1

Intermediate Microeconomics W3211

Lecture 6: The Effect of Changing Prices

Columbia University, Spring 2016

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Introduction

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The Story So Far….

- We have now learned how to solve the consumer's problem
- Used this to identify the **demand function**
  - How the consumer's choice depends on prices and income
- Discussed how demand changes with income
  - Introduced the concept of the income elasticity of demand

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Today's Aims

1. Discuss how demand for a good is affected by a change in its own price
   - Giffen Goods
   - Income and substitution effects
   - Compensated demand and the Slutsky equation
     - Varian Ch. 6, and 8 Feldman and Serrano Ch. 4
2. Discuss how demand for a good changes with the price of other goods
   - complements and substitutes
   - Varian Ch. 6, Feldman and Serrano Ch. 4

Demand and Own Price Changes

1: Introduction

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Own-Price Changes

- We have seen how demand changes with income
- How about with prices?
- Suppose only p_1 increases, from p_1' to p_1'' and then to p_1'''.

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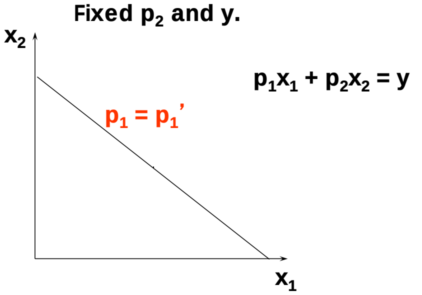
Own-Price Changes

Fixed p_2 and y.



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Own-Price Changes

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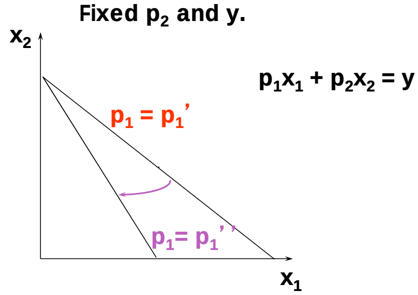

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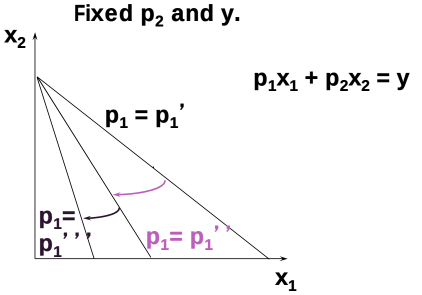

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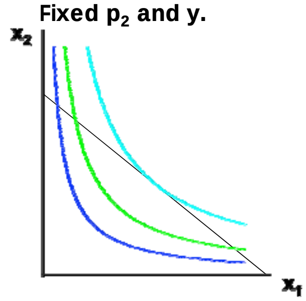

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Own-Price Changes

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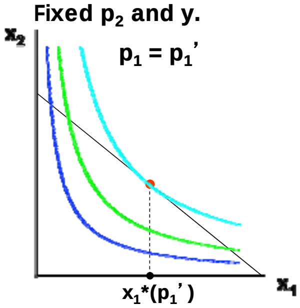

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Own-Price Changes

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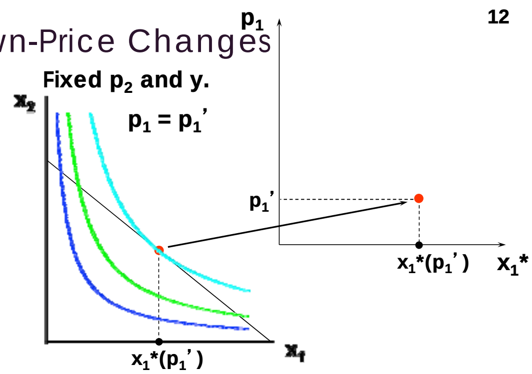

