Intermediate Microeconomics W3211

Lecture 21: Game Theory 3 – Back to the Firms

Columbia University, Spring 2016
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Introduction

The Story So Far….

- We now have a number of tools to analyze strategic situations
- Defined a game
- Thought about how to solve a game
- Dealt with issues of
  - Multiple equilibria (equilibrium selection)
  - Existence (mixed strategies)
- Thought about the difference between sequential and simultaneous move games

Today

- Go back to thinking about firms
- In particular, we are going to relax the assumption that firms are **price takers**
- Think about firms that are big enough to affect price
- Take this into account when deciding what to do
- First consider the case of Monopoly
  - Only one firm
  - Very boring game
- Then consider the case of a small number of firms
  - Oligopoly
  - More interesting game
- Varian Ch. 25/26/28
- Feldman and Serrano Ch 12/13

Monopoly

Monopolies

- Up until now, we have assumed that firms have been small
- In particular, they treat prices as given
- However this is clearly not always the case
- Sometimes firms are **large**
- Get to set their own prices

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Monopolies

- How does this happen?
- Presumably monopolies make big profits
- From previous lectures, we know that excess profits should encourage new firms to enter
- Why does that not always happen?

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Why Monopolies?

- a legal fiat; e.g. US Postal Service
- a patent; e.g. a new drug
- sole ownership of a resource; e.g. a toll highway
- formation of a **cartel**: many firms colluding to restrict output to affect prices. 2 examples:
  - OPEC : think of the '73 oil crisis
  - De Beers : controls 80% of world production of diamonds
- large economies of scale; e.g. local utility companies.

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How do Monopolies Behave?

- We will assume that monopolies act to maximize profits
- Remember, that for a perfectly competitive firm, the profit maximization problem was to choose output to maximize

$$\pi = py - c(y)$$

- How does this change with the monopolist?
- Well, we said that a monopolist gets to **choose prices**
- But, there will still be a relationship between price they choose and the quantity they can sell
- Given by the **demand function**

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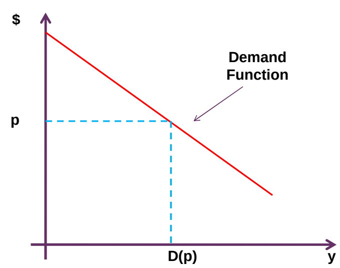
The Demand Curve



- The Demand curve tells the firm how much people will buy given price p

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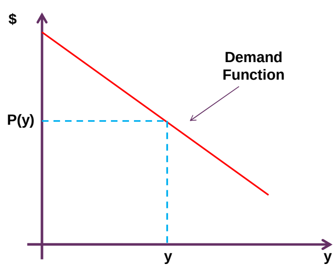
The Demand Curve



- Equivalently it tells us how much they can charge if they want to sell the amount y

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How do Monopolies Behave?

- The latter formulation will be more convenient

$$\pi = p(y)y - c(y)$$

- So, what should the firm do to maximize profits?
- Let's take first order conditions!

Profit=revenue-cost

- First order conditions

$$p(y) + y\frac{dp}{dy} = \frac{dc}{dy}$$

Marginal Revenue=Marginal Cost

- (Remember, you have to worry about corner solutions etc)

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How do Monopolies Behave?

$$p(y) + y\frac{dp}{dy} = \frac{dc}{dy}$$

Marginal Revenue=Marginal Cost

- This is the same condition as in the perfect competition case
- But now marginal revenue has an additional term: $y\frac{dp}{dy}$
- This is **exactly** the firm taking into account the effect that increased production has on prices
- Side note: at the moment, we are assuming that the only thing the monopolist can do is charge a constant price per unit output
  - Later in the course we will consider some sneaky other things that a monopolist might want to do

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How do Monopolies Behave?

$$p(y) + y\frac{dp}{dy} = \frac{dc}{dy}$$

Marginal Revenue=Marginal Cost

- What does Marginal Revenue look like?
- Well, note that $\frac{dp}{dy}$ is negative, so the marginal revenue at y will be less than the price

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How do Monopolies Behave?

