G5212: Game Theory

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More on Nash Equilibrium

- So now we know
  - That (almost) all games have a Nash Equilibrium in mixed strategies
  - How to find these equilibria by calculating best responses
- Over the next lecture (or so) we will discuss further aspects of NE
  - Justification
  - Refinements
  - Experimental Evidence

Justification of Nash Equilibrium

- Sometime, Nash equilibrium can seem such a natural concept that we don't think too hard about its justification
- However, this can be problematic
  - If we don't think about **why** Nash Equilibrium is a compelling notion, perhaps we don't have a good idea about **when** to apply it
- It is particularly worth thinking about in the context of mixed strategies
- How do we interpret an equilibrium in mixed strategies?

Justification 1: Mixed Strategies as Objects of Choice

- Perhaps the most obvious justification is that mixed strategies are **objects of choice**
  - In the same way that people choose pure strategies as a best response
- After all, in some games it would be a really bad idea to play a pure strategy
  - e.g. Matching pennies
- Is this a good justification?

Justification 1: Mixed Strategies as Objects of Choice

	A_p	B_{1-p}
A_p	$9,9$	$0,5$
B_{1-p}	$5,0$	$7,7$

- NE (σ_i) : If player i plays a mixed strategy σ_i, each s_i such that $\sigma_i(s_i) > 0$ is a best response to σ^*_{-i}. Then why bother to randomize?
- Look for symmetric mixed strategy equilibrium (both players play A with the same probability p)
- Player 1 is indifferent between A and B : $9p + 0(1-p) = 5p + 7(1-p) \Longrightarrow p = \frac{7}{11}$
- Player 2 randomizes just to make player 1 indifferent?

Justification 1: Mixed Strategies as Objects of Choice

- Treating mixed strategies as objects of choice has two related problems

1. The games as we have written them do not capture the benefits of randomization
2. Players are indifferent between different randomizations which have the same support
   - Why would they pick the precise randomization which supports the Nash Equilibrium?

Justification 2: Mixed Strategies as A Steady State

- A justification for Nash that can be used more generally is that it represents a steady state
  - Players interact repeatedly
  - Ignore any strategic link between play
  - Receive information on the historical frequency with which their opponent has played each strategy
  - Uses this to pick their strategy
- A (mixed strategy) Nash equilibrium is a steady state of this system in the sense that, if people stick with their mixing
  - These frequencies remain fixed over time
  - No one has incentive to deviate
- Note that **only** Nash Equilibria have this property
- Alternative interpretation: Playing against players drawn from a population, whose behavior is described by the mixed strategy

Justification 2: Mixed Strategies as A Steady State

- A follow up question: Is is this steady state stable?

Justification 2: Mixed Strategies as A Steady State

Example

	X	Y
X	3, 3	3, 0
Y	0, 3	10, 10

- It is a Nash equilibrium for each player to play X with probability 0.3 and Y with probability 0.7
  - Value of playing X : 3
  - Value of playing Y : $(1 - p_x)10$
- But what happened if there is a shock which makes it more likely that X was played?
  - Decrease value of playing Y relative to playing X
  - Make it less likely that people will play Y in the future
- Arguably the equilibrium is unstable

Justification 2: Mixed Strategies as A Steady State

Example

	Hawk	Dove
Hawk	$-1, -1$	$2, 0$
Dove	$0, 2$	$1, 1$

- It is a Nash equilibrium for each player to play Hawk with probability 0.5 and Dove with probability 0.5
  - Value of playing Hawk: $-p_H + 2(1 - p_H) = 2 - 3p_H$
  - Value of playing Dove:$= 1 - p_H$
- But what happened if there is a shock which makes it more likely that Hawk was played?
  - Decrease the value of playing Hawk relative to Dove
  - Make it less likely that people will play Hawk in future
- Arguably the equilibrium is stable

Justification 2: Mixed Strategies as A Steady State

- We formalizes this idea for games in which all players have the same strategy set

