ECO 514 Economics and Psychology

Mark Dean

Problem Set - Autumn 2012

Due 23rd January 2013

Question 1 Consider a decision maker (DM) who will live for two periods. In period 2 the DM must choose an action from set $A \subset \mathbb{R}$ The period 2 agent will make choices in order to maximize

$$u_2(a, s, \beta) = s\beta U(a) - a$$

where s is the state of the world drawn from some $S \subset \mathbb{R}_{++}$, $\beta \in \mathbb{R}_{++}$ and U is an increasing function which is twice differentiable with $U'' < 0$ on the interior of A . From the point of view of the DM in period 1, the value of taking action a in state s given by

$$u_1(a, s, \beta, t) = sU(a) - a - t$$

s is unknown at time 1, and assumed to be drawn from some distribution F. t is any monetary transfers that occur in period 1 (for example any payment for commitment devices). The DM in period 1 wishes to maximize the expected value of u_1.

1. Assume that $U(a) = \log a$ and $A = \mathbb{R}_{++}$. If F is degenerate at some s^*, how much would the DM in period 1 pay to fully commit to an action (relative to having the DM in period 2 choose the action)? How does it depend on β?

   Answer If the DM in period 2 chooses the action, then they will do so in order to maximize

$$s^*\beta \log a - a$$

As this problem is well behaved, we can solve it by taking first order conditions, meaning that the DM will choose a_2^* to solve

$$\begin{aligned}\frac{s^*\beta}{a_2^*} &= 1 \\ &\Rightarrow a_2^* = s^*\beta\end{aligned}$$

If the DM could commit to an action in period 1, they would choose to set

$$a_1^* = s^*$$

Thus we can calculated the value of t such that

$$s^* \log a_2^* - a_2^* = s^* \log a_1^* - a_1^* - t$$

by plugging in for a_1^* and a_2^*

$$\begin{aligned}s^* \log s^*\beta - s^*\beta &= s^* \log s^* - s^* - t \\ &\Rightarrow t = -s^* \left(\log \beta + (1-\beta)\right)\end{aligned}$$

Notice that $t = 0$ for $\beta = 1$

Taking the derivative with respect to β gives

$$-s^*(\frac{1}{\beta} - 1)$$

which is negative for $\beta < 1$ and positive for $\beta > 1$. Thus, the DM in period 1 is prepared to pay 0 for commitment if $\beta = 1$, with the amount they are prepared to pay increasing both as β falls below 1 and as it increases above 1.

2. Assume again that $U(a) = \log a$ and $A = \mathbb{R}_{++}$, but now assume that F is uniform on $(0, \bar{s}]$ and that $\beta = 1$. How much would the DM in period 1 pay to leave the choice until period 2 when s is revealed (relative to the alternative of full commitment to a course of action in period 1)? How does this depend on $\bar{s}$?

   Answer If the DM could delay their choice until period 2, then they would always choose to set $a(s)$ equal to s (as we have seen in the answer to the above question). Therefore the expected utility is given by

$$\begin{aligned}&\int_0^{\bar{s}} [s \ln s - s]\, f(s) ds \\ =& \int_0^{\bar{s}} [s \ln s - s]\, \frac{1}{\bar{s}} ds\end{aligned}$$

Integration by parts tells us that $\int x \ln x = \frac{x^2 \ln x}{2} - \frac{x^2}{4}$, thus, the above integral becomes

$$\begin{aligned} & \frac{1}{\bar{s}}\left[\frac{s^2 \ln s}{2} - \frac{3s^2}{4}\right]_0^{\bar{s}} \\ = & \bar{s}\left(\frac{1}{2}\ln \bar{s} - \frac{3}{4}\right) \end{aligned}$$