- For example, if the inverse demand curve is given by

$$p = a - by$$

- Then revenue is given by

$$py = (a - by)y$$

- And so marginal revenue is given by

$$\frac{d(py)}{dy} = a - 2by$$

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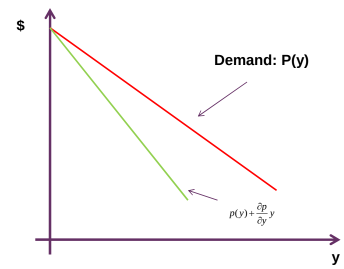
Marginal Revenue and Demand



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How do Monopolies Behave?

- So what does the optimal output look like for a monopolist?

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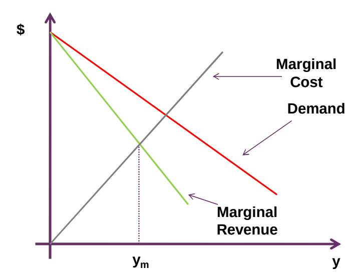
Monopolistic Output



- They will choose y in order to get marginal cost to equal marginal revenue

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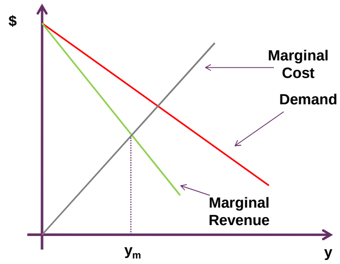
Monopolistic Output



- What price will they charge?

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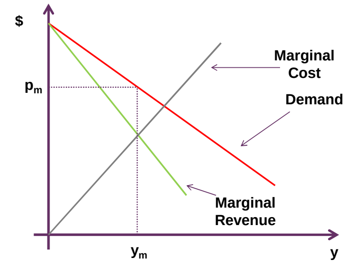
Monopolistic Output



- The price that allows them to sell this amount of output
- Which we can read off from the demand curve

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How do Monopolies Behave?

- How does the behavior of a monopolist compare to that of perfectly competitive firms
- Remember, for a perfectly competitive firm
  - Price equals marginal cost
  - The supply curve is given by the marginal cost
  - Equilibrium happens when supply equals demand

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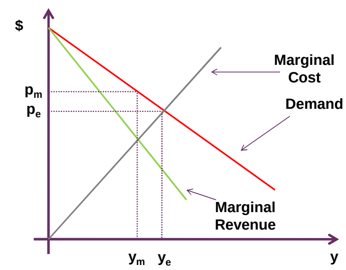
Monopolistic Output



- Monopolists will supply **less** and charge **higher prices** than a perfectly competitive industry

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How do Monopolies Behave?

- What happens to surplus in the monopoly case?

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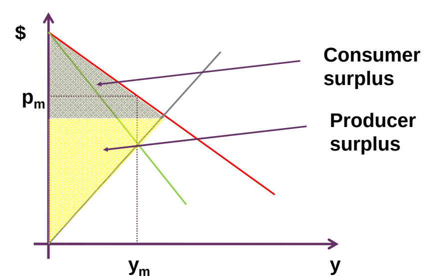
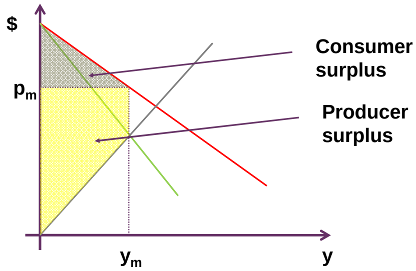
Surplus Under Perfect Competition



Surplus under Monopoly



How do Monopolies Behave?

