Bounded Rationality Lecture 2

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The Story So Far.....

- Last week we introduced on model of costly information search/attention: Satisficing
- Examined optimal behavior with search costs
- Assumed a particular form of information search
    - Sequential Search
- Seems unnecessarily restrictive

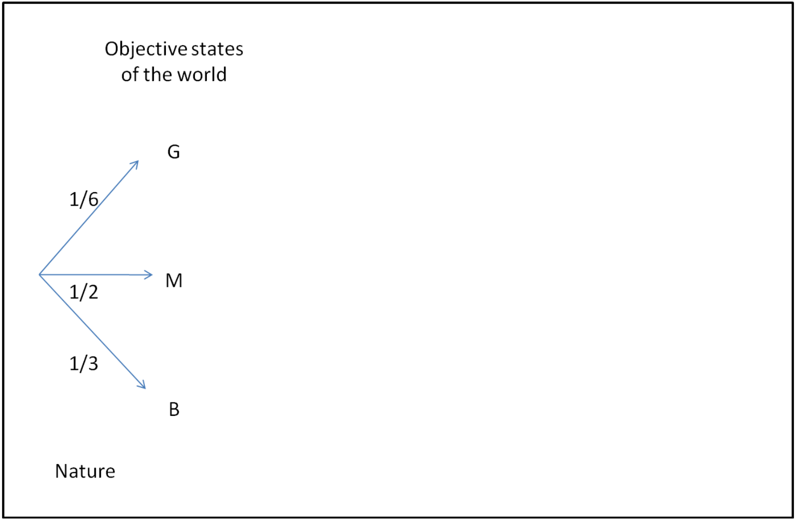
- People choose information to acquire to maximize utility net of information costs
- People free to choose
  - *How much* information to acquire
  - *What type* of information to acquire
- Has been used to examine
  - Consumption and Savings behavior - e.g: Sims 2006
  - Price Setting - e.g. Matejka 2010, Martin 2012
  - Portfolio Choice - Modria 2010

The Plan for Today

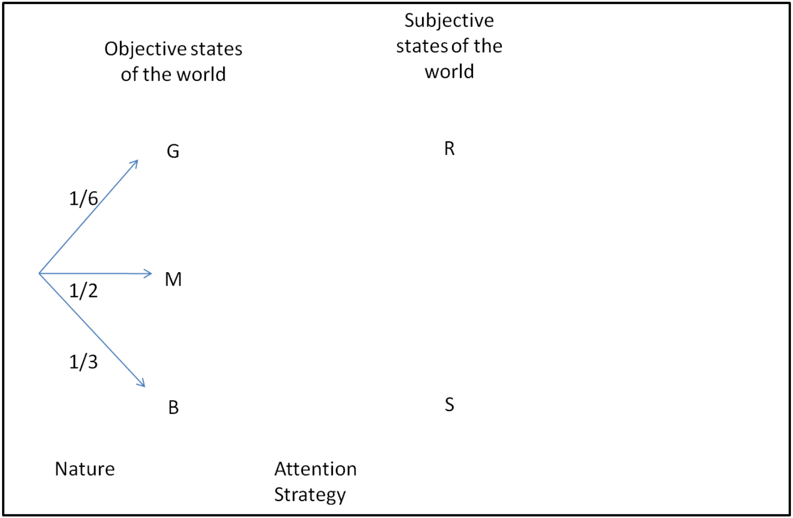
- A canonical model of rational inattention
- Implications of the canonical model

Set Up

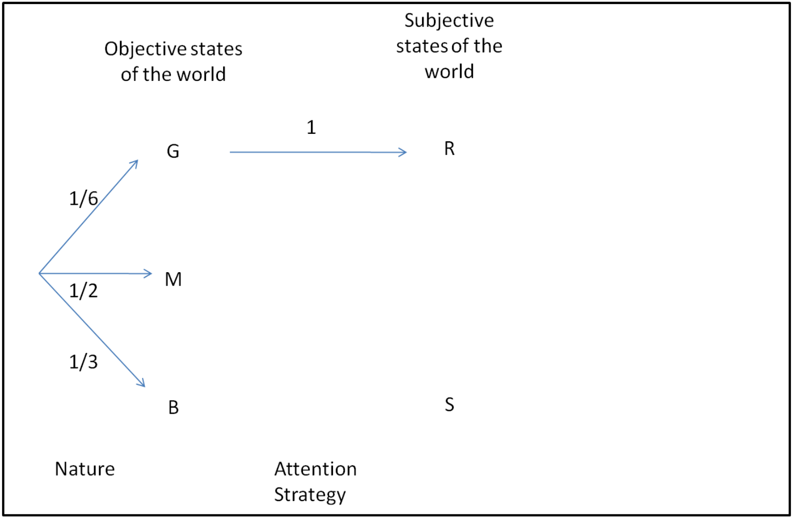
- Objective states of the world
  - e.g. Demand could be 'good', 'medium' or 'bad'
- Decision maker chooses an action
  - Set price to be high, medium or low
- Gross payoff depends on act and state
  - Quantity sold depends on price and demand


