Bounded Rationality Lecture 4

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The Story So Far.....

- Introduced the concept of bounded rationality
- Described some behaviors that might want to explain with bounded rational models
- Discussed two models of costly information search
  - Sequential Search/Satisficing
  - Rational Inattention
- Discussed pricing behavior with rationally inattentive consumer

Plan for Today

- Describe a new model of 'costly contemplation'
  - Bolton and Faure-Grimaud [2008, 2010]
  - Understanding the state of nature takes time
  - Have to decide when to make decisions given this constraint
  - Apply this to a model of contracting
- Revisit the behaviors from lecture 1, think about which ones can be well described by our model

The General Problem [BFG 2008]

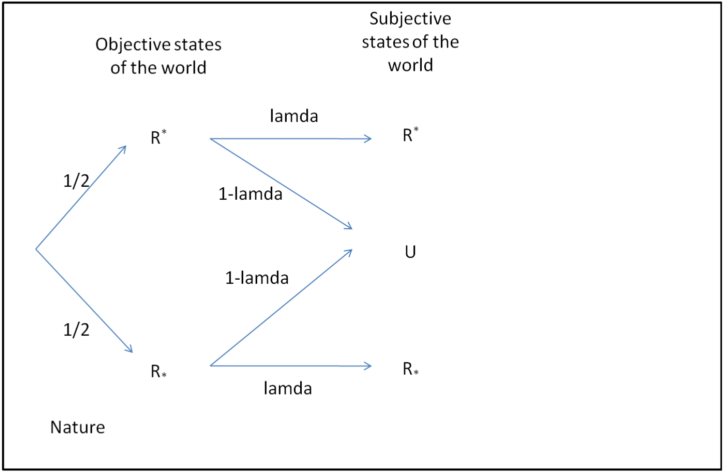
- Decision maker facing an investment option
- Cost of investing is I
- If decision maker invests at time t then at $t+1$ project ends up in 1 of two states

$$\{\theta_1, \theta_2\}$$

- Once state is realized, DM must choose between risky option which pays either R^* or R_*, safe option which pays S
- Ex-ante probability of R^* is v_i in each state

The General Problem [BFG 2008]

- ‘Bounded Rationality’: Agents can indulge in thought experiments
- Every period, can think about one state θ_i
- With probability λ, uncover whether payoff is R^* or R_* in θ_i
- Otherwise learn nothing


Objective states
of the world
Subjective
states of the
world
R*
lamda
R*
1/2
1-lamda
U
1-lamda
1/2
R*
R*
lamda
Nature

The Trade Off

- Cost of acquiring information is *delay*
  - Future payments discounted at rate δ
- Central trade off
- Acquire information before or after making initial investment I?
- Before:
  - Delays completion of project
    May acquire information on states of the world that do not obtain
- After:
  - Only acquire information on states of the world that actually obtain
  - May make unwise investments

The Bargaining Problem

- We focus on BFG [2010]
- Embed this problem inside a bargaining framework
- Aims: to show that certain types of contract can emerge *endogenously*
  - **Incomplete:** Do not condition on all available information, but instead assign control rights
  - **Coarse:** Specify act in each state of the world, but specify same act in different states
  - **Preliminary:** Initial contract to go ahead, followed by more exploration, followed by final contracting stage

Set Up

- Two agents A and B
- Project requires funding $I > 0$ from each agent
- If both agents invest in period t, then in period $t+1$ state $\theta \in \{\theta_1, \theta_2\}$ obtains (equally likely)
- In state θ_1 payoff π for both parties
- In state θ_2 must choose between risky and safe asset

$$\begin{aligned} R^* &= R_A^* + R_A^* > S = S_A + S_B \\ &> R_{*A} + R_{*B} = R_* \end{aligned}$$

- In each period, each agent gets a signal that reveals payoff with probability λ_i

Set Up

- Simplification of the BFG 2008 set up
- Only one state in which information is important
- Have to decide only on how much information
- Not what to get information on

