Homework 3

Intermediate Micro - Fall 2009 – Mark Dean

Due Thursday 1st October

Please remember to answer each question on a separate sheet of paper

Please put your name and Banner ID on each question

Question 1 (Utility Functions)

(a) *In the lecture notes I show that, if preferences are complete and transitive then we can find a utility function that represents those preferences (in the sense that u(x)>u(y) if and only if x is preferred to y.) It is also true that, if preferences can be represented by a utility function, then they must be complete and transitive. Show that this is the case.*

(b) *If you were paying attention in the lectures, then you might be worried about the fact that (a) the marginal rate of substitution is equal to the ratio of the marginal utility of good one to the marginal utility of good two and (b) if a utility function u represents a set of preferences, then so does any strictly increasing transform of that utility function (i.e if $u(x_1,x_2)$ represents preferences, then so does $v(x_1,x_1)=g(u(x_1,x_1))$ where g is strictly increasing). In order to assuage your fears show that the ratio of the marginal utilities in the same in both cases: i.e*

$$. \frac{\frac{\partial u(x_1,x_2)}{\partial x_1}}{\frac{\partial u(x_1,x_2)}{\partial x_2}} = \frac{\frac{\partial v(x_1,x_2)}{\partial x_1}}{\frac{\partial v(x_1,x_2)}{\partial x_2}}$$

Question 2 (Optimal Consumption and Cobb Douglas)

Two economics professors are having an argument. They both want to model how Ermintrude chooses between bundles of spam and chocolate. Professor Hirst claims that Ermintrude maximizes the utility function $u(x_s,x_c)= (x_s)^{1/3}(x_c)^{2/3}$. Professor Nilsson claims that Ermintrude maximizes the utility function $v(x_s,x_c)= 2\ln x_s+4\ln x_c$

(a) *Calculate the marginal utility with respect spam and chocolate according to each professor*

(b) *Calculate Ermintrude's MRS according to each professor*

(c) *For any budget constraint $p_s x_s + p_c x_c = M$, find the points of tangency with the indifference curve*
(d) *Find Ermintrude's optimal consumption bundle according to each professor*
(e) *Conclude that Professors Hirst and Nilssen are idiots*
(f) *Explain why the fact that $v(x_s,x_c)=3/2\ln(u(x_s,x_c)$ could have saved you a lot of trouble*
(g) *According to either professor, what proportion of Ermintrude's income does she spend on chocolate, and what proportion on spam? Does this depend on the price of chocolate and spam?*

Question 3 (Optimal consumption and quasi-linear preferences)

Ezekiel has preferences between guns and banjos that are described by the utility function $u(x_g,x_b)= x_g+(x_b)^{1/2}$.

(a) *Write down the equation for an indifference curve (i.e. for some utility u, write down the number of guns that give that level of utility as a function of the number of banjos*
(b) *Using this indifference curve, determine whether these preferences are monotone? Are they convex?*
(c) *Write down an equation for the marginal rate of substitution*
(d) *Show that, for a budget constraint $p_b x_b + p_g x_g = M$, at any point of tangency it must be the case that $x_b=1/4(p_g/p_b)^2$*
(e) *What is the optimal consumption bundle if $p_g=2$, $p_b=1$ and $m=8$?*
(f) *What is the optimal consumption bundle if $p_g=5$, $p_b=1$ and $m=4$?*