Mathematics For Economists

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Homework 4

Due October 14th

Question 1 Which of the following problems is guaranteed to have a solution? Either prove, or give a counter-example

1. A firm chooses a level of input $t \in \mathbb{R}_+$ to buy in order to maximize profits. For input t, the firm produces output $f(t)$, which is concave (and continuous) (i.e. $f(\alpha t + (1-\alpha)s) \geq \alpha f(t) + (1-\alpha)f(s) \ \forall \ t, s \in \mathbb{R}_+$). Output can be sold at the price of 1 per unit, while input can be bought at c per unit

2. As above, but there exists some x_0 such that $f(x_0) \leq cx_0$ (hint - draw a graph)

3. A consumer chooses a commodity bundle $x \in \mathbb{R}_+$ in order to maximize a continuous utility function $u : \mathbb{R}_+ \to \mathbb{R}$ subject to the budget constraint $B(p, I) = \{x \in X | px \leq I\}$ for some $p \in \mathbb{R}_+$

4. The object $x \in \mathbb{R}^2 \cap S(a, b, c, d)$ that is closest to the 0 vector, where $S(a, b, c, d) = \{(x_1, x_2) \in \mathbb{R}^2 | a \leq x_1 < b, \ c \leq x_2 < d\}$ for $a, b, c, d \in \mathbb{R}_+$

Question 2 Let X and Y be metric spaces, $\Gamma : X \Rightarrow Y$, and $f : X \times Y \to \mathbb{R}$. In each case, assume that the conditions of the theorem of the maximum hold unless explicitly stated. Find examples of the following:

1. y^* is not continuous

2. Γ is not UHC, and so f^* is not continuous

3. f is not continuous, and so y^* is not UHC

Question 3 Prove sections 4 and 5 of Theorem 1 in the Linear Algebra notes