1. Nature chooses 1 player to be the proposer and the other to be the receiver (WLOG A is the proposer)
2. A offers contract to B
3. B either accepts or rejects
4. If no investment, both players receive private signal about payoff of R
5. Choose whether or not to reveal this information
6. Based on result of 3 and 4, investment occurs or does not
7. If investment take place, state $\{\theta_1, \theta_2\}$ revealed
8. If in state θ_2 choice either to invest in R or S, or gather more information
9. Repeat from step 2

Illustrative Example

- Two investors deciding whether to invest in a software product (I)
- Research and Design continuing to solve a possible security flaw (λ)
- Security flaw may turn out to be unimportant or important $\{\theta_1, \theta_2\}$
- If it is important current version may be immune or may not be
- Can release the current version $\{R^*, R_*\}$
- Or an older version that is definitely immune (S)

- Expected payoff under preferred ex-post action choice

$$\rho_k^* = v \max\{R_k^*, S_k) + (1-v) \max\{R_{*,k}, S_k\}$$

- Expected payoff of risky action

$$\rho_k = vR_k^* + (1-v)R_{*k}$$

- Assumption:1:

$$\begin{aligned} \delta\frac{(\pi + S_k)}{2} &> I \\ \rho_k &> S_k \end{aligned}$$

Project is ex-ante desirable if safe options are considered , and expected value of risky option higher than that of safe option

Modelling Choices

- Preference Alignment - does $R_A^* \geq R_{*A} \Leftrightarrow R_B^* \geq R_{*B}$?
  - Consider both
- Is information cheap talk?
  - Consider both in the paper - we focus only on verifiable information
- Is utility transferrable?
  - Consider both in the paper - we focus on non-transferrable utility
- Symmetry
- Bargaining Structure
- No (non-time) costs to experimentation.

Solving the Model - Types of Contract

- C_R: R is immediately chosen in state θ_2 following investment
- C_S: S is immediately chosen in state θ_2 following investment
- C_A: A gets to make all post-investment decisions
- C_B: B gets to make all post-investment decisions
- C_{AB}: choice of S or R must be unanimous post investment
- C_α : Preliminary contract - agents agree to find out payoff or R then invest only once they have agreed a final contract $C \in \{C_R, C_S\}$

Solving the Model - Case 1: Congruent Objectives

- Assumption A2: A and B have same ranking over states of the world

$$\begin{aligned} R_A^* &> S_A > R_{*A} \\ R_B^* &> S_B > R_{*B} \end{aligned}$$

- Agents can still disagree about whether it is worthwhile resolving uncertainty
- Compare the strategy of deciding between R and S immediately, or waiting for uncertainty to be resolved
- Define Λ as the probability that the payoff of the risky asset will be uncovered in any given period under information sharing

$$\Lambda = 1 - (1 - \lambda_A)(1 - \lambda_B)$$

- Consider the payoff of waiting until the true state is realized before making decision

$$\begin{aligned} & \Lambda\rho_k^* + \delta(1-\Lambda)\Lambda\rho_k^* + \delta^2(1-\Lambda)^2\Lambda\rho_k^* + .. \\ = \; & \bar{\Lambda}\rho_k^* \end{aligned}$$

- where

$$\bar{\Lambda} = \frac{\Lambda}{1-(1-\Lambda)\delta}$$

- It could be the case that

$$\begin{aligned} \bar{\Lambda}\rho_A^* &< \rho_A \\ \bar{\Lambda}\rho_B^* &> \rho_B \end{aligned}$$

Benchmark Case: Unbounded Rationality

- Either $v = 0$ or $v = 1$, or $\lambda_A = \lambda_B = 1$
- There is always an optimal contract which specifies
    - Investment occurs immediately
    - Action S is risky asset is worth R_*, and action R otherwise.
- In this contract, actions are specified in all contingencies

Solving the Bounded Rationality Case

- **Lemma 1: Full Disclosure:** Under Assumption A1 and A2, full disclosure is subgame optimal
- Proof
  - Agents have same objectives post revelation, so revelation will immediately result in optimal action given true state of the world
  - Non-revelation cannot increase payoffs and may delay resolution

