Intermediate Microeconomics W3211

Lecture 23: Uncertainty and Information 1: Expected Utility Theory

Columbia University, Spring 2016
Mark Dean: mark.dean@columbia.edu

Introduction

The Story So Far....

- We have spent a lot of time modelling the choices made by
  - People
  - Firms
- In doing so, we have always assumed that people knew for sure what the outcome of those choices were
  - If you buy a commodity bundle, you get those things for sure
  - If a firm produces y output, they are 100% certain that they will be able to sell them at price p

Today

- In many cases the outcome of our choices are **uncertain**
- For example
  - You are deciding whether or not to buy shares in Apple
  - You are deciding whether to gamble your student loan on black on the roulette table
  - You are deciding whether or not to buy beachfront property in Miami
- The aim of this lecture is to think about how we model such choices
- Chapter 12 Varian, Chapter 19 Feldman and Serrano

Choice under Uncertainty

What should I maximize?

What Should I Maximize?

- When we thought about how consumers behave, we had a very specific model
  - They should make choices to maximize their preference (or utility)
- How can we extend this model to choices over things which are uncertain?

What Should I Maximize?

- Here is an example of a very boring fairground game
  - You pay an amount x to play the game
  - If you play, the fairground person flips a (fair) coin
  - If it comes down heads you win $10
  - If it comes down tails, you win $0
- So
  - If you don't play the game you get $0 for sure
  - If you play the game, with 50% chance you get $10-x and with 50% chance you get –x
- What is the most you would pay in order to play such a game?
  - i.e. how big an x?

What Should I Maximize?

- $5?
- This is the **expected (or mean) value** of the game

Prob($10).10+Prob($0).0

What Should I Maximize?

- Okay, here is a new game
  - The fairground person flips a coin
  - If it comes down tails you get $2
  - If it comes down heads, the coin gets flipped again
  - If it comes down tails, you get $4
  - If it comes down heads, the coin gets flipped again
  - If it comes down tails you get $8
  - If it come down heads, the coin gets flipped again
  - Etc. etc

- How much would you pay to play the game?

What Should I Maximize?

- Well, what is the expected value?

Prob($2).2+Prob($4).4+Prob($8).8…

$$\frac{1}{2}.2+\frac{1}{4}.4+\frac{1}{8}.8+\frac{1}{16}.16$$

1+1+1+1….= ∞!

- Would you pay a million dollars to play this game?
- Would you pay a billion?

What Should I Maximize?

- Here is another question
- Let's say a pauper finds a magic lottery ticket, which pays $1,000,000 with 50% chance and $0 otherwise
- A wealthy toff offers them $475,000 for the lottery ticket
- Should they accept the offer?
- If they are maximizing expected value they should not!
- Is this sensible?

Expected Utility

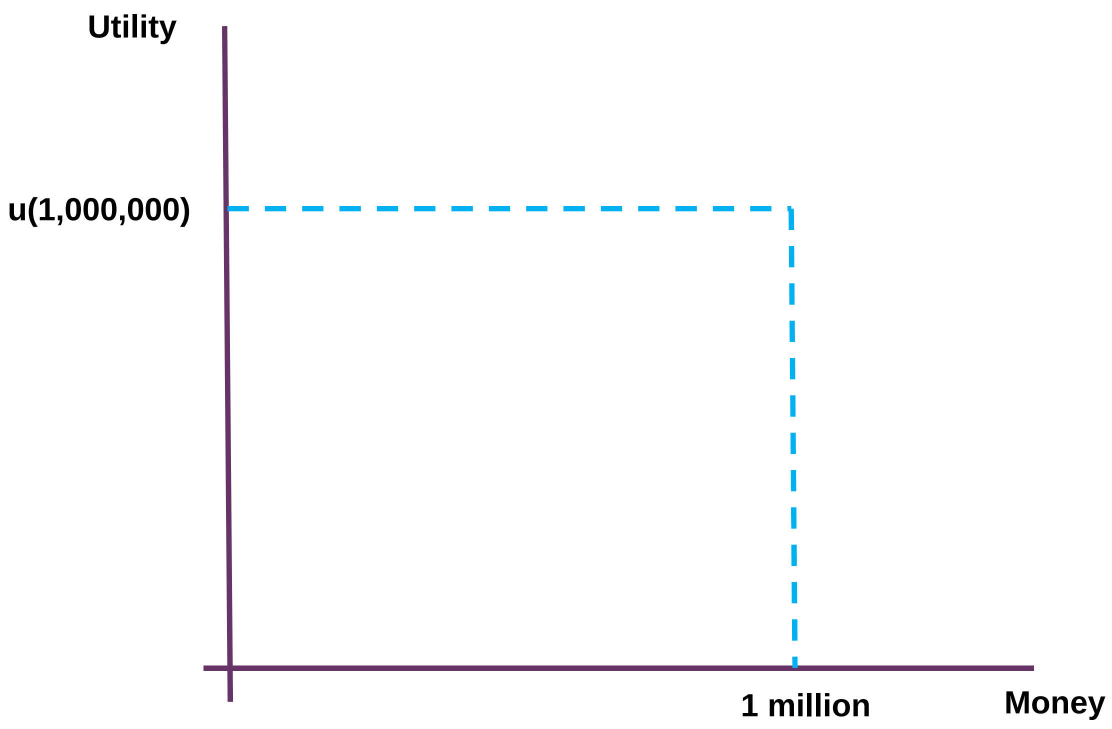
- Most of economics assumes that what you maximize is not expected **value**, but expected **utility**
- So the pauper should figure out the utility they get from $0, the utility they get from $475,000 and the utility they get from $1,000,000
- Then compare $\frac{1}{2}u(0) + \frac{1}{2}u(1{,}000{,}000)$ to $u(475{,}000)$
- Compare the **expected utility** of the gamble to the **expected utility** of the sure thing

