Microeconomic Analysis

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Homework 3

Due Thursday 26th September

Question 1 Consider a binary relation $\succeq$ on some finite set X which is transitive and reflexive but not necessarily complete (which I will call an incomplete preference relation).

1. Show that there exists a utility function $u : X \to R$ that represents $\succeq$ in the sense that

$$\begin{aligned} x &\succeq y \to u(x) \geq u(y) \\ x &\succ y \to u(x) > u(y) \end{aligned}$$

2. Clearly this representation is 'worse' than the standard one, in the sense that we cannot recover the preference relation from the utility function. To get round this problem, we can use a **multi-utility representation**. A multi-utility representation of the relation $\succeq$ on X is is a set of functions $\mathcal{U}$, where each $u \in \mathcal{U}$ is a function $u : X \to \mathbb{R}$, and these functions represent $\succeq$ in the sense that

$$x \succeq y \text{ if and only if } u(x) \geq u(y) \ \forall \ u \in \mathcal{U}$$

   Show that a multi utility representation has the same information as the original incomplete preference relation -i.e. there is a unique incomplete preference relation that is consistent with any multi-utility representation

3. One interpretation of the multi utility representation is that each object can be ranked on a number of dimensions, and you are only prepared to say that x is better than y if it is at least as good along all dimensions, and better on one. With that in mind, show how you can construct a multi-utility representation for the partial order $\geq$ on $\mathbb{R}^n$. (i.e. $x \geq y$ if and only if every element x_i of x is weakly greater than the associated y_i of y)

4. Show that any transitive, reflexive relation on a finite set X admits a multi utility representation.

Question 2 One modifcation of the standard economic model used to capture some idea of bounded rationality is to introduce the notion of consideration sets: rather than maximizing across all possible alternatives, the decision maker only considers a subset, and chooses the utility maximizing option in that subset.

Definition 1 *We say a set of choice data can be explained as choice with consideration sets if there is (i) a utility function $u : X \rightarrow R$ and (ii) a consideration set correspondence $\Gamma : 2^X/\emptyset \rightarrow 2^X/\emptyset$ such that $\Gamma(A) \subset A$ and*

$$C(A) = \arg\max_{x \in \Gamma(A)} u(x)$$

For simplicity, let's assume that we are dealing with choice functions (not correspondences), there is no indifference and X is finite

1. Show that a model of choice from consideration sets can explain any choice function
2. Now add the restriction

$$\Gamma(A) = \Gamma(A/x) \text{ if } x \notin \Gamma(A)$$

   We will call the model in Definition 1 with this restriction added 'Model A'.

   Determine whether the following methods of constructing consideration sets would satisfy this property

   (a) Top N: The decision maker is choosing between cars, and in any choice set considers the top 3 according to safety rating

   (b) The decision maker is choosing between jobs and has three criteria: wage, holiday and proximity. In any choice set they consider only the alternatives that are best in each category

   (c) The decision maker is choosing between hats, and considers all which are at or below the median price

3. Find a set of choices that cannot be explained by model A.
4. Show that, under model A, if $x = C(A)$ and $y \in A$, then it is not necessarily the case that $u(x) > u(y)$. Define the relation xPy such that if, for some choice set A such that

$x, y \in A$ and $x \neq y$ it is the case that $C(A) = x \neq C(A/y)$. Show that under model A xPy implies $u(x) > u(y)$

5. Consider the following condition: For any non-empty set S, there exists $x^* \in S$, such that, for any set T including x^*

$$\begin{aligned} C(T) &= x^* \text{ whenever} \\ \text{(i) } C(T) &\in S \text{ and} \\ \text{(ii) } C(T) &\neq C(T/x^*) \end{aligned}$$

   Show that this condition guarantees that the relation P is acyclic

6. Show that, if choices are produced by model A then they must satisfy the condition introduced in part 5.

7. Show that property α implies this property, but not visa versa

8. Show that the condition in part (5) is enough to guarantee the existence of a representation of the form of model A

Question 3 Some questions on random utility

1. Recall the example of a random choice rule we gave in class:

$$\begin{aligned} p(x, \{x, y\}) &= \frac{3}{4} \\ p(y, \{y, z\}) &= \frac{3}{4} \\ p(z, \{x, z\}) &= \frac{3}{4} \end{aligned}$$

   Show that, if we were also to observe choices probabilities from $\{x, y, z\}$ then we would have a violation of monotonicity

2. Show that, if $|X| = 3$ and we observe choices probabilities from all subsets then a data set satisfies the Block Marschak inequalities if and only if it satisfies monotonicity

3. Show that if choice probabilities have a Luce representation then they must satisfy Stochastic Independence of Irrelevant Alternatives