Case 1: Complete Satisficing Contracts

- Assume
  - Both agents prefer to wait: $\bar{\Lambda}\rho_k^* > \rho_k$
  - Delay is not costly: $I > \frac{\delta\pi}{2}$
- Then equilibrium involves thinking ahead of investing followed by either contract C_R or C_S

- Strategy of immediately investing and then thinking dominated by thinking then investing

$$-I + \frac{\delta}{2}\pi + \frac{\delta}{2}\bar{\Lambda}\rho_k^* \;<\; \bar{\Lambda}\left(-I + \frac{\delta}{2}\pi + \frac{\delta}{2}\rho_k^*\right)$$

$$\text{as } -I + \frac{\delta}{2}\pi \;<\; 0$$

- Implies waiting dominates C_A, C_B or $C_{A,B}$
- Waiting also dominates immediately signing up for C_R

$$-I + \frac{\delta}{2}\pi + \frac{\delta}{2}\rho_k < \bar{\Lambda}\left(-I + \frac{\delta}{2}\pi + \frac{\delta}{2}\rho_k^*\right)$$

As $\bar{\Lambda}\rho_k^* > \rho_k$

Case 2: Incomplete Satisficing Contracts

- Assume
  - Both agents prefer to wait: $\bar{\Lambda}\rho_k^* > \rho_k$
  - Delay **is** costly: $I < \frac{\delta\pi}{2}$
- Then equilibrium involves immediate investment and assignment of contract rights (C_A, C_B, C_{AB})
- In State θ_2, thinking will occur before investment

Proof

- Clearly, either party will wait in state θ_2 before investing if assigned contract , as

$$\bar{\Lambda}\rho_k^* > \rho_k$$

- This means assigning contract rights & investing immediate;it is better than waiting

$$\begin{aligned} -I + \frac{\delta}{2}\pi + \frac{\delta}{2}\bar{\Lambda}\rho_k^* &> \bar{\Lambda}\left(-I + \frac{\delta}{2}\pi + \frac{\delta}{2}\rho_k^*\right) \\ \text{as } -I + \frac{\delta}{2}\pi &> 0 \end{aligned}$$

- Also, assigning contract rights is better than deciding on the risky asset immediately as

$$-I + \frac{\delta}{2}\pi + \frac{\delta}{2}\bar{\Lambda}\rho_k^* > -I + \frac{\delta}{2}\pi + \frac{\delta}{2}\rho_k$$

Case 3: Conflict over Cautiousness

- Up untill now, our agents have agreed about everything
- We now consider the case where one part would like to delay and the other would not
- Assume
  - Agent A prefers not to wait: $\bar{\Lambda}\rho_A^* < \rho_k$
  - Agent B prefers to wait : $\bar{\Lambda}\rho_A^* \geq \rho_k$
  - Delay is costly: $I < \frac{\delta\pi}{2}$
- Also

$$-I + \frac{\delta}{2}\pi + \frac{\delta}{2}\bar{\Lambda}\rho_B^* < \bar{\Lambda}\left(-I + \frac{\delta}{2}\pi + \frac{\delta}{2}\rho_B^*\right)$$

(Agent B would rather delay investing)

Case 3: Conflict over Cautiousness

- Equilibrium: A offers a contract which plays C_B with probability y^* and C_R with probability $(1-y^*)$, where

$$-I+\frac{\delta}{2}\pi+y^*\frac{\delta}{2}\bar{\Lambda}\rho_B^*+(1-y^*)\frac{\delta}{2}\rho_B=\bar{\Lambda}\left(-I+\frac{\delta}{2}\pi+\frac{\delta}{2}\rho_B^*\right)$$

- A offers enough probability of property rights to B to make B indifferent between delaying or not

Proof

- The receiver can always guarantee themselves

$$\bar{\Lambda}\left(-I+\frac{\delta}{2}\pi+\frac{\delta}{2}\rho_B^*\right)$$

  in equilibrium by rejecting any offer until nature resolves itself
- Can they guarantee any more? Not if sender is not getting their best option
- Say receiver getting $\bar{U}_t > \bar{\Lambda}\left(-I+\frac{\delta}{2}\pi+\frac{\delta}{2}\rho_B^*\right)$
- If receiver cannot do better, must be that can do as well next period $\Rightarrow \bar{U}_t = \delta \bar{U}_{t+1}$
- Implies $\lim \bar{U}_t = \infty$

