Level K Thinking

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Introduction

- Game theory: The study of strategic decision making
  - Your outcome depends on your own actions and the actions of others
- Standard tool for prediction: Nash Equilibrium
  - No player has incentive to deviate given the actions of others
- But Nash Equilibrium has some problems
  - Play of experimental subjects systematically violate its predictions
  - Can be very complex to calculate
    - Assumes a high degree of rationality on the part of subject
    - Assumes that THEY assume a high degree of rationality on the part of others
- Level K model tries to deal with both of these problems

An Example: The p Beauty Contest Game

- n players
- Each player chooses $s_i \in \{1, 2....100\} = S_i$
- Earn \$10 if you are closes to p times average choice

$$s_i \in \arg\min_{s_1 \ldots s_n} |s_j - p\frac{\sum_{k=1}^{n} s_k}{n}|$$

- Earn zero otherwise
  - Split the prize in event of the tie
- $p \in (0, 1)$
- This defines $u_i(s_i, \{s_1, ..s_{i-1}, s_{i+1}, ..s_n\}) = u(s_i, s_{-i})$
  - The utility if you play s_i and others play s_{-i}

Nash Equilibrium

- Nash Equilibrium: A strategy profile $\{s_1^*, ...s_n^*\}$ such that no player has an incentive to deviate

$$u(s_i^*, s_{-i}^*) \geq u(s, s_{-i}^*) \ \forall \ s \in S_i$$

- What is the Nash Equilibrium of the p-beauty contest game?
- The (almost) unique Nash equilibrium is $s_i^* = 1 \ \forall \ i$
  - No gain by deviating for any player
  - For any other strategy profile, any player with $s_i \geq \frac{\sum_{k=1}^n s_k}{n}$ has incentive to deviate

Nash Equilibrium

- Do people play Nash Equilibrium strategies?
- Makes strong rationality assumptions
  - That players can figure out what the Nash Equilibrium is
  - They assume that others can figure out what the Nash equilibrium is
- Nash Equilibria may also come about through a process of learning
  - We will focus on one shot games

Depth of Reasoning

- Consider the following sequence of reasoning for the $\frac{2}{3}$ beauty contest

  1. I think the other players will play 50, so I will play the best response to 50, i.e $33\frac{1}{3}$
  2. I think the other players think everyone will play 50 and so will play $33\frac{1}{3}$. I will therefore play the best response to this, i.e. $22\frac{2}{9}$
  3. I think that the other players will initially think that everyone will play 50, and will consider playing $33\frac{1}{3}$. However, they will think that others have done the same reasoning, and will therefore play $22\frac{2}{9}$. I will best respond to this and play $14\frac{22}{27}$
  4. ......

- More generally (in the case of two players)
  1. I assume that the other player will play $\bar{s}$, so I will play $s_i^1 \in \arg\max_{s \in S_i} u_i(s, \bar{s})$
  2. I assume that other players will best respond to $\bar{s}$ and so play $s_j^1 \in \arg\max_{s \in S_j} u_j(s, \bar{s})$. I will therefore play $s_i^2 \in \arg\max_{s \in S_i} u_i(s, s_j^1)$
  3. I assume that other players will best respond to s_j^1 and so play $s_j^2 \in \arg\max_{s \in S_j} u_j(s, s_i^1)$. I will therefore play $s_i^3 \in \arg\max_{s \in S_i} u_i(s, s_j^2)$

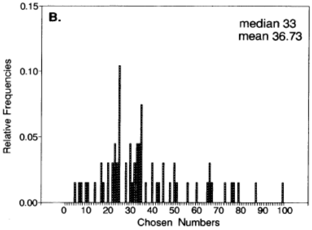
- Notice that a Nash equilibrium is a *fixed point* of this type of reasoning
    - I assume that other players will best respond to s_i^* and so play $s_j^* \in \arg\max_{s \in S_j} u_j(s, s_i^*)$. I will therefore play $s_i^* \in \arg\max_{s \in S_i} u_i(s, s_j^*)$
- In the case of the p-beauty contest game this type of reasoning will *converge* to the Nash Equilibrium
    - This is not always true

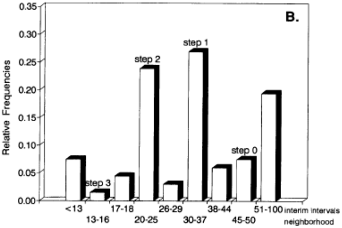
Level-K Thinking

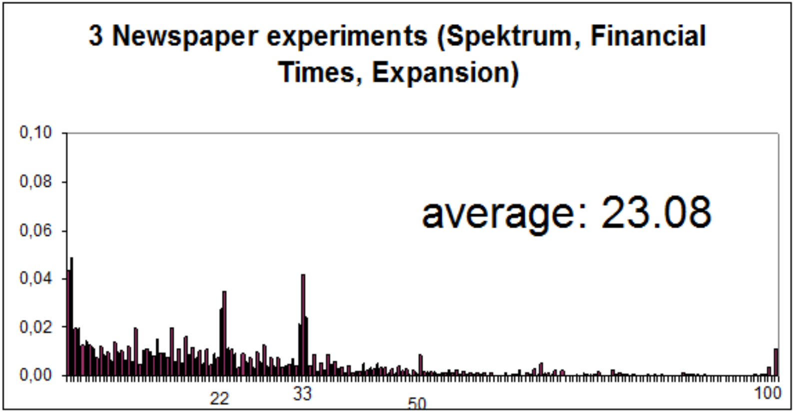
- What if you are constrained in how many steps of this type of reasoning that you can do?
- You have a ‘type’ equal to the
  - Level 0: Non-strategic (play at random)
  - Level 1: Best respond to level 0
  - Level 2: Best respond to level 1
  - Level 3: Best respond to level 2
  - ...
- There is a distribution of types in the population: π_i probability of level i
- Generally assumed that $\pi_i = 0$
  - ‘Anchor’ for remaining levels

