

Intermediate Microeconomics W3211

Lecture 15: Perfect Competition 5 The Short Run and the Long Run

Columbia University, Spring 2016
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Introduction

The Story So Far...

- We have now modeled the perfectly competitive firm in some detail
- Set up the firm's problem
- Discussed how to split the problem into two
 - Cost minimization
 - Profit maximization
- Solved both parts
- Thought a bit about how firm behavior will change as prices change

Today

- Think more about the behavior of the firm in the **short** and the **long** run

The Short and the Long Run

Technology in the Short and the Long Run

The Long-Run and the Short-Runs

- We now introduce the distinction between **long run** and **short run**
- The **long-run** is the circumstance in which a firm is unrestricted in its choice of **all input levels**.
 - In the long run a firm can choose how many workers to hire **and** how many machines to use
- The **short-run** is a circumstance in which a firm is **restricted** in some way in its choice of **at least one input level**.
 - They have already purchased machines, and can now only decide how many workers to hire
 - Notice that there are **many possible** short runs

The Long-Run and the Short-Runs

- Notice, there are other possible causes of the firm being in a 'short run' situation
- i.e. being unable to change one of its inputs:
 - temporarily being unable to install, or remove, machinery
 - being required by law to meet affirmative action quotas
 - having to meet domestic content regulations.

The Long-Run and the Short-Runs

- What do short-run restrictions imply for a firm's technology?
- Suppose the short-run restriction is fixing the level of input 2.
- Input 2 is thus a **fixed input** in the short-run. Input 1 remains **variable**.

The Long-Run and the Short-Runs

$$y = x_1^{1/3} x_2^{1/3}$$

is the long-run production function (both x_1 and x_2 are variable).

The short-run production function when

$x_2 = 1$ is

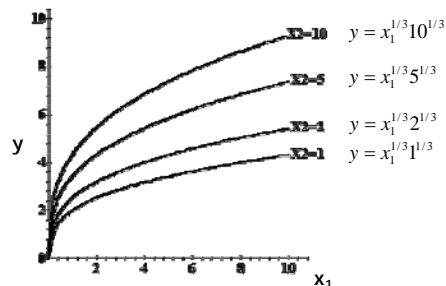
$$y = x_1^{1/3} 1^{1/3} = x_1^{1/3}.$$

The short-run production function when

$x_2 = 10$ is

$$y = x_1^{1/3} 10^{1/3}.$$

The Long-Run and the Short-Runs



Four short-run production functions

The Short and the Long Run

Cost Minimization in the Short and the Long Run

Short-Run & Long-Run Total Costs

- In the long-run a firm can vary all of its input levels.
- Consider a firm that cannot change its input 2 level from x_2' units.
- How does the short-run total cost of producing y output units compare to the long-run total cost of producing y units of output?

Short-Run & Long-Run Total Costs 13

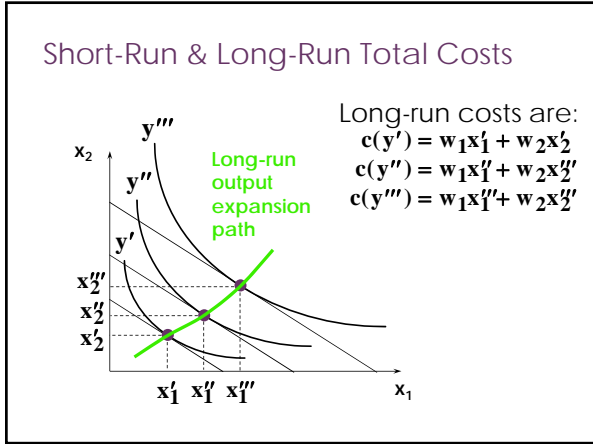
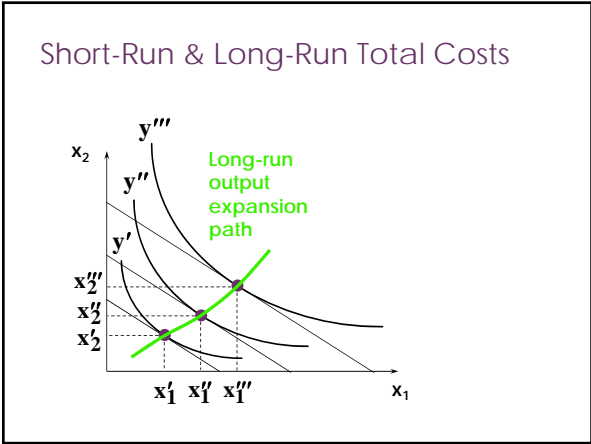
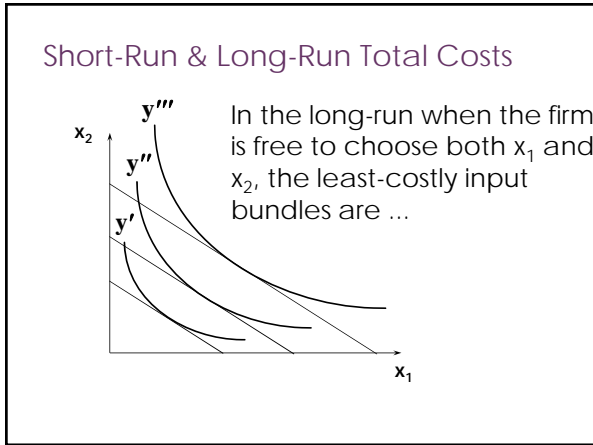
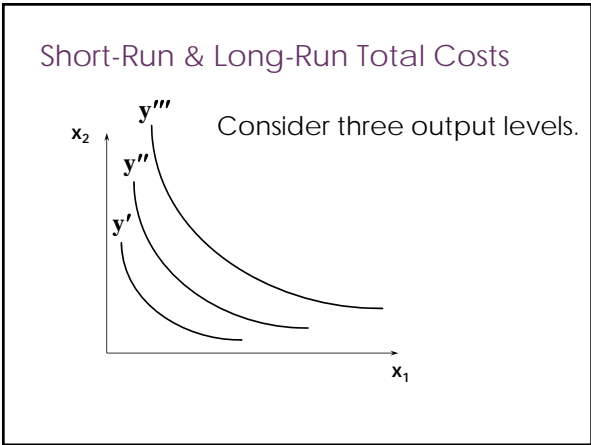
- The long-run cost-minimization problem is

$$\min_{x_1, x_2 \geq 0} p_1 x_1 + p_2 x_2$$
 subject to $f(x_1, x_2) = y$.
- The short-run cost-minimization problem is

$$\min_{x_1 \geq 0} p_1 x_1 + p_2 x'_2$$
 subject to $f(x_1, x'_2) = y$.

Short-Run & Long-Run Total Costs

- The short-run cost-min. problem is the long-run problem subject to the extra constraint that $x_2 = x'_2$.
- How does this affect costs?
 - If the long-run choice for x_2 was x'_2 then the extra constraint $x_2 = x'_2$ is not really a constraint at all
 - Long-run and short-run total costs of producing y output units are the same.
 - But, if the long-run choice for $x_2 \neq x'_2$ then the extra constraint $x_2 = x'_2$ prevents the firm from achieving its long-run production cost
 - Short-run total cost **exceed** the long-run total cost of producing y output units.



