## MA Game Theory

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Homework 3

Due Weds 22nd February

**Question 1** Consider the following extensive-form game:



- 1. Solve the game by backward induction.
- 2. Write down the normal form of the game. Find all pure strategy Nash equilibria of this extensive-form game.
- Question 2 There are *n* firms: 1, 2, ..., *n*. Each firm has zero marginal cost. In the given order firms choose their production level, knowing what the previous firms have chosen. That is, Firm 1 chooses  $q_1 \ge 0$  first; Firm 2 observes  $q_1$  and chooses  $q_2 \ge 0$ ; Firm 3 observes  $q_1$  and  $q_2$  and chooses  $q_3 \ge 0$ , and so on. They sell their good at market price  $P = 1 (q_1 + \cdots + q_n)$ .

Each firm *i* gets the payoff of  $q_i P$ . Compute a Nash equilibrium of this game using backward induction for the following two cases:

- n = 3.
- For an arbitrary n.
- Question 3 Consider the following variant of the Bach or Stravinski game in which each player gets 3 if they co-ordinate on their preferred action and 1 if the co-ordinate on their less preferred action. First, the 'Bach' player moves. This player can chose either to (a) burn a dollar (and therefore reduce all the subsequent payoffs of the Bach player by 1) or (b) not burn a dollar. Having observed this decision, the players then play a simultaneous version of the BoS game.
  - 1. Draw the game tree for this game
  - 2. Find the set of subgame perfect Nash equilibria
  - 3. Write out the Normal form of the game. Does the SPNE survive the iterated deletion of weakly dominated strategies?
  - 4. What would the outcome of the game be if we applied the intuitive notion of forward induction introduced in the lectures?
  - 5. Show that there exists a sequence of deletions of weakly dominated strategies which leaves this as a unique outcome
  - 6. Consider a variant of the game where, after observing the money burning action of the Bach player, the Stravinski player decides whether or not to set fire to a dollar. after which the players play a BoS game. Find one stategy which survives IDWDS in this game.