Expected Utility and Risk Aversion

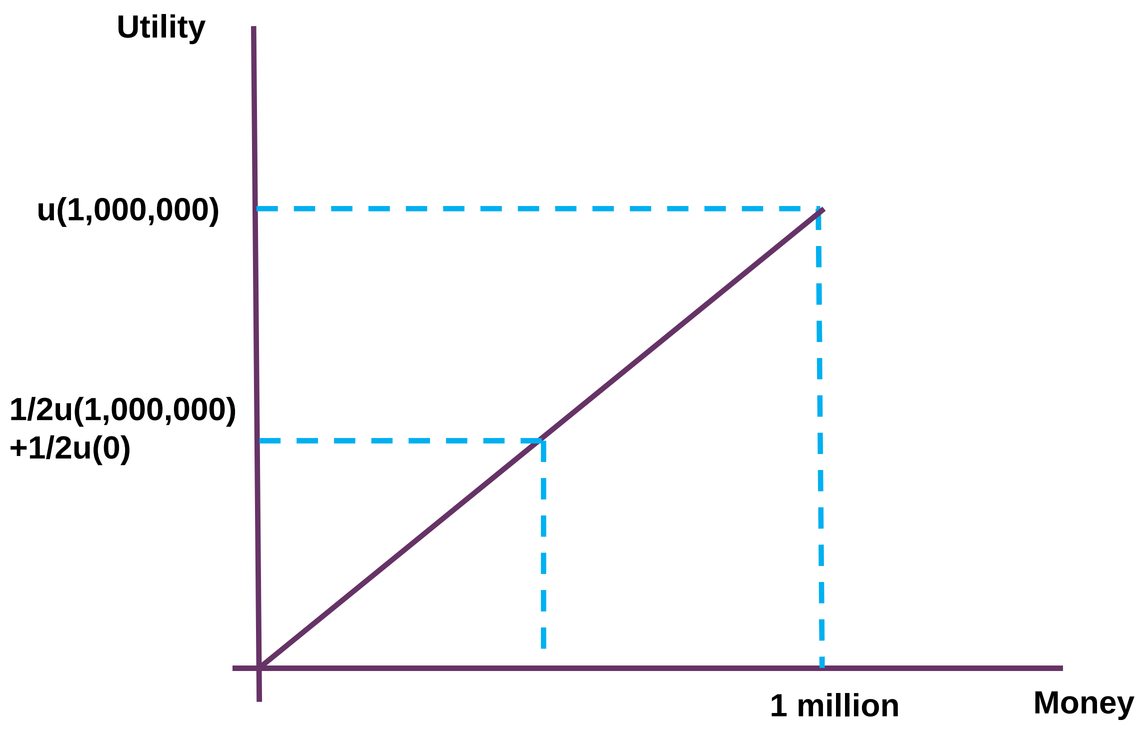
- Question: when would they prefer the sure thing to the gamble, even though the gamble has a higher **expected value?**
- Answer: when their utility function is **concave**