- What happens to surplus in the monopoly case?
- Three effects
  - Producer surplus **increases**
  - Consumer surplus **decreases**
  - Total surplus **decreases**
- i.e. monopoles are inefficient

Price Mark-Up and Elasticity

- Let's go back to the first order conditions for the monopolist

$$p(y) + y\frac{dp}{dy} = \frac{dc}{dy}$$

- Rearranging gives

$$p(1 + \frac{y}{p}\frac{dp}{dy}) = \frac{dc}{dy}$$

$$p = \frac{\frac{dc}{dy}}{(1 + \frac{y}{p}\frac{dp}{dy})}$$

- But $\frac{dc}{dy}$ is marginal cost , and $\frac{y}{p}\frac{dp}{dy}$ is the reciprocal of the price elasticity of demand

Price Mark-Up and Elasticity

$$p = \frac{MC(y)}{(1 - \frac{1}{\varepsilon})}$$

- In the perfectly competitive case, price would equal marginal cost
- Monopolists **mark up** the price above marginal cost
- The degree of the mark up depends on the price elasticity of demand
  - The **lower** the elasticity, the **higher** the mark up
- This makes sense: the less responsive demand is to prices, the higher the price that monopolists will charge

Oligopoly

Oligopoly

- We now want to move from one firm (monopoly) to a small number of firms (oligopoly)
  - Concentrate on the case of two firms (duopoly)
- It turns out there are different ways to analyze this case, depending on how we assume firms interact
  - Sometimes called the market structure
- We are going to think first about the case in which firms **compete on quantity**
  - Sometimes called Cournot competition
- In order to analyze the market, we will make use of game theoretic concepts

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Quantity Competition

- Assume that firms compete by choosing output levels.
- If firm 1 produces y_1 units and firm 2 produces y_2 units then total quantity supplied is $y_1 + y_2$. The market price will be $p(y_1 + y_2)$.
  - This is what makes this a game
  - Output of firm 1 affects the price of firm 2, and visa versa
- The firms' total cost functions are $c_1(y_1)$ and $c_2(y_2)$.

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Quantity Competition

- Suppose firm 1 takes firm 2's output level choice y_2 as given. Then firm 1 sees its profit function as

$$\Pi_1(y_1; y_2) = p(y_1 + y_2)y_1 - c_1(y_1).$$

- Given y_2, what output level y_1 maximizes firm 1's profit?

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Quantity Competition; An Example

- Suppose that the market inverse demand function is

$$p(y_T) = 60 - y_T$$

- and that the firms' total cost functions are

$$c_1(y_1) = y_1^2 \quad \text{and} \quad c_2(y_2) = 15y_2 + y_2^2.$$

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Quantity Competition; An Example

Then, for given y_2, firm 1's profit function is

$$\Pi(y_1; y_2) = (60 - y_1 - y_2)y_1 - y_1^2.$$

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Quantity Competition; An Example

Then, for given y_2, firm 1's profit function is

$$\Pi(y_1; y_2) = (60 - y_1 - y_2)y_1 - y_1^2.$$

So, given y_2, firm 1's profit-maximizing output level solves

$$\frac{\partial \Pi}{\partial y_1} = 60 - 2y_1 - y_2 - 2y_1 = 0.$$

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Quantity Competition; An Example

I.e., firm 1's best response to y_2 is

$$y_1 = R_1(y_2) = 15 - \frac{1}{4}y_2.$$

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Quantity Competition; An Example

I.e., firm 1's best response to y_2 is

$$y_1 = R_1(y_2) = 15 - \frac{1}{4}y_2.$$

Key point: Optimal output of firm 1 depends on the output of firm 2

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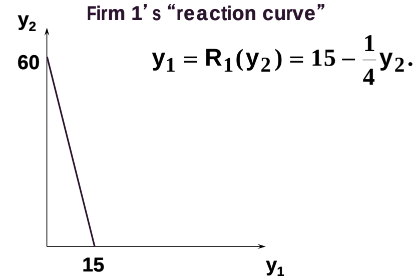
Quantity Competition; An Example



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Quantity Competition; An Example