Short-Run & Long-Run Total Costs

- Now suppose the firm becomes subject to the short-run constraint that $x_2 = x_2''$
- Denote by $c_s(y)$ the corresponding short run cost function

Short-Run & Long-Run Total Costs

Long-run costs are:
 $c(y') = w_1x_1' + w_2x_2'$
 $c(y'') = w_1x_1'' + w_2x_2''$
 $c(y''') = w_1x_1''' + w_2x_2''$

Short-Run & Long-Run Total Costs

Long-run costs are:
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Short-Run & Long-Run Total Costs

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Short-run costs are:
 $c_s(y') > c(y')$

Short-Run & Long-Run Total Costs

Long-run costs are:
 $c(y') = w_1x_1' + w_2x_2'$
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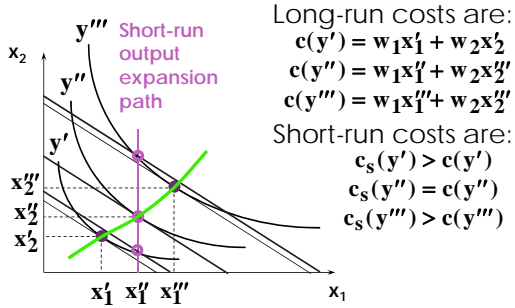
Short-run costs are:
 $c_s(y') > c(y')$
 $c_s(y'') = c(y'')$

Short-Run & Long-Run Total Costs

Long-run costs are:
 $c(y') = w_1x_1' + w_2x_2'$
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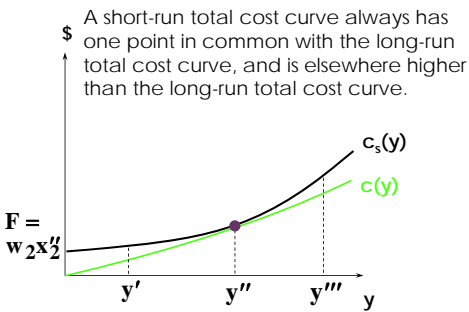
Short-Run & Long-Run Total Costs



Short-Run & Long-Run Total Costs

- Short-run total cost exceeds long-run total cost except for the output level where the short-run input level restriction is the long-run input level choice.
- This says that the long-run total cost curve always has **one point in common** with any particular short-run total cost curve.

Short-Run & Long-Run Total Costs



The Short and the Long Run

Cost Curves in the Short and the Long Run

Types of Cost Curves

- We are now going to think a little bit more about the **cost curves** of a firm
- In order to do so, we are going to differentiate between two different types of cost
 - Fixed Costs: these do not change regardless of how much the firm produces
 - Variable Costs: these do change depending on how much the firm produces
- Typically, in the long run, all costs are variable
 - If the firm produces no output it uses no input
- In the short run, the firm may have some fixed costs
 - If they are 'forced' to use a certain amount of one input, then they have to pay for that input regardless of how much they produce

Types of Cost Curves

- We now have lots of different types of costs
 - Total vs Fixed vs Variable
 - Long run vs Short run
 - Costs vs Average Costs vs Marginal Costs
- How are these cost curves related to each other?

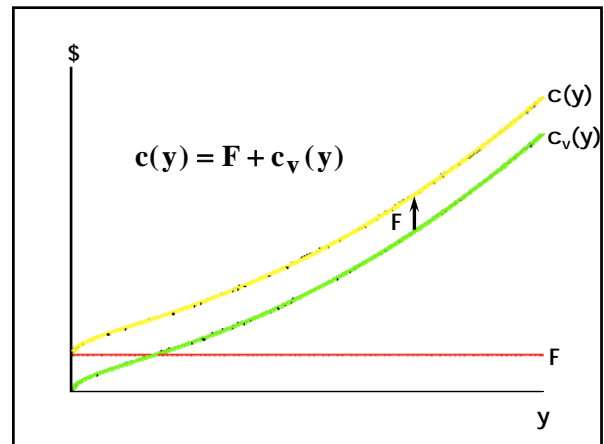
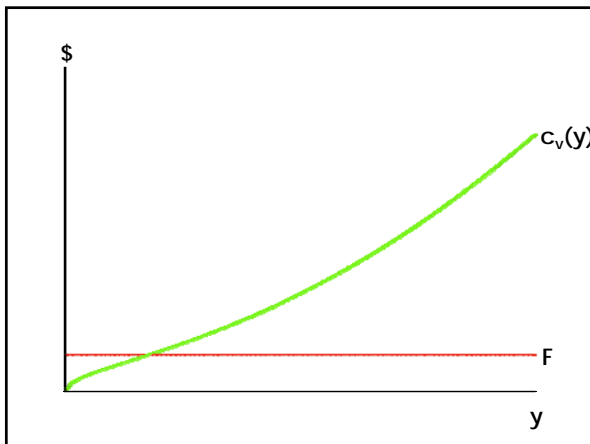
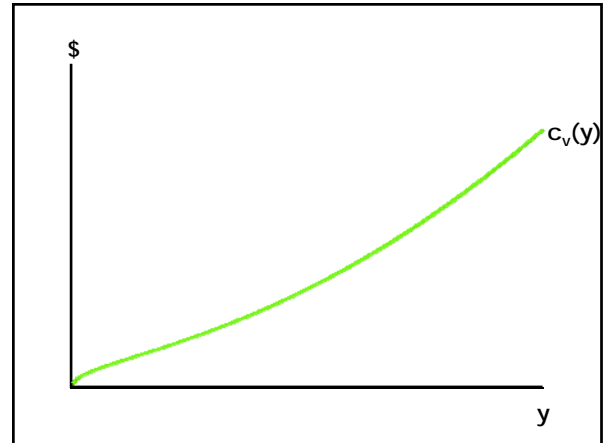
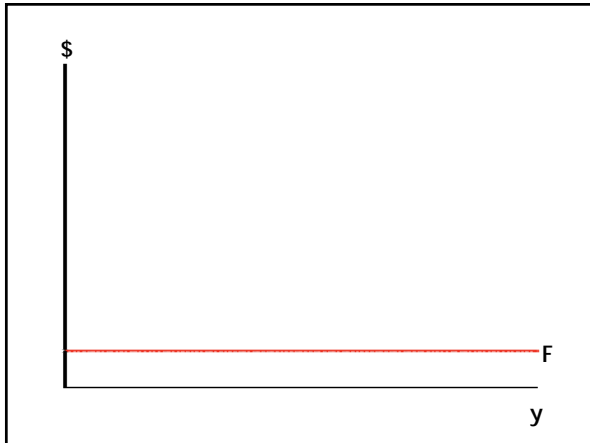
Fixed, Variable & Total Cost Functions

