A striking example is provided by Kaplan (in an earlier work). Recall Russell's friend who said to a boat owner, "I thought your boat was longer than it is"; upon which the irate owner replied "My boat cannot be longer than it is". Kaplan suggests that Russell's friend would have made his point more effectively had he said: "Let Russell be the length of your boat. I thought your boat was longer than a Russell." Now consider the following example:

(25) In the future people enrolling in top business schools will be rich
[because only rich people will be able to pay the tuition fees].

(26) In the future people enrolling now in top business schools will be rich
[because having graduated they will get high paying jobs].

The difference emerges more clearly when the sentences are recast as:

(25') Sometime after *now* people enrolling in top business schools will be rich.
(26') Sometime after *now* people enrolling *now* in top business schools will be rich.

And the final step of applying the substitution makes it as clear as possible:

[‡‡‡‡‡‡] It is expressible by different sentences in different contexts: on a date later than the two given dates, we can access the earlier dates by iterating the "yesterday"-operator provided in LD. The number of needed iteration will change with the date of utterance.

(25") Sometime after 2008 people enrolling in top business schools will be rich.
(26") Sometime after 2008 people enrolling in 2008 in top business schools will be rich.

A more formal regimentation (given here only as an illustration) is as follows. Let 'x' range over people, let '$\forall^{W}x$' stands for something like: 'many or most x', and let us ignore the vagueness of 'in the future' (it means here some reasonable future, say 6 – 10 years). Then (25') and (26') are rewritten as:

(25*) $\mathcal{F}(now)\ (\forall^{W}x)\{x$ enrolls in a top business school $\rightarrow x$ is rich$\}$

(26*) $\mathcal{F}(now)\ (\forall^{W}x)\{\mathcal{A}(now)\ x$ enrolls in a top business school $\rightarrow x$ is rich$\}$

Applying to these the context operator [*now*/2008] yields more formal versions of (25") and (26").

The examples used by Kaplan in [1989] are similarly handled by the present machinery. The truth of every possible utterance of 'I am now here' (its truth in every context in which it exists) is seen to be contingent when the sentence is subjected to the context operators $[I/p_1]$, $[now/p_2]$, $[here/p_3]$, where p_1, p_2, p_3 are permanent names of the speaker, the time of the utterance and the speaker's location at the time. What the resulting sentence says is: p_1 is at time p_2 in location p_3. And surely this could have been otherwise. Similarly, for the truth of all utterances of 'I am speaking now', or of 'I exist'. If there are any puzzles, they are completely resolved by the substitution mechanism. Kaplan's explanations appeal to the semantics, defined in the metalanguage, since LD does not provide means for making the situation explicit.

The notation [C] is also useful generally, in calling attention to and displaying the context. The operators can be applied to other units, besides sentences, such as terms, predicates or functions symbols (or, in a natural language, various phrases). In the case of textual contextuality, the contexts include larger pieces of text wherein the given unit is embedded. Now structured text provides by itself context. Therefore the meaning (denotation) of some composite construct requires, as a rule, less context than the context required for smaller components. In a compositional semantics the small components are, so to speak, self-sustaining. Therefore the degree of compositionality is shown by the extent to which the components, $X_1,\ldots,X_k$, of a construct, X, do not require bigger contexts than the context required by X. In other words, the degree of compositionality is indicated by the following kind of distributivity. Here $\mathcal{F}(X_1,\ldots,X_k)$ is a construct, composed of $X_1,\ldots,X_k$. The equivalence means that the two sides have the same denotation, where 'denotation' can be some abstract entity introduced for the purpose of the semantics.

(27) $[C]\ \mathcal{F}(X_1, X_2, \ldots,X_k) \equiv \mathcal{F}([C]X_1, [C]X_2\ldots,[C]]X_k)$

This type of analysis sheds light on various phenomena in natural languages. The topic is beyond the scope of the paper.[§§§§§§]

[§§§§§§] In a previous draft of the paper, the subject has been elaborated and some examples were discussed. I decided to omit that section for reasons of length..

Direct Reference?

So far, the framework I have outlined fits very well the direct reference theory. The individual constants that serve as context-independent names can be viewed as directly referring terms. And so presumably can the indexicals and the English proper names, which are substituted by them. Actually, in the debate between descriptivists and direct reference theorists (in which the latter have prevailed to a large extent) my proposal is neutral. The debate concerns the nature of meaning. Should meanings be construed as propositions that contain the references of indexicals and proper names — the very objects themselves — as constituents, or should they be construed as the *ways the objects are presented*, something like Fregean senses? The framework proposed here does not presuppose any particular theory of propositions, or senses. It obviously sits well with direct reference. But it can also fit a descriptivist view, provided that we have decided on the definite descriptions that express the Fregean senses of indexical-occurrences (including demonstrations), and the senses of proper names. One should also give an account that explains the failures of substitutivity, in modal contexts, of the indexical, or the proper name, by the corresponding description. Such a project is motivated by the goal of finding a solution to Frege's puzzle. The direct reference theorist resorts at this point to hand waving; he or she speaks generally and vaguely of different *ways of presenting the same proposition*, which amounts to shifting the problem to another domain, where it can languish indefinitely. I think such a project is promising. But this does not belong to the subject of the paper.

