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Intermediate Microeconomics W3211

Lecture 24: Uncertainty and Information 2

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Introduction

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The Story So Far....

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- Last lecture we started to think about how to model choice over options the outcomes of which are **uncertain**
  - Includes obvious cases such as investing or gambling
  - But almost all choices contain **some** uncertainty
- Our first model was that people should make choices to maximize the expected (or average) value of the outcomes
  - However, we showed that this lead to some bad predictions
- We therefore suggested that people should maximize **expected utility**
  - Figure out the utility of each possible outcome
  - Figure out the expected (or average) utility of each option
  - Choose the option which gives the highest average

Today

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- We will think about what has to be true about preference for them to behave like an expected utility maximizer
- Discuss what happens when there is uncertainty in economic markets
- Varian Ch. 38, Feldman and Serrano Ch. 20

Choice under Uncertainty

Preferences and Expected Utility

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Preferences and Expected Utility

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- Think back to the very start of the course
- When we asked what it is a consumer should maximize, we said that the should choose to maximize preferences
  - Choose the bundle x such that $x \succsim y$ for all available y
- We demanded that preferences be **well behaved:**
  - Reflexivity: $x \succsim x$
  - Transitivity: $x \succsim y$ and $y \succsim z$ implies $x \succsim z$
  - Completeness: $x \succsim y$ or $y \succsim x$ or both

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Preferences and Expected Utility

- In such cases we could represent preferences by a utility function
  - There is a function u such that $x \gtrsim y$ if and only if $u(x) \geq u(y)$
- The consumer could be modelled as a utility maximizer
- But these utility numbers didn't 'mean' very much
  - **Theorem:** Take two utility functions u and v. They both represent the same preferences if and only if there is a strictly increasing function f such that

$$\boldsymbol{v}(\boldsymbol{x}) = \boldsymbol{f}(\boldsymbol{u}(\boldsymbol{x}))$$

for all x

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Preferences and Expected Utility

- We now want to ask the same questions for **expected utility**

1. What has to be true about preferences for them to be represented by an expected utility function?
2. How unique are those utility numbers

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Preferences and Expected Utility

- In order to answer these questions we need to be more precise about what we mean by expected utility
- First, what is it that people are choosing between?
- They are choosing between **lotteries**
- What is a lottery?

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Preferences and Expected Utility

- First, fix a set of possible prizes
  - Amounts of money,
  - Types of fruit
- We will use three prizes a(pple) b(anana) c(anteloupe)
- A lottery is just a **probability of getting each of these prizes**
- Example:

$$p = \begin{pmatrix} p_a \\ p_b \\ p_c \end{pmatrix} = \begin{pmatrix} 0.2 \\ 0.6 \\ 0.2 \end{pmatrix}$$

Is a 20% chance of getting an apple, 60% chance of getting a banana and 20% chance of getting a cantaloupe

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Preferences and Expected Utility

- So what is the expected utility model?
- There is a utility function which assigns utility to **prizes**
  - u(a), u(b), u(c)
- Such that preferences over **lotteries** are represented by the **expectation** of those utilities
- i.e. p $\gtrsim q$ if and only if

$$p_a u(a) + p_b u(b) + p_c u(c) \geq q_a u(a) + q_b u(b) + q_c u(c)$$

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Preferences and Expected Utility

- When will preferences have an expected utility representation?
- Well they must be reflexive, transitive and complete
  - An expected utility representation is still a utility representation
- Is that enough?
- No, we need one more thing: Independence!
- if p $\gtrsim q$ then

$$\alpha p + (1-\alpha) r \gtrsim \alpha p + (1-\alpha) r$$

For any α between 0 and 1, and any other lotter r

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Independence

- What does this mean?
- Well, take two lotteries

$$p = \begin{pmatrix} p_a \\ p_b \\ p_c \end{pmatrix} = \begin{pmatrix} 0.75 \\ 0 \\ 0.25 \end{pmatrix}, \text{q} = \begin{pmatrix} q_a \\ q_b \\ q_c \end{pmatrix} = \begin{pmatrix} 0.25 \\ 0 \\ 0.75 \end{pmatrix}$$