- F is the total cost to a firm of its short-run fixed inputs. F , the firm's fixed cost, does not vary with the firm's output level.
- $c_v(y)$ is the total cost to a firm of its variable inputs when producing y output units. $c_v(y)$ is the firm's variable cost function.
 - $c_v(y)$ depends upon the levels of the fixed inputs.
- $c(y)$ is the total cost of all inputs, fixed and variable, when producing y output units. $c(y)$ is the firm's total cost function;

$$c(y) = F + c_v(y).$$

Fixed, Variable & Total Cost Functions

- What do these various cost curves look like?
 - Fixed
 - Variable
 - Total



Av. Fixed, Av. Variable & Av. Total Cost Curves

- What about average costs? (Remember $AC(y) = c(y)/y$)
- For $y > 0$, the firm's average total cost function is

$$AC(y) = \frac{F}{y} + \frac{c_v(y)}{y}$$

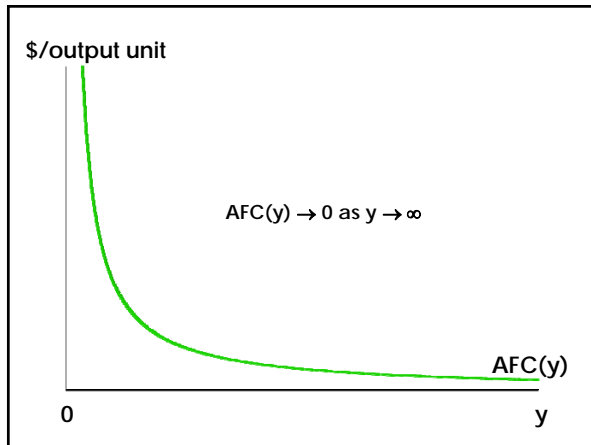
$$= AFC(y) + AVC(y).$$

Av. Fixed, Av. Variable & Av. Total Cost Curves

- What does an average fixed cost curve look like?

$$AFC(y) = \frac{F}{y}$$

- AFC(y) is a rectangular hyperbola so its graph looks like ...



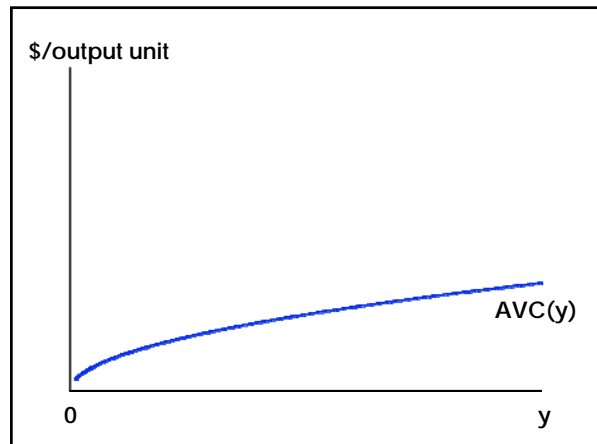
Av. Fixed, Av. Variable & Av. Total Cost Curves

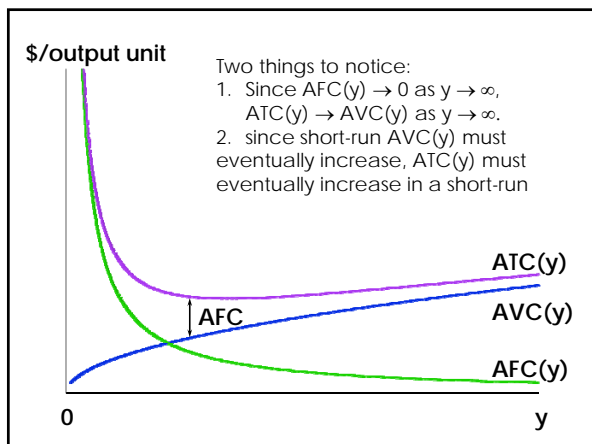
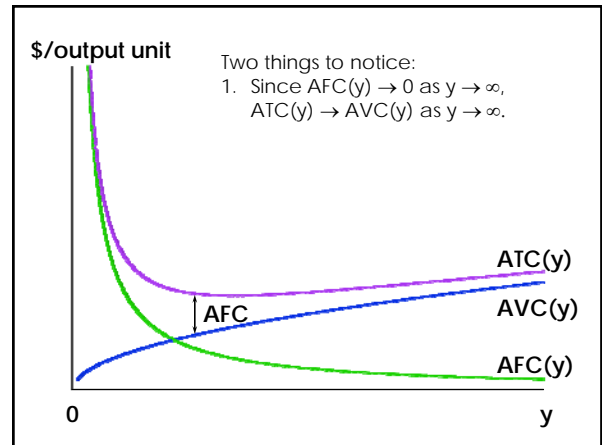
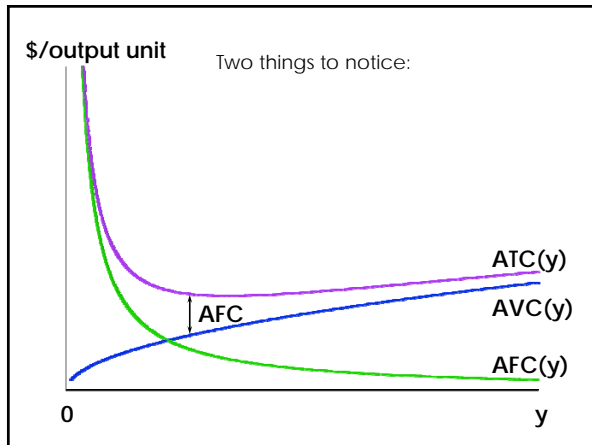
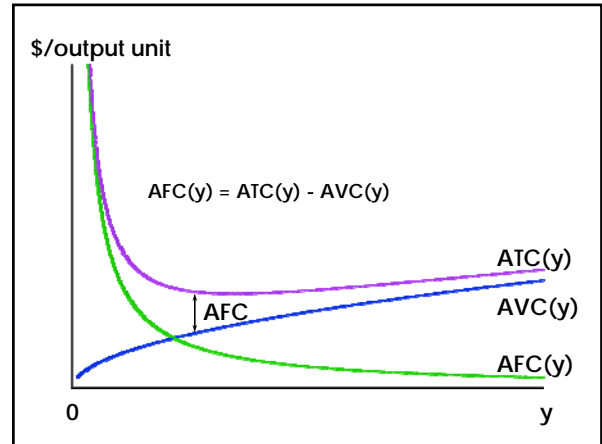
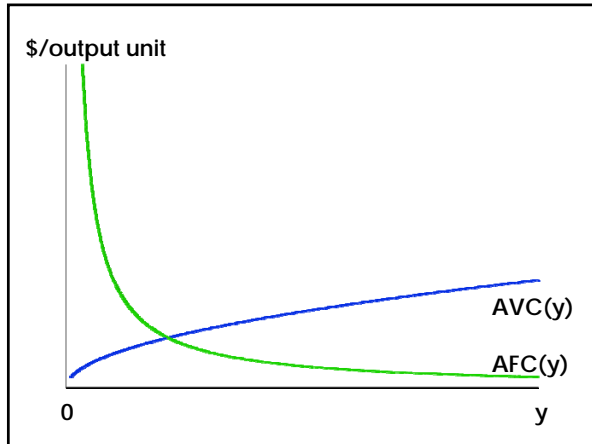
- What about average variable costs?
- Well, as we have seen, this will depend on whether the firm has **increasing, decreasing, or constant returns to scale**
- In the short run, at least some of the inputs are fixed
- We therefore typically assume that there will be **diminishing returns to scale** (at least eventually)
 - If we fix the number of computers, at some point we will have decreasing returns to scale if we keep adding economists

Av. Fixed, Av. Variable & Av. Total Cost Curves

- Think of a Cobb Douglas Production function

$$y = x_1^{a_1} x_2^{a_2}$$
- If $a_1 < 1$ then the firm will exhibit decreasing returns to scale in the short run
 - i.e. if x_2 is fixed at \bar{x}_2
 - $f(kx_1) = (kx_1)^{a_1} \bar{x}_2^{a_2} = k^{a_1} x_1^{a_1} < kf(x_1)$
- This is true even if the firm exhibits increasing returns to scale in the long run
 - i.e. $a_1 + a_2 > 1$