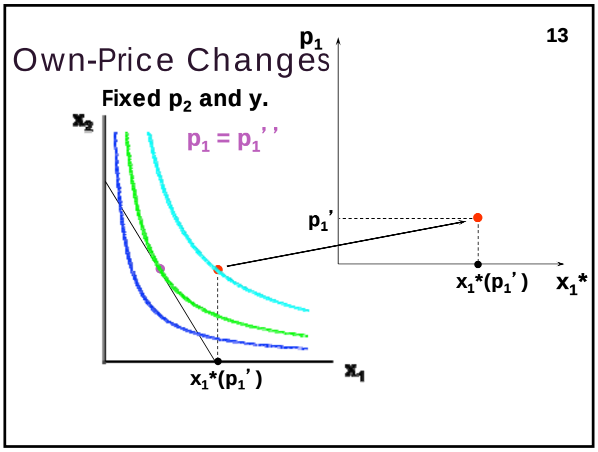
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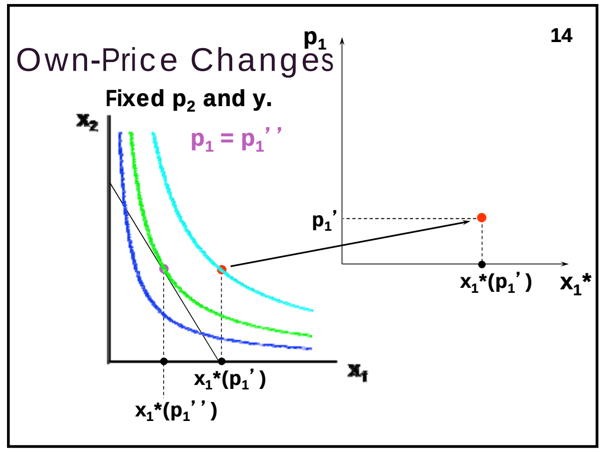
Own-Price Changes
Fixed p_2 and y.
$p_1 = p_1''$
p_1
p_1'
$x_1^*(p_1')$
x_1^*
x_2
x_1
$x_1^*(p_1')$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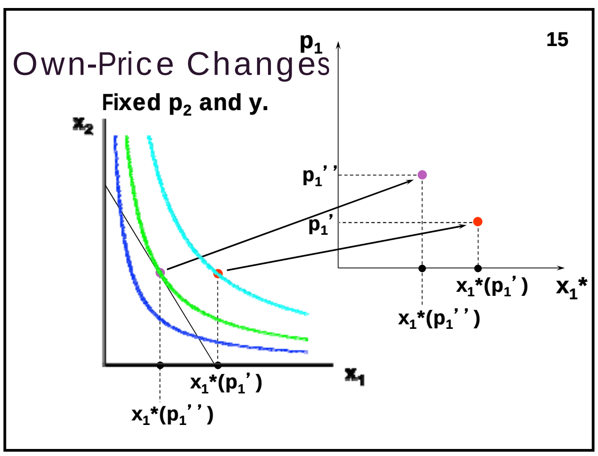
$x_1^*(p_1'')$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Own-Price Changes
Fixed p_2 and y.
p_1
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$x_1^*(p_1')$
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x_2
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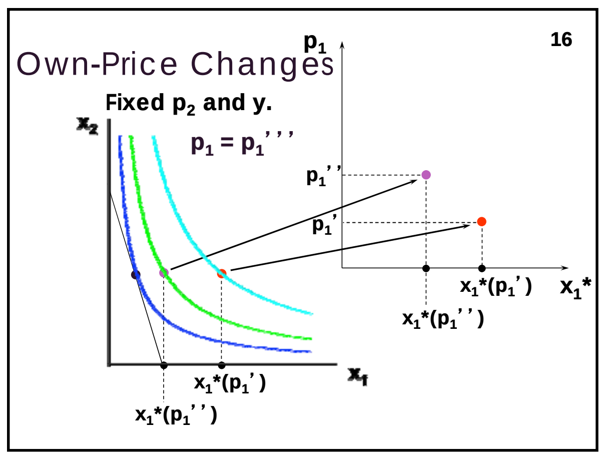
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Own-Price Changes
Fixed p_2 and y.
$p_1 = p_1'''$
p_1
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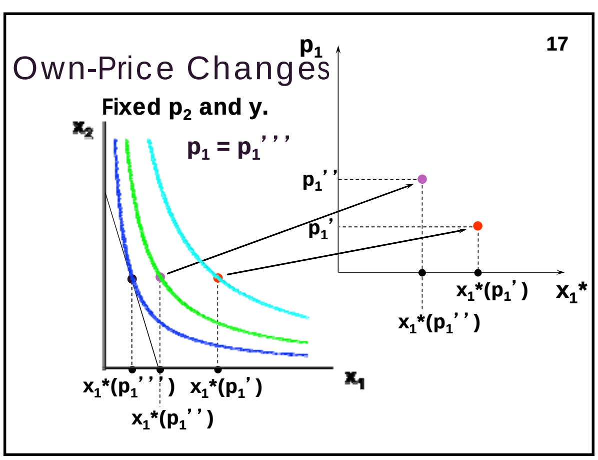
$x_1^*(p_1'')$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Own-Price Changes
Fixed p_2 and y.
$p_1 = p_1'''$
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$x_1^*(p_1')$
x_1^*
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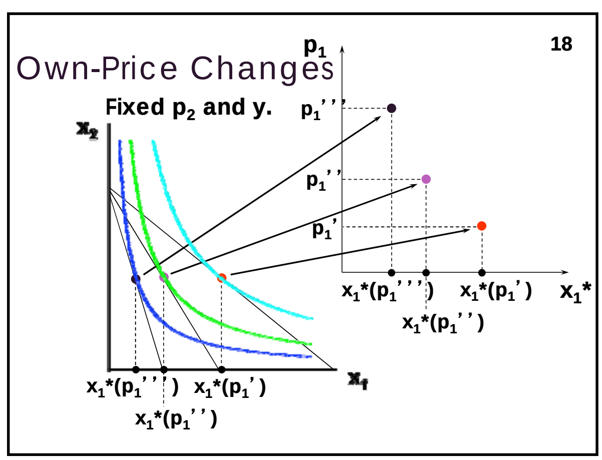
$x_1^*(p_1'')$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Own-Price Changes
Fixed p_2 and y.
p_1
p_1'''
p_1''
p_1'
$x_1^*(p_1''')$
$x_1^*(p_1')$
x_1^*
$x_1^*(p_1'')$
x_2
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Own-Price Changes

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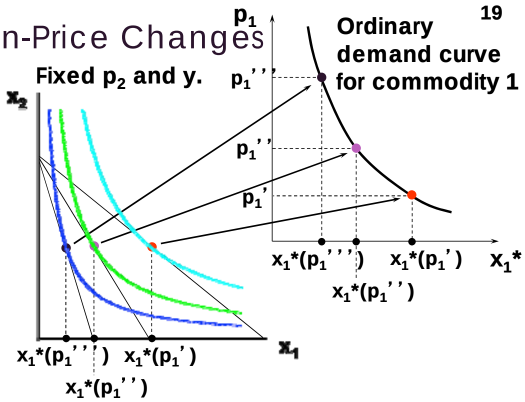

Own-Price Changes

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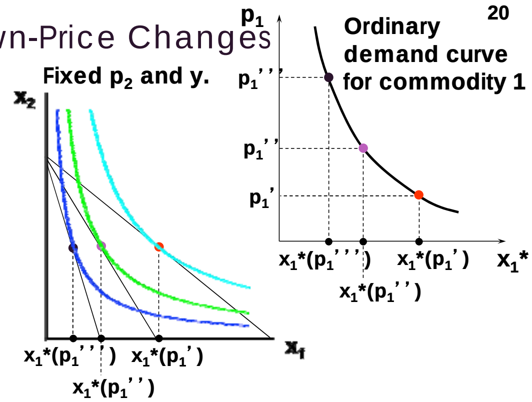

Own-Price Changes

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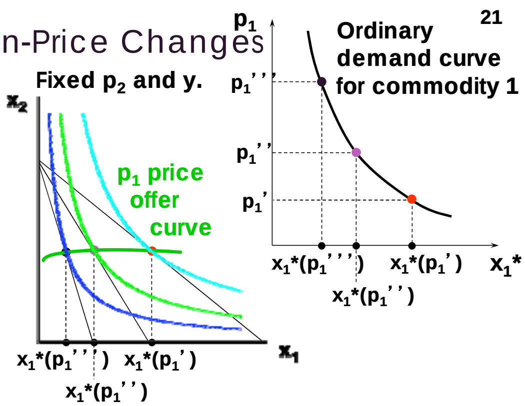

Own-Price Changes

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- The curve containing all the utility-maximizing bundles traced out as p_1 changes, with p_2 and y constant, is the p_1- **price offer curve**.
- The plot of the x_1-coordinate of the p_1- price offer curve against p_1 is the **ordinary demand curve** for commodity 1.

Own-Price Changes

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- What does a p_1 price-offer curve look like for Cobb-Douglas preferences?

