Utility Maximization

Mark Dean
Spring 2017
Columbia University

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Introduction

- Before we get on to some behavioral stuff, we are going to do something that doesn't seem to be very behavioral
- We will ask how you can test the model of utility maximization
- You are not going to enjoy this (particularly)
  - It involves some proofs
- Why am I making you do this?
  - It's a pretty foundational model
  - It will introduce a problem, and some techniques for solving it, that will be very useful
- This lecture should be in part revision from your IM class

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Testing Utility Maximization

- In order to understand how to test the model of utility maximization (or indeed any model) we need two elements

1. The data we are going to use
2. A precise description of the model

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The Data

- We observe:
  - The choices someone makes
  - What they were choosing from
- Example: choices from different sets of snack foods

Available Snacks	Chosen Snack
Jaffa Cakes, Kit Kat	Jaffa Cakes
Kit Kat, Lays	Kit Kat
Lays, Jaffa Cakes	Jaffa Cakes
Kit Kat, Jaffa Cakes, Lays	Jaffa Cakes

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The Model

- We want to test the model of utility maximization
- Every object has a fixed utility value attached to it
- For example:
  - U(jaffa cakes)=10
  - U(kit kat) =5
  - U(lays)=2
- In any choice set, choose the object with highest utility

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The Question

- Is our data set consistent with the model of utility maximization?
- Problem: Our model contains 'unobservables'
  - We do not observe utilities
  - Kit Kats do not come with utility numbers stamped on them
  - Model says that people maximize utility, but as the experimenter I do not observe utility
- Notice that this is a problem that will get worse as our models get more behavioral
  - We will add more psychological features that we can't observe directly
  - Temptation, self control, beliefs, envy, etc
- How can we proceed?

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Approach 1

- Pick a particular utility function
  - e.g. utility=calories
- Test whether this utility function can explain the data
  - e.g. Do people pick the option with the most calories?
- This is now a testable prediction
- Problem: What does failure tell us?
  - Perhaps people do not maximize utility
  - Or perhaps utility is not equal to calories

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Approach 2

- Ask the question: Is there ANY utility function that can explain the data?
- i.e. we are agnostic about what utility is
- We require only that the person has some consistent utility function that they are using to make their choices

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Algenon's Choices

Choice	Available Snacks	Chosen Snack
1	Jaffa Cakes, Kit Kat	Jaffa Cakes
2	Kit Kat, Lays	Kit Kat
3	Lays, Jaffa Cakes	Lays
4	Kit Kat, Jaffa Cakes, Lays	Jaffa Cakes

- Is there any utility function that can explain Algenon's choices
- No!
  - Choice 1 implies u(jaffa cake)>u(kit kat)
  - Choice 2 implies u(kit kat)>u(lays)
  - Choice 3 implies u(lays)>u(jaffa cakes)
- Implies u(jaffa cake)>u(jaffa cake): Contradiction

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Brittney's Choices

Choice	Available Snacks	Chosen Snack
1	Jaffa Cakes, Kit Kat	Jaffa Cakes
2	Kit Kat, Lays	Kit Kat
3	Lays, Jaffa Cakes	Jaffa Cakes
4	Kit Kat, Jaffa Cakes, Lays	Kit Kat

- What about Brittney's Choices?
- No!
  - Choice 1 implies u(jaffa cake)>u(kit kat)
  - Choice 4 implies u(kit kat)>u(jaffa cakes)
- Contradiction

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Colvin's Choices

Choice	Available Snacks	Chosen Snack
1	Jaffa Cakes, Kit Kat	Jaffa Cakes
2	Kit Kat, Lays	Kit Kat
3	Lays, Jaffa Cakes	Jaffa Cakes
4	Kit Kat, Jaffa Cakes, Lays	Jaffa Cakes

- How about Colvin's Choices?
- Yes!
  - u(jaffa cakes)>u(kit kat)>u(lays)
- Eg
  - u(jaffa cakes)=3
  - u(kit kat)=2
  - U(lays)=1

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A General Rule

- Question: Is there a general rule that differentiates data sets that can be explained by some utility function from those that can't?

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The Independence of Irrelevant Alternatives
Say x is chosen from a set of alternatives A
B is a subset of A that contains x
Then x must be chosen from B

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A General Rule

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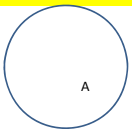

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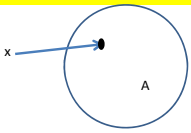

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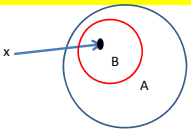

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Independence of Irrelevant Alternatives

Choice	Available Snacks	Chosen Snack
1	Jaffa Cakes, Kit Kat	
2	Kit Kat, Lays	
3	Lays, Jaffa Cakes	
4	Kit Kat, Jaffa Cakes, Lays	

- In our example, whatever is chosen in set 4 must always be chosen when it is available

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Algenon's Choices

Choice	Available Snacks	Chosen Snack
1	Jaffa Cakes, Kit Kat	Jaffa Cakes
2	Kit Kat, Lays	Kit Kat
3	Lays, Jaffa Cakes	Lays
4	Kit Kat, Jaffa Cakes, Lays	Jaffa Cakes

- Algenon's choices violate these condition
  - Jaffa cakes chosen in set 4
  - Lays chosen in set 3

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Brittney's Choices

Choice	Available Snacks	Chosen Snack
1	Jaffa Cakes, Kit Kat	Jaffa Cakes
2	Kit Kat, Lays	Kit Kat
3	Lays, Jaffa Cakes	Jaffa Cakes
4	Kit Kat, Jaffa Cakes, Lays	Kit Kat

- Also violated by Brittney's choices
  - Kit Kat chosen in set 4
  - Jaffa cakes chosen in set 1

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Colvin's Choices

Choice	Available Snacks	Chosen Snack
1	Jaffa Cakes, Kit Kat	Jaffa Cakes
2	Kit Kat, Lays	Kit Kat
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- Colvin's choices satisfy IIA
  - Jaffa cakes chosen in 4
  - Also chosen in 3 and 1

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A Necessary Condition

The Independence of Irrelevant Alternatives
Say x is chosen from a set of alternatives A
B is a subset of A that contains x
Then x must be chosen from B

- If we observe a utility maximizer, then they must satisfy IIA
  - If x is chosen from A, must have a higher utility than anything in A
  - B is a subset of A
  - X must have higher utility than anything in B
  - Should be chosen from B

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A Sufficient Condition?

The Independence of Irrelevant Alternatives
Say x is chosen from a set of alternatives A
B is a subset of A that contains x
Then x must be chosen from B

- Is it the case that, if IIA holds, there exists some utility function such that choices maximize utility according to that utility function?
- This would be great!
  - It means testing the condition is the same as testing the model of utility maximum
  - If the condition is satisfied then the person looks like a utility maximizer
  - If not, then they don't

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