4. CLST, Contextual Logic with Spatiotemporal Modalities

Some features of the contextual operators, namely, the slots for optional additional information, are left as implicit elements in the formalism. These notational features play no role in the truth definition. Also context operators that transform predicates are left out of the picture, though they can easily be incorporated in it.

The Language $\mathfrak{L}$

(I.1) $\mathfrak{L}$ comprises three sublanguages, $\mathfrak{L}_B$, $\mathfrak{L}_T$, $\mathfrak{L}_S$. The first is the basic language, which does not handle any temporal and spatial items. The second is the temporal language and the third is the spatial language. $\mathfrak{L}$ is thus many-sorted. The three languages have distinct non-logical vocabularies and distinct types of variables. $\mathfrak{L}_T$ is the language for version 0 of temporal logic, hence it has no variables and its vocabulary, specified below, is chosen accordingly.

All predicates and function symbols of $\mathfrak{L}$ come from these languages. Terms are generated as usual from individual constants and variables; each belongs to exactly one of the languages and can stand under predicates of that language only.

The equality sign '=' is part of the shared logical vocabulary, but it can be flanked by terms belonging to the same sublanguage only.

Interaction between the languages is done by modal operators, which can apply to any wff. Also context operators can apply to any wff.

(I.3) All individual constants of $\mathfrak{L}$ are divided into *permanent names* and *contextual names*. They are specified below for each of the three languages.

(II.1) $\mathfrak{L}_B$ contains any number of predicates and function symbols of arbitrary arities. The permanent names of $\mathfrak{L}_B$ are constants denoting persons and any other objects that the language can refer to.

(II.3) The contextual names of $\mathfrak{L}_B$ are:

- Standard indexicals (and demonstratives): *I*, *you, he-she*, *this, that*
- English names.

(III.1) The non-logical vocabulary of $\mathfrak{L}_T$ consists of

(i) Permanent names for time points,

(ii) One contextual name: *now*.

(iii) A binary predicate $<$ for temporal order

(iv) Monadic function symbols, *next* ( ) and *prev*( ), for forward and backward time shifts.

(IV.1) $\mathfrak{L}_S$ has any number of predicates and function symbols, which are supposed to describe space.

(IV.2) The individual constants of $\mathfrak{L}_S$ are:

(i) permanent names of locations (spatial points, and possibly regions)

(ii) one contextual name: *here*.

(V.1) The temporal modalities are:
$\mathcal{A}(\tau)$ [at time τ], $\mathcal{F}(\tau)$ [sometime after τ], $\mathcal{P}(\tau)$ [sometime before τ], where τ is any temporal term.

(V. 2) The spatial modality is: $\mathfrak{L}(\sigma)$ [in location σ] , where σ is any spatial term.

(IV) The context operators of $\mathfrak{L}$ are:

- [*n/p*] where *n* is a contextual name and *p* is a permanent term, in the same sublanguage. If *n* = *here*, *p* denotes a spatial point.

- $[\ell{:}\sigma]$, where σ is a permanent term in $\mathfrak{L}_S$, for specificating contexts that indicate locations.

Wffs and Terms of $\mathfrak{L}$

(1) Terms and atomic wffs are defined in the usual way, in each of $\mathfrak{L}_B$, $\mathfrak{L}_T$ and $\mathfrak{L}_S$. These are the terms and the atomic wffs of $\mathfrak{L}$.

(2) Wffs are generated by the recursive rules:

(2.1) The set of wffs is closed under combinations by sentential connectives.

(2.2) If α is a wff, so is $\forall x\alpha$, for any variable of $\mathfrak{L}_B$ or $\mathfrak{L}_S$.

(2.3) If α is a wff and τ is a temporal term, then $\mathcal{A}(\tau)\alpha$, $\mathcal{F}(\tau)\alpha$, $\mathcal{P}(\tau)\alpha$ are wffs,

(2.4) If α is a wff and σ is a spatial term, then $\mathfrak{L}_S(\sigma)\alpha$ is a wff.

(2.5) If α is a wff and [C] is any context operator than [C]α is a wff

(3) A variable x in a wff α is free iff it is not in the scope of some $\forall x$.