Similarly, given y_1, firm 2's profit function is

$$\Pi(y_2; y_1) = (60 - y_1 - y_2)y_2 - 15y_2 - y_2^2.$$

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Quantity Competition; An Example

Similarly, given y_1, firm 2's profit function is

$$\Pi(y_2; y_1) = (60 - y_1 - y_2)y_2 - 15y_2 - y_2^2.$$

So, given y_1, firm 2's profit-maximizing output level solves

$$\frac{\partial \Pi}{\partial y_2} = 60 - y_1 - 2y_2 - 15 - 2y_2 = 0.$$

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Quantity Competition; An Example

Similarly, given y_1, firm 2's profit function is

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So, given y_1, firm 2's profit-maximizing output level solves

$$\frac{\partial \Pi}{\partial y_2} = 60 - y_1 - 2y_2 - 15 - 2y_2 = 0.$$

I.e., firm 1's best response to y_2 is

$$y_2 = R_2(y_1) = \frac{45 - y_1}{4}.$$

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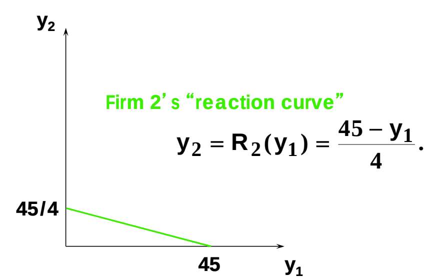
Quantity Competition; An Example



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Quantity Competition; An Example

- What do we think will happen in this case?
- Well, we can think of this as a game with
  - Two players
  - The action of each player is to choose a quantity
  - Payoff is given by the profit for each firm
- How do we solve games?
- Nash equilibrium!
- This is an action for each player which is the best for them, given what the other player is doing

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Quantity Competition; An Example

- An equilibrium is when each firm's output level is a best response to the other firm's output level, for then neither wants to deviate from its output level
- A pair of output levels (y_1*,y_2*) is a Cournot-Nash equilibrium if

$$y_1^* = R_1(y_2^*) \text{ and } y_2^* = R_2(y_1^*)$$

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Quantity Competition; An Example

$$y_1^* = R_1(y_2^*) = 15 - \frac{1}{4}y_2^* \text{ and } y_2^* = R_2(y_1^*) = \frac{45 - y_1^*}{4}.$$

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Quantity Competition; An Example

$$y_1^* = R_1(y_2^*) = 15 - \frac{1}{4}y_2^* \text{ and } y_2^* = R_2(y_1^*) = \frac{45 - y_1^*}{4}.$$

Substitute for y_2* to get

$$y_1^* = 15 - \frac{1}{4}\left(\frac{45 - y_1^*}{4}\right)$$

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Quantity Competition; An Example

$$y_1^* = R_1(y_2^*) = 15 - \frac{1}{4}y_2^* \text{ and } y_2^* = R_2(y_1^*) = \frac{45 - y_1^*}{4}.$$

Substitute for y_2* to get

$$y_1^* = 15 - \frac{1}{4}\left(\frac{45 - y_1^*}{4}\right) \Rightarrow y_1^* = 13$$

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Quantity Competition; An Example

$$y_1^* = R_1(y_2^*) = 15 - \frac{1}{4}y_2^* \text{ and } y_2^* = R_2(y_1^*) = \frac{45 - y_1^*}{4}.$$

Substitute for y_2* to get

$$y_1^* = 15 - \frac{1}{4}\left(\frac{45 - y_1^*}{4}\right) \Rightarrow y_1^* = 13$$

Hence $$y_2^* = \frac{45 - 13}{4} = 8.$$

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Quantity Competition; An Example

$$y_1^* = R_1(y_2^*) = 15 - \frac{1}{4}y_2^* \text{ and } y_2^* = R_2(y_1^*) = \frac{45 - y_1^*}{4}.$$

Substitute for y_2^* to get

$$y_1^* = 15 - \frac{1}{4}\left(\frac{45 - y_1^*}{4}\right) \Rightarrow y_1^* = 13$$

