G5212: Game Theory

Mark Dean

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Recap

- So far we have
  - Explained what game theory is trying to do
  - Formally defined what we mean by a (strategic form) game
  - Introduced some classic games
  - Discussed the concept of mixed strategies
- Today we will start to think about how to 'solve' a game
  - i.e. make some predictions about how people will play the game

Rationality

- To begin with, let's see how far we can get with the assumption of **rationality**
  - Should be familiar from last semester
- In particular, let's assume that
  - Each player has some belief about what the other player will do
  - Chooses their action in order to maximize expected utility given their beliefs
- Does this allow us to make predictions about how players will play the game?

Example

Prisoner's Dilemma

		Bob: Confess	Bob: Don't Confess
Anne	Confess	$-6, -6$	$0, -9$
	Don't Confess	$-9, 0$	$-1, -1$

- 'Confess' provides higher utility than 'Don't Confess' regardless of what Bob thinks Anne will do
- If we assume Anne and Bob are rational, then the only possible outcome is that both play 'Confess'
- 'Confess' dominates 'Don't Confess'

Definition

A strategy $s_i' \in S_i$ strictly dominates s_i'' if $\forall s_{-i} \in S_{-i}$,

$$u_i(s_i', s_{-i}) > u_i(s_i'', s_{-i}).$$

A strategy s_i is a strictly dominant strategy for player i if s_i strictly dominates every strategy $s_i'' \in S_i \setminus \{s_i\}$.

Lemma

*If i has a **strictly dominant strategy**, then $\arg\max_{s_i} u_i(s_i, s_{-i})$ is independent of s_{-i} and is unique.*

Definition

A strategy $s_i' \in S_i$ weakly dominates s_i'' if $\forall s_{-i} \in S_{-i}$,

$$u_i(s_i', s_{-i}) \geq u_i(s_i'', s_{-i})$$

and $\exists s'_{-i} \in S_{-i}$,

$$u_i(s_i', s'_{-i}) > u_i(s_i'', s'_{-i}).$$

A strategy $s_i' \in S_i$ is a **weakly dominant strategy** for player i if s_i weakly dominates every strategy $s_i'' \in S_i \setminus \{s_i\}$

Dominance and Mixed Strategies

Example

Is D dominated in the following game?

		Bob	
		L	R
Anne	U	3, 0	0, 0
	M	0, 0	3, 0
	D	1, 0	1, 0

- Not using the definition we have so far
- But what about playing U half the time and M half the time?

Dominance and Mixed Strategies

Definition

A strategy $s_i' \in S_i$ is **strictly dominated** by a mixed strategy $\sigma_i \in \Delta(S_i)$ if

$$u_i(s_i', s_{-i}) < u_i(\sigma_i, s_{-i}) \ \forall s_{-i} \in S_{-i}.$$

Solving Games Using Dominance

- If a game has a strictly dominant strategy for each player, then using only rationality we can predict what will happen
- Note that games that can be solved in this way are in some sense not very interesting
  - Strategic element is effectively switched off
- What can we say about games that cannot be solved in this way?

Prisoner's Dilemma

Example

Prisoner's Dilemma

		Bob		
		Confess	Don't Confess	Go Bananas
Anne	Confess	$-6, -6$	$0, -9$	$-100, -100$
	Don't Confess	$-9, 0$	$-1, -1$	$-1, -100$

- Bob now has the strategy 'Go Bananas"
- Confess is no longer dominant for Anne
- BUT note that Go Bananas is strictly dominated for Bob
- If Anne thinks that Bob is rational, then can conclude that he will never go bananas
- Once this strategy has been ruled out, Confess is once once again strictly dominant for Anne

Common Knowledge of Rationality

- So far we have assumed that players are rational
- We can additionally assume that rationality is **common knowledge**
  - A knows that B is rational
  - B knows that A knows that B is rational
  - A knows that B knows that A knows that B is rational....
- This justifies solving games by **iterated deletion of strictly dominated strategies**
  - However - note that common knowldege can be quite a strong assumption....

