Microeconomic Analysis

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Homework 2

Due Thursday 19th September

Question 1 Show that the two definitions of continuity we introduced in class are identical. Show they are also identical to the following conditions

- For any x the upper and lower contours $\{y|y \succeq x\}$ and $\{y|x \succeq y\}$ are closed
- For any x the sets $\{y|y \succ x\}$ and $\{y|x \succ y\}$ are open

Question 2 Some more continuity questions:

1. Can a continuous preference relation be represented by a discontinuous utility function?
2. Let $X = \mathbb{R}$. Show that the preferences represented by the utility function $u(x) = \max\{z \in \mathbb{Z}|x \geq z\}$ is not continuous. What about the preferences represented by $v(x) = \max\{z \in \mathbb{Z}|x > z\}$?

Question 3 A preference relation $\succeq$ on $\mathbb{R}^N_+$ is strictly increasing if $x > y$ (i.e. $x_n \geq y_n\ \forall\ n$ and $x_n > y_n$ some n) implies $x \succ y$ for all $x, y \in \mathbb{R}^N_+$. Let $\succeq$ be a continuous and strictly increasing preference relation on $\mathbb{R}^N_+$. Define $u(x) = \max[\alpha \geq 0|x \succeq (\alpha, \alpha, ..., \alpha)]$. Show that u is well defined, strictly increasing (i.e. $x > y$ implies $u(x) > u(y)$), continuous and represents $\succeq$.

Question 4: In class we defined the transitive closure of a binary relation in the following way

Definition 1 *A transitive closure of a binary relation R is a binary relation $T(R)$ that is the smallest transitive binary relation that contains R.*

1. In class we defined the notion of an extension for a partial order, but the same notion can be applied to any binary relation: a binary relation B is an extension to a binary relation R if

   (a) xRy implies xBy

   (b) xRy and not yRx implies xBy and not yBx

   Give an example of a binary relation for which its transitive closure is not an extension

2. We also defined a transitive closure in the following way

   Definition 2 *For a binary relation R on a set X define the set of relations $\{R_i\}_{i=0}^{\infty}$ by*

   (a) $R_1 = R$

   (b) For any $i \geq 2$, $x, y \in X$ then xR_iy if there exists a sequence $z_1, ...z_{i-1}$ such that $xRz_1R....Rz_{i-1}Ry$

   Then the transitive closure of R is defined as $T(R) = \cup_{i \in \mathbb{N}} R_i$

   Show that these two definitions are equivalent

Question 5 Question 6 in problem set 2 from the Rubinstein book (note he isn't joking about part 3 being difficult. Have a go but if you get stuck ask for a hint).