A contextual name, n, is free in any term in which it occurs. It is free in a wff α if it has an occurrence in α that is not in the scope of a context operator of the form $[n/p]$

(4) A term or a wff is *permanent* if it has no free contextual names. Note that it can contain free variables.

Note: The inverted-ι operator, *the unique x such that* …, can be added in the usual way.

Models for $\mathfrak{L}$

A model $\mathcal{M}$ is a structure of the form: $(U, T, S, R_j, f_k , c_q, L)_{j\in J, k\in K, q\in Q}$, such that:

(1) U, T, S are non-empty disjoint sets.

- U is the universe for the language $\mathfrak{L}_B$
- T is the time line, for which we shall consider two possibilities: (i) $T = \mathbb{Z}$, the set of integers (ii) $T = \mathbb{Q}$ the set of rationals. (There are of course other possibilities.)
- S is the set of spatial locations (space points and possibly regions).

(2) Each R_j is the interpretation of the corresponding predicate $\underline{R}_j$ of $\mathfrak{L}$, where $(\underline{R}_j)_{j\in J}$ includes all the relation symbols; similarly, for function symbols. c_q is the object denoted by $\underline{c}_q$, where $(\underline{c}_q)_{q\in Q}$ is the set of all permanent individual constants.

For convenience we treat relations as functions that map tuples to truth values, **T**, **F**.

(3) The relation and function symbols of $\mathfrak{L}_B$ are interpreted, as relations over U, and functions from Cartesian powers of U into U.

(4) The relation and function symbols of $\mathfrak{L}_S$ are interpreted, as relations over S, and functions from Cartesian powers of S into S.

(5) The relation symbol, $<$, of $\mathfrak{L}_T$ is interpreted as $<_T$, the ordering of T, i.e., the ordering of $\mathbb{Z}$, or of $\mathbb{Q}$.

The function symbols $\underline{next}$ ( ) and $\underline{prev}$ ( ), are interpreted, respectively, as $\lambda x(x+1)$ and $\lambda x(x-1)$.

(6) If $\underline{R}$ is an n-ary function symbol of $\mathfrak{L}_B$, then its interpretation, R, is a function:

$$R\colon\ D_{1,\underline{R}} \cup D_{2,\underline{R}} \times T \longrightarrow \{\mathbf{T}, \mathbf{F}\},$$

where $D_{1,\underline{R}} \cup D_{2,\underline{R}} = U^n$, is a partition of U^n that depends on $\underline{R}$. $D_{2,\underline{R}}$ consists of those n-tuples $(a_1,\ldots a_n)$ for which time can be relevant for the satisfaction of $\underline{R}$.

Explanation: This interpretation of relation-symbols is motivated by natural language examples, where the same predicate gives rise to time-dependent, as well as and to “eternal” sentences; e.g., ‘John is heavier than Mary’ and ‘Gold is heavier than lead’. Formally, we can simplify the semantics by construing the time-independent cases as special cases in which the value of the function is constant in time. This would be philosophically wrong, but would make no difference in the truth-definition and the formal properties of the system.

(7) Similarly, if $\underline{f}$ is an n-ary function symbol of $\mathfrak{L}_B$, its interpretation, f, is a function:

$$f\colon\ D_{1,\underline{f}} \cup D_{2,\underline{f}} \times T \longrightarrow U,$$

where $D_{2,\underline{f}}$ consists of those n-tuples whose value under f can depend on time.

(8) L is the interpretation of the spatial modality, $\mathcal{L}(\ )$. It correlates with each n-ary relation symbol, $\underline{R}$, of $\mathfrak{L}_B$, a mapping:

$$L_{\underline{R}}\colon U^n \times S \longrightarrow \{\mathbf{T}, \mathbf{F}\},$$

that satisfies, for all $(a_1,\ldots,a_n) \in U^n$:

(‡) $R(a_1,\ldots,a_n) = \mathbf{T}$ iff $L_{\underline{R}}(a_1,\ldots, a_n, s) = \mathbf{T}$, for some $s \in S$.

$L_{\underline{R}}$ is called $\underline{R}$–in-location; it determined the dependence of R on the location.

(‡) means that $\underline{R}$ is satisfied iff it is satisfied somewhere.

Truth Definition

A word on the notation: The satisfaction of wffs depends on assignment of values to free variables, hence one uses creatures like: $\alpha\left[\begin{smallmatrix} x_1 \dots x_n \\ a_1 \dots a_n \end{smallmatrix}\right]$. I shall simplify the notation by using $\alpha(a_1,\dots,a_n)$ instead, with the understanding that α has the free variables $x_1,\dots,x_n$. Formally this amounts to treating members of the universe as names of themselves and adding them to the language. This simplifying device is often used in mathematical logic. (no philosophical position is intended).