Marginal Cost Function

- What about marginal cost?
- Well, marginal fixed cost is zero
- So marginal costs are just equal to marginal variable costs

$$MC(y) = \frac{\partial c_v(y)}{\partial y}$$

Relationship Between Marginal and Total Cost

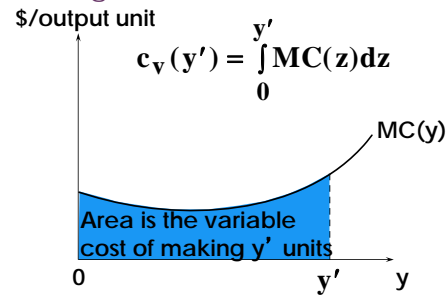
- Since $MC(y)$ is the derivative of $c_v(y)$, $c_v(y)$ must be the integral of $MC(y)$.
- Fundamental Theorem of Calculus

$$MC(y) = \frac{\partial c_v(y)}{\partial y}$$

$$\Rightarrow c_v(y) = \int_0^y MC(z) dz + c_v(0)$$

$$\Rightarrow c(y) = \int_0^y MC(z) dz + F$$

Marginal and Variable Cost



Marginal & Average Cost Functions

- How is marginal cost related to average variable cost?

Marginal & Average Cost Functions

Since $AVC(y) = \frac{c_v(y)}{y}$,

$$\frac{\partial AVC(y)}{\partial y} = \frac{y \times MC(y) - 1 \times c_v(y)}{y^2}.$$

Marginal & Average Cost Functions

Since $AVC(y) = \frac{c_v(y)}{y}$,

$$\frac{\partial AVC(y)}{\partial y} = \frac{y \times MC(y) - 1 \times c_v(y)}{y^2}.$$

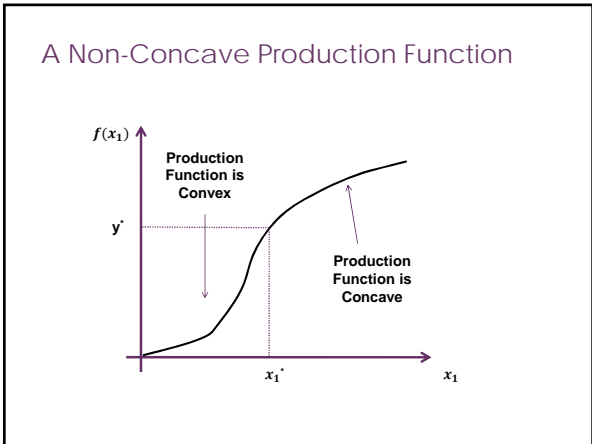
Therefore,

$$\frac{\partial AVC(y)}{\partial y} > 0 \text{ as } MC(y) > \frac{c_v(y)}{y} = AVC(y).$$

$$\frac{\partial AVC(y)}{\partial y} < 0 \text{ as } MC(y) < \frac{c_v(y)}{y} = AVC(y).$$

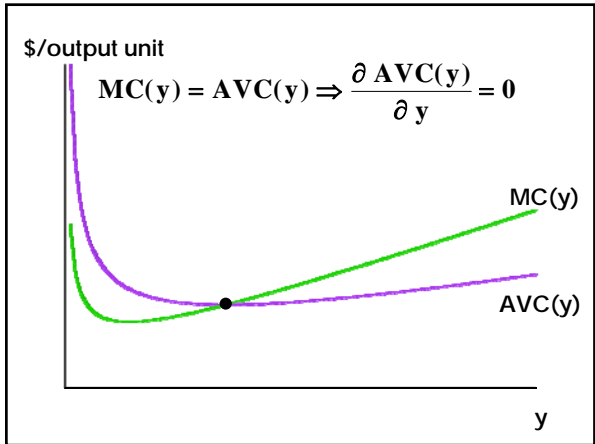
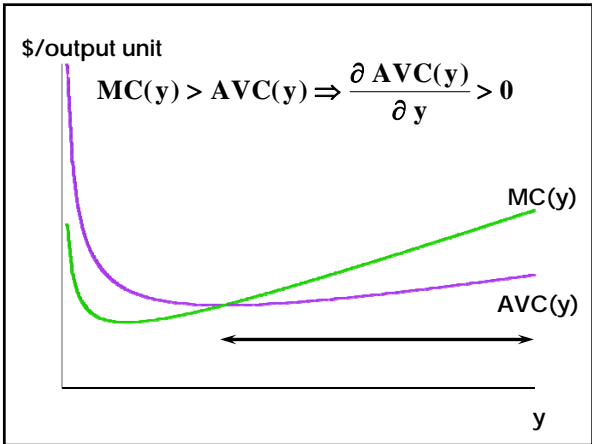
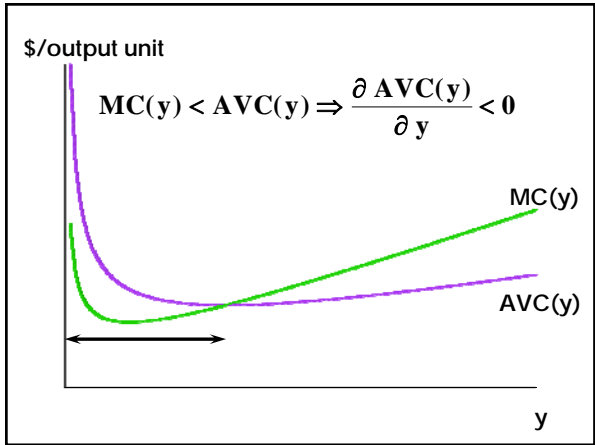
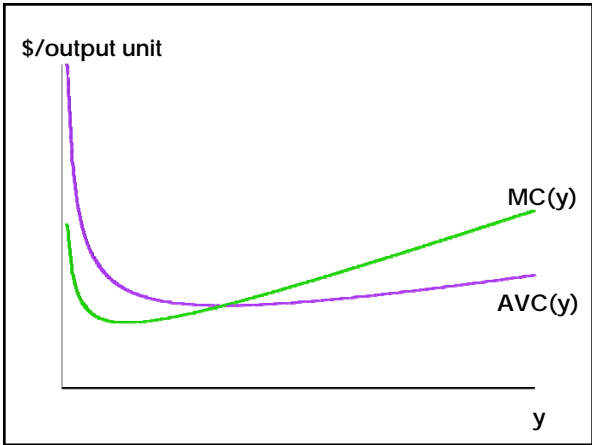
Av. Fixed, Av. Variable & Av. Total Cost Curves