Objective states
of the world
G
1/6
M
1/2
1/3
B
Nature

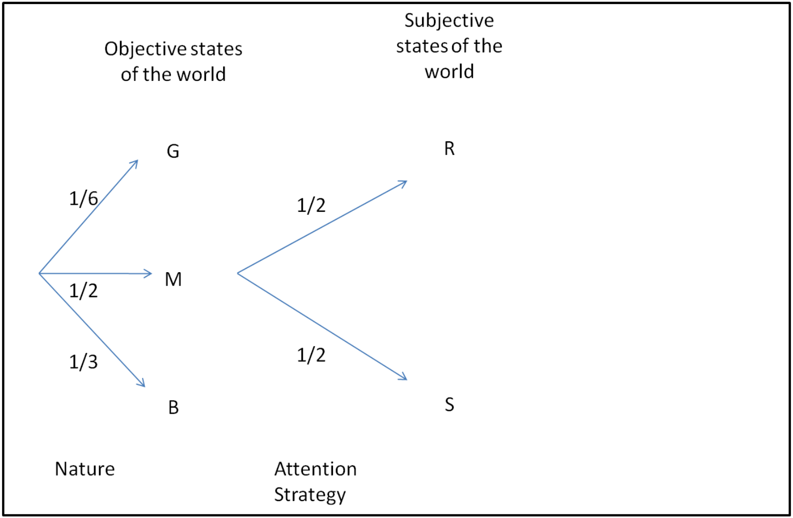
The Choice Problem



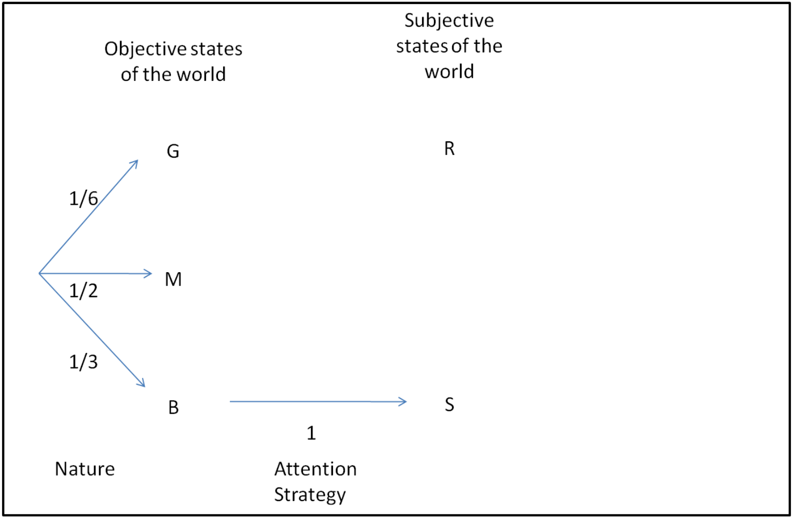
The Choice Problem

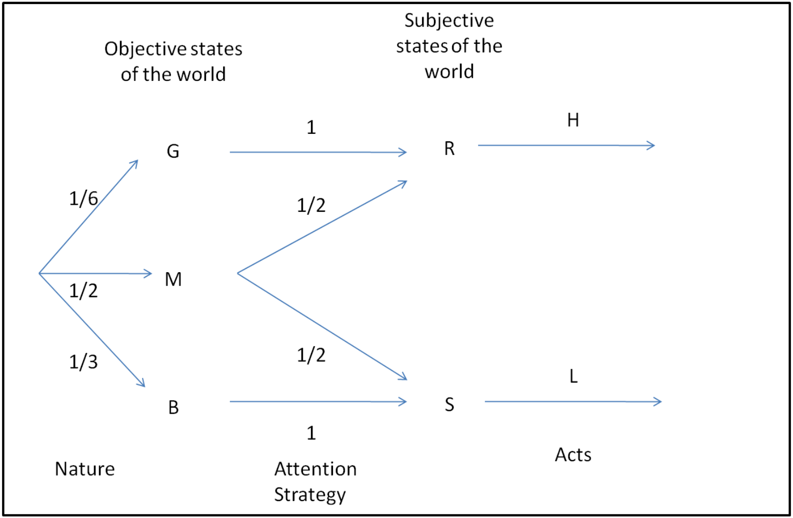


The Choice Problem

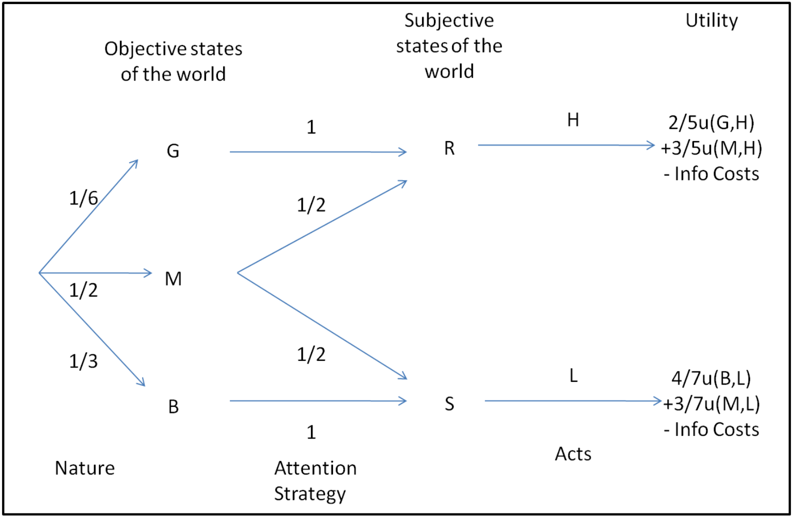


The Choice Problem




Objective states of the world
Subjective states of the world
G
M
B
R
S
1/6
1/2
1/3
1
1/2
1/2
1
H
L
Nature
Attention Strategy
Acts

The Choice Problem



- $\Omega = \{\omega_1, ...\omega_M\}$: Objective States
- $\Delta(\Omega)$: Probability distributions over Ω
- X : Finite set of outcomes
- $U : X \rightarrow \mathbb{R}$: (Expected) utility function over outcomes
- $f : \Omega \rightarrow X$: Act, with F the set of all acts and $\mathcal{F}$ non-empty subsets of F
- $\{\beta, A\}$: Decision problem: $\beta \in \Delta(\Omega)$, $A \in \mathcal{F}$

Attention Strategy and Costs

- A strategy consists of a set of subjective states

$$\left\{t^1, ...t^N\right\} = T(\lambda) \in \Delta(\Omega)^N$$

- and conditional probabilities linking objective and subjective states

$$\lambda : \Omega \times T(\lambda) \rightarrow [0,1]$$

  $\lambda_m(t^n)$ is the probability of subjective state t^n conditional on objective state ω_m

- Which obey Bayes law