Definition

A mixed strategy profile σ^* is **Evolutionary Stable** if

1. $u_i(\sigma_i^*, \sigma_{-i}^*) \geq u_i(\sigma_i, \sigma_{-i}^*)$ for all i and $\sigma \in \Delta(S_i)$
2. if $u_i(\sigma_i^*, \sigma_{-i}^*) = u_i(\sigma_i, \sigma_{-i}^*)$ then $u_i(\sigma_i^*, \sigma_i) > u_i(\sigma_i, \sigma_i)$

- Called 'evolutionarily stable' because it can 'fight off' invaders
  - i.e. players that always play some other strategy
- This is our first example of an **equilibrium refinement**
  - Rules out some Nash Equilibria as 'unlikely'

Justification 3: Mixed Strategies as Pure Strategies in a Perturbed Game

	A	B
A	$9+\varepsilon t_1, 9+\varepsilon t_2$	$0,5$
B	$5,0$	$7,7$

- Suppose $\varepsilon > 0$ is a fixed known number. $t_1, t_2 \sim U[0,1]$, independent.
- Each player i privately observes $t_i > 0$.
- Now intuitively, player i's strategy will depend on what he/she observes.
- Say player i uses a "cutoff" strategy:

$$s_i(t_i) = \begin{cases} \text{A} & \text{if } t_i > k \\ \text{B} & \text{if } t_i \leq k \end{cases}$$

Justification 3: Mixed Strategies as Pure Strategies in a Perturbed Game

	A	B
A	$9+\varepsilon t_1, 9+\varepsilon t_2$	$0, 5$
B	$5, 0$	$7, 7$

- Say player 2 uses a "cutoff" strategy:

$$s_i(t_i) = \begin{cases} \text{A} & \text{if } t_i > k \\ \text{B} & \text{if } t_i \leq k \end{cases}$$

- Player 1's exp. payoff when he observes t_1 and plays A is

$$\begin{aligned} & (9+\varepsilon t_1)\Pr(\text{player 2 plays A}) + 0\Pr(\text{player 2 plays B}) \\ = \; & (9+\varepsilon t_1)(1-k) \end{aligned}$$

- Player 1's exp. payoff when he observes t_1 and plays B is

$5\Pr(\text{player 2 plays A}) + 7\Pr(\text{player 2 plays B}) = 5(1-k) + 7k$

- A is optimal for player 1 iff

Justification 3: Mixed Strategies as Pure Strategies in a Perturbed Game

	A	B
A	$9+\varepsilon t_1, 9+\varepsilon t_2$	$0, 5$
B	$5, 0$	$7, 7$

- Say player 2 uses a "cutoff" strategy:

$$s_i(t_i) = \begin{cases} \text{A} & \text{if } t_i > k \\ \text{B} & \text{if } t_i \leq k \end{cases}$$

- A is optimal for player 1 iff
  $(9+\varepsilon t_1)(1-k) \geq 5(1-k) + 7k \Longrightarrow t_1 \geq \frac{11k-4}{\varepsilon(1-k)}$.
- By symmetry $k = \frac{11k-4}{\varepsilon(1-k)}$;
- Solution is $k = \frac{1}{2\varepsilon}\left(\varepsilon + \sqrt{-6\varepsilon + \varepsilon^2 + 121} - 11\right)$
- $k \to \frac{4}{11}$ as $\varepsilon \to 0$

Justification 3: Mixed Strategies as Pure Strategies in a Perturbed Game

	A	B
A	$9+\varepsilon t_1, 9+\varepsilon t_2$	$0, 5$
B	$5, 0$	$7, 7$

- Say player 2 uses a "cutoff" strategy:

$$s_i(t_i) = \begin{cases} A & \text{if } t_i > k \\ B & \text{if } t_i \leq k \end{cases}$$

- $k \to \frac{4}{11}$ as $\varepsilon \to 0$.

$$s_i(t_i) = \begin{cases} A & \text{if } t_i > \frac{4}{11} \\ B & \text{if } t_i \leq \frac{4}{11} \end{cases}$$

- So from an outside observer's perspective (who does not know t_2), A is played by player 2 with probability 7/11 and B is played with probability 4/11. The same is true for player 2.

Justification 4: Mixed Strategies as Beliefs

Player 2's mixed strategy is player 1's **conjecture** about player 2's play.