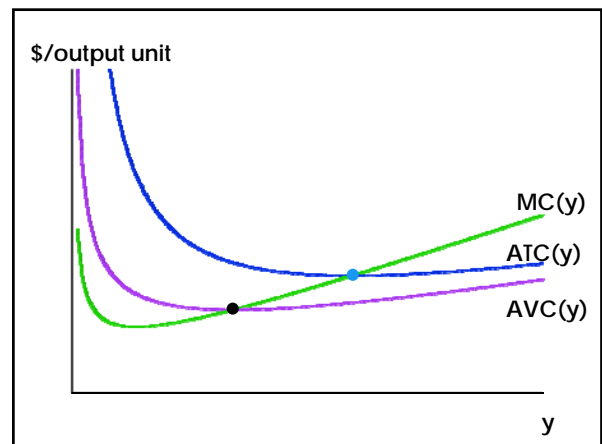
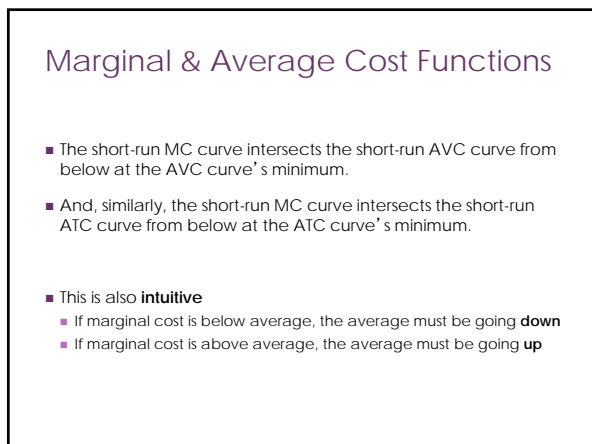
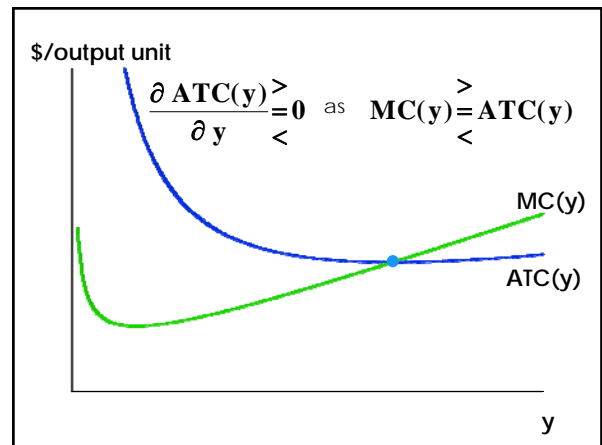
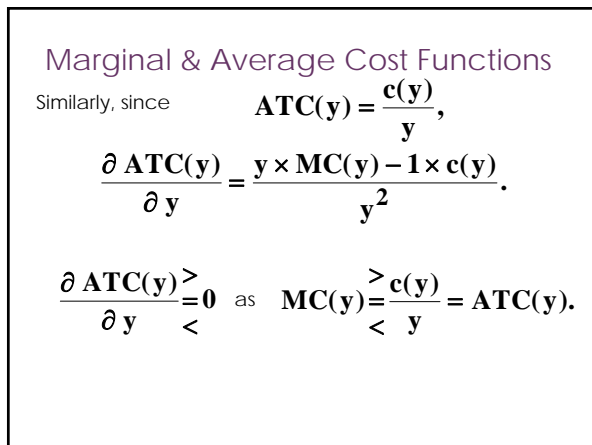
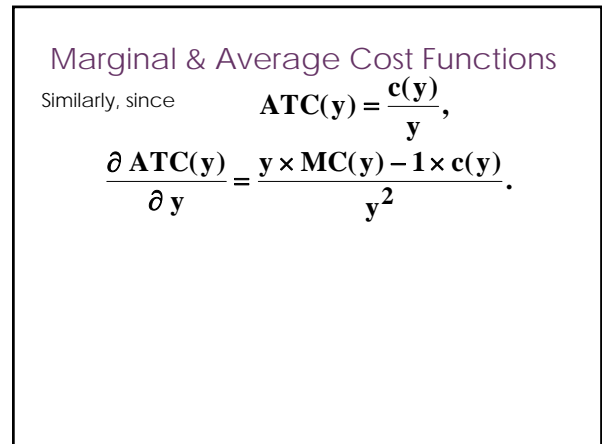
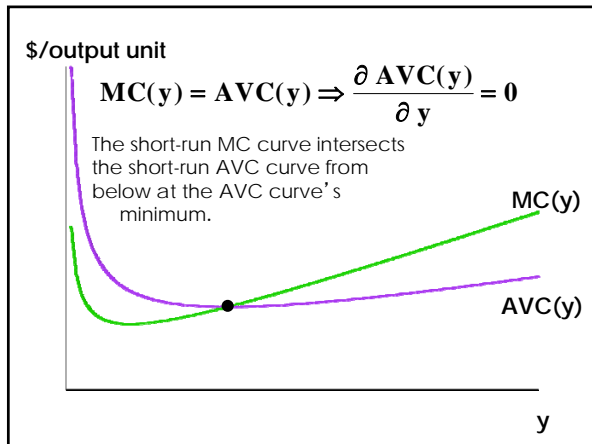
- What does this look like in practice?
- To make things more interesting, let's think about a production function which has both **increasing** and **decreasing** returns to scale



Av. Fixed, Av. Variable & Av. Total Cost Curves

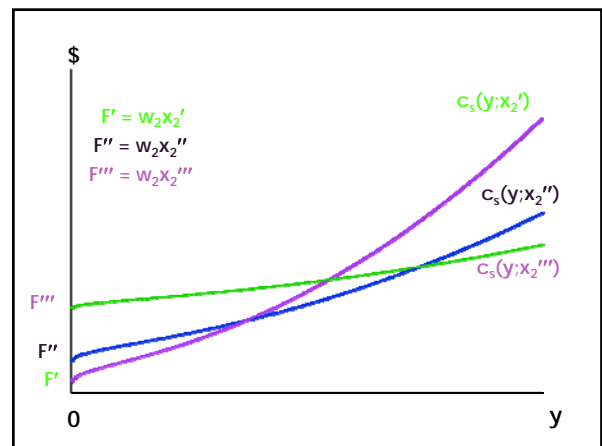
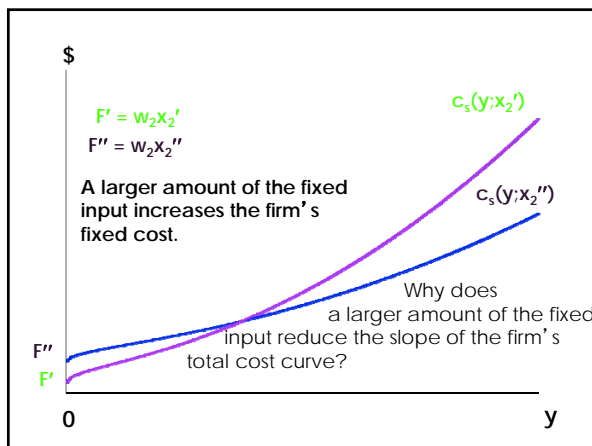
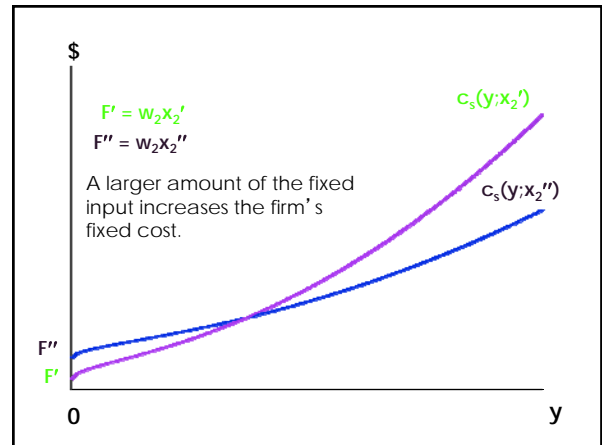
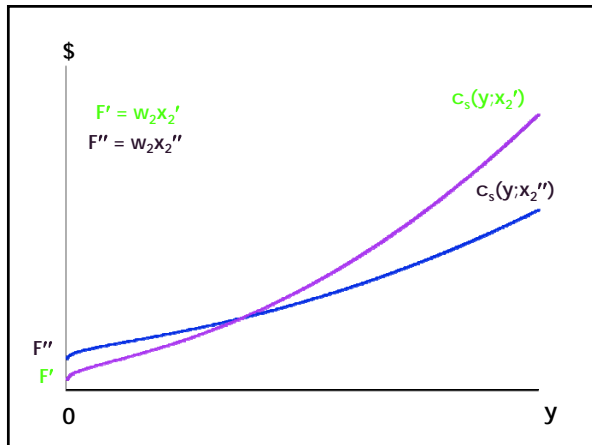
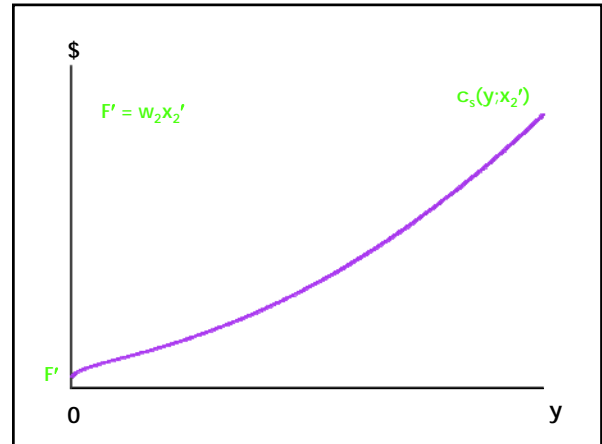
- What does this look like in practice?
- To make things more interesting, let's think about a production function which has both **increasing** and **decreasing** returns to scale
- Implies Marginal Cost first decreases then increases