$$t^n_m = \frac{\beta_m \lambda_m(t^n)}{\sum_{k=1}^M \beta_k \lambda_k(t^n)}$$

- For a decision problem $\{\beta.A\}$ decision maker must chose
  - An attention strategy $\lambda \in \Lambda(\beta)$
  - A choice function for acts $C : T(\lambda) \rightarrow A$
- In order to maximize

Expected utility from acts - cost of information

$$\sum_{m=1}^{M} \beta_m \sum_{t \in T(\lambda)} \lambda_m(t) U(C_t(\omega_m)) - K(\lambda, \beta)$$

- where
  - $\Lambda(\beta)$: set of attention strategies available from prior β
  - $K : \beta \times \Lambda(\beta) \rightarrow \mathbb{R}$: cost of attention strategy

Comments

- Decision maker can choose any form of information structure
- Nests other models of information acquisition
    - Shannon Entropy
    - Fixed signals
    - Partitions
    - Fixed capacity
    - Sequential Search

The Data Set

- Data: State dependant stochastic choice
- For each $\omega_i \in \Omega$ and $f \in A$:
- $D_i(f)$: probability of choosing act f in state ω_i
- Easy to observe in the laboratory
- With assumptions, can be observed outside the lab

- Question: If we don't make explicit assumptions about the costs of information, can we make any predictions about rationally inattentive behavior?
- Answer: Yes, if we assume that more information is more costly

