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Intermediate Microeconomics W3211

Lecture 19: Game Theory 1

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Introduction

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The Story So Far….

- So far, we have analyzed the decisions of consumers and firms
- However, the economic agents we have studied have not had to take into account the actions of others
- Prices have been given
- They just have to choose what to do given those prices
- What other people do is not relevant to their optimization problem

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The Actions of Others

- Sometimes this is not a very sensible assumption
- E.g. consider two hotdog vendors next to each other in central park
- Each is selling an identical hot dog
- The cost of the hot dog to the vendor is c
- Customers will always go to the vendor with the lowest price
- What price should each vendor set?

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The Actions of Others

- The key feature of this problem is that it involves **strategic interaction**
- The best action for each hot dog vendor depends on the actions of the other
- The study of strategic interactions is called Game Theory

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The Actions of Others

- Other examples of strategic situations

1. Dell, HP and Apple are deciding on how to set the prices of their laptops
2. Two nations are deciding how much to invest in military equipment
3. A penalty taker in soccer is deciding whether to go left or right, and the goalkeeper is deciding whether to dive left or right
4. Terrorists are deciding which target to hit, and the Department of Homeland Security is trying to decide which targets to defend
5. Your evil professor is trying to decide which questions to set to get you to fail your exam, and you are trying to decide what to revise.

Game Theory is Fun!

- Examples of questions you might be able to answer:

This story involves a village high up in the Italian Alps. The occupants of this village confirm to all currently available stereotypes. First, the men are lotharios, in the sense that some of them are cheating on their wives with the wives of other men. Second, they are dreadful gossips, so every man in the village knows whether every other man in the village is being cheated on by his wife (but he does not know about his own wife). Third, they are fiercely proud (and sexist hypocrites) - and each man declares that if he catches his own wife cheating, he will shoot her in the town square at midnight. Fourth, they are very religious, and all attend mass every Sunday. One Sunday, a new young firebrand priest turns up to give a sermon. As part of his sermon he condemns the town as a den of wickedness, with the words "everywhere I look in this village, I see sin. I know for a fact that some of the men in this village are lying with the wives of other men".

For the first night after the preacher leaves, all is quiet, as is the second night. On the third night, shots are heard in the square at midnight. The question is, how many shots were fired, and how many husbands were cheating on their wives

Game Theory is Fun!

- Examples of questions you might be able to answer:

The beauty contest game: This week I will run the following contest: each of you can email me a number between 1 and 100. Whoever sends in the number that is closest to 2/3 of the average of all the numbers sent in gets the prize.

Today

- Today we will

1. Define what a game is
2. Think of some ways of **solving** a game
   - i.e. make predictions about how we think people behave in the game

- Varian Ch 29.
- Feldman and Serrano Ch 14

What is a Game?

What is A Game?

- A game is a way of modelling strategic interaction between players
- What do I have to tell you in order to **define** a game?

1. A list of players
2. A list of actions that each player can take
3. The payoff that each player gets, depending on the actions of all players

- For simple games (with 2 players and a small number of actions) we can use a **matrix** to describe the game

An Example of a Two-Player Game

- The players are called A and B.
- Player A has two actions, called “Up” and “Down”.
- Player B has two actions, called “Left” and “Right”.
- The table showing the payoffs to both players for each of the four possible action combinations is the game’s payoff matrix.

An Example of a Two-Player Game

Player B

		L	R
Player A	U	(3,9)	(1,8)
	D	(0,0)	(2,1)

This is the game's payoff matrix.

Player A's payoff is shown first.
Player B's payoff is shown second.

An Example of a Two-Player Game

Player B

		L	R
Player A	U	(3,9)	(1,8)
	D	(0,0)	(2,1)

A play of the game is a pair such as (U,R) where the 1st element is the action chosen by Player A and the 2nd is the action chosen by Player B.

An Example of a Two-Player Game

Player B

		L	R
Player A	U	(3,9)	(1,8)
	D	(0,0)	(2,1)

This is the game's payoff matrix.

E.g. if A plays Up and B plays Right then A's payoff is 1 and B's payoff is 8.

An Example of a Two-Player Game

Player B

		L	R
Player A	U	(3,9)	(1,8)
	D	(0,0)	(2,1)

This is the game's payoff matrix.

And if A plays Down and B plays Right then A's payoff is 2 and B's payoff is 1.