	A_p	B_{1-p}
A_p	9, 9	0, 5
B_{1-p}	5, 0	7, 7

- NE $p = \frac{7}{11}$
- Player 1's **conjecture** about player 2's play: $\frac{7}{11}A \oplus \frac{4}{11}B$
- Player 2's **conjecture** about player 1's play: $\frac{7}{11}A \oplus \frac{4}{11}B$
- Conjectures are **correct**: actions in the support of the conjectures are played.

Justification 4: Mixed Strategies as Beliefs

Lemma

A strategy profile $(\sigma_1^*, ..., \sigma_n^*)$ *is a Nash Equilibrium if and only if, for every player* i*, every action* s_i *such that* $\sigma_i^*(s_i) > 0$ *is a best response to* σ_{-i}^*

Refinements

- As I mentioned in a previous lecture, there are two things that economists love in a solution concept
  - Existence
  - Uniqueness
- Nash deals with the first issue very well, as we have seen
- Less well with the second
- In general there may be multiple NE
  - Though we can put some structure on the number

How Many Nash Equilibria?

- **Theorem** (Wilson): Almost all finite games have a finite and odd number of equilibria.(idea: the number of fps of continuous functions)
- Two Nash equilibria:

	A	B
A	$1, 1$	$0, 0$
B	$0, 0$	$0, 0$

- For $\varepsilon > 0$, one Nash equilibrium:

	A	B
A	$1, 1$	$0, 0$
B	$0, 0$	$-\varepsilon, -\varepsilon$

- Three Nash equilibria:

	A	B
A	$1, 1$	$0, 0$
B	$0, 0$	ε, ε

Refinements

- In order to make predictions more precise, we use **refinements** to the definition of NE
  - These are additional conditions that allow us to rule out some equilibria as implausible
- We have already come across one refinement
  - Evolutionary Stability
- We will discuss two other types
  - Trembling Hand Perfect
  - Refinement in co-ordination games

Trembling-hand perfect equilibrium

- A game u with two Nash equilibria (A, A) and (B, B) :

	A	B
A	$1, 1$	$0, -3$
B	$-3, 0$	$0, 0$

- Are both of these equally convincing?
- Arguable (B, B) does not look very promising
  - Both players playing weakly dominated strategies
  - If they had any uncertainty about what the other player was doing then this seems like a bad thing to do
  - Equilibrium is not robust to slight changes in beliefs
- Can we formalize this idea?

Trembling-hand perfect equilibrium

- A mixed strategy σ_i is **completely mixed** if $\sigma_i(s_i) > 0$ for all $s_i \in S_i$.
- A strategy profile σ is a **perfect equilibrium** if there exists a sequence of completely mixed strategy profiles $\sigma^k \to \sigma$ such that $\sigma_i \in \phi_i\left(\sigma_{-i}^k\right)$ for each i, i.e.,

$$u_i\left(\sigma_i, \sigma_{-i}^k\right) \geq u_i\left(s_i, \sigma_{-i}^k\right)$$

  for all k and $s_i \in S_i$.
- **Theorem** (Selten): A perfect equilibrium exists in all finite games

Trembling-hand perfect equilibrium

	A	B
A	$1, 1$	$0, -3$
B	$-3, 0$	$0, 0$

- (A, A) is a perfect equilibrium
  - Let $\sigma_R^k(A) = 1 - \frac{1}{k}$ and $\sigma_R^k(B) = \frac{1}{k}$
  - $u_C(A, \sigma_R^k) = 1 - \frac{1}{k} > -3(1 - \frac{1}{k}) = u_C(B, \sigma_R^k)$
- (B, B) is not
  - Take any fully mixed strategy σ_R^k
  - $u_C(B, \sigma_R^k) = -3\sigma_R^k(A) < \sigma_R^k(A) = u_C(A, \sigma_R^k)$

Trembling-hand perfect equilibrium

- Is Trembling-hand perfect the same as ruling out weakly dominated strategies?
  - In any THP equilibrium, no weakly dominated strategies will be used
  - A NE in which no weakly dominated strategies are used is THP in two player games