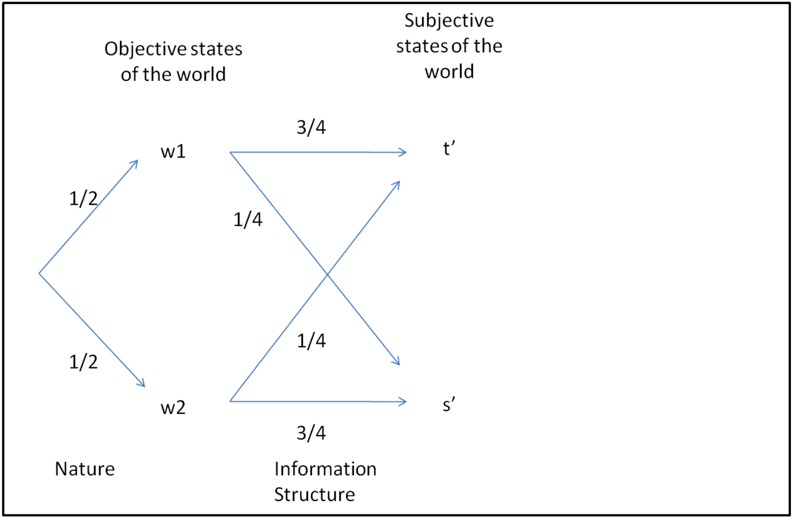
An Aside: Blackwell Information Ordering

- λ is sufficient for λ' if there exists a $|T(\lambda)| \times |T(\lambda')|$ matrix B that

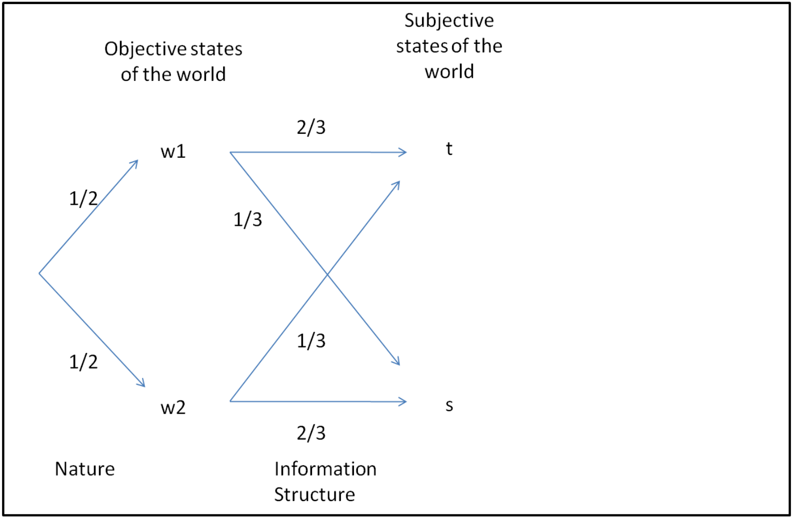
$$\begin{aligned} \sum_j B^{ij} &= 1 \;\forall\; j \\ \lambda'_m(t^j) &= \sum_i B^{ij} \lambda_m(t^i) \;\forall\; j \end{aligned}$$

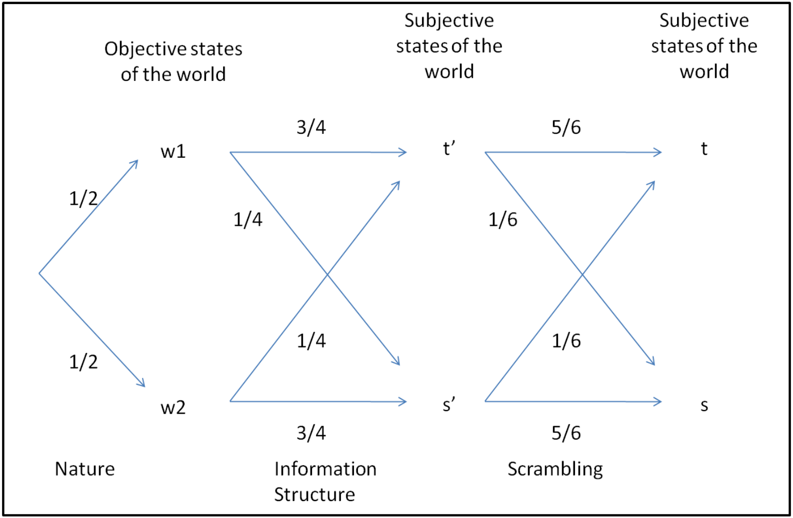
- The matrix B 'scrambles' the information in λ in order to get to λ'
- B^{ij} probability of going from subjective state t^i to the subjective state t'^j