When I speak of terms and wffs, I mean terms and wffs *under assignments of values to their free variables*, i.e., closed terms and sentences of the extended language.

Satisfaction is expressed by giving the wffs truth values, e.g., $\mathcal{M} \models \alpha\left[\begin{smallmatrix} x_1 \dots x_n \\ a_1 \dots a_n \end{smallmatrix}\right]$ is rewritten as $|\alpha(a_1,\dots,a_n)|_{\mathcal{M}} = \mathbf{T}$. Since $\mathcal{M}$ is fixed throughout the definition, I shall omit the subscript. And since the relations in $\mathcal{M}$ are conveniently treated as functions with values in $\{\mathbf{T}, \mathbf{F}\}$, the condition for an atomic wffs is expressed as an equality of the form: $|\underline{P}\,(a_1,\dots,a_n)| = P(a_1,\dots,a_n)$. Similarly, the value of a term, ρ, in the model is $|\rho|$. In the following definition, we start by giving the truth conditions for permanent wffs.

Let $\mathcal{M} = (U, T, S, R_j, f_k, c_q, L)_{j\in J,\, k\in K,\, q\in Q}$ be some fixed model. The universe of $\mathcal{M}$ is $U \cup T \cup S$. Since the values of terms, and the truth values of wffs, may in some cases depend on an implicit time parameter, we define these values as functions of t, where 't' is a fixed meta-variable ranging over T. Thus the truth definition defines, for every permanent term ρ and wff α, the values $|\rho|(t)$ and $|\alpha|(t)$, as functions of t.

We use 'ρ', 'ρ'',… for terms of $\mathfrak{L}$;
'τ', 'τ'',… — for terms of $\mathfrak{L}_T$;
'σ', 'σ'',… — for terms of $\mathfrak{L}_S$

In case of a permanent time-independent wff α (i.e., one whose truth-value does not depend on time) $|\alpha|(t)$ is the same for all t. Similarly for terms. If τ and σ are permanent constant terms, we use '$|\tau|$' and '$|\sigma|$' also for the constant values of these terms.

Note: The definition assigns values to all permanent terms and truth values to all permanent wffs. This is all we need. A non-permanent wff becomes permanent after applying the relevant context operator and this is handled in clause (6).

(1) Terms are given values via the following recursive clauses:

(a) If f is in $\mathfrak{L}_T$ or in $\mathfrak{L}_S$, $|\underline{f}\,(a_1,\dots,a_n)|(t) = f\,(a_1,\dots,a_n)$ for all t.

(b) If f is in $\mathfrak{L}_B$, then:

i. If $(a_1,\dots,a_n)\in D_{1,f}$ then $|\underline{f}(a_1,\dots,a_n)|(t) = f(a_1,\dots,a_n)$ for all $t \in T$

ii. If $(a_1,\ldots,a_n) \in D_{2,\underline{f}}$ then $|\underline{f}(a_1,\ldots,a_n)|(t) = f(a_1,\ldots,a_n, t)$

(c) $|\underline{c}_i|(t) = c_i$ for every permanent name $\underline{c}_i$.

(d) $|\underline{f}(\rho_1,\ldots,\rho_n)|(t) = f(|\rho_1|(t),\ldots,|\rho_n|(t))(t)$; if $\rho_i = a_i$, $|\rho_i|(t) =_{\mathrm{Df}} a_i$.|

(2) If $\underline{R}(\rho_1,\ldots,\rho_n)$ is an atomic wff, , then,

(e) If $\underline{R}$ is in $\mathfrak{L}_T$ or in $\mathfrak{L}_S$, then $|\underline{R}(\rho_1,\ldots,\rho_n)|(t) = R((|\rho_1|(t),\ldots,|\rho_n|(t))$

(f) If $\underline{R}$ is in $\mathfrak{L}_B$, then:

i. If $(|\rho_1|,\ldots,|\rho_n|)) \in D_{1,\underline{R}}$ then
$|\underline{R}((|\rho_1|,\ldots,|\rho_n|)|(t) = R(|\rho_1|(t),\ldots,|\rho_n|(t))$

ii. If $(|\rho_1|,\ldots,|\rho_n|) \in D_{2,\underline{R}}$ then
$|\underline{R}(|\rho_1|,\ldots,|\rho_n|)|(t) = R(|\rho_1|(t),\ldots,|\rho_n|(t))(t)$

(3) The clauses for sentential connectives and quantifiers are the usual ones, e.g.