Trembling-hand perfect equilibrium

- Selten viewed "tremble" as a modelling device.
- Later refinenement concepts build on this idea
- Sequential equilibria
  - Used in extensive form games
  - Will come back to this
- Proper equilibrium:
  - Players make mistakes
  - Less likely to mistakenly take a strategy that leads to terrible outcomes (i.e., you tremble, but put probability 1/100 on some non-best response good strategy and put probability 1/10000 on very bad strategies)
  - This is a refinement of THP

Co-ordination Games

- The next set of refinements we consider apply to co-ordination games

Example

Coordination Game

	A	B	C	D
A	1, 1	0, 0	0, 0	0, 0
B	0, 0	1, 1	0, 0	0, 0
C	0, 0	0, 0	1, 1	0, 0
D	0, 0	0, 0	0, 0	1, 1

- All players benefit if the same action in played
- Multiple Nash Equilibria
- Which will be played?

Focal Points

- Schelling (RIP) suggested a theory of focal points
- Something outside the description of the game that will help us to determine what will be played
- Example: Meeting in New York
    - You have to meet a friend somewhere in New York at 6pm
    - You cannot communicate with your friend
    - Where do you go?

Focal Points

Example

Coordination Game

	A	B	C	D
A	1, 1	0, 0	0, 0	0, 0
B	0, 0	1, 1	0, 0	0, 0
C	0, 0	0, 0	1, 1	0, 0
D	0, 0	0, 0	0, 0	1, 1

What would you play now?

Risk Dominance and Payoff Dominance

- Focal points often rely on things external to the game description
- Can we use the payoffs of the game to determine what will be played?

	A	B
A	80, 80	80, 0
B	0, 80	100, 100

- What do you think would be played?
- Two concepts
    - Payoff dominance
    - Risk dominance

Risk Dominance and Payoff Dominance

- Can we use the payoffs of the game to determine what will be played?

	A	B
A	80, 80	80, 0
B	0, 80	100, 100

- Payoff dominance is simply any Nash equilibrium which Pareto dominates all others
- In this case (B, B)

Risk Dominance and Payoff Dominance

- Can we use the payoffs of the game to determine what will be played?

	A	B
A	80, 80	80, 0
B	0, 80	100, 100

- Risk dominance is more complex
  - Imagine I don't know which equilibrium will be played
  - Say that my opponent thinks we are playing (B, B) but I think we are playing (A, A)
  - How much do I lose (relative to playing B)
  - (100-80)=20
  - This is the opportunity cost of deviating from B
  - Opportunity cost of deviating from A is 80

Risk Dominance and Payoff Dominance

Definition

In a two player game we say (s_i, s_{-i}) risk dominates (s_j, s_{-j}) if

$$\begin{aligned} & (u_1(s_i, s_{-i}) - u_1(s_j, s_{-i}))\,(u_2(s_i, s_{-i}) - u_2(s_j, s_{-i})) \\ \geq\ & (u_1(s_j, s_{-j}) - u_1(s_i, s_{-j}))\,(u_2(s_j, s_{-j}) - u_2(s_i, s_{-j})) \end{aligned}$$

- Interpretation: it is more costly to mistakenly think we are playing B than to mistakenly think we are playing A
- In previous game, (A, A) was the risk dominant equilibrium

Experimental Evidence

- Before we move on to different types of games, let's have a quick look at the experimental evidence
- We are interested in whether experimental subjects do indeed play Nash
- To keep things manageable, we will focus largely on the literature on one shot games
- There is also a big literature on whether people learn to play Nash, which we will have less to say about

Sources

- General Nash:
  - Goeree, Jacob K., and Charles A. Holt. "Ten little treasures of game theory and ten intuitive contradictions." American Economic Review (2001): 1402-1422.
- Beauty Contest Games
  - Nagel, Rosemarie. "Unraveling in guessing games: An experimental study." The American Economic Review 85.5 (1995): 1313-1326.