Solving a Game 1

Equilibrium in Dominant Strategies

Solving a Game

- As with the other bits of analysis we have done so far, we are interested in making **predictions** about how people play a game
- This is sometimes called **solving** a game
- How can we solve games?
- To give you one idea, let me tell you the story of Bonny and Clyde

The Prisoner's Dilemma

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- Bonny and Clyde have both been arrested after their latest nefarious doings
- Kept in separate rooms, both have the opportunity to keep silent, or confess
- If both keep silent, then both will receive a relatively short jail term (5 years)
- If both confess, then they both get a long jail term (10 years)
- If one confesses and the other keeps silent, then the snitch gets a reduced term (1 year) and the patsy gets a long stretch (30 years)
- What does this look like as a game?

The Prisoner's Dilemma

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Clyde

Bonnie	S	C
S	(-5,-5)	(-30,-1)
C	(-1,-30)	(-10,-10)

What plays are we likely to see for this game?

The Prisoner's Dilemma

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Clyde

Bonnie	S	C
S	(-5,-5)	(-30,-1)
C	(-1,-30)	(-10,-10)

If Bonnie plays Silence then Clyde's best reply is Confess.

The Prisoner's Dilemma

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Clyde

Bonnie	S	C
S	(-5,-5)	(-30,-1)
C	(-1,-30)	(-10,-10)

If Bonnie plays Silence then Clyde's best reply is Confess.
If Bonnie plays Confess then Clyde's best reply is Confess.

The Prisoner's Dilemma

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Clyde

Bonnie	S	C
S	(-5,-5)	(-30,-1)
C	(-1,-30)	(-10,-10)

So no matter what Bonnie plays, Clyde's best reply is always Confess. Confess is a dominant action for Clyde.

The Prisoner's Dilemma

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Clyde

Bonnie	S	C
S	(-5,-5)	(-30,-1)
C	(-1,-30)	(-10,-10)

Similarly, no matter what Clyde plays, Bonnie's best reply is always Confess. Confess is a dominant action for Bonnie also.

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The Prisoner's Dilemma

Clyde

Bonnie		S	C
	S	(-5,-5)	(-30,-1)
	C	(-1,-30)	(-10,-10)

We would expect the outcome of this game to be confess, confess

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Equilibrium in Dominant Strategies

- A strictly dominant action for one player is a action s which pays more than all other strategies s' regardless of what the other players do
  - A weakly dominant action pays at least as much as all other strategies, regardless of what the other players do, and strictly more in some case
- An **equilibrium in (strictly/weakly) dominant strategies** is a (strictly/weakly) dominant action for each player
- These are typically thought of as very robust predictions
- But notice that the game is not strategic in a very interesting way
  - Best action of each player doesn't depend on the actions of others

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The Prisoner's Dilemma

Clyde

Bonnie		S	C
	S	(-5,-5)	(-30,-1)
	C	(-1,-30)	(-10,-10)

What else do you notice about the solution of the prisoner's dilemma game?

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Equilibrium and Pareto Optimality

- Equilibria in games are not necessarily **Pareto optimal**
  - Bonny and Clyde could have spent 5 years in jail
  - Instead they spend 10 years
- This is in stark contrast to what we have found in non strategic settings
- This is an Important Result
- Especially as prisoner's dilemmas crop up quite a lot in economics

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Other Examples of Prisoner's Dilemma

- You are part of a small tribe which is about to be attacked by another tribe.
- If everyone fights, there is a small chance that any one person will be killed, but you will fight off the invaders.
- If no one fights then your tribe will be taken into slavery.
- However, if you are the only one that fights, then something really nasty is going to happen to you,
- If everyone else fights and you don't then you will certainly survive.

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Other Examples of Prisoner's Dilemma

- The USA and USSR are deciding whether or not to nuke each other.
- If neither country nukes the other, then there is no nuclear war, but they have to continue sharing resources.
- If both countries nuke, then there will be a war, but neither country will be wiped out.
- However, if one country nukes and the other does not, then the nuking country gets to take over the world, while the other country gets wiped off the face of the map

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Other Examples of Prisoner's Dilemma

- China and Europe are deciding whether or not to reduce their carbon footprint
- If both countries reduce their carbon footprint, then both get a high payoff from the world
- If neither do, then the world sinks, but neither gain a competitive advantage.
- If only one country reduces emissions, then that country loses all competitive advantage, while the other country gains competitive advantage and benefits from less sinking

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Getting out of the Prisoners Dilemma

- The Prisoner's dilemma is of interest to economists, environmentalists, political scientists and anthropologists
- Not least because we see people avoiding the bad outcome
  - Societies band together in times of war
  - Countries don't nuke each other
  - Countries do sign environmental deals
- How do we escape the prisoner's dilemma?