- And say you prefer apples to cantaloupes
- You would prefer p to q
- The independence axiom says that if we mix p with another lottery r, and q with the same lottery r, then you should prefer the first mixture to the second

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Independence

- For example, lets pick

$$r = \begin{pmatrix} r_a \\ r_b \\ r_c \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

- And set α=1/2
- Then

$$\frac{1}{2}p + \frac{1}{2}r = \frac{1}{2}\begin{pmatrix} 0.75 \\ 0 \\ 0.25 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0.375 \\ 0.5 \\ 0.125 \end{pmatrix}$$

$$\frac{1}{2}q + \frac{1}{2}r = \frac{1}{2}\begin{pmatrix} 0.25 \\ 0 \\ 0.75 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0.125 \\ 0.5 \\ 0375 \end{pmatrix}$$

- You should prefer the former to the latter

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Independence

- Does this sound sensible?
- Here is one justification
- You tell me you prefer p to q
- Now I offer you the following choice
  - Option A: I am going to flip a coin. If it comes down heads, you get to play lottery p. If it comes down tails, you get to play lottery r
  - Option B: I am going to flip a coin. If it comes down heads, you get to play lottery q. If it comes down tails, you get to play lottery r
- You **should** prefer A to B
- What happens when you get tails (as long as the same thing happens in each case) should not affect how you feel about A or B

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Independence

- Theorem: preferences can be represented by an expected utility function **if and only if** they satisfy
  - Reflexivity
  - Transitivity
  - Completeness
  - Independence
- Do people's preferences generally satisfy the independence axiom?

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The Common Ratio Effect

100% $8	**vs**	**80% $10**	**$0**

- Which would you choose?

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The Common Ratio Effect

25% $8	**75 % $0**	**vs**	**20% $10**	**80% $0**

- Which would you choose?

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The Common Ratio Effect

C1 | 100% $8 | vs | 80% $10 | $0 | C2

D1 | 25% $8 | 75 % $0 | vs | 20% $10 | 80% $0 | D2

- Many people choose C1 and D2

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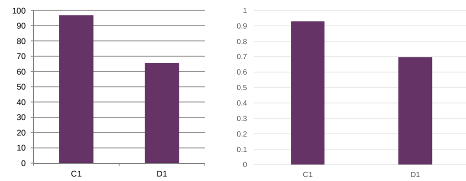
The Common Ratio Effect

C1 | 100% $8 | vs | 80% $10 | $0 | C2

D1 | 25% $8 | 75 % $0 | vs | 20% $10 | 80% $0 | D2



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The Common Ratio Effect

C1 | 100% $8 | vs | 80% $10 | $0 | C2

D1 | 25% $8 | 75 % $0 | vs | 20% $10 | 80% $0 | D2

- Claim: Choosing C1 and D2 violates the independence axiom
- D1 is C1 mixed with 100% 0
- D2 is C2 mixed with 100% 0
- Independence: C1 preferred to C2 implies D1 preferred to D2

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How Much do Expected Utility Numbers Mean?

- Recall that, for 'standard' utility, take two utility functions u and v. They both represent the same preferences if and only if there is a strictly increasing function f such that

$$\boldsymbol{v}(\boldsymbol{x}) = \boldsymbol{f}(\boldsymbol{u}(\boldsymbol{x}))$$

for all x

- Is that still true for expected utility?
- Let's say

$$u(a) = 1, u(b) = 2, u(c) = 3$$
$$v(a) = 2, v(b) = 3, v(c) = 1000$$

- Do these represent the same preferences over lotteries?