Own-Price Changes

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- What does a p_1 price-offer curve look like for Cobb-Douglas preferences?
- Take $$U(x_1,x_2) = x_1^a x_2^b.$$

  Then the ordinary demand functions for commodities 1 and 2 are

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Own-Price Changes

$$x_1^*(p_1,p_2,y) = \frac{a}{a+b} \times \frac{y}{p_1}$$

and

$$x_2^*(p_1,p_2,y) = \frac{b}{a+b} \times \frac{y}{p_2}.$$

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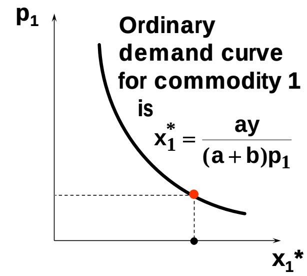
Own-Price Changes



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Own-Price Changes

- What does a p_1 price-offer curve look like for a perfect-complements utility function?

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Perfect Complements

- What does a p_1 price-offer curve look like for a perfect-complements utility function?

$$U(x_1,x_2) = \min\{x_1,x_2\}.$$

Then the ordinary demand functions for commodities 1 and 2 are

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Own-Price Changes

$$x_1^*(p_1,p_2,y) = x_2^*(p_1,p_2,y) = \frac{y}{p_1+p_2}.$$

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Own-Price Changes

$$x_1^*(p_1,p_2,y) = x_2^*(p_1,p_2,y) = \frac{y}{p_1+p_2}.$$

With p_2 and y fixed, higher p_1 causes smaller x_1* and x_2*.

Own-Price Changes

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$$x_1^*(p_1,p_2,y) = x_2^*(p_1,p_2,y) = \frac{y}{p_1+p_2}.$$

With p_2 and y fixed, higher p_1 causes smaller x_1* and x_2*.

As $p_1 \to 0, \quad x_1^* = x_2^* \to \dfrac{y}{p_2}.$

Own-Price Changes

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$$x_1^*(p_1,p_2,y) = x_2^*(p_1,p_2,y) = \frac{y}{p_1+p_2}.$$

With p_2 and y fixed, higher p_1 causes smaller x_1* and x_2*.

As $p_1 \to 0, \quad x_1^* = x_2^* \to \dfrac{y}{p_2}.$

As $p_1 \to \infty, \quad x_1^* = x_2^* \to 0.$

Own-Price Changes

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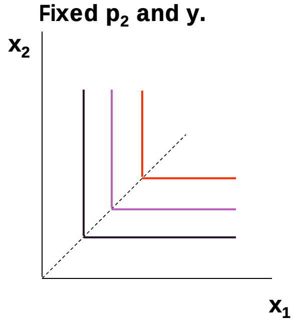
Fixed p_2 and y.



Own-Price Changes

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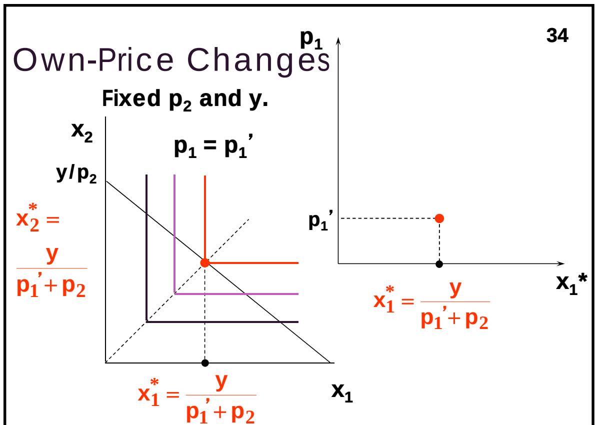

Own-Price Changes

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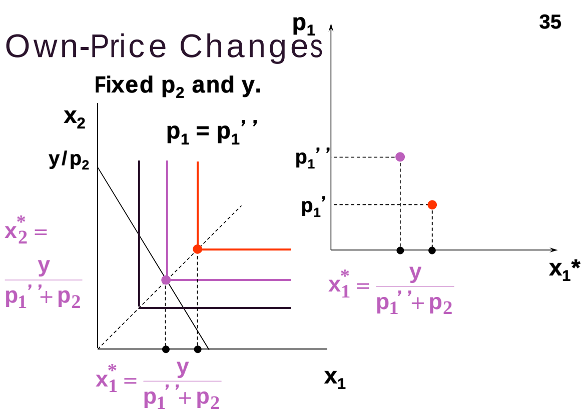
Fixed p_2 and y.



Own-Price Changes

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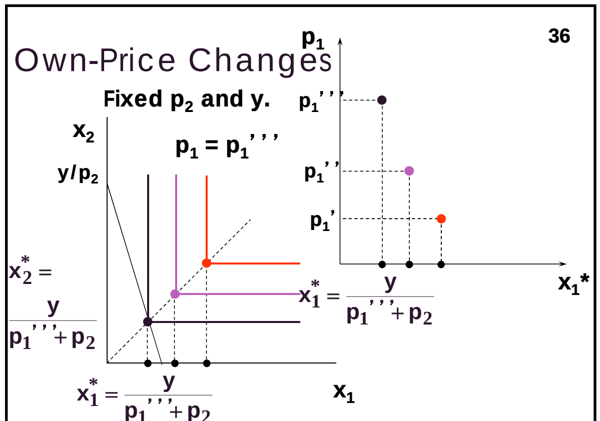

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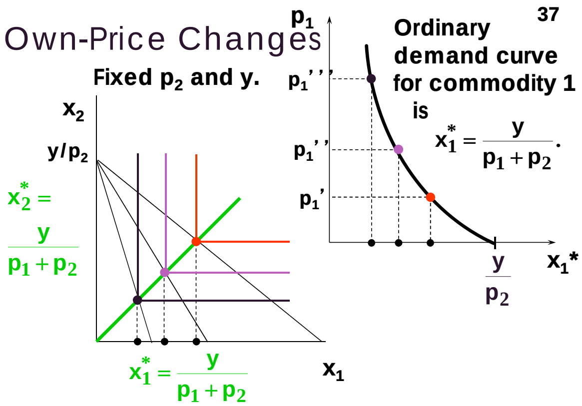
Own-Price Changes



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Ordinary Goods

- So far, all the demand curves have been downward sloping
- Question: does the quantity demanded of a good have to increase as a price decreases?
- **Surely** it does?
- Sigh.
- A good is called **ordinary** if the quantity demanded of it always **increases** as its own price **decreases**.