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Getting out of the Prisoners Dilemma

- Largely beyond the scope of this course, but here are some ideas
- **Repetition**
- **Punishment**
- **Social Preferences**
- **Commitment**

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Iterated Deletion of Strictly Dominated Strategies

- Do all games have an equilibrium in dominated strategies?
- What about the following?
  - We allow Bonny a new action 'Go Bezerk', in which case she can break out of her cell and get to Clyde
  - If she goes Bezerk, then she will get set down for 50 years for sure
  - But if Clyde has ratted she will take him down with her

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Iterated Deletion of Strictly Dominated Strategies

Clyde

Bonnie

	S	C
S	-5,-5	-30,-1
C	-1,-30	-10,-10
B	-50,0	-50,-50

- Does Clyde have a dominant action?

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Iterated Deletion of Strictly Dominated Strategies

Clyde

Bonnie

	S	C
S	-5,-5	-30,-1
C	-1,-30	-10,-10
B	-50,0	-50,-50

- Does Clyde have a dominant action?
- No – S is better than C if Bonnie goes Bezerk
- There is no equilibrium in dominant strategies

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Iterated Deletion of Strictly Dominated Strategies

Clyde

Bonnie

	S	C
S	-5,-5	-30,-1
C	-1,-30	-10,-10
B	-50,0	-50,-50

- However, let's look at Bonnie
- What do we notice about the action B?

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Iterated Deletion of Strictly Dominated Strategies

Clyde

Bonnie

	S	C
S	-5,-5	-30,-1
C	-1,-30	-10,-10
B	-50,0	-50,-50

- However, let's look at Bonnie
- What do we notice about the action B?
- It is **dominated** by C

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Iterated Deletion of Strictly Dominated Strategies

Clyde

Bonnie

	S	C
S	-5,-5	-30,-1
C	-1,-30	-10,-10
B	-50,0	-50,-50

- Will Bonnie ever play B?
- No!
- And Clyde should realize this
- Clyde should ignore action B

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Iterated Deletion of Strictly Dominated Strategies

Clyde

Bonnie

	S	C
S	-5,-5	-30,-1
C	-1,-30	-10,-10
B	~~-50,0~~	~~-50,-50~~

- Will Bonnie ever play B?
- No!
- And Clyde should realize this
- Clyde should ignore action B

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Iterated Deletion of Strictly Dominated Strategies

Clyde

Bonnie

	S	C
S	-5,-5	-30,-1
C	-1,-30	-10,-10
B	~~-50,0~~	~~-50,-50~~

- Ignoring B, does Clyde have a dominant action?

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Iterated Deletion of Strictly Dominated Strategies

Clyde

Bonnie

	S	C
S	-5,-5	-30,-1
C	-1,-30	-10,-10
B	~~-50,0~~	~~-50,-50~~

- Ignoring B, does Clyde have a dominant action?
- Yes, C is now dominant
- C is also dominant for Bonnie

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Iterated Deletion of Strictly Dominated Strategies

Clyde

Bonnie

	S	C
S	-5,-5	-30,-1
C	-1,-30	-10,-10
B	~~-50,0~~	~~-50,-50~~

- C,C is still what we would expect to happen in this game
- This is equilibrium in **iterated deletion of dominated strategies**

Solving a Game 2

Nash Equilibrium

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Iterated Deletion of Strictly Dominated Strategies

- Can we always solve a game using iterated deletion of dominated strategies?

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An Example of a Two-Player Game

Player B

Player A		L	R
	U	(3,9)	(1,8)
	D	(0,0)	(2,1)

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An Example of a Two-Player Game

Player B

Player A		L	R
	U	(3,9)	(1,8)
	D	(0,0)	(2,1)

- Player A prefers U if B plays L or D if B Plays R
- Player B prefers L if A plays U or R if A plays D
- No dominant strategies. What to do!

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An Example of a Two-Player Game

Player B

Player A		L	R
	U	(3,9)	(1,8)
	D	(0,0)	(2,1)

What plays are we likely to see for this game?

An Example of a Two-Player Game

Player B

Is (U,R) a likely play?

Player A		L	R
	U	(3,9)	(1,8)
	D	(0,0)	(2,1)

An Example of a Two-Player Game

Player B

Is (U,R) a likely play?

Player A		L	R
	U	(3,9)	(1,8)
	D	(0,0)	(2,1)

If B plays Right then A's best reply is Down since this improves A's payoff from 1 to 2. So (U,R) is not a likely play.

An Example of a Two-Player Game

Player B

Is (D,R) a likely play?

Player A		L	R
	U	(3,9)	(1,8)
	D	(0,0)	(2,1)

An Example of a Two-Player Game

Player B

Is (D,R) a likely play?