Expected Utility and Risk Aversion

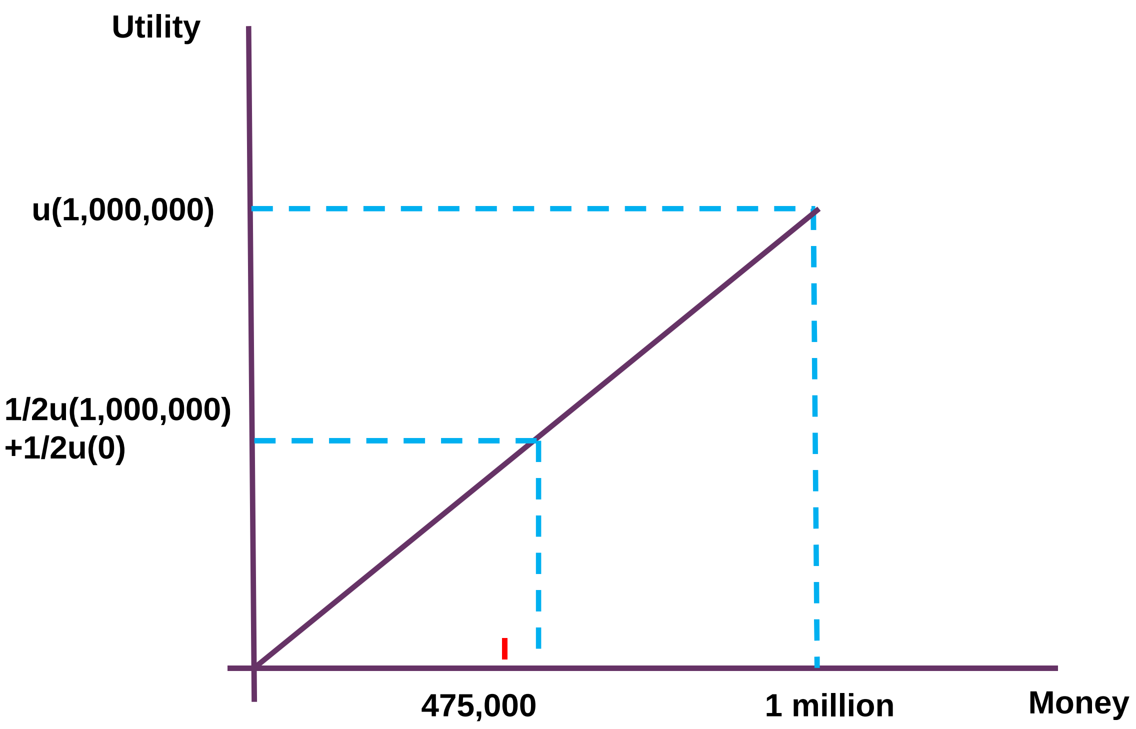
Expected Utility and Risk Aversion



Expected Utility and Risk Aversion

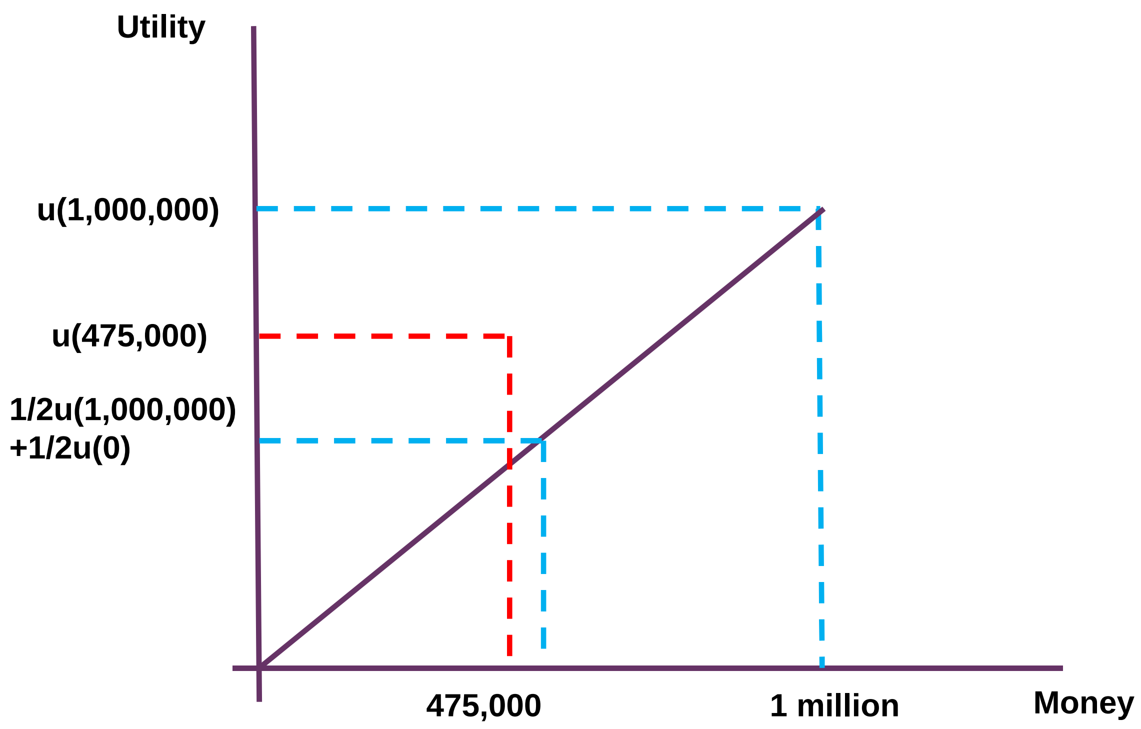


Expected Utility and Risk Aversion

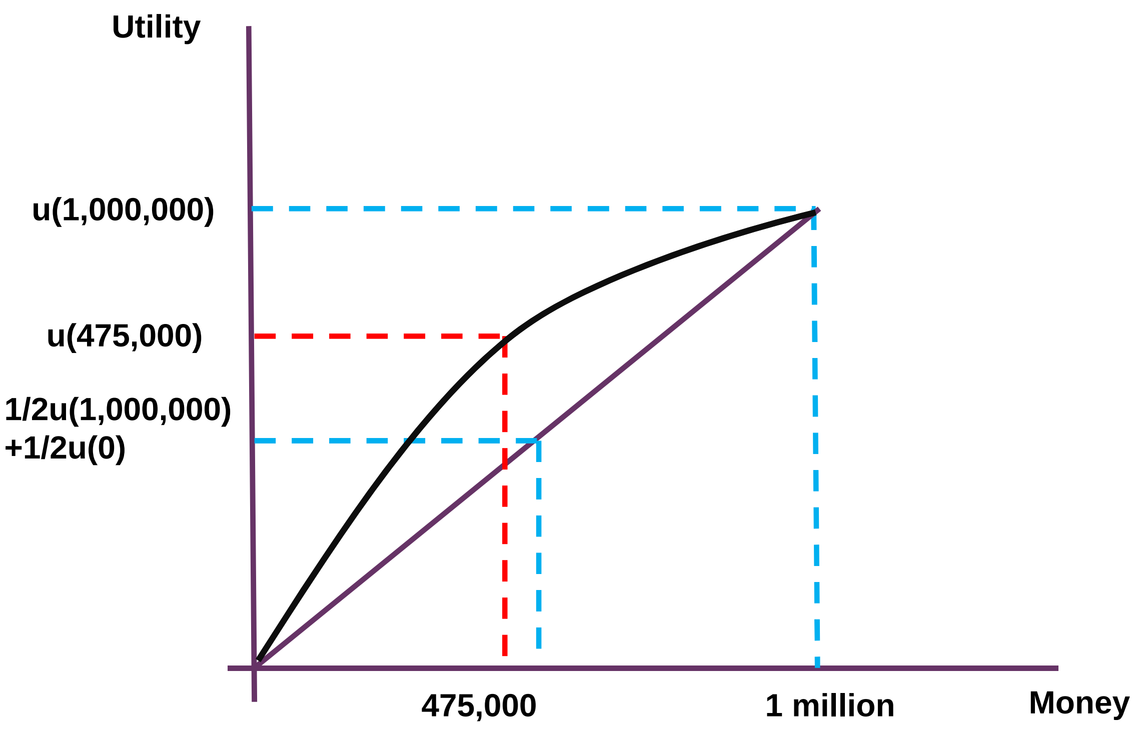


- If pauper prefers to get the money for sure, u(475,000) must be above 1/2u(1,000,000)+1/2u(0)

Expected Utility and Risk Aversion



Expected Utility and Risk Aversion



- Requires utility function to be concave

Expected Utility and Risk Aversion

- Intuition is as follows
- The additional utility from getting $475,000 relative to $0 is huge
- The additional utility from getting $1,000,000 relative to $475,000 is much smaller
- So the additional **utility** gained from winning the lottery is relatively small
- Not worth the additional risk

Expected Utility and Risk Aversion

- This is the idea of **diminishing marginal utility of wealth**
  - The utility from an additional dollar is lower when you are rich than when you are poor
- Diminishing marginal utility is exactly the same as saying the utility function is concave
  - Slope of the utility function is decreasing

Expected Utility and Risk Aversion

- Definition: we say someone is **risk averse** if, for any lottery, they prefer to receive the expected value of that lottery for sure than play the lottery
  - E.g., for a lottery which pays 1/3 $30, 1/3 $15, 1/3 $0,
  - They would prefer $15 for sure
- An expected utility maximizer is risk averse if and only if they have a concave utility function
  - i.e. decreasing marginal utility of money

Summary

Summary

- We have introduced the idea of expected utility as a way of modelling the way people make choices over risky options