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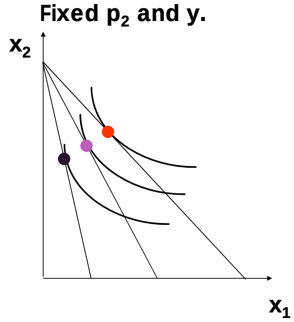
Ordinary Goods



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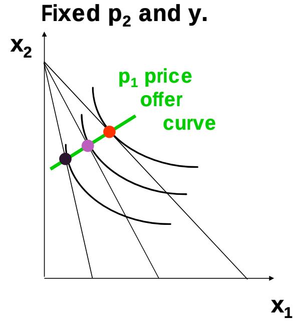
Ordinary Goods



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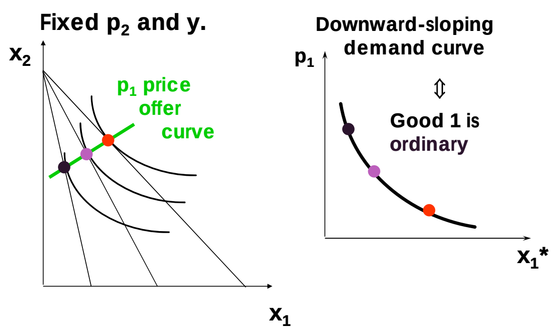
Ordinary Goods



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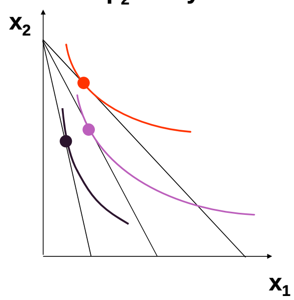
Giffen Goods

- If, for some values of its own price, the quantity demanded of a good rises as its own-price increases then the good is called **Giffen**.

Ordinary Goods

Fixed p_2 and y.



Ordinary Goods

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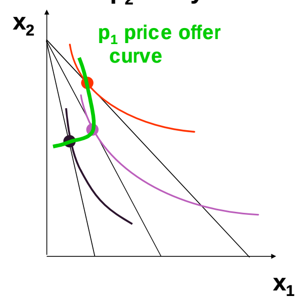

Ordinary Goods

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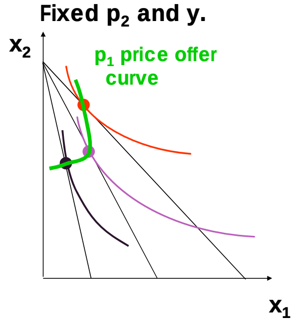
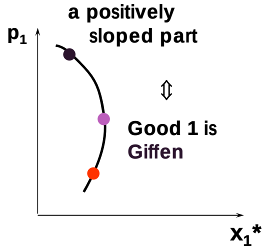


Demand curve has a positively sloped part



Giffen goods

- Ok, so some goods are weird!
  - Prices **increases** and so does **demand!!!**
- What is going on?
  - Increase in the price of the good should cause the consumer to switch to the other good and consume less **(substitution effect)**
  - However, price increases make the consumer poorer **(income effect)**
  - If the good is inferior, they will consume more
  - If they are **really** inferior, then the income effect may outweigh the substitution effect
- Sounds crazy, and realistically this is probably an extreme effect
- Could you think of some examples?
  - Potato in XIX century Ireland
  - Rice/Sorghum in some developing countries

Demand and Own Price Changes

2: Income and Substitution Effects

More about price changes

- In order to understand the effect of price changes on demand, we can "decompose them"
- What happens when a commodity's price decreases?

1. **Substitution effect**: the commodity is relatively cheaper, so consumers substitute it for now relatively more expensive other commodities.
2. **Income effect**: the consumer's budget of $y can purchase *more than before*, *as if the consumer's income rose*, with consequent income effects on quantities demanded.

- We will now analyze these two effects separately

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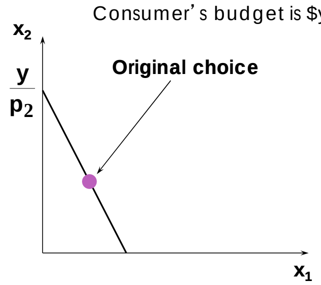
Effects of a Price Change

Consumer's budget is $y



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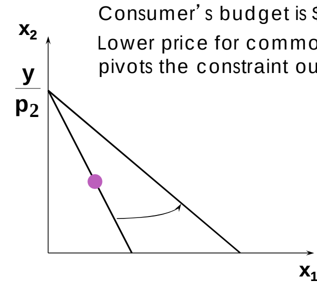
Effects of a Price Change

Consumer's budget is $y

Lower price for commodity 1 pivots the constraint outwards



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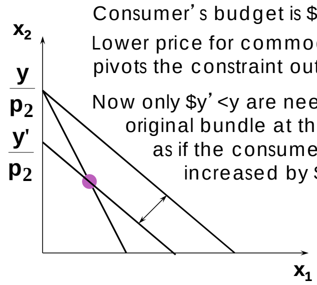
Effects of a Price Change

Consumer's budget is $y

Lower price for commodity 1 pivots the constraint outwards

Now only $y'<y are needed to buy the original bundle at the new prices, as if the consumer's income has increased by $y - $y'



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Effects of a Price Change

- Changes to quantities demanded due to this 'extra' income are the **income effect** of the price change
- It's as if the agent became "wealthier"
- That's because she can buy more with same money

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Pure Substitution Effect

- We will now calculate these two effects separately
- How?
- We 'break up' the movement of the budget line in 2
- First, we move the budget line with the **new slope** but so that the **original bundle was just affordable**
- This is gives us a new budget set
- We compute the bundle that would be chosen by the decision maker for this set

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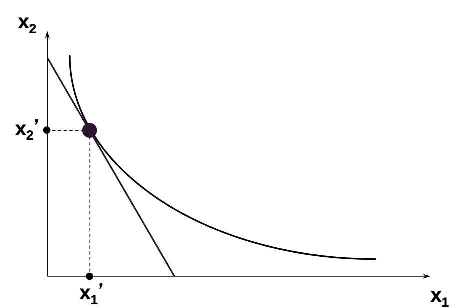
Pure Substitution Effect Only



Pure Substitution Effect Only

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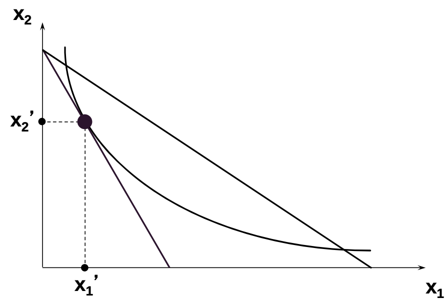