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- Zero sum games
  - Palacios-Huerta, Ignacio. "Professionals play minimax." The Review of Economic Studies 70.2 (2003): 395-415.
  - Palacios-Huerta, Ignacio, and Oscar Volij. "Experientia docet: Professionals play minimax in laboratory experiments." Econometrica 76.1 (2008): 71-115.
  - Levitt, Steven D., John A. List, and David H. Reiley. "What happens in the field stays in the field: Exploring whether professionals play minimax in laboratory experiments." Econometrica 78.4 (2010): 1413-1434.
  - Kovash, Kenneth, and Steven D. Levitt. Professionals do not play minimax: evidence from major League Baseball and the National Football League. No. w15347. National Bureau of Economic Research, 2009.

Experiment 1: Traveler's Dilemma

- An airline loses two suitcases belonging to two different travelers. Both suitcases happen to be identical and contain identical antiques. An airline manager tasked to settle the claims of both travelers explains that the airline is liable for a maximum of \$300 per suitcase—he is unable to find out directly the price of the antiques.

Experiment 1: Traveler's Dilemma

- To determine an honest appraised value of the antiques, the manager separates both travelers so they can't confer, and asks them to write down the amount of their value - no less than \$180 and no larger than \$300. He also tells them that if both write down the same number, he will treat that number as the true dollar value of both suitcases and reimburse both travelers that amount. However, if one writes down a smaller number than the other, this smaller number will be taken as the true dollar value, and both travelers will receive that amount along with a bonus/malus: \$R extra will be paid to the traveler who wrote down the lower value and a \$R deduction will be taken from the person who wrote down the higher amount. The challenge is: what strategy should both travelers follow to decide the value they should write down?

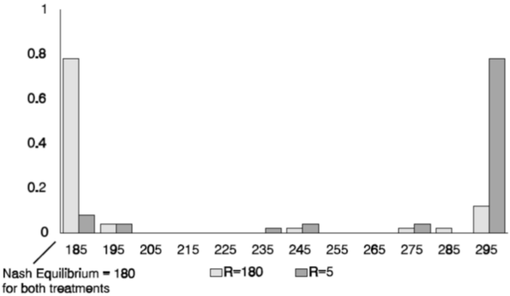
Experiment 1: Traveler's Dilemma

- Two players
- Pick a number between 180 and 300
- Both players receive
  - the lower of the two numbers
  - Plus an amount $R > 1$ transferred from the person who played the higher number to the person who played the lower
- What is the Nash Equilibrium of this game?

Experiment 1: Traveler's Dilemma

- 180 is the only NE of this game
- In fact it is the only strategy that survives IDSDS
- And so the only rationalizable strategy
- What do you think people do?
- Do you think it depends on R?

Experiment 1: Traveler's Dilemma



- When R is high (180) then almost 80% of subjects play the nash equilibrium
- When R is low (5) less than 10% do

Experiment 2: Matching Pennies

- Variant 1:

	L	R
T	80, 40	40, 80
B	40, 80	80, 40

- Nash Equilibrium?
- $\sigma(L) = \sigma(T) = 0.5$

Experiment 2: Matching Pennies

- Variant 2:

	L	R
T	320, 40	40, 80
B	40, 80	80, 40

- Nash Equilibrium?
- $\sigma(T) = 0.5$, $\sigma(L) = \frac{1}{8}$

Experiment 2: Matching Pennies

- Variant 3:

	L	R
T	44, 40	40, 80
B	40, 80	80, 40

- Nash Equilibrium?
- $\sigma(T) = 0.5$, $\sigma(L) = \frac{10}{11}$

Experiment 2: Matching Pennies

Symmetric Matching Pennies		*Left* (48%)	*Right* (52%)
	***Top* (48%)**	80, 40	40, 80
	***Bottom* (52%)**	40, 80	80, 40

Asymmetric Matching Pennies		*Left* (16%)	*Right* (84%)
	***Top* (96%)**	320, 40	40, 80
	***Bottom* (4%)**	40, 80	80, 40

Reversed Asymmetry		*Left* (80%)	*Right* (20%)
	***Top* (8%)**	44, 40	40, 80
	***Bottom* (92%)**	40, 80	80, 40