Sufficiency: An Example



Sufficiency: An Example




Objective states of the world
Subjective states of the world
Subjective states of the world
w1
w2
t'
s'
t
s
1/2
1/2
3/4
1/4
1/4
3/4
5/6
1/6
1/6
5/6
Nature
Information Structure
Scrambling

An Aside: Blackwell's Theorem

- Let $V(\beta, A, \lambda)$ be the *value* of using attentional structure λ in environment $\{A, \lambda\}$

$$V(\beta, A, \lambda) = \max_{\{C_t\}_{T(\lambda)} \in A} \sum_{m=1}^{M} \beta_m \sum_{t \in T(\lambda)} \lambda_m(t) U(C_t(\omega_m))$$

- An information structure λ is sufficient for information structure λ' if and only if

$$V(\beta, A, \lambda) \geq V(\beta, A, \lambda') \ \forall \ \{\beta, A\}$$

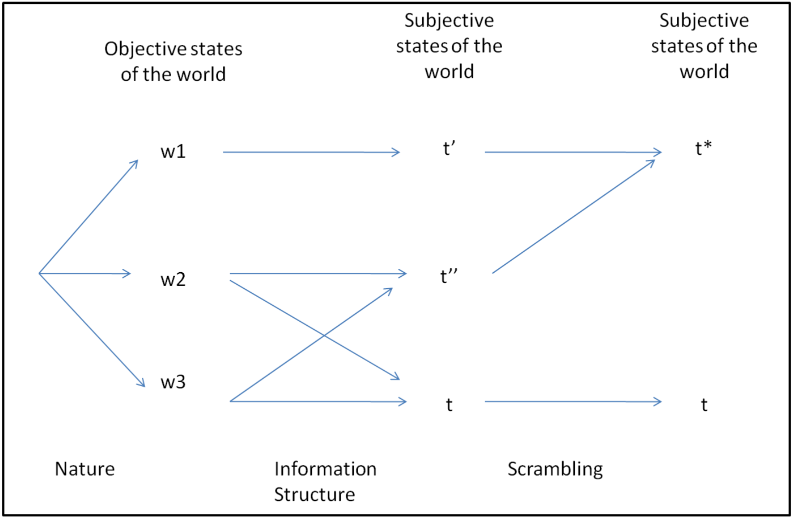
Observing Attentional Strategies

- Observation: Assuming more information is more costly, one subjective state per act
  - Will never have t' and t' such that $C(t') = C(t'')$
- Why? Let $\{\lambda, C\}$ be such a strategy, with $C(t') = C(t'')$ for $t', t'' \in T(\lambda)$
- Now consider alternative strategy $\{\lambda', C\}$ such that
  - $\lambda(t) = \lambda'(t)\ \forall\ t \neq t', t''$
  - λ' 'merges' t and t'' to state t^* defined by

$$t^*_m = \frac{\beta_m \left(\lambda_m(t') + \lambda_m(t'')\right)}{\sum_{k=1}^{M} \beta_k \left(\lambda_k(t') + \lambda_k(t'')\right)}$$

- Clearly λ is sufficient for λ'

Observing Attentional Strategies



Observing Attentional Strategies

- Means λ' is cheaper than λ,yet

$$V(\beta, A, \lambda) = V(\beta, A, \lambda')$$

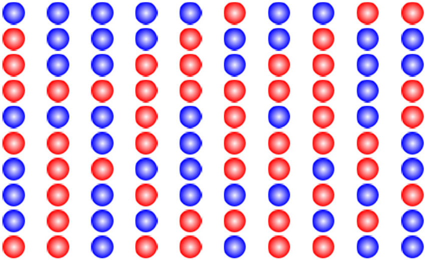
  (assuming choices were optimal under λ)
- Thus, never optimal to choose the same act in two states
- Means that we can recover attentional strategies from stochastic choice
  - For each chosen act f a subjective state t^f such that