Hence $y_2^* = \frac{45 - 13}{4} = 8.$

So the Cournot-Nash equilibrium is

$$(y_1^*, y_2^*) = (13,8).$$

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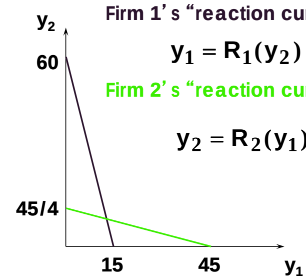
Quantity Competition; An Example



Firm 1's "reaction curve"

$$y_1 = R_1(y_2) = 15 - \frac{1}{4}y_2.$$

Firm 2's "reaction curve"

$$y_2 = R_2(y_1) = \frac{45 - y_1}{4}.$$

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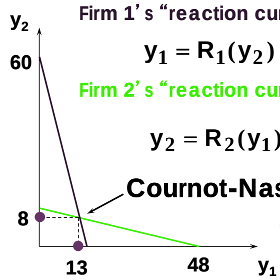
Quantity Competition; An Example



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$$y_2 = R_2(y_1) = \frac{45 - y_1}{4}.$$

Cournot-Nash equilibrium

$$(y_1^*, y_2^*) = (13,8).$$

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Quantity Competition

Generally, given firm 2's chosen output level y_2, firm 1's profit function is

$$\Pi_1(y_1; y_2) = p(y_1 + y_2)y_1 - c_1(y_1)$$

and the profit-maximizing value of y_1 solves

$$\frac{\partial \Pi_1}{\partial y_1} = p(y_1 + y_2) + y_1\frac{\partial p(y_1 + y_2)}{\partial y_1} - c_1'(y_1) = 0.$$

The solution, $y_1 = R_1(y_2)$, is firm 1's Cournot-Nash reaction to y_2.

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Quantity Competition

Similarly, given firm 1's chosen output level y_1, firm 2's profit function is

$$\Pi_2(y_2; y_1) = p(y_1 + y_2)y_2 - c_2(y_2)$$

and the profit-maximizing value of y_2 solves

$$\frac{\partial \Pi_2}{\partial y_2} = p(y_1 + y_2) + y_2\frac{\partial p(y_1 + y_2)}{\partial y_2} - c_2'(y_2) = 0.$$

The solution, $y_2 = R_2(y_1)$, is firm 2's Cournot-Nash reaction to y_1.

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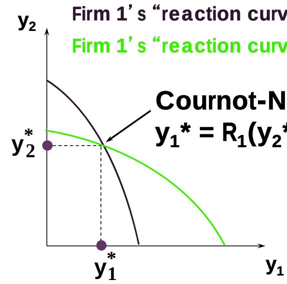
Quantity Competition



Firm 1's "reaction curve" $y_1 = R_1(y_2).$

Firm 1's "reaction curve" $y_2 = R_2(y_1).$

Cournot-Nash equilibrium

$y_1^* = R_1(y_2^*)$ and $y_2^* = R_2(y_1^*)$

Collusion

- Q: Are the Cournot-Nash equilibrium profits the largest that the firms can earn in total?
- Not if they are allowed to co-operate!

Collusion

- Suppose the two firms want to maximize their total profit and divide it between them. Their goal is to choose cooperatively output levels y_1 and y_2 that maximize

$$\Pi^m(y_1, y_2) = p(y_1 + y_2)(y_1 + y_2) - c_1(y_1) - c_2(y_2).$$

This is similar to the monopolist problem!

In fact, if the firms have constant costs, it **is** the monopolist problem

Typically, this will give the firm higher profit

Sometimes called forming a cartel

Collusion

- Suppose two firms face an inverse market demand of $p(y_T) = 24 - y_T$ and have total costs of $c_1(y_1) = y^2{}_1$ and $c_2(y_2) = y^2{}_2$.
- Can they make more in a cartel or if they compete?