Experiment 2: Matching Pennies

- Changing the payoff of the row player should not impact the choice of that player
  - This does not hold up in the experiment
- Column player's seem to anticipate this, and adjust their play
- Given empirical distributions in variant 2

$$\begin{aligned} u_R(T, \sigma_{-i}) &= 84.8 \\ u_R(B, \sigma_{-i}) &= 73.6 \end{aligned}$$

$$\begin{aligned} u_C(L, \sigma_{-i}) &= 41.6 \\ u_C(R, \sigma_{-i}) &= 78.4 \end{aligned}$$

Experiment 3: Extended Coordination game

	L	H	S
L	$90, 90$	$0, 0$	$x, 40$
H	$0, 0$	$180, 180$	$0, 40$

- Nash Equilibrium?
  - S is dominated for the column player
  - So turns into a standard co-ordination game
  - Three equilibria
    - LL, HH, and $\sigma_R(L) = \sigma_H(L) = \frac{2}{3}$
  - HH is risk and payoff dominant

Experiment 3: Extended Coordination game

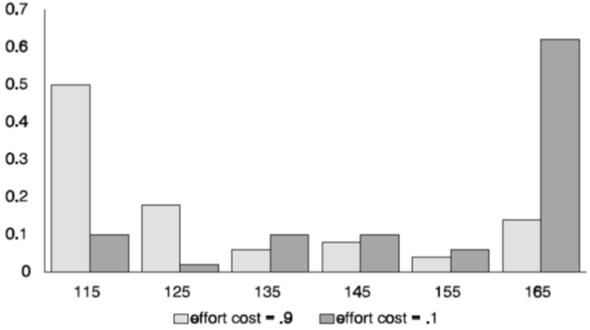
	L	L	S
L	$90, 90$	$0, 0$	$x, 40$
H	$0, 0$	$180, 180$	$0, 40$

- If $x = 0$ 96% of row and 84% of column players play H
- If $x = 400$, 64% of row and 76% of column players choose H

Experiment 4: Extended Coordination game

- Each player chooses an effort level between 110 and 170
- Payoff is the minimum effort level chosen minus c times own effort
- Nash Equilibrium?
- If $c \in (0,1)$ any pair of equal effort levels is an equilibrium
  - Deviating upwards brings $-c$
  - Deviating downwards brings $-(1-c)$
- This is independent of c

Experiment 4: Extended Coordination Game



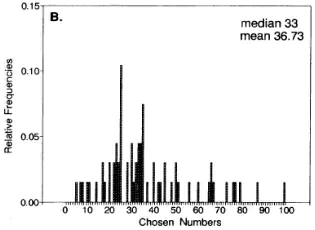
Summary of Experiments 1-4

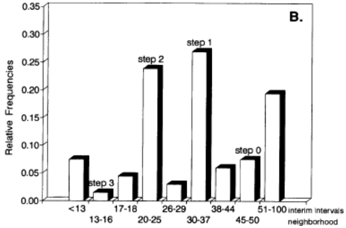
- Summary
  - Nash equilibrium sometimes makes correct predictions
  - Often it does not
  - One theme: 'Irrelevant' changes in payoff affect play
  - Irrelevant in the sense that they don't matter in equilibrium
- Caveat - we have only looked at 1 shot games here
- One model that has been introduced to deal with this type of problem is 'Quantal Response Equilibrium'
  - People make mistakes when they play
  - More likely to play strategies that have higher payoffs given play of others
  - But not guaranteed to play the best
  - Makes 'irrelevant' payoffs relevant
- McKelvey, Richard D., and Thomas R. Palfrey. "Quantal Response Equilibria for Normal Form Games." Games and Economic Behavior (1995).

