Mathematics For Economists

Mark Dean

Midterm

Stay Calm!

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Prove all statements unless told otherwise. Feel free to make use of results that we have proved in class, but tell me what results you are using.

Question 1 (5pts) Find an example of a closed, bounded set and continuous function such that the image of that set is not compact. For good measure, give an example of a continuous function on a closed and bounded set that does not obtain a maximum on that set

Question 2 Consider the following problem: You are trying to decide where to sit in the bar. There are currently two people in the bar, one of which you really like (l) and one of which you really dislike (d). Let x_l be the location of the person that you like and x_d the location of the person you dislike. Your happiness is going to be a decreasing function of distance from the person you like and an increasing function of the distance from the person you dislike

1. (5 pts) Write this problem formally. Give conditions on the elements of the problem that guarantee it to have a solution
2. (20 pts) Let x_l be the location of the person that you like and x_d the location of the person you dislike, $P(x_l, x_d)$ be the optimal location given x_l and x_d and $V(x_l, x_d)$ be your maximal happiness given x_l and x_d. Give conditions on the underlying problem such that P is upper hemi-continuous and V is continuous
3. (10pts) Give an example of this problem in which a solution is guaranteed to exist for any x_l and x_d, but P is not continuous

4. (10pts) Give an example of the problem in which a solution is guaranteed to exist, P is upper hemi continuous, but V is not continuous

5. (10pts) Now consider the problem of the barman who can choose where l and d sit in the bar. The barman wants to maximize his or her revenue. The happier each customer is, the longer they will stay and the more they spend. l and d don't care where they are in relation to other people, but both have a favourite spot. Their happiness depends on their distance from that spot. You (as in the person from the first four parts of the question) get to choose your position in the bar having seen the location of l and d. Provide conditions on the primitives of the problem that ensure that the barman's problem has a solution (assume that the barman knows everyones utility function).

Question 3 We say a collection of closed sets $\mathcal{A}$ of some space X has the finite intersection property if any finite collection of sets in $\mathcal{A}$ have a non-empty intersection

1. (5pts) Show that, in general, if a collection $\mathcal{A}$ has the finite intersection property it can be the case that $\cap\mathcal{A}$ is empty

2. (15pts) Show that if X is compact, then if a collection $\mathcal{A}$ has the finite intersection property it can be the case that $\cap\mathcal{A}$ is non-empty

Question 4 (20pts) Let V be a linear space, and L and K be linear operators on V. Show that, $LK(x) = L(x)K(x)$ is a linear operator if and only if $LK(x) = \emptyset \; \forall \; x \in V$