In contrast, if the DM had to choose in period 1, then they would select a_1^* in order to maximize

$$\begin{aligned} & \int_0^{\bar{s}} \left[s \ln a_1^* - a_1^*\right] \frac{1}{\bar{s}} ds \\ = & \frac{1}{\bar{s}}\left[\frac{s^2}{2} \ln a_1^* - a_1^* s\right]_0^{\bar{s}} \\ = & \frac{\bar{s}}{2} \ln a_1^* - a_1^* \end{aligned}$$

Taking first order conditions, it is trivial to show that $a_1^* = \frac{\bar{s}}{2}$, and so we can solve for the maximal t that the DM would be willing to pay for flexibility by solving for t in

$$\begin{aligned} \bar{s}\left(\frac{1}{2}\ln \bar{s} - \frac{3}{4}\right) - t &= \frac{\bar{s}}{2} \ln \frac{\bar{s}}{2} - \frac{\bar{s}}{2} \\ &\Rightarrow \quad t = \frac{\bar{s}}{2}(\ln 2 - \frac{1}{2}) \end{aligned}$$

which goes to zero as $\bar{s}$ goes to zero

3. Now assume that $\beta \in (0,1)$ and that F is such that there is a 50% chance probability that $s = s_1$ and a 50% chance that $s = s_2$ with $s_1 < s_2$. Assume that the DM in period 1 is going to offer the DM in period 2 the choice of 2 actions a_1 and a_2. Write down the incentive compatibility constraints that ensure that each type in period 2 chooses the correct action. (i.e. if the state of the world is s_1 then the DM in period 2 chooses a_1, and if it is s_2 then they choose a_2). For what values of β (as a function of s_1 and s_2) can the DM in period 1 support their first best outcome? (Note, while you will get two conditions, only one is binding. Why is that?).

Answer To ensure that type s_1 wants to choose a_1 and s_2 wants to choose a_2, it must be the case that

$$\begin{aligned} s_1 \beta \log a_1 - a_1 &\geq s_1 \beta \log a_2 - a_2 \\ s_2 \beta \log a_2 - a_2 &\geq s_2 \beta \log a_1 - a_1 \end{aligned}$$

The first best solution from the point of view of the DM in period 1 is to set $a_1 = s_1$ and $a_2 = s_2$. For this to be incentive compatible, it must be the case that

$$\begin{aligned} s_1\beta \log s_1 - s_1 &\geq s_1\beta \log s_2 - s_2 \\ &\Rightarrow \frac{s_2 - s_1}{s_1 (\ln s_2 - \ln s_1)} \geq \beta \end{aligned}$$

And also

$$\begin{aligned} s_2\beta \log s_2 - s_2 &\geq s_2\beta \log s_1 - s_1 \\ &\Rightarrow \beta \geq \frac{s_2 - s_1}{s_2 (\ln s_2 - \ln s_1)} \end{aligned}$$

While this second condition is binding, the first as not, as

$$\frac{s_2 - s_1}{s_1 (\ln s_2 - \ln s_1)} = \frac{1}{\ln(\frac{s_2}{s_1})}(\frac{s_2}{s_1} - 1)$$

.This function is increasing in $\frac{s_2}{s_1}$ for $\frac{s_2}{s_1} \geq 1$, and it is easy to show (using l'hopital's rule) that $\lim_{x\to 1} \frac{1}{\ln x}(x-1) = 1$, Thus, this constraint is never more restrictive than requiring that that β be less than or equal to 1

4. Continuing with the example from part 3, now assume that $s_1 = 1$, $s_2 = 2$ and $\beta = 0.6$. Calculate the optimal incentive compatible commitment contract (hints: only one constraint is binding, and that is the constraint that in state s_2 the DM does not want to use the contract designed for state $a1$. Having used first order conditions to solve for a_1 as a function of a_2, you will probably want to use Matlab to solve explicitly for the optimal contract.)