Pure Substitution Effect Only

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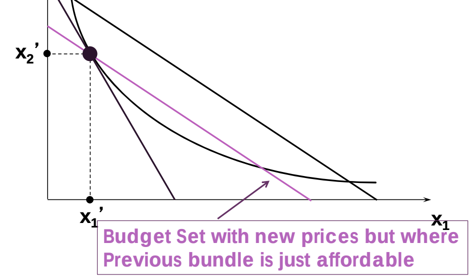
We break up the movement of the budget set in **two steps**



Pure Substitution Effect Only

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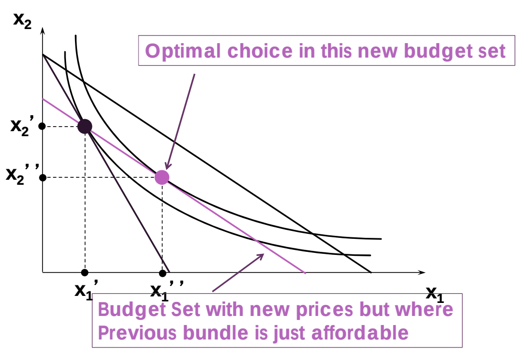

Pure Substitution Effect Only

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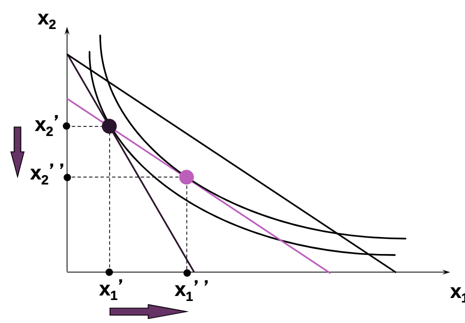

Pure Substitution Effect Only

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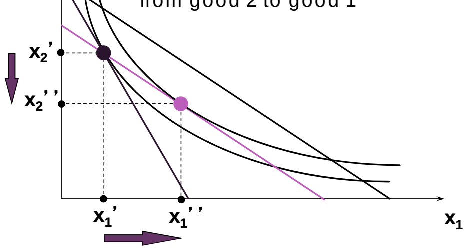
Lower p_1 makes good 1 relatively cheaper and causes a substitution from good 2 to good 1



Pure Substitution Effect Only

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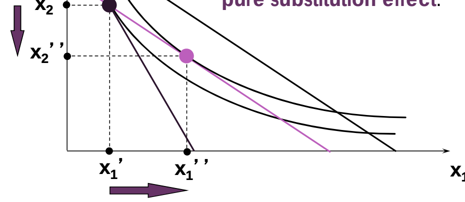
Lower p_1 makes good 1 relatively cheaper and causes a substitution from good 2 to good 1.

$(x_1', x_2') \rightarrow (x_1'', x_2'')$ is the **pure substitution effect**.



And Now The Income Effect

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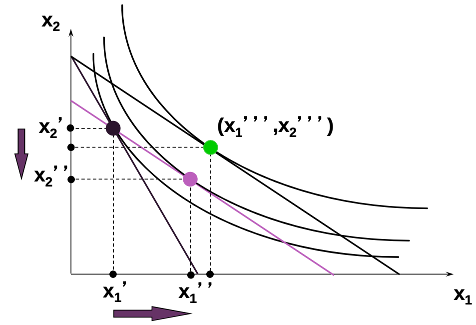

And Now The Income Effect

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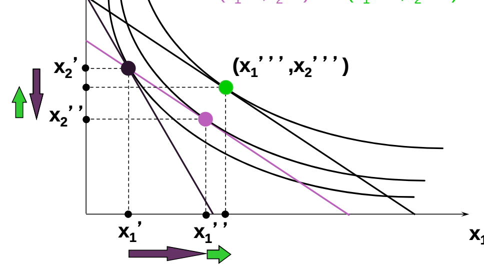
The **income effect** is $(x_1'', x_2'') \rightarrow (x_1''', x_2''')$.



It is just like a change in income!

The Overall Change in Demand

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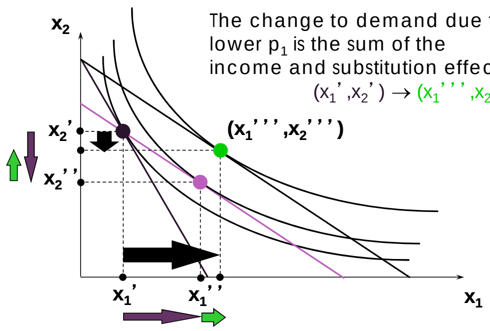
The change to demand due to lower p_1 is the sum of the income and substitution effects, $(x_1', x_2') \rightarrow (x_1''', x_2''')$.



So:

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- We can separate the change due to a price change into:
  - Substitution effect: computed as the change to the optimal choice in a budget set with **new prices** but in which **old bundle is just affordable**
  - Income effect: the change from that optimal choice to the new one – just like a change in income
- Total effect is **just the sum of the two effects**

What are the signs of these effects?

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- Are these effects always positive, negative ?
- First, how about the substitution effect?
  - It must be **always positive:** lower price lead to more demand in the substitution effect
  - Why? See graph

Pure Substitution Effect Only

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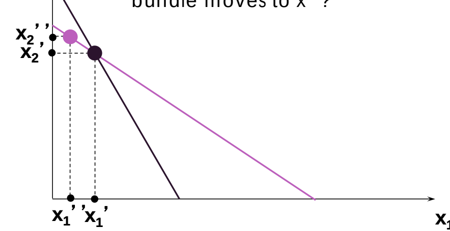
Can the substitution effect mean that the optimal bundle moves to x''?



Pure Substitution Effect Only

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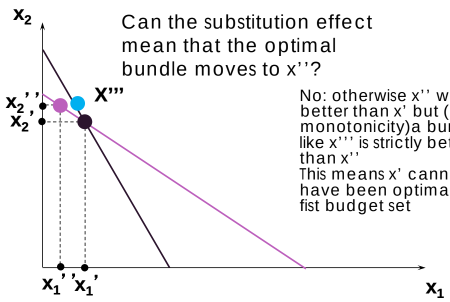

Can the substitution effect mean that the optimal bundle moves to x''?

No: otherwise x'' weakly better than x' but (by monotonicity)a bundle like x''' is strictly better than x''
This means x' cannot have been optimal in the fist budget set

What are the signs of these effects?

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- Are these effects always positive?ky negative, ?
- First, how about the substitution effect?
  - It must be **always positive:** lower price lead to more demand in the substitution effect
  - Why? See graph
- How about the income effect?
  - It's just like a change in income
  - Does demand go up or down with income?
  - If good is **inferior** -> negative
  - If good is **normal** -> positive

What are the signs of these effects?