$|\forall x\, \alpha(x)|(t) = \mathbf{T}$ iff $|\alpha(a)|(t) = \mathbf{T}$ for all a in the range of x, as determined by x's type.

(4) If α is a wff and τ is a permanent temporal term then:

(g) $|\mathcal{A}(\tau)\alpha|(t) = |\alpha|(|\tau|)$

(h) $|\mathcal{F}(\tau)\alpha|(t) = \mathbf{T}$, if for some $t > |\tau|$, $|\alpha|(t) = \mathbf{T}$; $|\mathcal{F}(\tau)\alpha|(t) = \mathbf{F}$, otherwise.

(i) $|\mathcal{P}(\tau)\alpha|(t) = \mathbf{T}$, if for some $t < |\tau|$, $|\alpha|(t) = \mathbf{T}$; $|\mathcal{P}(\tau)\alpha|(t) = \mathbf{F}$, otherwise.

From now on, we shall not display the temporal parameter; '$|\alpha|$', '$|\rho|$',,, are understood as '$|\alpha|(t)$', '$|\rho|(t)$',…etc.

(5) Truth conditions for spatial modality.
In the following σ, σ' are any (permanent) spatial terms.

(j) If $\underline{R}(\rho_1,\ldots,\rho_n)$ is an atomic wff, where $\underline{R}$ is in $\mathfrak{L}_B$, then
$|\mathcal{L}(\sigma)\,\underline{R}(\rho_1,\ldots,\rho_n)| = L_{\underline{R}}(|\rho_1|,\ldots,|\rho_n|, |\sigma|)$

(k) If $\underline{R}(\rho_1,\ldots,\rho_n)$ is an atomic wff, where $\underline{R}$ is in $\mathfrak{L}_T$ or $\mathfrak{L}_B$, then
$|\mathcal{L}(\sigma)\,\underline{R}(\rho_1,\ldots,\rho_n)| = |\underline{R}(\rho_1,\ldots,\rho_n)|$

(l) $|\mathcal{L}(\sigma)(\neg\alpha)| = |\neg\mathcal{L}(\sigma)\alpha|$,

(m) $|\mathcal{L}(\sigma)(\alpha\star\beta)| = |(\mathcal{L}(\sigma)\alpha)\star(\mathcal{L}(\sigma)\beta)|$, where $\star$ is any sentential connective.

(n) $|\mathcal{L}(\sigma)(\forall x\, \alpha)| = |\forall x(\mathcal{L}(\sigma)\alpha)|$, provided that *x* is not free in σ.

(o) $|\mathcal{L}(\sigma)(\mathcal{T}\alpha)| = |\mathcal{T}(\mathcal{L}(\sigma))\alpha|$,
where $\mathcal{T}$ is any temporal modality, $\mathcal{A}(\tau)$, $\mathcal{F}(\tau)$, $\mathcal{P}(\tau)$

(p) $|\mathcal{L}(\sigma)\mathcal{L}(\sigma')\alpha| = |\mathcal{L}(\sigma)\alpha \wedge \mathcal{L}(\sigma')\alpha|$

(6) For the context operators the rules are:

(q) $|[n/p]\alpha| = |S^n_p(\alpha)|$,
where $S^n_p\,\alpha$ is obtained by substituting every occurrence of *n* by *p*.

(r) $|[\ell{:}\sigma](\alpha)| = |\mathcal{L}(\sigma)\alpha|$

Applied recursively. (1) – (6) assign values to all permanent wffs. Some of the clauses can be restricted to more basic cases, e.g., (p) can be restricted to the case where α is atomic, the other cases follow from it and the other clauses in (5).

Note: The behavior of the spatial modality $\mathcal{L}(\sigma)$ is determined by *L* (item (8) in the definition in the model), according to the clauses in (5) of the truth definition. Now *L* is subject to condition (‡) , which says that an atomic wff is true iff it is true in some location. It is not difficult to see that the clauses in (5) imply that this holds for all wffs. Moreover, if α is a true wff in $\mathfrak{L}_S \cup \mathfrak{L}_T$ (for which locations are irrelevant) then it is true in all locations.

Deductive System

The setup, which is based on a 3-sorted language and is loaded with context operators and modalities, looks more complicated than it actually is. The difficulty of getting a sound and complete deductive system is due to the combination of FOL (first-order logic) with temporal modalities, where time points are modeled as standard integers. The other features, which are motivated by philosophical concerns, are not substantial mathematically speaking, though there is potential for complication if we are more specific in the modeling of space.

The deductive system is based on a standard Hilbert type system for FOL, adapted in the obvious way for a many-sorted language. The inference rules are modus ponens and universal generalization. All instances of valid first-order wffs are provable, by virtue of the standard completeness theorem for FOL.

