Intermediate Microeconomics - Spring 2016

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Homework 1

Due Wednesday 3rd February

Question 1 (Budget Sets 1) This question concerns a consumer who is choosing how many of two goods to buy: Footballs (the round ones, that you kick with your foot) and cricket balls (like baseballs, but better). The consumer has an income of \$20, and the cost of a football is \$4 and a cricket ball is \$2

1. Write down the equation for the consumer's budget constraint and graph it in the commodity space
2. The government decides that football is evil and needs to be taxed. They introduce a 50% tax on each football sold. Rewrite and re-graph the budget constraint.
3. A new government is elected that hates all sports. They now tax both footballs and cricket balls at 50%. What does the budget constraint look like now?
4. Due to a threat of revolt amongst sports fans, the government hands out a subsidy of \$10 to the consumer. What does their new budget constraint look like? How would you expect consumer behavior to differ between this situation and the no-tax, no-subsidy situation described in part (1)
5. Revolution comes, and all taxes and subsidies are abolished. Even better, the consumer finds a new shop that offers bulk discounts. In this shop, footballs cost \$4 each if you buy 3 or fewer. However, the cost of any additional football after 3 is \$2. What does the budget set look like now?

Question 2 (Budget Sets 2) Edmund consumes two commodities, garbage and punk rock video cassettes. He doesn't eat garbage of course, but he gets paid for taking it away at \$2 per sack. Edmund can accept as much garbage as he wishes at that price. He has no other source of income. Video cassettes cost him \$6 each. He has a utility function $u(g, v)$ on garbage (g) and videos (v) which is decreasing in g and increasing in v.

1. If Edmund's accepts 0 sacks of garbage, how many video cassettes can he buy?
2. Write down Edmund's constrained optimization problem
3. Draw Edmund's budget line and shade his budget set.

Question 3 (Preferences and Transitivity) In lectures, I claimed that well behaved preferences satisfied transitivity, i.e.

$$\begin{aligned} x &\succcurlyeq y \\ y &\succcurlyeq z \\ \text{implies } x &\succcurlyeq z \end{aligned}$$

it seems like it would also be sensible for preferences to obey the following two properties

1. $x \succ y$, $y \succ z$ implies $x \succ z$ (i.e. if one **strictly** prefers x to y and y to z then one **strictly** prefers x to z)
2. $x \sim y$, $y \sim z$ implies $x \sim z$ (i.e. if one **is indifferent** between x to y and y to z then one **is indifferent** between x to z)

Show that we don't need to additionally assume these properties: if preferences are complete, and satisfy transitivity, then these two other properties are implied.

Question 4 (Utility Representations) Here are some questions based on the idea that there are many different utility representations of the same preferences

1. In class I claimed that two utility functions u and v represent the same preferences if and only if there is a strictly increasing function f such that $u(x) = f(v(x))$ for all x. I would like you to prove half of this statement: if there is such a function f, then u and v represent the same preferences. Also, does the statement remain true if I drop the word 'strictly'? (note, a strictly increasing function is one in which $x > y \rightarrow f(x) > f(y)$ An increasing function is one such that $x > y \rightarrow f(x) \geq f(y)$)
2. Do the following utility functions represent the same preferences?

(a) $u(x_1, x_2) = x_1 x_2$, $v(x_1, x_2) = 3(x_1 x_2)^2 + 6$

(b) $u(x_1, x_2) = x_1 x_2$, $v(x_1, x_2) = -3(x_1 x_2)^2 + 6$

(c) $u(x_1, x_2) = x_1 x_2$, $v(x_1, x_2) = \ln x_1 + \ln x_2$

(d) $u(x_1, x_2) = x_1 x_2$, $v(x_1, x_2) = x_1 + x_2$

3. In class I said that you should not 'trust' marginal utility too much, because different utility functions which represent the same preferences may give rise to different marginal utilities. However, it is true that if two utility functions represent the same preferences then the marginal utility must always have the same sign at each bundle (x_1, x_2) (i.e. if u and v represent the same preferences then $\frac{\partial u(x_1,x_2)}{\partial x_i} > 0$ if and only if $\frac{\partial v(x_1,x_2)}{\partial x_i} > 0$) Show that this is the case