MA Game Theory

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Homework 4

Due Mon 6th March

Question 1 Two players bargain over a pie of size 1. There are two rounds. In the first round, both players *simultaneously* propose an offer/demand. If the proposals are consistent with each other (i.e., the sum of demands is no larger than 1), they split the pie accordingly. Otherwise, they continue to the next round, and the size of the pie shrinks to 0.7. In the second round, player 1 proposes a division, and player 2 decides either to accept or reject it; If the proposal is rejected, the game ends and both get 0; if the proposal is accepted, the pie is allocated accordingly. Consider the concept of subgame perfect equilibria.

1. In the second round, what will be the equilibrium outcome?
2. Can there be a subgame perfect equilibrium such that an agreement is reached in the first round with a division $(0.8, 0.2)$, i.e., 0.8 to player 1 and 0.2 to player 2?
3. Can there be a subgame perfect equilibrium such that, in the first round, an agreement is reached with the division $(0.6, 0.4)$?
4. Can there be a subgame perfect equilibrium in which there is a disagreement in the first round?

Question 2 Here is an alternative, equivalent formulation of Bayesian games to the one that we gave in class: A Bayesian game consists of

- N: Set of players
- $S = \{S_n\}_{n \in N}$: Strategy space for each player

- Ω: A set of states
- $P \in \Delta(\Omega)$: prior probabilities over states
- $\Pi = \{\pi_n\}_{n \in N}$: Signal function $\pi_n : \Omega \to \Delta(T_n)$ for each player, where T_n is the set of possible signals that that player n could receive and $\pi_n(t_n|\omega)$ is the probability that player n gets signal t_n given state ω. Beliefs are then formed using Bayes rule
- $u : S \times \Omega \to \mathbb{R}^N$: Payoffs function for each player

Consider the following description of a set up: A suspect in a murder trial is in front of three judges. The judges initially think that the defendant is equally likely to be guilty or innocent. During the trial, each judge interprets the information they get, meaning they get a private 'message' about the guilt or innocence of the defendant which is accurate with probability $p > \frac{1}{2}$. This message is independent for the three judges, conditional on the guilt or innocence of the defendant. At the end of the trial the three judges must vote simultaneously for whether the defendant is guilty or innocent. The defendant is convicted if all three judges vote guilty. Each judge gets a payoff of 1 if the correct verdict occurs (i.e. a guilty defendant is convicted or an innocent one aquitted) or 0 otherwise. NOTE THAT I AM NOT ASKING YOU TO SOLVE THE GAME AS PART OF THIS QUESTION

1. Formalize the above as a Bayesian game using both the set up from class and the new set up above
2. Show that your two formulations are equivalent in that
   (a) Both formulations induces the same distribution over payoffs for each player
   (b) Both formulations induce the same beliefs over the payoffs of other players for each player
   (c) Both formulations induce the same beliefs over the beliefs of other players for each player

Question 2 Two risk neutral firms are competing in the Cournot style. Price is given by $P = 3 - Q$ where Q is total output. The marginal cost of firm 2 is 0. The marginal cost of firm 1 is equally likely to be 1 or 0. If the cost of firm 1 is 0, with probability $\frac{1}{2}$ firm 2 gets a signal saying that this is the case, and with probability $\frac{1}{2}$ no signal is sent. If the cost of firm 1 is 1, no signal is sent. Firm 1 doesn't know whether a signal is sent or not.

1. Write this situation as a Bayesian game (you can use the formulation from question 2 above)
2. Write down the equations that characterize a Bayesian equilibrium of this game (In this formulation, a strategy is a mapping from signals to probability distributions over actions. A BNE is a set of strategies which are best responses given beliefs generated by Bayes rule and the strategies of all other players).
3. Find a Bayesian equilibrium