$$\lambda_i(t^f) = D_i(f)$$

Optimal Behavior Under Blackwell Costs

- Choice of act optimal given attentional strategy

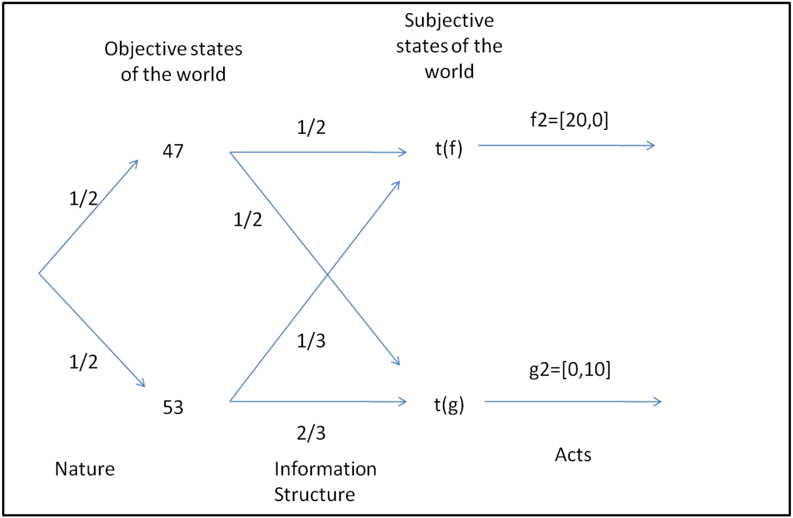
- Choice of attention strategy optimal



Act	Payoff 47 red dots	Payoff 53 red dots
$\mathbf{f}_2$	20	0
$\mathbf{g}_2$	0	10

Prior: $\{0.5, 0.5\}$

Optimal Choice of Acts



- Posterior probability of 47 red balls when act g was chosen

$$
\begin{aligned}
t_{47}^{g} &= \frac{P(\omega = 47, g \text{ chosen})}{P(g \text{ chosen})} \\
&= \frac{\frac{1}{4}}{\frac{1}{4} + \frac{2}{6}} = \frac{3}{7}
\end{aligned}
$$

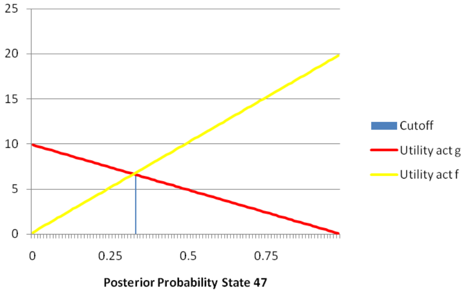
- But for this posterior

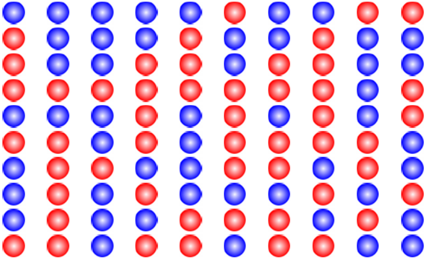
$$
\begin{aligned}
\frac{3}{7}U(f_{47}) + \frac{4}{7}U(f_{53}) &= \frac{3}{7}20 + \frac{4}{7}0 = 8.6 \\
\frac{3}{7}U(g_{47}) + \frac{4}{7}U(g_{53}) &= \frac{3}{7}0 + \frac{4}{7}10 = 5.7
\end{aligned}
$$