Since we do not restrict the interpretation of $\mathfrak{L}_B$ in any way, FOL covers the clauses in (1), (2), (3) in the truth-definition which relate to $\mathfrak{L}_B$. The possible time-dependence of the functions and the relations that interpret the vocabulary of $\mathfrak{L}_B$ comes into play only when we consider the temporal modalities that link $\mathfrak{L}_B$ to $\mathfrak{L}_T$. Let us look first at the other parts of the system.

Clauses (k) – (r) of the truth definition are of the form $|\varphi| = |\psi|$; they translate into biconditionals, $\varphi \leftrightarrow \psi$, and these will be axioms. We get an infinite number of axioms that fall under a finite number of schemes. (Actually, we can economize considerably, since the biconditionals can be derived from those relating to atomic wffs, by using biconditionals that express distributivity laws, and other laws regarding the application of context operators, such as, $[C](\neg\alpha) \leftrightarrow \neg[C]\alpha$, $[C](\alpha\star \beta) \leftrightarrow [C]\alpha\star[C]\beta$, $[C]\forall x\alpha \leftrightarrow \forall x[C]\alpha$,)

Clause (j) determines the behavior of $\mathcal{L}(\sigma)$ with respect to atomic wffs. This is subject to condition (‡), in item (8) of the definition of the model. It is not difficult to see that the required condition is expressed by the following wff, where α is an atomic wff and u is a variable of $\mathfrak{L}_S$:

(Somewhere) $\alpha \leftrightarrow \exists\, u\, (\mathcal{L}(u)\, \alpha)$,

(See also the note following the truth-definition). We therefore add (Somewhere) to the list of axiom, which, together with the other axioms takes care of the semantics of spatial modalities.

Since the spatial language $\mathfrak{L}_S$ is largely unspecified it requires no particular axioms. If our conditions regarding space are expressible in a first-order language, then we can choose it as our $\mathfrak{L}_S$ and add these conditions as axioms. Otherwise, there is no possibility of incorporating such a modeling into a first-order axiomatizable system. The same is true with regard to $\mathfrak{L}_B$, which is so far uninterpreted and which is to serve as an all-purpose language.

It remains to take care of $\mathfrak{L}_T$ and the temporal modalities. For this purpose we can combine $\mathfrak{L}_B$ and $\mathfrak{L}_S$ into a single first-order language, disregard the contextual operators whose treatment via axioms is straightforward (as indicated above) and also disregard the spatial modalities, which in this setting pose no problem. In this cleaned up setting the problem reduces to the following one.

Let $\mathfrak{L}_1$ be any first-order language. Let $\mathfrak{L}_T$ be a temporal language, to be interpreted in the standard integers, $\mathbb{Z}$, based on equality, =, a relation symbol, $<$, for the ordering, a name $\underline{0}$ for 0, and two function symbols *next*( ) and *prev*( ) for the successor and predecessor function; $\mathfrak{L}_T$ has sentential connectives but no variables. The two languages share only the logical vocabulary. Let $\mathfrak{L} = \mathfrak{L}_1 \oplus \mathfrak{L}_T$ be the two-sorted language generated by $\mathfrak{L}_1$ and $\mathfrak{L}_T$, together with the temporal modalities, $\mathcal{A}(\tau)$, $\mathcal{F}(\tau)$, $\mathcal{P}(\tau)$, where τ ranges over the terms of $\mathfrak{L}_T$.

The models of $\mathfrak{L}$ are two-sorted, of the form $\mathcal{M} = \mathcal{M}_1 \oplus \mathbb{Z}$, where $\mathcal{M}_1$ is a model of $\mathfrak{L}_1$, and $\mathbb{Z}$ is a copy of the integers disjoint from $|\mathcal{M}_1|$ (the universe of $\mathcal{M}_1$). The interpretation of the predicates and function symbols of $\mathfrak{L}_1$ in $\mathcal{M}$ involves a time parameter ranging over $\mathbb{Z}$; i.e., an n-ary relation symbol $\underline{R}$ and an n-ary function symbol $\underline{f}$ are interpreted as:

$$R : |\mathcal{M}_1|^n \times \mathbb{Z} \longrightarrow \{\mathbf{T}, \mathbf{F}\} \qquad f : |\mathcal{M}_1|^n \times \mathbb{Z} \longrightarrow |\mathcal{M}_1|$$

The temporal modalities, which act on wffs of $\mathfrak{L}$ are interpreted in the obvious way, (i.e., as in (4) of the truth-definition above). Let $\mathrm{Mod}_{\mathfrak{L}}$ be the class of all models of $\mathfrak{L}$ and let $\mathrm{Th}(\mathrm{Mod}_{\mathfrak{L}})$ be the theory of this set, i.e., the set of all wff that are true in all such models. A deductive system that is sound and complete with respect to this theory consists of: (i) A Hilbert-type system for FOL , (ii) the axioms and inference rules listed below.