Experiment 5: Beauty Contest Game

- Nagel [1995] ran a variant of the beauty contest game we studied in class
  - Pick a number between 0 and 100
  - Win a prize if amongst the closest to some fraction $p \in (0, 1)$ of the mean
  - Show results for $p = \frac{2}{3}$
  - Other values reported in the paper
- In all cases Nash equilibrium is for all players to bid 0

Experiment 5: Beauty Contest Game





Experiment 5: Beauty Contest Game

- Experiments of this type have been used to justify the 'level k' model of non-equilibrium reasoning
    - Level 0: pick at random
    - Level 1: best respond to level 0
    - Level 2: best respond to level 1
    - And so on
- In this game
    - Level 0 play 50 on average
    - Level 1 play $2/3 \times 50 \simeq 33.3$
    - Level 2 play $2/3 \times 2/3 \times 50 \simeq 22.2$

Issues with Level K Model

- Lots of additional degrees of freedom/Low predictive power
- Consistency of k across games?
- Consistency of k due to change in incentives/experience

Experience and Zero Sum Games

- The experiments we have looked at so far have looked at one shot games with inexperienced subjects
- Maybe this is the problem?
- What about if we look at experienced subjects?
- Where could we find experienced subjects?
- On the football field!

Palacios-Huerta (2003 Restud)

- 1417 penalty kicks from five years of professional soccer matches among European clubs.
- The success rates of penalty kickers given the decision by both the keeper (row player) and the kicker (left or right) are as follows:

	Left	Right
Left	58%	93%
Right	95%	70%

Palacios-Huerta (2003 Restud)

- If the ball goes into the net, the keeper's payoff is -1, while the kicker's payoff is $+1$. Otherwise, keeper $+1$, kicker -1.
- So keeper's payoff from (Left, Left): $-1 \times 58\% + 1 \times (1 - 58\%) = -0.16$.

	Left	Right
Left	$-0.16, +0.16$	$-0.86, +0.86$
Right	$-0.9, +0.9$	$-0.4, +0.4$

Palacios-Huerta (2003 Restud)

- In the mixed strategy equilibrium, players should get the same payoff from playing each action
- Notice that this game is ‘zero sum’
- Maximizing your payoff is the same as minimizing the payoff your opponent
- Want to pick the mixed strategy which minimizes the maximal payoff your opponent can get
    - This is the ’minimax’ strategy
    - See homework
- Means that there is pressure on both sides to equalize payoffs
    - If the GK randomizes in such a way to make Left better than Right, taker will play left and win more often
    - Means that the GK loses more often

Palacios-Huerta (2003 Restud)

	Left	Right
Left	$-0.16, +0.16$	$-0.86, +0.86$
Right	$-0.9, +0.9$	$-0.4, +0.4$

- NE proportion of kicks to the left: 38%
  - Observed proportion of kicks to the left: 40%
- NE proportion of jumps to the left: 42%
  - Observed proportion of jumps to the left: 42%

Palacios-Huerta (2008 ECMA)

- Are these skills transferrable?
- In a follow up study, Palacios-Huerta took professional football players into the lab
- See whether they still play minimax with a similar game in a lab setting

Palacios-Huerta (2008 ECMA)

TABLE I

RELATIVE FREQUENCIES OF CHOICES AND WIN PERCENTAGES IN PENALTY KICK EXPERIMENT WITH PROFESSIONAL PLAYERS[a]

A. Frequencies

		Column Player Choice: L	Column Player Choice: R	Marginal frequencies for row player
Row Player Choice	L	0.152 (0.165) [0.0068]	0.182 (0.198) [0.0073]	0.333 (0.364) [0.0088]
	R	0.310 (0.289) [0.0083]	0.356 (0.347) [0.0087]	0.667 (0.636) [0.0088]
Marginal frequencies for column player		0.462 (0.455) [0.009]	0.538 (0.545) [0.009]	

B. Win Percentages

Observed row player win percentage	0.7947
Minimax row player win percentage	0.7909
Minimax row player win std. deviation	0.0074

[a]In panel A the numbers in parentheses represent minimax predicted relative frequencies, whereas those in brackets represent standard deviations for observed relative frequencies under the minimax hypothesis. In panel B, minimax row player win percentage and std. deviation are the mean and the standard deviation of the observed row player mean percentage win under the minimax hypothesis.

Comment

- Both these results have been questioned in subsequent work
- Kovash and Levitt
  - Study choice of pitch in baseball and run vs pass in American Football
  - Find deviations from minimax play
  - Not all options give equal payoff
  - BUT, defining outcome variables is much harder
- Levitt, List and Reilley
  - Unable to replicate the result that professionals are better at Minimax in the lab