Iterated Deletion of Strictly Dominated Strategies

Definition

Iterated Deletion of Strictly Dominated Strategies is described by the following procedure

1. Delete all strategies that are strictly dominated. Go to 2.
2. In the remaining game, are there any strictly dominated strategies? If Yes, go to 1. If No, done.

A game is called **dominance solvable**, or **solvable by strict dominance**, if this procedure leads to a *unique* outcome.

Example

- Beauty Contest
  - Occurred in UK in 1930s.
  - Newspaper prints 100 photographs of women. People choose the six "most beautiful" faces. Those who pick the most popular face are eligible for a prize

Example

The "Miss Rheingold" campaign, run by the J. Walter Thompson Co. for Liebmann Breweries, Inc. for over 25 years, is the best-known American example of a Keynesian beauty contest.

At the height of its popularity, between 15 and 20 million votes were cast per year—a turnout second only to the Presidential elections.

Example

- Keynes (General Theory, 1936): "It is not a case of choosing those [faces] that, to the best of one's judgment, are really the prettiest, nor even those that average opinion genuinely thinks the prettiest. We have reached the third degree where we devote our intelligences to anticipating what average opinion expects the average opinion to be. And there are some, I believe, who practice the fourth, fifth and higher degrees."

Example

- There are $n > 1$ players.
- Simultaneously, each player submits a number x_i between 0 and 100.
- Payoff: the player(s) whose number is closest to two thirds of the average $\overline{x}$ receives a positive prize, everyone else gets 0.

$$\overline{x} = \frac{x_1 + x_2 + ... + x_n}{n}.$$

$$\begin{aligned} u(x_i, x_{-i}) &= 100 - |x_i - \frac{2}{3}\bar{x}| \text{ if } x_i \text{ is closest to } \frac{2}{3}\bar{x} \\ &= 0 \text{ otherwise} \end{aligned}$$

- Does this game have a dominant strategy?

Example

- Not quite
- Although rationality does give us some predictive power
  - The average cannot be larger than 100.
  - Two thirds of the average cannot be larger than $\frac{2}{3} \times 100$.
  - $x_i > \frac{2}{3} \times 100$ have a payoff of 0 for sure, and is strictly dominated (by what?).
- So we can use rationality to guess than no-one will play hiver than $\frac{2}{3} \times 100$
- Can we do better?

Example

- Round 1:
  - The average cannot be larger than 100.
  - Two thirds of the average cannot be larger than $\frac{2}{3} \times 100$.
  - $x_i > \frac{2}{3} \times 100$ is strictly dominated and so we can delete this
- Round 2:
  - The average cannot be larger than $\frac{2}{3} \times 100$.
  - Two thirds of the average cannot be larger than $\frac{2}{3} \times \frac{2}{3} \times 100$.
  - $x_i > \left(\frac{2}{3}\right)^2 \times 100$ is strictly dominated.
- Leads to a unique solution.....

Problems with IDSDS?

Example

Does IDSDS always give a sensible answer?

		Bob	
		L	R
Anne	U	$3, 4$	$4, 5$
	D	$2, 1000000$	$0, -100000$

- IDSDS predicts U,R
- Is this reasonable?
- Bob would have to be REALLY sure that Anne is rational

IDWDS

Example

Iterated Deletion of Weakly Dominated Strategies

		Bob	
		L	R
Anne	U	1, 1	0, 0
	D	0, 0	0, 0

- This game is not solvable by IDSDS
- But {U,L} is the only strategy profile that survives deletion of **weakly** dominated strategies
- IDWDS allows us to make sharper predictions, but....

Order Matters in IDWDS

Example

Order Matters in IDWDS

		Bob	
		L	R
Anne	U	1, 1	0, 0
	M	1, 1	2, 1
	D	0, 0	2, 1

- One could first eliminate U, then L, leaving {M,R} or {D,R}
- Or one could first eliminate D, then R, leaving {U,L} or {M,L}