Condition 1 At every state $s \in T(\lambda)$, it must be the case that

$$C(s) \in \arg\max_{f \in A} \sum_{m=1}^{M} s_m U(f(\omega_m))$$

- In 2×2 case generates a cutoff c such that $t^f \geq c$ and $t^g \leq c$





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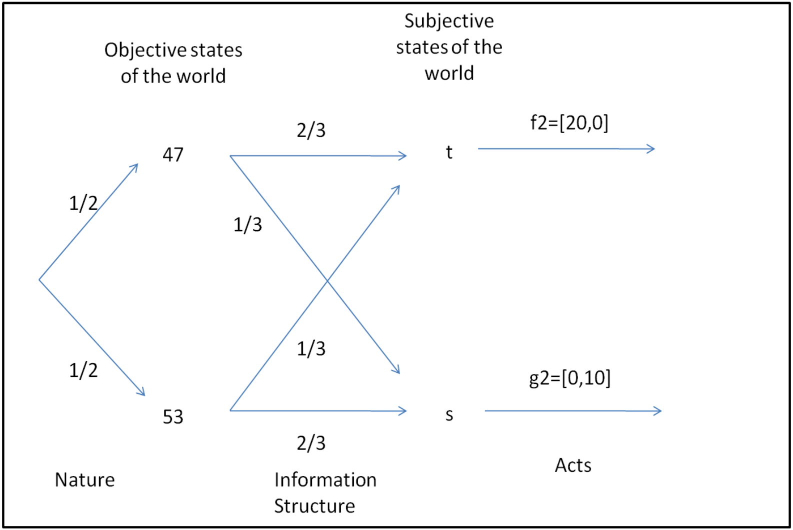
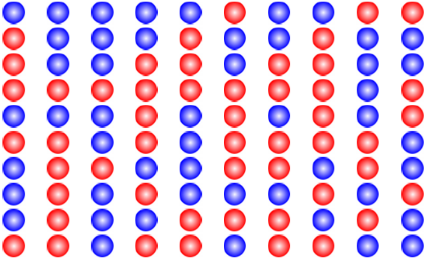
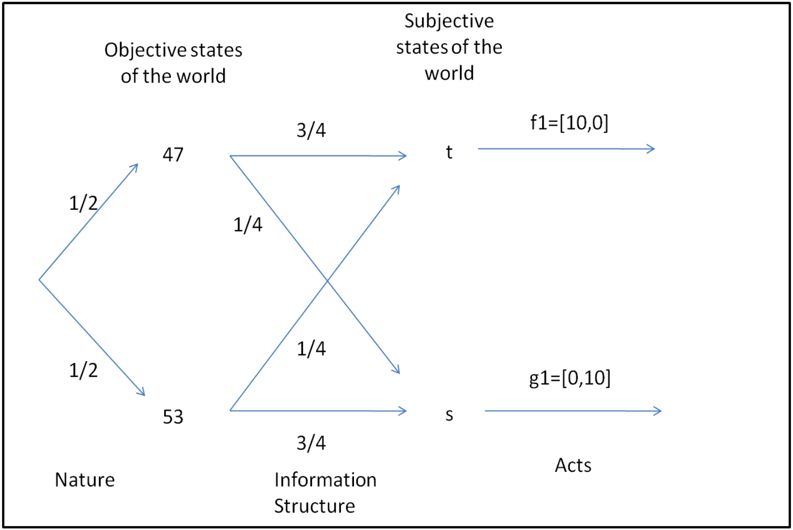

Objective states of the world
Subjective states of the world
47
53
1/2
1/2
2/3
1/3
1/3
2/3
t
s
f2=[20,0]
g2=[0,10]
Nature
Information Structure
Acts



Act	Payoff 47 red dots	Payoff 53 red dots
$\mathbf{F}_1$	10	0
$\mathbf{G}_1$	0	10

Prior: $\{0.5, 0.5\}$



Objective states of the world
Subjective states of the world
3/4
f1=[10,0]
47
t
1/2
1/4
1/4
1/2
g1=[0,10]
53
s
3/4
Acts
Nature
Information Structure

V	λ_1	λ_2
$\{f_1, g_1\}$	$7\frac{1}{2}$	$6\frac{2}{3}$
$\{f_2, g_2\}$	$11\frac{1}{4}$	10

- Cost function must satisfy

$$\begin{aligned} V(\beta, \{f_1, g_1\}, \lambda_1) - K(\lambda_1, \beta) &\geq V(\beta, \{f_1, g_1\}, \lambda_2) - K(\lambda_2, \beta) \\ V(\beta, \{f_2, g_2\}, \lambda_2) - K(\lambda_2, \beta) &\geq V(\beta, \{f_2, g_2\}, \lambda_1) - K(\lambda_1, \beta) \end{aligned}$$

- Which implies

$$\frac{5}{6} = V(\beta, \{f_1, g_1\}, \lambda_1) - V(\beta, \{f_1, g_1\}, \lambda_2) \geq$$
$$K(\lambda_1, \beta) - K(\lambda_2, \beta) \geq$$
$$V(\beta, \{f_2, g_2\}, \lambda_1) - V(\beta, \{f_2, g_2\}, \lambda_2) = 1\frac{1}{4}$$

Optimal Choice of Attention Strategy

- Surplus must be maximized by correct assignments

$$\begin{aligned} & V(\beta, \{f_1, g_1\}, \lambda_1) - V(\beta, \{f_1, g_1\}, \lambda_2) \\ & + V(\beta, \{f_2, g_2\}, \lambda_2) - V(\beta, \{f_2, g_2\}, \lambda_1) \\ \geq \; & 0 \end{aligned}$$

- To guarantee the existence of a cost function require a stronger condition

Condition 2 For any β and observed sequence of acts $A^1.....A^K$ and associated information structures $\lambda^1...\lambda^K$

$$\begin{aligned} & V(\beta, A^1, \lambda^1) - V(\beta, A^1, \lambda^2) \\ & + V(\beta, A^2, \lambda^2) - V(\beta, A^2, \lambda^3) \\ & + ... \\ & + V(\beta, A^K, \lambda^K) - V(\beta, A^K, \lambda^1) \\ \geq \; & 0 \end{aligned}$$