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- So:
- **Substitution is always positive and income could be positive or negative (in the case of inferior goods)**
- Total effect is always the sum
- Means that: for normal goods, total effect is positive
- For inferior goods: if income effect is stronger than substitution effect, then total effect can be negative
- This is what happens with Giffen goods
  - **Question: Can Giffen goods be normal?**

Example, normal good

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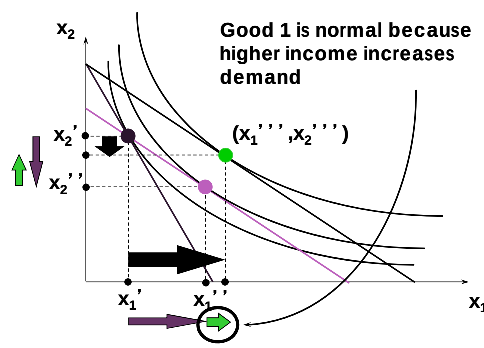
Good 1 is normal because higher income increases demand



Effects for Normal Goods

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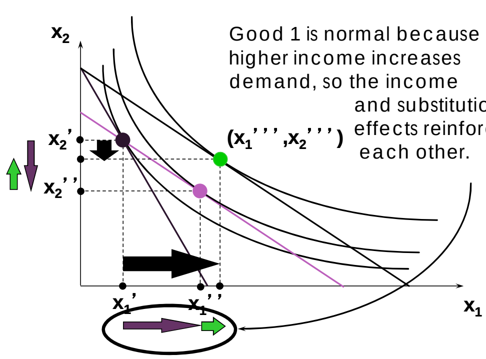
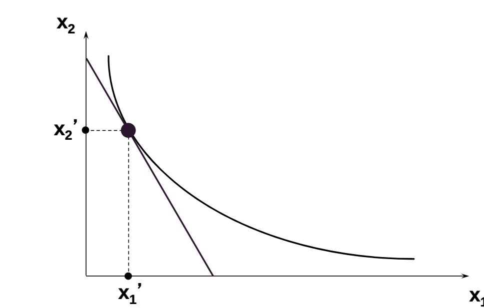
Good 1 is normal because higher income increases demand, so the income and substitution effects reinforce each other.



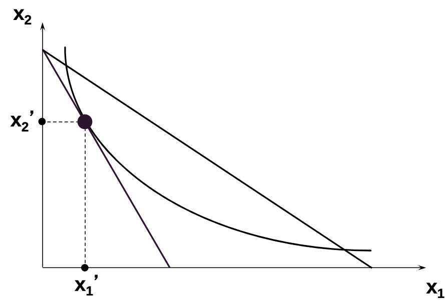
Example: Inferior Goods

72



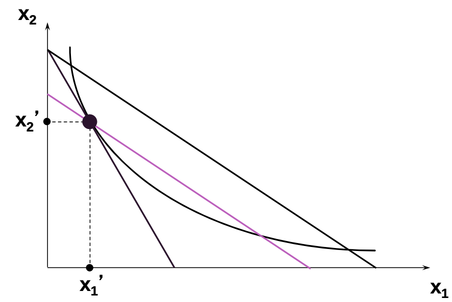
Example: Inferior Goods

73



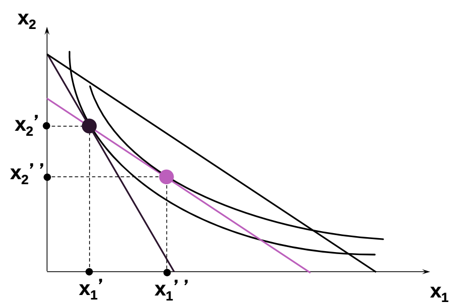
Example: Inferior Goods

74



Example: Inferior Goods

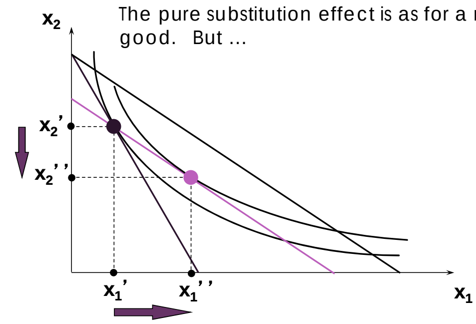
75



Example: Inferior Goods

76

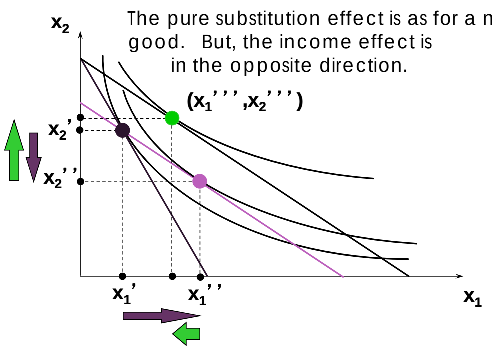
The pure substitution effect is as for a normal good. But ...



Example: Inferior Goods

77

The pure substitution effect is as for a normal good. But, the income effect is in the opposite direction.



Example: Inferior Goods

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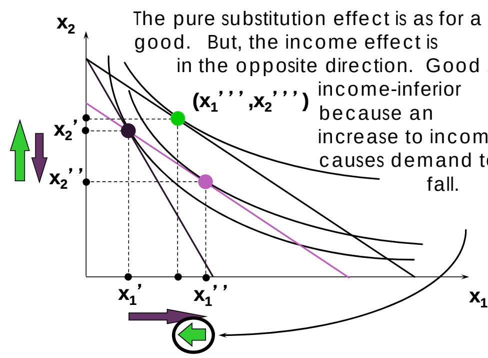
The pure substitution effect is as for a normal good. But, the income effect is in the opposite direction. Good 1 is income-inferior because an increase to income causes demand to fall.



Example: Inferior Goods

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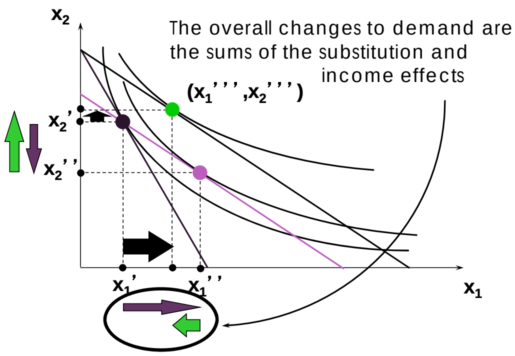
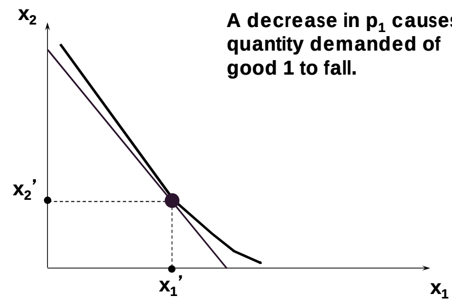


Effects for Giffen Goods

80

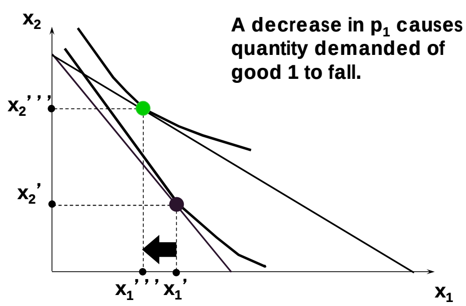
A decrease in p_1 causes quantity demanded of good 1 to fall.