- What would this imply for the data in the $\frac{2}{3}$ beauty contest game?
- We would see a focus of responses at the following levels:
    - π_0 : 50
    - π_1 : $33\frac{1}{3}$
    - π_2 : $22\frac{2}{9}$
    - π_3 : $14\frac{22}{27}$

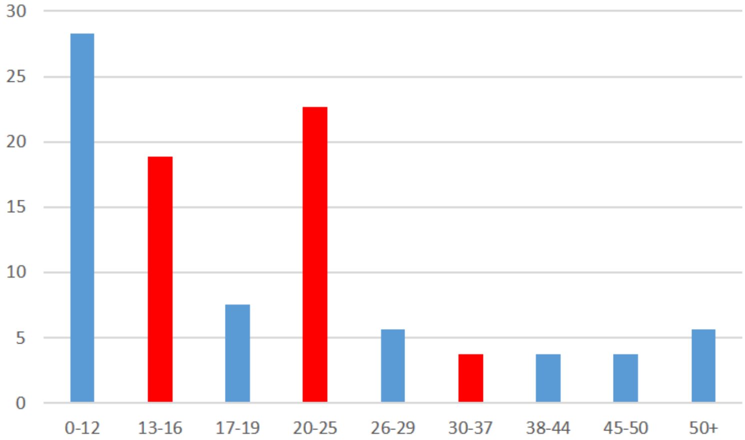
Nagel [1995]

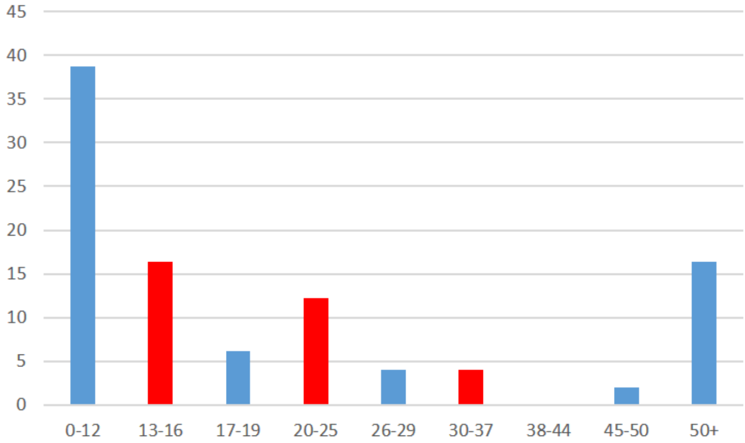

B.
median 33
mean 36.73
Relative Frequencies
0.15
0.10
0.05
0.00
0
10
20
30
40
50
60
70
80
90
100
Chosen Numbers


B.
Relative Frequencies
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
step 3
step 2
step 1
step 0
<13
13-16
17-18
20-25
26-29
30-37
38-44
45-50
51-100
interim intervals
neighborhood


3 Newspaper experiments (Spektrum, Financial Times, Expansion)
average: 23.08
0,10
0,08
0,06
0,04
0,02
0,00
22
33
50
100

Data from my Last Class at Brown




45
40
35
30
25
20
15
10
5
0
0-12
13-16
17-19
20-25
26-29
30-37
38-44
45-50
50+

Class Data

- You are much more ‘rational’ than the Nagel sample
  - Class mean 23.5 vs 36.7 in the Nagal Data
  - Brown Students: 22.6
- Many more ‘Nash’ players
  - But also more 'high' players
- Some evidence that there are a number of level 3 players
  - 7 players played 22
- 2/3 of the mean: 15.7
- Winning guess
  - Zoey Chopra
  - Pranav Balan
  - Shambhavi Tiwari
  - Ahana Maken

Issues with Level K Model

- Lots of additional degrees of freedom
  - What is level 0?
  - What is the distribution of types?
- The model has low predictive power
  - Consistent with any choice pattern
  - Needs more (ad hoc) assumptions in order to constrain it

Issues with Level K Model

- Are types fixed?
- Should be able to use estimated type in one game to predict play in others
- Georganas et al [2013] get same subject to play 'undercutting' and 'guessing games'
- Estimate type in each case
- Find no correlation in estimated type or estimated rank

Issues with Level K Model

- Response to learning and incentives?
- There is also evidence that types change predictably
  - Nagel [1995] - bidding in the p-beauty contest game falls with experience
  - Alaoui and Penta [2013] - subjects change their level of play with incentives