Answer Using the hint, we can set up Lagrangian with one constraint

$$\begin{aligned} & L(a_1, a_2) \\ = & \frac{1}{2}\left[s_1 \ln a_1 - a_1 + s_2 a_s - a_s\right] \\ & +\lambda\left[\beta s_2 \ln a_1 - a_1 - \beta s_2 \ln a_2 + a_2\right] \end{aligned}$$

Taking first order conditions with respect to a_1 and a_2 gives

$$\begin{aligned} \frac{s_1}{a_1} - 1 &= -\lambda\left[\frac{\beta s_2}{a_1} - 1\right] \\ \frac{s_2}{a_2} - 1 &= \lambda\left[\frac{\beta s_2}{a_2} - 1\right] \end{aligned}$$

rearranging these two equations gives

$$a_1 = \frac{a_2(s_1 + \beta s_2) - \beta s_2(s_1 + s_2)}{2a_2 - s_2(1 + \beta)}$$

Along with the binding contratint

$$\beta s_2 \ln a_1 - a_1 = \beta s_2 \ln a_2 - a_2$$

This gives two equations in two unknowns. Throwinging these equations into Matlab and using the parameter values above gives the solutions $a_1 = 0.79$ annd $a_2 = 1.73$

5. Repeat the procedure for $\beta = 0.5$ How does this solution differ from that in part 4?

Answer If we assume that the sam constraint is binding as in part 3 above, we get that

$$a_1 = \frac{2a_2 - 3}{2a_2 - 3} = 1 = a_2$$

This is clearly dominated by the optimal pooling solution of $a_1 = a_2 = 1.5$, which is in fact the optimal contract. The Amador, Angeletos and Werning paper we studied in class in fact tells us that separation is optimal if and only if $\beta > \frac{s_1}{s_2}$

6. Consider a monopolist supplier of commitment devices - that is a firm that can offer a menu of contracts, each consisting of a pair (a_1, a_2). If an individual purchases a commitment contract, then in period 2 they can only choose from (a_1, a_2).. if they buy no contract then the period 2 agent is free to choose anything they wish. The firm faces no cost of supplying commitment contracts, and charges a price p for each contract in their menu. What is the optimal menu of contracts if the monopolist is facing a buyer who is in the situation described in section 4? What about if $\beta = 0.9$? If $\beta = 0.5$? (hint, the supplier is a monopolist, so they can extract ALL consumer surplus)

Answer As the supplier is a monopolist, the optimal course of action is to offer the period 1 agent their best contract, then extract all the surplus in the price. We figured out the optimal contract for an agent with $\beta = 0.6$ in part 4, an agent with $\beta = 0.5$ in part 5 and (from part 3) we know that for $\beta = 0.9$, the optimal contract is the period 1 agent's first best contract. The following table calculates for each β the period 1 value of the optimal contract and having no contract (i.e. where the period 2 agent gets to choose freely, and so therefore the price that the monopolist can charge as the difference between the two

β	Value, Optimal Contract	Value, No Contract	Price
0.6	-0.83	-0.97	0.14
0.5	-0.89	-1.10	0.20
0.9	-0.80	-0.81	0.01

7. Now assume that the monopolist is facing a population of consumers with different levels of β in the set $\{0.5, 0.6, 0.9\}$. If they were able to differentiate between consumers with different levels of β then the optimal thing to do would be to offer each consumer the optimal menu you calculated in part 6. Will this still work if the monopolist cannot differentiate between consumers with different $\beta's$? (i.e. would consumers self select to the correct contracts?, Or would consumers with one level of β want to choose the contracts designed for other levels of β)

Answer CLARIFICATION: what I meant is for you to consider the case where the firm offers the three optimal contracts for each level of β, and charges for each of them the price calulated in section 6

Let's first think about the consumer with $\beta = 0.9$. It should be clear that, for such a contract, all three contracts are incentive compatible. Thus, the can choose between consuming 2 in state 2 and 1 in state 1 at price 0.1, consuming 1.73 in state 2 and 0.79 in state 1 and paying 0.14, or consuming 1.5 in both states and paying 0.2. As the contract designed for them is both their first best and the cheapest, then they will clearly do this.