Theorem

For a sequence of decision problems $\{\beta, A_l\}_{l=1}^{L}$, *attention strategies* $\left\{\lambda^l\right\}_{l=1}^{L}$ *and choice functions* $\left\{C^l\right\}_{l=1}^{L}$ *such that* $C^l : T\left(\lambda^l\right) \longrightarrow A^l$ *the following two statements are equivalent*

1. $\{\beta, A^l\}_{i=1}^{L}$ *and* $\left\{\lambda^l\right\}_{l=1}^{L}$ *satisfy conditions 1 and 2*
2. *there exists a* $K : \beta \times \Lambda(\beta) \longrightarrow \mathbb{R}$ *such that* λ^l *and* C^l *solve the decision problem for each* $\{\beta, A^l\}$

Proof.

$2 \longrightarrow 1$ *Trivial*

$1 \longrightarrow 2$ *Rochet [1987]* □

Proof

- This problem is familiar from the implementation literature
- Say there were a set of environments $X_1....X_N$ and actions $B_1....B_M$ such that the utility of each environment and each state is given by

$$u(X_i, B_j)$$

- Say we want to implement a mechanism such that action $Y(X_i)$ is taken at in each environment.
- We need to find a taxation scheme $\tau : B_1....B_M \rightarrow \mathbb{R}$ such that

$$\begin{aligned} u(X_i, Y(X_i)) - \tau(Y(X_i)) &\geq u(X_i, B) - \tau(B) \\ &\forall\ B_1....B_M \end{aligned}$$

- This is the same as our problem.

Proof

- Taxation theorem: this is the equivalent problem to the following:
  - Find $\theta : \{A_i\}_{i=1}^{L} \to \mathbb{R}$ such that, for all A_i, A_j

$$V(\beta, A_i, \lambda^i) - \theta(A_i) \geq V(\beta, A_i, \lambda^j) - \theta(A_j)$$

  - Just define $K(\lambda) = \theta(A_i)$ if $\lambda = \lambda^i$ for some i, or $= \infty$ otherwise

- Now, pick some arbitrary A_0 and define

$$T(A) = \sup_{\textit{all chains s.t } A_0 \textit{ to } A = A_M} \sum_{n=1}^{M-1} V(\beta, A_{i+1}, \lambda^i) - V(\beta, A_i, \lambda^i)$$

- Condition 2 implies that $T(A_0) = 0$
- It also implies that

$$T(A_0) \geq T(A_i) + V(\beta, A_0, \lambda^i) - V(\beta, A_i, \lambda^i)$$

- So $T(A_i)$ is bounded

Proof

- Furthermore, for any A_i A_j we have

$$T(A_i) \geq T(A_j) + V(\beta, A_i, \lambda^j) - V(\beta, A_j, \lambda^j)$$

- So, setting $\theta(A_j) = V(\beta, A_j, \lambda^j) - T(A_j)$, we get

$$V(\beta, A_i, \lambda^i) - \theta(A_i) \geq V(\beta, A_i, \lambda^j) - \theta(A_j)$$

Cost Function and Blackwell Ordering

- Observation: if λ is sufficient for λ', then

$$V(\beta, A, \lambda) \geq V(\beta, A, \lambda')$$

- for all β, A
- Thus, for any cost function that rationalizes behavior

$$\begin{aligned} V(\beta, A', \lambda') - K(\beta, \lambda') &\geq V(\beta, A', \lambda) - K(\beta, \lambda) \\ 0 &\geq V(\beta, A', \lambda') - V(\beta, A', \lambda) \geq \\ & \quad K(\beta, \lambda') - K(\beta, \lambda) \\ &\Rightarrow K(\beta, \lambda) \geq K(\beta, \lambda') \end{aligned}$$

- Cost function will weakly obey Blackwell
- For unchosen information structures, assume cost is equal to lowest cost chosen Blackwell dominant option

Cost Function and Blackwell Ordering

- Does not guarantee strict observance with Blackwell
- Example: two states, $\beta_1 = 0.5$, 3 acts

Act	**Payoff 47 red dots**	**Payoff 53 red dots**
f	10	0
g	0	10
h	7.5	7.5

- Strategy $\lambda_1(t^f) = 0.75$, $\lambda_2(t^f) = 0.75$.
- $V(\beta, A, \lambda) = 7.5$