Mathematics For Economists

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Homework 6

Due Tuesday 11th Nov

Question 1 Determine which of the following is convex

1. A non-empty, connected subset of $\mathbb{R}^2$

2. $\{t, y\} \in \mathbb{R}^2$ such that $f(t) \geq y$ for a concave real valued function f

3. An open ball in $\mathbb{R}^4$

4. the set of bounded functions on $[0, 1]$ such that $f(x) \geq 0$

5. the set of continuous functions in $[0, 1]$ such that $f(1) \neq 0$

Question 2 This question has three parts

1. Prove that, if C is convex and $C^0 \neq \{\}, C^o = (cl(C))^o$

2. If a set is closed, does it's convex hull have to be closed? Either prove or provide a counterexample.

3. Provide examples of the following in $\mathbb{R}^n$

   (a) Two closed convex sets that can be strictly separated but not strongly separated

   (b) A convex set and a point outside that set that can be properly separated, but not strictly separated

Question 3 Say that that C_1 is closed and convex, and C_2 is compact and convex and the two sets are disjoint. Show that $K = C_1 - C_2$ is closed and convex. Show by example that if C_2 is not compact, then this is not necessarily the case.

Question 4 Let $y = \{5, 4, 3\}$ and $C = \bar{B}(2, \{0, 0, 0\})$

1. Find the orthogonal projection of y onto C

2. Identify a hyperplane that separates y from C Can you identify a supporting hyperplane (i.e. one that is tangent to C)