Now consider the case of a consumer with $\beta = 0.6$. They can clearly pay 0.2 in order to consume 1.5 in each state, but this is both more expensive and worse than their optimal contract. What about paying 0.01 for the contract $\{2, 1\}$? We know that this is not incentive compatible. First consider the state 2 agent. They must choose between

$$1.2 \ln 2 - 2 = -1.16$$

and

$$1.2 \ln 1 - 1 = -1$$

So they will choose to consume 1. Clearly the state 1 agent will also choose to consume 1. The gross utility of this contract is therefore -1, and if they additionally have to pay 0.01, this would give utility of -1.01. Choosing their optimal contract and paying the requisite price will give them their autarky utility of -0.97, so they will also select the contract designed for them.

Finally consider the agent with $\beta = 0.5$. Again, clearly neither of the other contracts are incentive compatible for them In both cases, the state 2 guy would choose the consumption level designed for the state 1 guy. Thus the $\beta = 0.5$ guy has the choice of

(a) Consuming 1.5 in both states and paying 0.20, giving utility -1.10

(b) Consuming 1 in both states and paying 0.01, giving utility -1.01

(c) Consuming 0.79 in both states and paying 0.14, giving utility -1.28

This consumer would therefore buy the contract designed for the $\beta = 0.9$ guy

8. Repeat parts 6 and 7, but now assume that there is a freely available contract which is the optimal commitment contract for the agent with $\beta = 0.6$. How do your results change?

Now the maximal amount that the monopolist can charge is the difference between the optimal contract for each agent and their best outside option, which is either allowing the state person free choice or choosing the optimal contract for $\beta = 0.6$. (which we calculated above). The following table summarizes these values

β	Value, Optimal Contract	Value, No Contract	Value, $\beta = 0.6$ Contract	Price
0.6	-0.83	-0.97	-0.83	0
0.5	-0.89	-1.10	-1.28	0.20
0.9	-0.80	-0.81	-0.83	0.01

In fact (because I worded the question incorrectly), this does not change the result of part 8 at all.

Question 2 In this question we will consider models of rational inattention with costs based on Shannon mutual information.

1. In the lecture notes (slide 26, Bounded Rationality Lecture 3), we showed that, in the

case of two acts $\{f, g\}$ and two underlying states $\{1, 2\}$, then

$$\frac{u_1^f - u_1^g}{\ln t_1^f - \ln t_1^g} = \frac{u_2^f - u_2^g}{\ln(t_2^f) - \ln(t_2^g)} = k$$

where u_i^j is the utility that act j pays in state u, t_i^j is the posterior probability of state i when j is chosen and k is the cost of information. Show this result is true more generally - i.e. if there are an arbitrary number of states and a number of chosen acts less than or equal the number of states. Would the result still be true if, rather than being able to purchase information at cost k, there was a fixed amount of information that the DM could choose to allocate.

Answer for an arbitrary number of states and arbitrary number of acts, we can set the Legrangian problem up as follows

$$\begin{aligned} L\left(\left\{P(f), t^f\right\}_{f\in A}\right) &= \sum_{f\in A} P(f) \sum_{m=1}^{m} t_m^f u_m^f \\ &-k\left[\sum_{f\in A} P(f) \sum_{m=1}^{M} t_m^f \ln t_m^f - \sum_{m=1}^{M} \mu_m \ln \mu_m\right] \\ &+\sum_{m=1}^{m} \lambda_m \left[\sum_{f\in A} P(f) t_m^f - \mu_m\right] \\ &+\sum_{f\in A} \chi^f \left[\sum_{m=1}^{m} t_m^f - 1\right] \end{aligned}$$