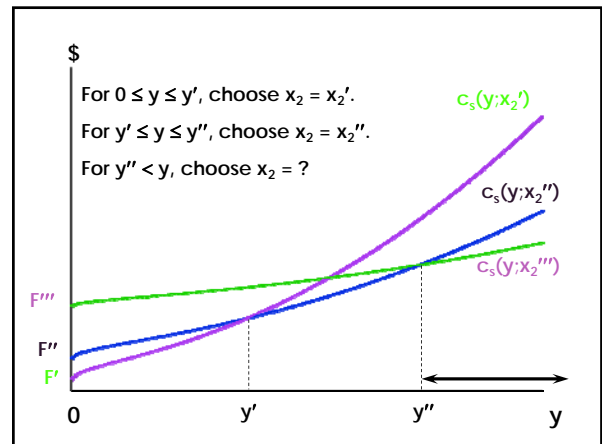
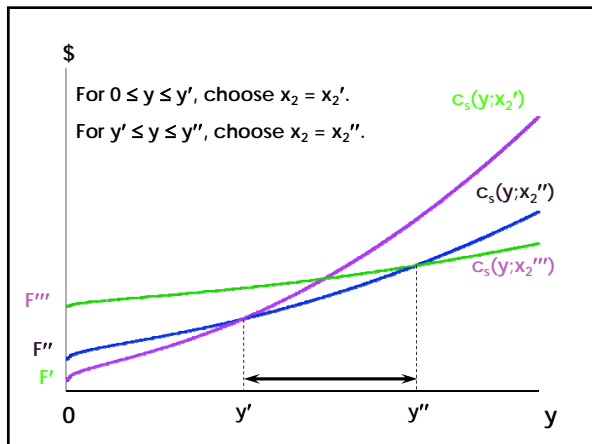
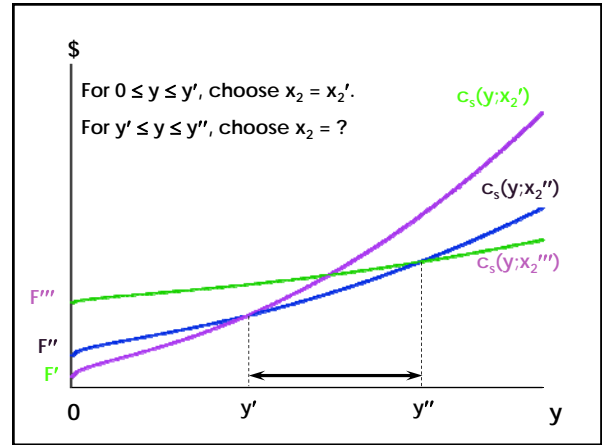
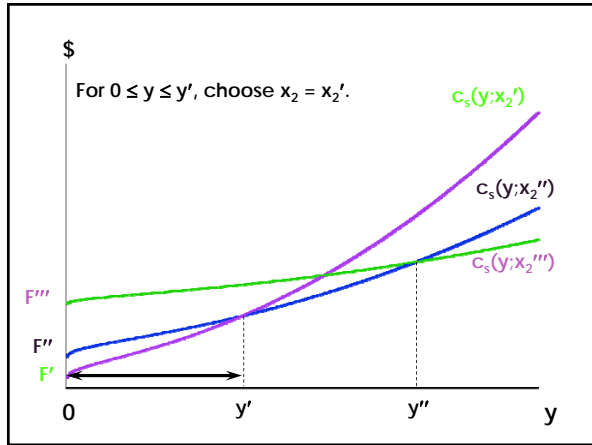
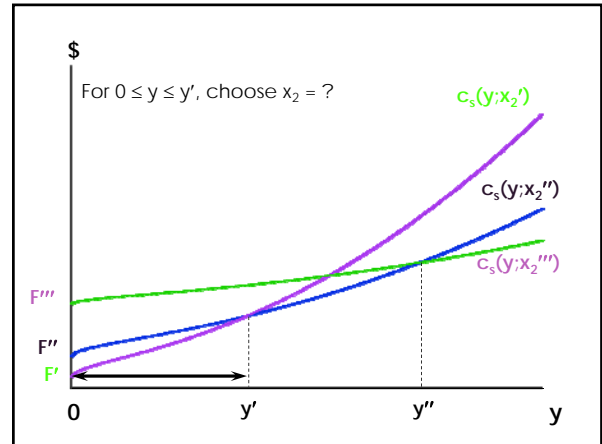
Short-Run & Long-Run Total Cost Curves