We shall use the following notation. For a term τ of $\mathfrak{L}_T$, we write $\underline{next}\,(\tau)$ as '$\tau+1$', $prev(\tau)$ — as '$\tau - 1$'. $|\tau|_{\mathbb{Z}}$ is the value of the term in $\mathbb{Z}$; α, β, are arbitrary wffs; '=' and '≠' are used ambiguously, as symbols of $\mathfrak{L}$ and in our metalanguage; similarly, for '$<$' and '$\not<$'.

The axioms are divided into the following groups.

(I) Correct Equality and Ordering Axioms:

$\tau = \tau'$, for all τ, τ' such that $|\tau|_{\mathbb{Z}} = |\tau'|_{\mathbb{Z}}$

$\tau \neq \tau'$, for all τ, τ' such that $|\tau|_{\mathbb{Z}} \neq |\tau'|_{\mathbb{Z}}$

$\tau < \tau'$, for all τ, τ' such that $|\tau|_{\mathbb{Z}} < |\tau'|_{\mathbb{Z}}$

$\tau \not< \tau'$, for all τ, τ' such that $|\tau|_{\mathbb{Z}} \not< |\tau'|_{\mathbb{Z}}$

(II) Axioms for wffs of $\mathfrak{L}_T$

For any wff α of $\mathfrak{L}_T$,

$\alpha \rightarrow (\mathcal{P}(\tau)\alpha \wedge \mathcal{A}(\tau)\alpha \wedge \mathcal{F}(\tau)\alpha)$

$(\mathcal{P}(\tau)\alpha \vee \mathcal{A}(\tau)\alpha \vee \mathcal{F}(\tau)\alpha) \rightarrow \alpha$

(III) Axioms Relating Temporal Modalities

$\alpha \rightarrow (\mathcal{P}(\tau)\alpha \vee \mathcal{A}(\tau)\alpha \vee \mathcal{F}(\tau)\alpha)$

$(\mathcal{P}(\tau)\alpha \vee \mathcal{A}(\tau)\alpha \vee \mathcal{F}(\tau)\alpha) \rightarrow (\mathcal{P}(\tau')\alpha \vee \mathcal{A}(\tau')\alpha \vee \mathcal{F}(\tau)\alpha)$

$\mathcal{F}(\tau)\alpha \leftrightarrow \mathcal{A}(\tau+1)\alpha \vee \mathcal{F}(\tau+1)\alpha$

$\mathcal{P}(\tau)\alpha \leftrightarrow \mathcal{A}(\tau-1)\alpha \vee \mathcal{P}(\tau)\alpha$

$\tau < \tau' \rightarrow (\mathcal{P}(\tau)\alpha \rightarrow \mathcal{P}(\tau')\alpha)$

$\tau < \tau' \rightarrow (\mathcal{F}(\tau')\alpha \rightarrow \mathcal{F}(\tau)\alpha)$

$\mathcal{T}_1(\tau_1)\mathcal{T}_2(\tau_2)\,\alpha \leftrightarrow \mathcal{T}_2(\tau_2)\alpha$ where $\mathcal{T}_1, \mathcal{T}_2$ are any of $\mathcal{P}, \mathcal{A}, \mathcal{F}$

(IV) Axioms for $\mathcal{A}(\tau)$

$\mathcal{A}(\tau)(\neg\,\alpha) \leftrightarrow \neg\mathcal{A}(\tau)(\alpha)$

$\mathcal{A}(\tau)(\alpha\star\beta) \leftrightarrow \mathcal{A}(\tau)\alpha\star\mathcal{A}(\tau)\beta$, where $\star$ is any sentential connective .

At this point it is convenient to use the dual modalities: $\mathcal{G}(\tau)\,\alpha =_{\mathrm{Df}} \neg\,\mathcal{F}(\tau)\neg\,\alpha$ $\mathcal{H}(\tau)\,\alpha =_{\mathrm{Df}} \neg\mathcal{P}(\tau)\neg\,\alpha$. The first expresses “always in the future of τ”, and the second – “always in the past of τ”. Let $\mathcal{M}$ be any of $\mathcal{A}(\tau)$, $\mathcal{G}(\tau)$, $\mathcal{H}(\tau)$, then we have:

(V) Standard axioms from modal logic

$\mathcal{M}(\alpha \rightarrow \beta) \rightarrow (\mathcal{M}\alpha\rightarrow\mathcal{M}\beta)$

$\mathcal{M}(\alpha \wedge \beta) \rightarrow (\mathcal{M}\alpha \wedge \mathcal{M}\beta)$

$\mathcal{M}(\forall x\alpha) \rightarrow \forall x\mathcal{M}\alpha$

(VI) Modal-logic inference rules

$$\frac{\vdash \alpha}{\mathcal{M}\alpha}$$

Preferring clarity, I have not aimed at an economical list. Thus, the second axiom in (V) is derivable from the first via the inference rules. Some, perhaps all, axioms in (I), can be derived from those in (II) and (III).