Proof

- Problem of sender is therefore to max their payoff subject to receiver payoff equal to $\bar{\Lambda}\left(-I + \frac{\delta}{2}\pi + \frac{\delta}{2}\rho_B^*\right)$
- Possible contracts
  - Give full control to B with some probability
  - Each period either give control to B or choose risky act every period (equivalent)
  - Choose safe action before learning state - (dominated for both players)
- Focus on the first type.
- Choice variables:
  - x : prob of thinking ahead before investing
  - y : prob of handing over control after investing

- Max

$$
\begin{aligned}
& x\bar{\Lambda}(-I + \frac{\delta}{2}\pi + \frac{\delta}{2}\rho_A^*) \\
& +(1-x)\left(-I + \frac{\delta}{2}\pi + y\frac{\delta}{2}\bar{\Lambda}\rho_A^* + (1-y)\frac{\delta}{2}\rho_A\right)
\end{aligned}
$$

- s.t

$$
\begin{aligned}
& \bar{\Lambda}\left(-I + \frac{\delta}{2}\pi + \frac{\delta}{2}\rho_B^*\right) \\
\leq \quad & x\bar{\Lambda}(-I + \frac{\delta}{2}\pi + \frac{\delta}{2}\rho_B^*) \\
& +(1-x)\left(-I + \frac{\delta}{2}\pi + y\frac{\delta}{2}\bar{\Lambda}\rho_B^* + (1-y)\frac{\delta}{2}\rho_B\right)
\end{aligned}
$$

- Rearranging last constraint gives

$$
\begin{aligned}
& (1-x)\bar{\Lambda}\left(-I+\frac{\delta}{2}\pi+\frac{\delta}{2}\rho_B^*\right) \\
\leq \quad & (1-x)\left(-I+\frac{\delta}{2}\pi+y\frac{\delta}{2}\bar{\Lambda}\rho_B^*+(1-y)\frac{\delta}{2}\rho_B\right)
\end{aligned}
$$

- As objective function is decreasing in x, set x to zero,
- As objective function is decreasing in y and constraint increasing in y, choose y^* such that

$$
\begin{aligned}
& -I+\frac{\delta}{2}\pi+y^*\frac{\delta}{2}\bar{\Lambda}\rho_B^*+(1-y^*)\frac{\delta}{2}\rho_B \\
= \quad & \bar{\Lambda}\left(-I+\frac{\delta}{2}\pi+\frac{\delta}{2}\rho_B^*\right)
\end{aligned}
$$

Solving the Model - Case 2: Conflicting Objectives

- Assumption A7: A and B have different ranking over states of the world

$$\begin{aligned} R_A^* &< S_A < R_{*A} \\ R_B^* &> S_B > R_{*B} \end{aligned}$$

- Begin by assuming extreme case

$$\begin{aligned} R_A^* &< 0 \\ R_{*B} &< 0 \end{aligned}$$

Agents No Longer Share Information

- Say Agent A has control
- If agent B learns that the true state is R_*
- If they tell agent A, will choose the risky asset straight away
- Would rather delay selection of risky asset, so will keep quiet
- However, A will update their beliefs in the face of B's silence

$$1 - v_\tau = \frac{1 - v}{1 - v + v(1 - \lambda_B)^\tau} \to 1$$

- At some point will stop experimenting and choose risky asset (at time v_{τ_A}

Agents No Longer Share Information

- What about if B learns true state is R^*
- If they reveal, then agent A will immediately choose S
- If v_τ is close to v_{τ_A} may want to keep quiet so that agent chooses risky asset
- No pure strategy equilibrium
  - If B is accurately reporting R^* then A updates if no report
  - If A is updating if no report, B wants to keep quiet about R^*
- There is, however, a mixed strategy equilibrium