Collusion

- What is each firm's per period profit in the cartel?
- $p(y_T) = 24 - y_T$, $c_1(y_1) = y^2{}_1$, $c_2(y_2) = y^2{}_2$.
- If the firms collude then their joint profit function is $\pi^M(y_1,y_2) = (24 - y_1 - y_2)(y_1 + y_2) - y^2{}_1 - y^2{}_2$.
- What values of y_1 and y_2 maximize the cartel's profit?

Collusion

- $\pi^M(y_1,y_2) = (24 - y_1 - y_2)(y_1 + y_2) - y^2{}_1 - y^2{}_2$.
- What values of y_1 and y_2 maximize the cartel's profit? Solve

$$\frac{\partial \pi^M}{\partial y_1} = 24 - 4y_1 - 2y_2 = 0$$

$$\frac{\partial \pi^M}{\partial y_2} = 24 - 2y_1 - 4y_2 = 0.$$

Collusion

- $\pi^M(y_1,y_2) = (24 - y_1 - y_2)(y_1 + y_2) - y^2{}_1 - y^2{}_2$.
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$$\frac{\partial \pi^M}{\partial y_2} = 24 - 2y_1 - 4y_2 = 0.$$

- Solution is $y^M{}_1 = y^M{}_2 = 4$.

Collusion

- $\pi^M(y_1,y_2) = (24 - y_1 - y_2)(y_1 + y_2) - y^2_1 - y^2_2$.
- $y^M_1 = y^M_2 = 4$ maximizes the cartel's profit.
- The maximum profit is therefore $\pi^M = \$(24 - 8)(8) - \$16 - \$16 = \$96$.
- Suppose the firms share the profit equally, getting \$96/2 = \$48 each per period.

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Collusion

- What are the firms' profits if they do not co-operate?
- $p(y_T) = 24 - y_T$, $c_1(y_1) = y^2_1$, $c_2(y_2) = y^2_2$.
- Given y_2, firm 1's profit function is $\pi_1(y_1;y_2) = (24 - y_1 - y_2)y_1 - y^2_1$.

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Collusion

- What are the firms' profits if they do not cooperate?
- $p(y_T) = 24 - y_T$, $c_1(y_1) = y^2_1$, $c_2(y_2) = y^2_2$.
- Given y_2, firm 1's profit function is $\pi_1(y_1;y_2) = (24 - y_1 - y_2)y_1 - y^2_1$.
- The value of y_1 that is firm 1's best response to y_2 solves

$$\frac{\partial \pi_1}{\partial y_1} = 24 - 4y_1 - y_2 = 0 \quad \Rightarrow \quad y_1 = R_1(y_2) = \frac{24 - y_2}{4}.$$

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Collusion

- What are the firms' profits if they do not cooperate?
- $\pi_1(y_1;y_2) = (24 - y_1 - y_2)y_1 - y^2_1$.

$$y_1 = R_1(y_2) = \frac{24 - y_2}{4}.$$

- Similarly,

$$y_2 = R_2(y_1) = \frac{24 - y_1}{4}.$$

- The C-N equilibrium (y^*_1,y^*_2) solves $y_1 = R_1(y_2)$ and $y_2 = R_2(y_1) \Rightarrow y^*_1 = y^*_2 = 4\cdot8$.

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Collusion

- What are the firms' profits if they do not cooperate?
- $\pi_1(y_1;y_2) = (24 - y_1 - y_2)y_1 - y^2_1$.
- $y^*_1 = y^*_2 = 4\cdot8$.
- So each firm's profit in the C-N equilibrium is $\pi^*_1 = \pi^*_2 = (14\cdot4)(4\cdot8) - 4\cdot8^2 \approx \$46$ each period.

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Collusion

- So both firms earn more if they form a cartel
- But is such a cartel stable?
- What would firm 1 want to do if firm 2 is producing at the cartel level?