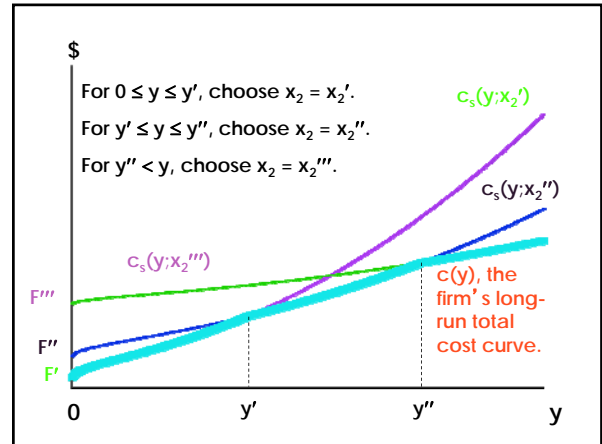
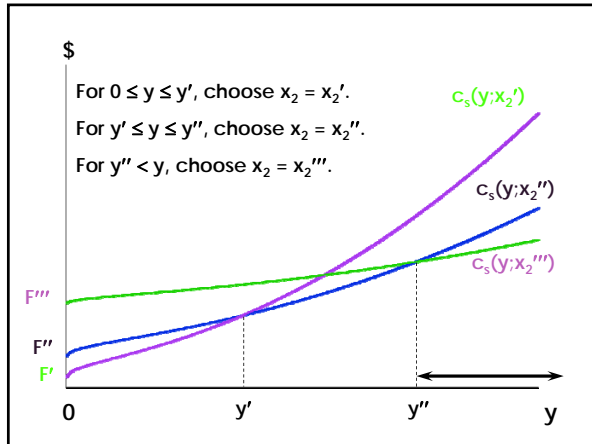
- Remember, a firm has a different short-run total cost curve for each possible short-run circumstance.
 - By 'circumstance' we mean 'level of the fixed input'
- Suppose the firm can be in one of just three short-runs:
 - $x_2 = x_2'$
 - or $x_2 = x_2''$ $x_2' < x_2'' < x_2'''$.
 - or $x_2 = x_2'''$.



Short-Run & Long-Run Total Cost Curves

- The firm has three short-run total cost curves.
- In the long-run the firm is free to choose amongst these three since it is free to select x_2 equal to any of x_2^1 , x_2^2 , or x_2^3 .
- How does the firm make this choice?



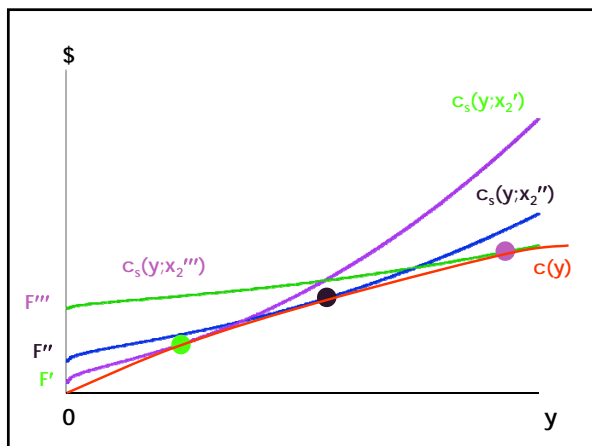


Short-Run & Long-Run Total Cost Curves

- The firm's long-run total cost curve consists of the lowest parts of the short-run total cost curves. The long-run total cost curve is the **lower envelope** of the short-run total cost curves.

Short-Run & Long-Run Total Cost Curves

- If input 2 is available in continuous amounts then there is an infinity of short-run total cost curves but the long-run total cost curve is still the lower envelope of all of the short-run total cost curves.

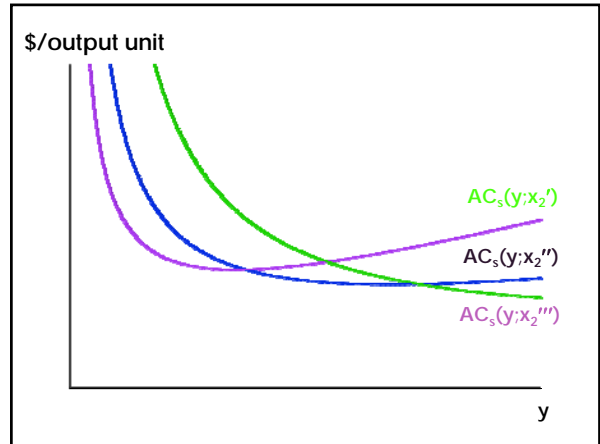


Short-Run & Long-Run Average Total Cost Curves

- For any output level y , the long-run total cost curve always gives the lowest possible total production cost.
- Therefore, the long-run av. total cost curve must always give the lowest possible av. total production cost.
- The long-run av. total cost curve must be the lower envelope of all of the firm's short-run av. total cost curves.

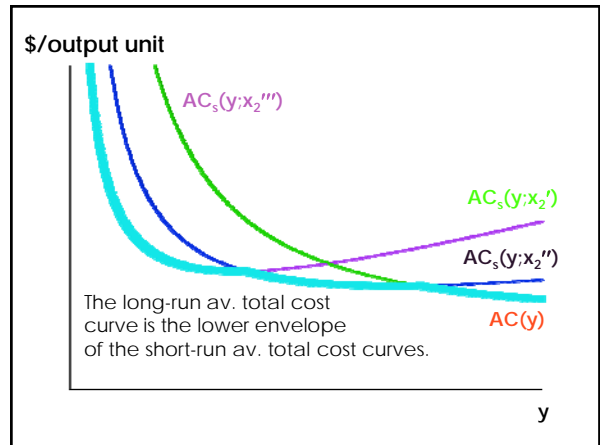
Short-Run & Long-Run Average Total Cost Curves

- E.g. suppose again that the firm can be in one of just three short-runs:
 $x_2 = x_2'$
 or $x_2 = x_2''$ ($x_2' < x_2'' < x_2'''$)
 or $x_2 = x_2'''$
 then the firm's three short-run average total cost curves are ...



Short-Run & Long-Run Average Total Cost Curves

- The firm's long-run average total cost curve is the lower envelope of the short-run average total cost curves ...



Short-Run & Long-Run Marginal Cost Curves

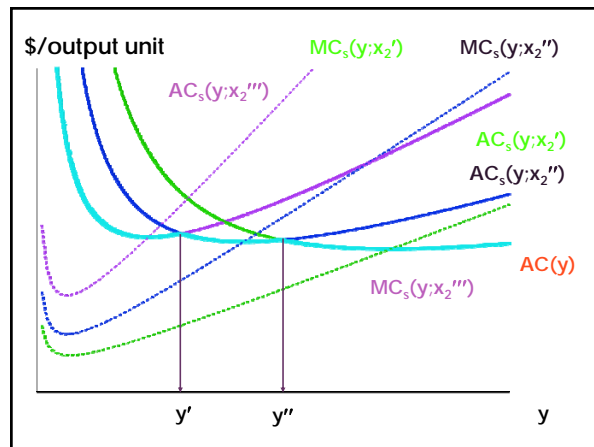
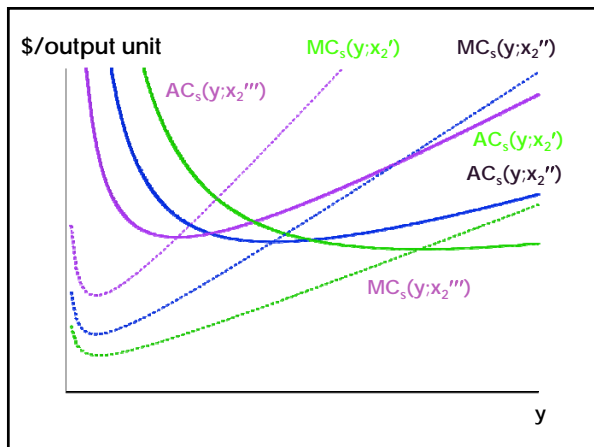
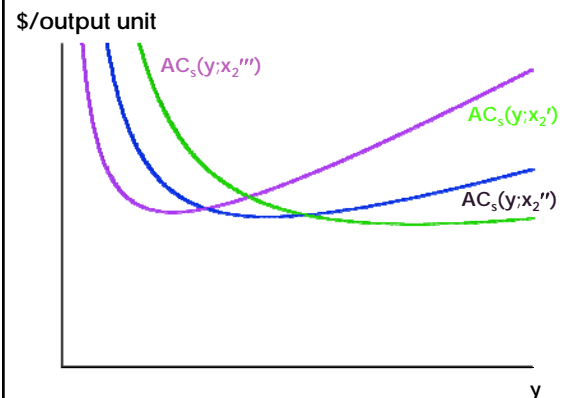
- Q: Is the long-run marginal cost curve the lower envelope of the firm's short-run marginal cost curves?