Effects for Giffen Goods

81

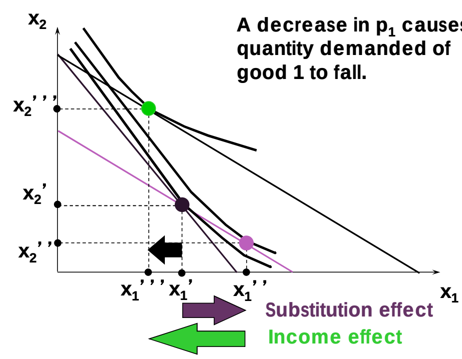
A decrease in p_1 causes quantity demanded of good 1 to fall.



Effects for Giffen Goods

82

A decrease in p_1 causes quantity demanded of good 1 to fall.



Demand and Own Price Changes

3: The Slutsky Equation

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The Slutsky Equation

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- There is another, more elegant way of separating out the income and substitution effects
- You will like it….
- But it means we will have to go through some new concepts a little quickly
- (Sorry)
- In my defense, these concepts which may seem a little weird now, will seem very sensible when we start talking about firms

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Ordinary and Compensated Demand

- Here is the standard consumer problem
1. **CHOOSE a consumption bundle**
2. **IN ORDER TO MAXIMIZE preferences**
3. **SUBJECT TO the budget constraint**
- This gives rise to demand functions: amount of the good consumed given prices and income
$$x_i(p,y)$$
- Can also give rise to the **indirect utility** function: the maximum utility that can be achieved given prices p and income y
$$U(p,y) = u(x_1(p,y), x_2(p,y))$$

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Ordinary and Compensated Demand

- Here is a related problem, sometimes called the 'dual' problem
1. **CHOOSE a consumption bundle**
2. **IN ORDER TO MINIMIZE expenditure**
3. **SUBJECT TO utility being equal to some u***

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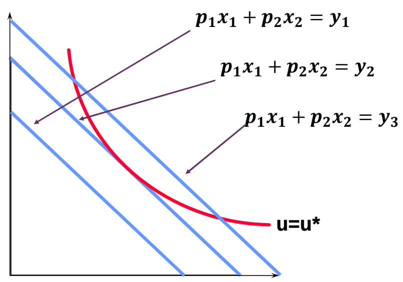
The Dual Problem



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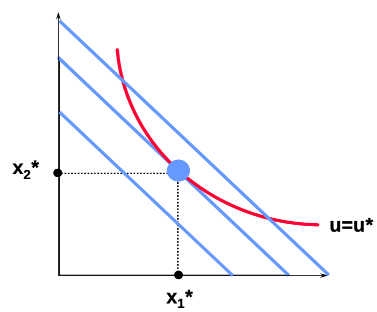
The Dual Problem



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Ordinary and Compensated Demand

- Here is a related problem, sometimes called the 'dual' problem
1. **CHOOSE a consumption bundle**
2. **IN ORDER TO MINIMIZE expenditure**
3. **SUBJECT TO utility being equal to some u***
- This gives rise to **compensated** demand functions: amount of the good consumed given prices and **utility**
$$x_i^h(p,u)$$
- Can also give rise to the **expenditure** function: the minimum expenditure that can be achieved given prices p and utility u
$$e(p,u) = p_1x_1^h(p,u) + p_2x_2^h(p,u)$$

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Relationship between the Two Problems

- The previous picture should give you the idea that there is a strong relationship between the two problems
- Fact:
$$x_i(p, e(p,u)) = x_i^h(p,u)$$
- In words:
  - Figure out the amount of x_1 I would use to minimize expenditure while achieving utility u
  - Figure out the amount of expenditure e I need to achieve u
  - Figure out the amount of x_1 I would use to maximize utility if you gave me e
  - These two demands are the same

The Dual Problem

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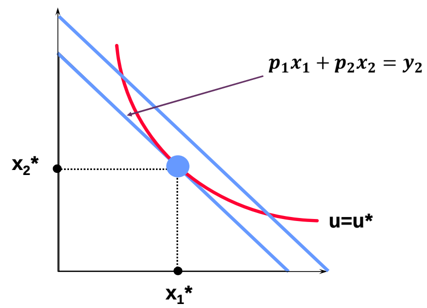

- Makes sense: Same bundle minimizes y with respect to u and maximizes u with respect to y

Relationship between the Two Problems

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$$x_i(p, e(p,u)) = x_i^h(p,u)$$

- We can use this to split out income and substitution effects

$$\frac{\partial x_1(p,y)}{\partial p_1} + \frac{\partial x_1(p,y)}{\partial y}\frac{\partial e(p,u)}{\partial p_1} = \frac{\partial x_1^h(p,u)}{\partial p_1}$$

- Where $y = e(p,u)$
- Rearranging

$$\frac{\partial x_1(p,y)}{\partial p_1} = \frac{\partial x_1^h(p,u)}{\partial p_1} - \frac{\partial x_1(p,y)}{\partial y}\frac{\partial e(p,u)}{\partial p_1}$$

- This already looks like an income and substitution effect
- However, we can do better

The Envelope Theorem

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- What about $\frac{\partial e(p,u)}{\partial p_1}$?
- Well, we know that $e(p,u) = p_1x_1^h(p,u) + p_2x_2^h(p,u)$, so

$$\frac{\partial e(p,u)}{\partial p_1} = x_1^h(p,u) + p_1\frac{\partial x_1^h(p,u)}{\partial p_1} + p_2\frac{\partial x_2^h(p,u)}{\partial p_1}$$

- Claim: these last two terms are zero,
- This means $\frac{\partial e(p,u)}{\partial p_1} = x_1^h(p,u)$
- Follows because x_1^h is the solution to an optimization problem
  - This is the 'envelope theorem'
  - You won't get this on your first time through
  - Make sure you review
  - Take a deep breath..