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How Much do Expected Utility Numbers Mean?

$$u(a) = 1, u(b) = 2, u(c) = 3$$
$$v(a) = 2, v(b) = 3, v(c) = 1000$$

- Do these represent the same preferences over lotteries?
- No! according to u, the following two lotteries are indifferent

$$p = \begin{pmatrix} p_a \\ p_b \\ p_c \end{pmatrix} = \begin{pmatrix} 0.5 \\ 0 \\ 0.5 \end{pmatrix}, \text{q} = \begin{pmatrix} q_a \\ q_b \\ q_c \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

- As

$$0.5u(a) + 0.5u(c) = 2 \text{ and } u(b) = 2$$

- But according to v

$$0.5v(a) + 0.5v(c) = 501 \text{ and v}(b) = 3$$

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How Much do Expected Utility Numbers Mean?

- **Theorem:** Take two **expected** utility functions u and v. They both represent the same preferences if and only if there is an a and $b > 0$ such that

$$v(x) = a + bu(x)$$

For every prize x

- Sometimes called a positive affine transformation
- You will see how this works for homework.

Asymmetric Information

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Information in Competitive Markets

- In purely competitive markets all agents are fully informed about traded commodities and other aspects of the market.
  - There is no uncertainty
- What about markets for medical services, or insurance, or used cars?

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Asymmetric Information in Markets

- A doctor knows more about medical services than does the buyer.
- An insurance buyer knows more about his riskiness than does the seller.
- A used car's owner knows more about the condition of a car than does a potential buyer.
- Markets with one side or the other imperfectly informed are markets with **incomplete information.**
- Imperfectly informed markets with one side better informed than the other are markets with **asymmetric information**.

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Asymmetric Information in Markets

- In what ways can asymmetric information affect the functioning of a market?
- Generally badly
- We will focus on one particular example: **adverse selection**

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Adverse Selection

- Consider a used car market.
- Two types of cars; "lemons" and "peaches".
- Car is owned by a 'seller', who can try to sell to a 'buyer'
  - Lemons are worth less than peaches
  - Seller values each type of car less than buyers
- E.g.
  - Each lemon seller will accept $1,000; a buyer will pay at most $1,200.
  - Each peach seller will accept $2,000; a buyer will pay at most $2,400.

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Adverse Selection

- If every buyer can tell a peach from a lemon, then lemons sell for between $1,000 and $1,200, and peaches sell for between $2,000 and $2,400.
- Gains-to-trade are generated when buyers are well informed.
  - Trade is efficient

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Adverse Selection

- Suppose no buyer can tell a peach from a lemon before buying.
- But the seller knows what type of car they are selling
- What is the most a buyer will pay for any car?
  - To make things easier, lets assume everyone maximizes expected value, not expected utility

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Adverse Selection

- Let q be the fraction of peaches.
- 1 - q is the fraction of lemons.
- Expected value to a buyer of any car is at most

$$\mathbf{EV = \$1200(1-q) + \$2400q.}$$

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Adverse Selection

- Suppose EV > $2000.
- Every seller can negotiate a price between $2000 and $EV (no matter if the car is a lemon or a peach).
- All sellers gain from being in the market.

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Adverse Selection

- Suppose EV < $2000.
- A peach seller cannot negotiate a price above $2000 and will exit the market.
  - Remember, they value the car at more than $2000
- All buyers are smart and realize this is happening
- So all buyers know that remaining sellers own lemons only.
- Buyers will pay at most $1200 and only lemons are sold.

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Adverse Selection

- Hence "too many" lemons "crowd out" the peaches from the market.
- Gains-to-trade are reduced since no peaches are traded.
- The presence of the lemons inflicts an external cost on buyers and peach owners.
- This is called 'market unravelling'

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Adverse Selection

- How many lemons can be in the market without crowding out the peaches?
- Buyers will pay $2000 for a car only if

$$EV = \$1200(1-q) + \$2400q \geq \$2000$$

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Adverse Selection

- How many lemons can be in the market without crowding out the peaches?
- Buyers will pay $2000 for a car only if

$$EV = \$1200(1-q) + \$2400q \geq \$2000$$
$$\Rightarrow q \geq \frac{2}{3}.$$

- So if over one-third of all cars are lemons, then only lemons are traded.