Short-Run & Long-Run Marginal Cost Curves

- Q: Is the long-run **marginal cost curve** the lower envelope of the firm's short-run **marginal cost curves**?
- A: No.

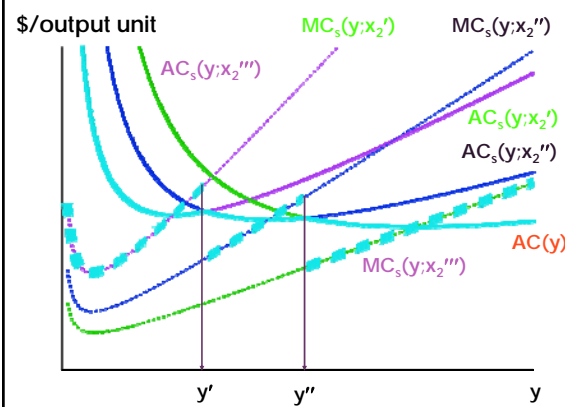
Short-Run & Long-Run Marginal Cost Curves

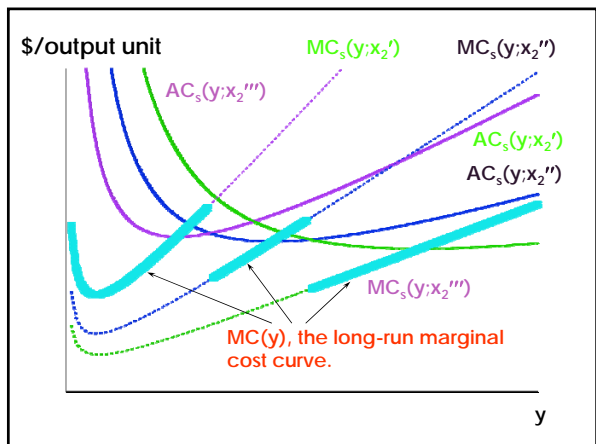
■ The firm's three short-run average total cost curves are ...



+ Short-Run & Long-Run Marginal Cost Curves

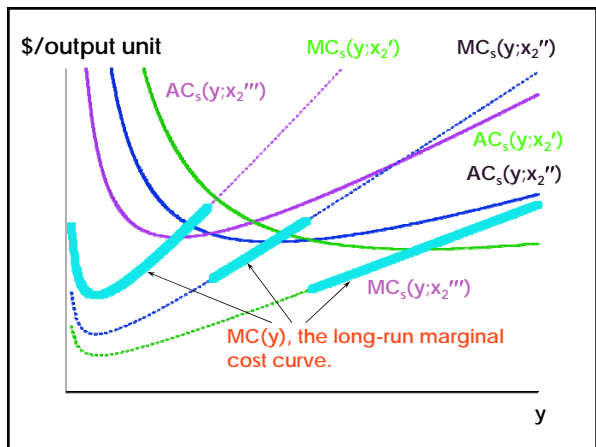
- Below y' , will choose x_1' , despite the fact it has higher marginal cost than either x_1'' or x_1'''
- Because it gives lower total cost





Short-Run & Long-Run Marginal Cost Curves

- For any output level $y > 0$, the long-run marginal cost of production is the marginal cost of production for the short-run chosen by the firm.

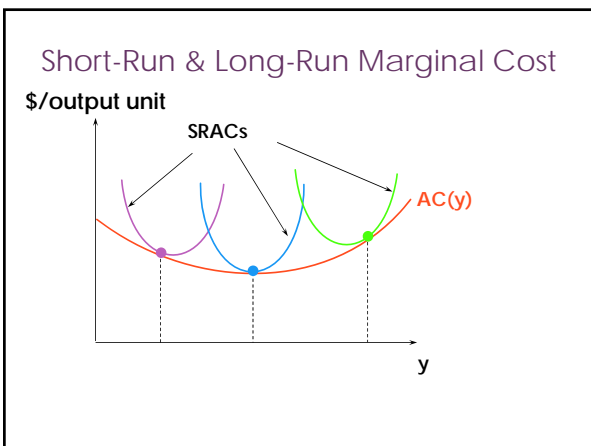


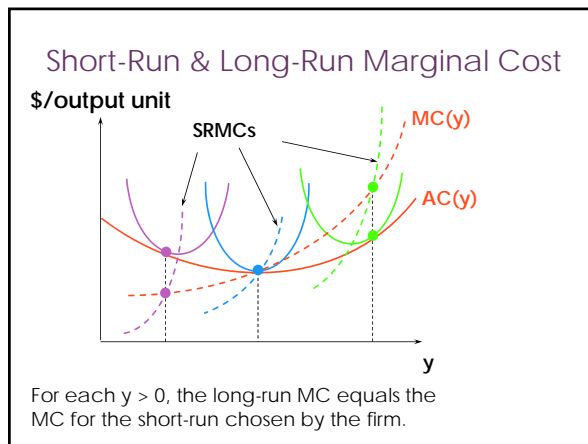
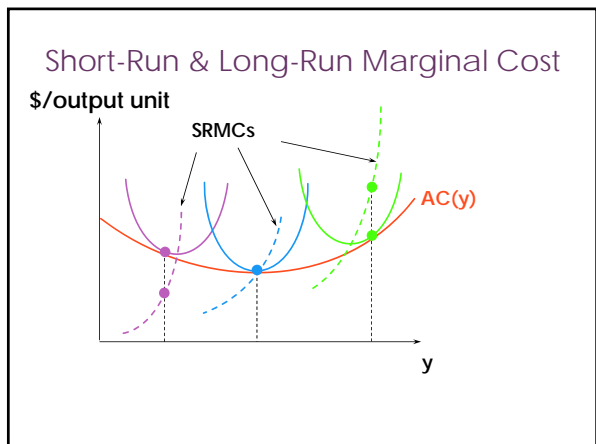
Short-Run & Long-Run Marginal Cost

- For any output level $y > 0$, the long-run marginal cost is the marginal cost for the short-run chosen by the firm.
- This is always true, no matter how many and which short-run circumstances exist for the firm.

Short-Run & Long-Run Marginal Cost

- For any output level $y > 0$, the long-run marginal cost is the marginal cost for the short-run chosen by the firm.
- So for the continuous case, where x_2 can be fixed at any value of zero or more, the relationship between the long-run marginal cost and all of the short-run marginal costs is ...





Summary

- Summary
- Today we focused on the difference between the short and the long run
 - How technology changes
 - How costs change