Let $\mathrm{ST}_0(\mathbb{Z})$ be this deductive system (the System for version 0 of Temporal modality, where time is modeled as $\mathbb{Z}$). Then we have:

Soundness and Completeness of $\mathbf{ST_0(\mathbb{Z})}$: $\alpha \in \mathrm{Th}(\mathrm{Mod}_{\mathfrak{L}}) \Leftrightarrow \mathrm{ST}_0(\mathbb{Z}) \vdash \alpha$.

For traditional temporal logic, which is based on the modalities *F*, *P* (which can be iterated), there can be no such result when time is modeled as $\mathbb{Z}$, for the set of valid wff is not c.e.

Note: Compactness fails for $\mathrm{Mod}_{\mathfrak{L}}$; that is there is a sequence $\alpha_1, \alpha_2, \ldots, \alpha_k, \ldots$ of wffs such that every finite conjunction is satisfied by some model in $\mathrm{Mod}_{\mathfrak{L}}$, but the whole set is not. To see this let β, be any sentence of $\mathfrak{L}_1$, such that each of β and $\neg\beta$ is consistent. Let $\alpha_n = \mathcal{H}(\underline{n})\neg\beta \wedge \mathcal{F}(\underline{n})(\beta)$. Then α_n says that β is never true before time-point *n* but is true sometime after *n*. It is not difficult to see that this sequence has the claimed property.

Consequently the completeness of $ST_0(\mathbb{Z})$ concerns only single wffs, what is sometimes called "weak completeness".

References

Bach, K. 1999, "The Semantics-Pragmatics Distinction: What It Is and Why It Matters," in *The Semantic-Pragmatic Interface from Different Points of View*, Turner (ed), Oxford: Elsevier, 65 – 84.

Burgess, J. 1984, "Basic Tense Logic" in *Handbook of Philosophical Logic*, ed. D. Gabbay and F. Guenthner, pp. 89 – 133, Kluwer Academic Publications.

Donnellan , K. 1966, "Reference and Definite Descriptions," *Philosophical Review* 75, pp. 281-304.

Emerson, E. 1990, "Temporal and modal logic" in *Handbook of Theoretical Computer Science*, vol. B, pp. 995 – 1067, MIT Press, 1990.

Gaifman, H. 2001 "Contextual Logic and Its Applications to Vagueness" *The Bulletin of Symbolic Logic*, September 2001, vol. 7 No 3, p.241 (Abstracts of Invited Talks of the Annual 2001 Meeting of the ASL)

Gaifman, H. 2002 "Vagueness, Tolerance and Contextual Logic," on the website http://www.columbia.edu/~hg17/, 41 pages.

Kamp, H. and Reyle, W. 1993, *From discourse to logic*, Dordrecht , Boston : Kluwer Academic.

Kaplan, D. 1989, 'Demonstratives' in *Themes from Kaplan*, ed J. Almog, H. Wettstein, and J. Perry, Oxford University Press, pp. 481-563.

Kripke, S. 1977, "Speaker's Reference and Semantic Reference," *Midwest Studies in Philosophy* 2, pp. 255-76.

Pnueli, A. 1977 "The temporal logic of programs" *Proc 18 Ann. IEEE on Foundations of Computer Science* pp. 46-57

Quine, W. 1951, *Mathematical Logic*, The Harvard University Press.

Quine, W. 1960, *Word and Object*, The M.I.T Press.

Recanati , F. 2004, *Literal meaning*, Cambridge University Press

Searle, J. 1992 *The Rediscovery of the Mind*, MIT Press.

Soames, S. 2002, *Beyond Rigidity*, Oxford University Press.

Soames, S. 2002, “Replies” *Philosophy and Phenomenological Research* 62, 429–452.

Tennant, N. 1981 “Formal Games and Forms for Games” *Linguistics and Philosophy*, pp. 311-320.

Vendler, Z. 1962 “Each and Every and Any and All”, *Mind*, v.71. A shorter version appears as the entry “Any and All” in *The Encyclopedia of Philosophy*, edited by P. Edwards, Macmillan and Free Press, 1967.