Player A		L	R
	U	(3,9)	(1,8)
	D	(0,0)	(2,1)

If B plays Right then A's best reply is Down.

An Example of a Two-Player Game

Player B

Is (D,R) a likely play?

Player A		L	R
	U	(3,9)	(1,8)
	D	(0,0)	(2,1)

If B plays Right then A's best reply is Down.
If A plays Down then B's best reply is Right.
So (D,R) is a likely play.

An Example of a Two-Player Game

Player B

Is (D,L) a likely play?

Player A		L	R
	U	(3,9)	(1,8)
	D	(0,0)	(2,1)

An Example of a Two-Player Game

Player B

		L →	R
Player A	U	(3,9)	(1,8)
	D	(0,0)	(2,1)

Is (D,L) a likely play?

If A plays Down then B's best reply is Right, so (D,L) is not a likely play.

An Example of a Two-Player Game

Player B

		L	R
Player A	U	(3,9)	(1,8)
	D	(0,0)	(2,1)

Is (U,L) a likely play?

An Example of a Two-Player Game

Player B

		L	R
Player A	U	(3,9)	(1,8)
	D	(0,0)	(2,1)

Is (U,L) a likely play?

If A plays Up then B's best reply is Left.

An Example of a Two-Player Game

Player B

		L	R
Player A	U	(3,9)	(1,8)
	D	(0,0)	(2,1)

Is (U,L) a likely play?

If A plays Up then B's best reply is Left.
If B plays Left then A's best reply is Up.
So (U,L) is a likely play.

Nash Equilibrium

- So in this game we said that there were two likely plays
  - (U,L) and (D,R)
- Why did we think they were likely plays?
- In both cases, the action of each player was optimal **given the actions of the other player**
- We call such a situation a **Nash Equilibrium** (of Beautiful Mind fame)
- **Definition:** A Nash Equilibrium is an action for each player such that each player has no incentive to deviate given what everyone else has done
- The bit in red is important

Nash Equilibrium

- Why do we think that we are likely to see Nash Equilibria?
- One justification is that it is a resting point of the system
- If we somehow ended up at one, then no one would have an incentive to change
  - i.e. if we all went to the bar after the game and revealed how we played, no one would have any regrets
- This makes the outcome 'stable'
- If we weren't at a Nash equilibrium this wouldn't be true

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Nash Equilibrium

- Is this enough?
- Not necessarily
- Think of the previous game
- There were two Nash equilibria (U,L) and (D,R)
- If I paired you with another person in the lecture hall, what would you play?
- So why do we think Nash equilibria are good predictions of behavior?
- Two (not completely satisfactory) answers
  - Pre play communication
  - Repeated play

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Uniqueness and Existence

- Setting these problems aside, there are two things that we look for in a theory in order for equilibrium to make good predictions
  - Existence
  - Uniqueness
- Without existence, the theory may make **no** predictions
- Without uniqueness, the theory may make **too many** predictions
- We will talk about existence next lecture
- But we already know that we don't have uniqueness
- What can we do?

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Uniqueness and Existence

- One possibility is to look for ways of ruling out some equilibria as implausible
  - This is sometimes called looking for **equilibrium refinements**
- Consider the following game
  - Two players are trying to decide which concert to go and see
  - They can either go to see Mahler or Mozart
  - Both prefer Mozart to Mahler
  - However, both prefer seeing a concert together than seeing music apart

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Uniqueness and Existence

	Mo	Ma
Mo	2,2	0,0
Ma	0,0	1,1

- What are the Nash Equilibria of this game?
- There are 2 – Mo,Mo and Ma,Ma
- However, do you think they are both equally likely?
- Most people think that Mo,Mo is more likely than Ma,Ma, because both players get a higher payoff
- Mo,MO **payoff dominates** Ma,Ma

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Uniqueness and Existence

- Do we always think that payoff dominant equilibria are the most likely?
- Modify the previous game so that, if either person turns up to Mozart on their own they are shot

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Uniqueness and Existence

	Mo	Ma
Mo	2,2	-1000,0
Ma	0,-1000	1,1

- Game still has two Nash equilibria
- Mo,Mo still payoff dominates Ma,Ma
- Do you still think it is more likely?
- If either player has any uncertainly about what the other will do, they will play Ma
- Ma,Ma **risk dominates** Mo,Mo

Summary

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Summary

- Today we

1. Defined what a game is
2. Thought of some ways of **solving** a game
   - i.e. make predictions about how we think people behave in the game
   - First we will use optimization
   - Then we will think about equilibrium