

Intermediate Microeconomics

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Homework 8

Due To be done at your leisure

Question 1 Consider the following game

	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>
1	2, 0	10, 0	1, 12	2, 15
2	10, 8	5, -1	2, 5	3, 3
3	1, 7	6, -3	1, 15	11, 2
4	1, 7	9, 0	11, 6	3, 1

1. Calculate the best responses for each player to each strategy of the other players. Does this game have a Nash Equilibrium?
2. Show that strategy B for the column player is strictly dominated by strategy C. Can this game be solved by the iterated deletion of dominated strategies?
3. Now consider the following game

	<i>A</i>	<i>B</i>
1	2, 2	1, 0
2	6, 1	0, 2
3	0, 1	6, 2

Does this game have a Nash Equilibrium? Is strategy 1 dominated for the row player?
Does your answer depend on whether the row player is allowed to play mixed strategies?

4. Now consider the following game

	<i>A</i>	<i>B</i>	<i>C</i>
1	7, 3	1, 7	5, 0
2	4, 6	10, 1	3, 0
3	0, 10	0, 15	1, 6

Does this game have a Nash Equilibrium in pure strategies? Can any strategies be removed by the iterated deletion of strictly dominated strategies. Show that there is a mixed strategy Nash equilibrium where the row player mixes between 1 and 2 and the column player mixes between A and B

Question 2 Thor Industries and Zeus Technology both produce Yo Yos. The cost function of Thor is $c(q_T) = c_T q_T$ and the cost function for Zeus is $c(q_Z) = c_Z q_Z$ where $c_T > c_Z$. The price that each firm can sell yoyos for is $p(q) = a - b(q_T + q_Z)$

1. Calculate Zeus's best response to Thor producing q_T
2. Calculate Thor's best response to Zeus producing q_Z
3. Find the Nash equilibrium level of output. Who produces more in the Nash equilibrium?
4. Say that Thor improves his technology, so his cost function is now the same as Zeus's. Moreover the two gods want to collude in the yo yo market (i.e. each produce half the monopoly output. In order to achieve this, the gods write a contract with each other such that each one has to pay the other F if they deviate from this level of output. How big does F have to be to ensure that neither god will cheat on the agreement

Question 3 Consider the Bertrand Competition game described in the lecture notes. So each firm has a cost of producing an item equal to c . Show that the ONLY Nash equilibrium is for them both to offer to sell the widget for c . Now imagine that one firm can produce the widget for d and the other for c , where $d > c$. Also say that firms can now only offer prices in whole dollars and cents (i.e. they can offer the price \$3.45, but not \$3.457). Can you find a Nash equilibrium of this game?

Question 4 Consider the following game: At present there is one monopoly firm in an industry, making profits p . Another firm is deciding whether or not to enter that industry. If they enter then the monopolist has two choices. They can either share the market (s) or they can

fight (f). If they share the market, then each firm gets $\frac{p}{2}$. If the monopolist fights, then each firm gets -1. If the entrant does not enter, then the monopolist gets p and the entrant gets 0.

1. Draw the game tree for this game. Find the Nash equilibria for this game. Find the subgame perfect Nash equilibrium using backwards induction.
2. Now imagine that before the entrant makes their decision, the monopolist makes a choice. They can either write a contract which says the following: "If an entrant enters the market and I do not fight, then we will give $2p$ to charity", or they can not write such a contract. Draw the new game tree. What is the subgame perfect Nash equilibrium now?
3. Now imagine that, before the monopolist chooses whether or not to write such a contract, the entrant can make a legally binding commitment to enter. If they do not enter, they get fined 1000. Write the game tree of the new game. What is the SPNE?