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Adverse Selection

- A market equilibrium in which both types of cars are traded and cannot be distinguished by the buyers is a pooling equilibrium.
- A market equilibrium in which only one of the two types of cars is traded, or both are traded but can be distinguished by the buyers, is a separating equilibrium.

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Adverse Selection

- What if there is more than two types of cars?
- Suppose that
  - car quality is uniformly distributed between $1000 and $2000
  - any car that a seller values at $x is valued by a buyer at $(x+300).
- Which cars will be traded?

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Adverse Selection

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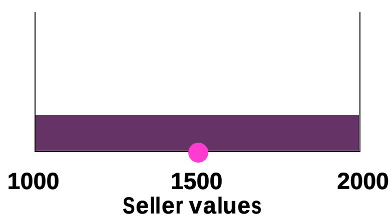
Adverse Selection



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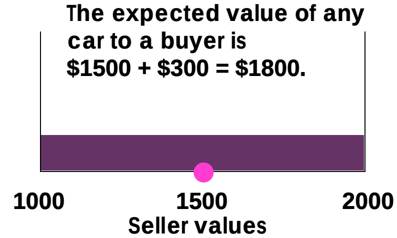
Adverse Selection



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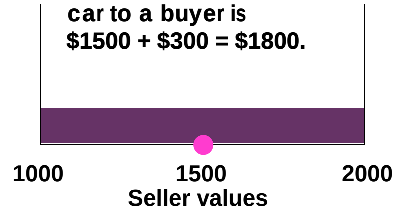
Adverse Selection

The expected value of any car to a buyer is $1500 + $300 = $1800.



So sellers who value their cars at more than $1800 exit the market.

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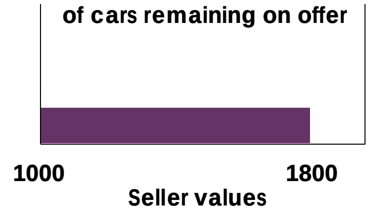
Adverse Selection

The distribution of values of cars remaining on offer



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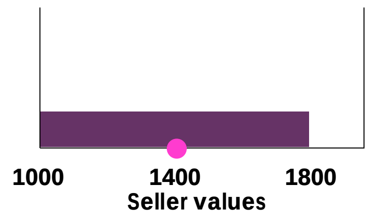
Adverse Selection



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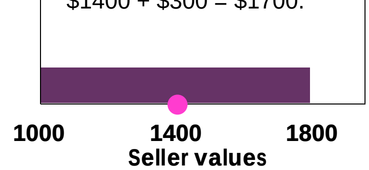
Adverse Selection

The expected value of any remaining car to a buyer is $1400 + $300 = $1700.



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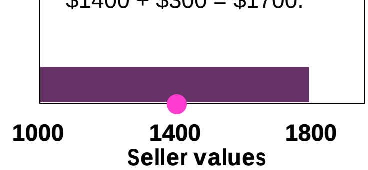
Adverse Selection

The expected value of any remaining car to a buyer is $1400 + $300 = $1700.



So now sellers who value their cars between $1700 and $1800 exit the market.

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Adverse Selection

- Where does this unraveling of the market end?
- Let v_H be the highest seller value of any car remaining in the market.
- The expected seller value of a car is

$$\frac{1}{2} \times 1000 + \frac{1}{2} \times v_H .$$

Adverse Selection

- So a buyer will pay at most

$$\frac{1}{2} \times 1000 + \frac{1}{2} \times v_H + 300.$$

Adverse Selection

- So a buyer will pay at most

$$\frac{1}{2} \times 1000 + \frac{1}{2} \times v_H + 300.$$

- This must be the price which the seller of the highest value car remaining in the market will just accept; i.e.

$$\frac{1}{2} \times 1000 + \frac{1}{2} \times v_H + 300 = v_H.$$

Adverse Selection

$$\frac{1}{2} \times 1000 + \frac{1}{2} \times v_H + 300 = v_H$$

$$\Rightarrow \; v_H = \$1600.$$

Adverse selection drives out all cars valued by sellers at more than $1600.