- The value of control is now lower, because agent gets less information
  - Only their own signal plus any info from the fact that the other person said nothing
- Characterizing stopping time (v_{τ_A}) difficult
- Focus on the case where the agent in control immediately chooses their preferred action

$$\begin{aligned} \rho_A \quad > \quad & v\Lambda S_A + (1-v)\lambda_A R_{*A} \\ & + (1 - v\Lambda + (1-v)\lambda_A)\rho_A \end{aligned}$$

- Implies A cannot do better under C_A than C_R
- Also cannot do better under C_{AB} or C_B

- Notice that B also prefers C_R to their outside option of waiting till the state is determined as

$$
\begin{aligned}
& -I + \frac{\delta}{2}\pi + \frac{\delta}{2}\rho_B \\
> \; & \bar{\Lambda}[v(-I + \frac{\delta}{2}\pi + \frac{\delta}{2}S_B) \\
& +(1-v)\max(0, -I + \frac{\delta}{2}\pi + \frac{\delta}{2}R_{*B})
\end{aligned}
$$

- Thus, as A prefers C_R to any incomplete contract, and C_B pays B above their outside option
- Incomplete contracts will not be part of any equilibrium
- Contracts may be coarse (rather than state contingent) if cost of delay is high enough to A

Preliminary Contract

- Agents agree to think ahead of investing
- Commit to an action contingent on R
- Can lead to higher ex ante payoffs that C_R by committing agents to ex post actions that are not optimal
- Can relax player B's participation constraint if state turns out to be $\bar{R}_*$
- Without pre-contracting, this constraint is

$$-I + \frac{\delta}{2}\pi + \frac{\delta}{2}\left(xS_B + (1-x)R_{*B}\right) \geq 0$$

where x is the probability of taking the safe action in state $\bar{R}_*$

- Consider the contract
- Commit to invest once they have discovered value of R
- If $R = R_*$ choose action r in state θ_2
- If $R = R^*$ choose action s with probability ξ and action r with probability $(1 - \xi)$
- where ξ is chosen to solve acgent B's participation constraint at time 0
- BFG give conditions under which this contract is the unique equilbrium

- In Lecture 1 we introduced these behaviors
  - Random Choice
  - Status Quo Bias
  - Failure to Choose the Best Option
  - Salience/Framing Effects
  - Statistical Biases
  - Too Much Choice
  - Compromise Effect
- Which can be explained by the models that we have discussed?

- Arguably Yes
  - Random Choice
  - Status Quo Bias
  - Failure to Choose the Best Option
  - Salience/Framing Effects
  - Statistical Biases
- Not so much
  - Too Much Choice
  - Compromise Effect

- Random Choice
  - We have seen that optimal response to attention costs may involve random choice
  - Links between rational inattention and logit choice
- Status Quo Bias
- Failure to Choose the Best Option
- Salience/Framing Effects
- Statistical Biases

- Random Choice
- Status Quo Bias
  - Status quo always searched in model of sequential search
  - get some information for free - will lead it to be chosen more by risk averse individual
  - Also - varying costs of attention
- Failure to Choose the Best Option
- Salience/Framing Effects
- Statistical Biases

Behaviors

- Random Choice
- Status Quo Bias
- Failure to Choose the Best Option
  - Emerges both from models of sequential search and rational inattention
- Salience/Framing Effects
- Statistical Biases

- Random Choice
- Status Quo Bias
- Failure to Choose the Best Option
- Salience/Framing Effects
    - Changes in environment that make some information ‘free’ can affect choice
- Statistical Biases

Behaviors

- Random Choice
- Status Quo Bias
- Failure to Choose the Best Option
- Salience/Framing Effects
- Statistical Biases
  - Can emerge from subjective states that 'merge' objective states

Too Much Choice?

- Stylized fact: people 'check out' of the decision problem in large choice sets
  - Choose status quo more often
  - Choose not to choose
- Hard to model with rational inattention
  - Benefits to search flat/increasing with choice set size
  - If costs are increasing, why not ignore some options?
- One alternative: contextual inference
  - Roland will go through this
- Can also explain compromise effect