The Envelope Theorem

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- Why is $p_1\frac{\partial x_1^h(p,u)}{\partial p_1} + p_2\frac{\partial x_2^h(p,u)}{\partial p_2} = 0$?
- First notice that, because $u(x_1^h(p,u), x_2^h(p,u)) = u$

$$\frac{\partial u}{\partial x_1^h}\frac{\partial x_1^h}{\partial p_1} + \frac{\partial u}{\partial x_2^h}\frac{\partial x_2^h}{\partial p_1} = 0$$

- Next, take the tangency condition

$$\frac{\partial u}{\partial x_1^h} \Big/ \frac{\partial u}{\partial x_2^h} = \frac{p_1}{p_2}$$

Implies $\frac{\partial u}{\partial x_2^h} = \frac{\partial u}{\partial x_1^h}\frac{p_2}{p_1}$

The Envelope Theorem

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- So

$$\frac{\partial u}{\partial x_1^h}\frac{\partial x_1^h}{\partial p_1} + \frac{\partial u}{\partial x_2^h}\frac{\partial x_2^h}{\partial p_1} = 0$$

Implies $\frac{\partial u}{\partial x_1^h}\frac{\partial x_1^h}{\partial p_1} + \frac{\partial u}{\partial x_1^h}\frac{p_2}{p_1}\frac{\partial x_2^h}{\partial p_1} = 0$

Implies $\frac{\partial u}{\partial x_1^h}\frac{\partial x_1^h}{\partial p_1} + \frac{\partial u}{\partial x_1^h}\frac{p_2}{p_1}\frac{\partial x_2^h}{\partial p_1} = 0$

Implies $\frac{\partial u}{\partial x_1^h}\frac{1}{p_1}\left(p_1\frac{\partial x_1^h}{\partial p_1} + p_2\frac{\partial x_2^h}{\partial p_1}\right) = 0$

- Assuming strict monotonicity $p_1\frac{\partial x_1^h}{\partial p_1} + p_2\frac{\partial x_2^h}{\partial p_1} = 0$

The Envelope Theorem

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- This tells us that

$$\frac{\partial e(p,u)}{\partial p_1} = x_1^h(p,u)$$

- And so

$$\frac{\partial x_1(p,y)}{\partial p_1} = \frac{\partial x_1^h(p,u)}{\partial p_1} - \frac{\partial x_1(p,y)}{\partial y}\frac{\partial e(p,u)}{\partial p_1}$$

- Becomes

$$\frac{\partial x_1(p,y)}{\partial p_1} = \frac{\partial x_1^h(p,u)}{\partial p_1} - \frac{\partial x_1(p,y)}{\partial y}x_1^h(p,u)$$

- This is the Slutsky Equation

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The Slutsky Equation

- This is the Slutsky Equation

$$\frac{\partial x_1(p,y)}{\partial p_1} = \frac{\partial x_1^h(p,u)}{\partial p_1} - \frac{\partial x_1(p,y)}{\partial y} x_1^h(p,u)$$

- Second term is the income effect
- First term is the 'substitution effect'
  - Impact of prices on Hicksian Demand
  - i.e. while keeping utility constant
  - (Slightly different from the previous substitution effect – keeping income constant)
  - Always negative

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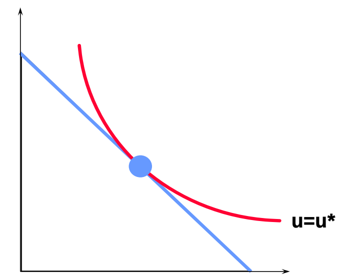
Hicksian Demand and Increase in Price of Good 1



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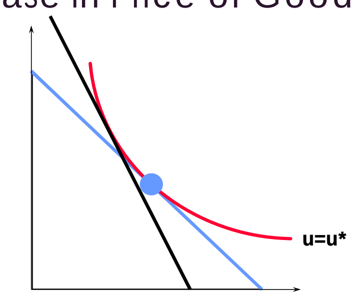
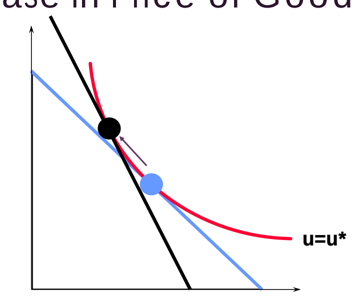
Hicksian Demand and Increase in Price of Good 1



100

Hicksian Demand and Increase in Price of Good 1



Demand and Own Price Changes

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Cross-Price Effects

- What happens to demand of one good as I change the price **of the other good?**
- Think about it:
  - If the price of cars increases does demand for petrol increase or decrease?
  - If the price of cars increases, does demand for bicycles increase or decrease?
- The first is the case of complements: things that are generally consumed together
- The second a case of substitutes: things that are generally consumed instead of each other

Cross-Price Effects

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- Formally
- Price increase for commodity 2 **increases** demand for commodity 1 then commodity 1 is a **gross substitute** for commodity 2.
  - Same as saying the cross-elasticity of demand is positive
    $$\frac{\partial x_1(p_1,p_2,y)}{\partial p_2}\frac{p_2}{x_1} > 0$$
- Price increase for commodity 2 **decreases** demand for commodity 1 then commodity 1 is a **gross compliment** for commodity 2.
  - Same as saying the cross-elasticity of demand is negative
    $$\frac{\partial x_1(p_1,p_2,y)}{\partial p_2}\frac{p_2}{x_1} < 0$$

Cross-Price Effects

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A perfect-complements example:

$$x_1^* = \frac{y}{p_1+p_2}$$

so

$$\frac{\partial x_1^*}{\partial p_2} = -\frac{y}{(p_1+p_2)^2} < 0.$$

Therefore commodity 2 is a gross complement for commodity 1.
That's why it's called **perfect complement**

Cross-Price Effects

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A Cobb- Douglas example:

$$x_2^* = \frac{by}{(a+b)p_2}$$

so

Cross-Price Effects

106

A Cobb- Douglas example:

$$x_2^* = \frac{by}{(a+b)p_2}$$

so

$$\frac{\partial x_2^*}{\partial p_1} = 0.$$

Therefore commodity 1 is neither a gross complement nor a gross substitute for commodity 2.

Summary

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Summary

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- Today we have done the following

1. Discuss how demand for a good is affected by a change in its own price
   - Giffen Goods
   - Income and substitution effects
   - Compensated demand and the Slutsky equation
2. Discuss how demand for a good changes with the price of other goods
   - complements and substitutes