Collusion

- Let's say that firm 1 knows that firm 2 will produce at the cartel level.
- What is their best response?
- Firm 1 produces quantity y^{CH}_1 that maximizes firm 1's profit given that firm 2 continues to produce $y^M_2 = 4$. What is the value of y^{CH}_1?
- $y^{CH}_1 = R_1(y^M_2) = (24 - y^M_2)/4 = (24 - 4)/4 = 5$.
- Firm 1's profit in the period in which it cheats is therefore
  $\pi^{CH}_1 = (24 - 5 - 4)(5) - 5^2 = \$50$.

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Collusion

- So a profit-seeking cartel in which firms cooperatively set their output levels is fundamentally unstable.
- This is basically a prisoner's dilemma
- Both firms would do better if they collude, but this is not an equilibrium
- *E.g.,* OPEC's broken agreements.

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Collusion

- So a profit-seeking cartel in which firms cooperatively set their output levels is fundamentally unstable.
- This is basically a prisoner's dilemma
- Both firms would do better if they collude, but this is not an equilibrium
- *E.g.,* OPEC's broken agreements.
- But is the cartel unstable if the game is repeated many times, instead of being played only once? Then there is an opportunity to punish a cheater.

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Collusion & Punishment Strategies

- Imagine a firm that is deciding between

1. Staying in the cartel forever
2. Cheating on the cartel, and then being punished forever by the other cartel member

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Collusion & Punishment Strategies

- To determine if such a cartel can be stable we need to know 3 things:
  - (i) What is each firm's per period profit in the cartel?
  - (ii) What is the profit a cheat earns in the first period in which it cheats?
  - (iii) What is the profit the cheat earns in each period after it first cheats?

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Collusion & Punishment Strategies

- (i) What is each firm's per period profit in the cartel?
  - This is just the cartel profits - $48 per period

Collusion & Punishment Strategies

- (ii) What is the profit a cheat earns in the first period in which it cheats?
- This is what firm 1 would get if they best responded to firm 2 producing the cartel output
- They would get $50

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Collusion & Punishment Strategies

- (iii) What is the profit the cheat earns in each period after it first cheats?
- This depends upon the punishment inflicted upon the cheat by the other firm.

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Collusion & Punishment Strategies

- (iii) What is the profit the cheat earns in each period after it first cheats?
- This depends upon the punishment inflicted upon the cheat by the other firm.
- Suppose the other firm punishes by forever after not cooperating with the cheat.
- Then each firm will earn the Cournot profits - $46 – in each period
- Note that this is a 'credible' threat

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Collusion & Punishment Strategies

- To determine if such a cartel can be stable we need to know 3 things:
  - (i) What is each firm's per period profit in the cartel? $48.
  - (ii) What is the profit a cheat earns in the first period in which it cheats? $50.
  - (iii) What is the profit the cheat earns in each period after it first cheats? $46.

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Collusion & Punishment Strategies

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- The present-value of firm 1's profits if it does not cheat is ??

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$$PV^{cheat} = \$50 + \frac{\$46}{1+r} + \frac{\$46}{(1+r)^2} + \cdots = \$50 + \frac{\$46}{r}.$$

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Collusion & Punishment Strategies

$$PV^{loyal} = \$48 + \frac{\$48}{1+r} + \frac{\$48}{(1+r)^2} + \cdots = \$\frac{(1+r)48}{r}.$$
$$PV^{cheat} = \$50 + \frac{\$46}{1+r} + \frac{\$46}{(1+r)^2} + \cdots = \$50 + \frac{\$46}{r}.$$

So the cartel will be stable if

$$\frac{(1+r)48}{r} > 50 + \frac{46}{r} \Rightarrow r < 1 \Rightarrow \frac{1}{1+r} > \frac{1}{2}.$$

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Collusion

- Cartels can be stable if

1. The game is played an infinite number of times
2. The players are patient enough

Summary

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Summary

- Today we have
- Modelled monopolies
- Modelled duopolies that compete on quantity
- Thought about when duopolies form cartels