$P(f)$ is the unconditional probability of chosing act f, t_m^f is the posterior probability of state ω_m when act f is chosen (these are the objects of choice) and μ_m is the prior probability of state m. The first term is the gross utility of the attentional strategy, the second the mutual information cost of the strategy, the third is the set of constraints that ensure that the posteriors are commensurate with the prior (with the Legrange multipliers indicated by the λ_m terms, while the fourth is the constraints that ensure that all posteriors sum to one (with the χ^f terms as the legrange multipliers). Notice that we are assuming that the constraint that $P(f) > 0$ is not binding, so all acts are being chosen with strictly positive probability

Taking first order conditions with respect to t_m^f gives

$$P(f)u_m^f - kP(f)[\ln t_m^f - 1] + P(f)\lambda_m - \chi^f \tag{1}$$

while taking first order conditions with respect to $P(f)$ gives

$$\sum_{m=1}^{m} t_m^f \left[u_m^f - k \ln t_m^f + \lambda_m\right] = 0$$

Equation 1 can be rewritten as

$$u_m^f - k \ln t_m^f + \lambda_m = \frac{\chi^f}{P(f)} - 1$$

multiplying by t_m^f and summing tells us that

$$\sum_{m=1}^{m} t_m^f \left[u_m^f - k \ln t_m^f + \lambda_m\right] = \sum_{m=1}^{m} t_m^f \left[\frac{\chi^f}{P(f)} - 1\right] = 0$$

Implying that $\frac{\chi^f}{P(f)} = 1$ for all $f \in A$

Again using equation 1 tells us that

$$\begin{aligned} u_m^f - k \ln t_m^f &= 1 - \lambda_m - \frac{\chi^f}{P(f)} \\ &= 1 - \lambda_m - \frac{\chi^g}{P(g)} \\ &= u_m^g - k \ln t_m^g \end{aligned}$$

Rearranging this final equation gives the desired result

2. Consider the problem of a consumer who faces a choice of whether or not to buy a good at price p. The good can be either of high quality or low quality. If a high quality good is purchased, then the consumer gets utility $u^* - p$. If they purchase a low quality good then they get utility $u_* - p$. if they do not purchase the good then they get utility 0. The consumer initially places probability β on the good being of high quality, and can gather more information on the quality of good, paying costs based on mutual information (i.e. there is a constraint on the amount of information that they can gather?)

   Consider a strategy in which t_j^i is the probability of true state j when action i is taken, where $j \in \{h, l\}$ for high or low quality goods and $i \in \{b, n\}$ for buy or not. Verify that the value of such a strategy is

$$P(t^b)[H(t^b) + U(t^b, b)] + P(t^n)[H(t^n) + U(t^n, n)] - H(\beta)$$

   where $P(t^i)$ is the unconditional probability of taking act i, $H(t^i)$/is the entropy of distribution t^i and $U(t^i, j)$ is the expected utility of taking act j under posterior distribution t^i. (Hint, make use of the reformulation of mutual information on slide 12 of bounded rationality lecture notes 3).

Answer This is immediate from the fact that we can rewrite the mutual information cost of the strategy from

$$\sum_{m=1}^{M}\sum_{f\in A}\mu_m\lambda_m(f)\log\frac{\mu_m\lambda_m(f)}{\mu_m P(f)}$$

to

$$\begin{aligned}&\sum_{f\in A}P(f)\sum_{m=1}^{1}t_m^f\log t_m^f-\sum_{m=1}^{M}\mu_m\ln\mu_m\\ =\;&\sum_{f\in A}P(f)H(t^f)-H(\mu)\end{aligned}$$

as shown in the notes

3. Assume that $p=2$, $u^*=4$, $u_*=0, k=1$ and $\beta=0.5$. Draw a graph which has as its horizontal axis the probability of the good being of high quality. Add the following lines to that graph (you will need to use excel, Matlab or somesuch)

   (a) The entropy associated with each distribution

   (b) The expected utility of taking act b at each distribution

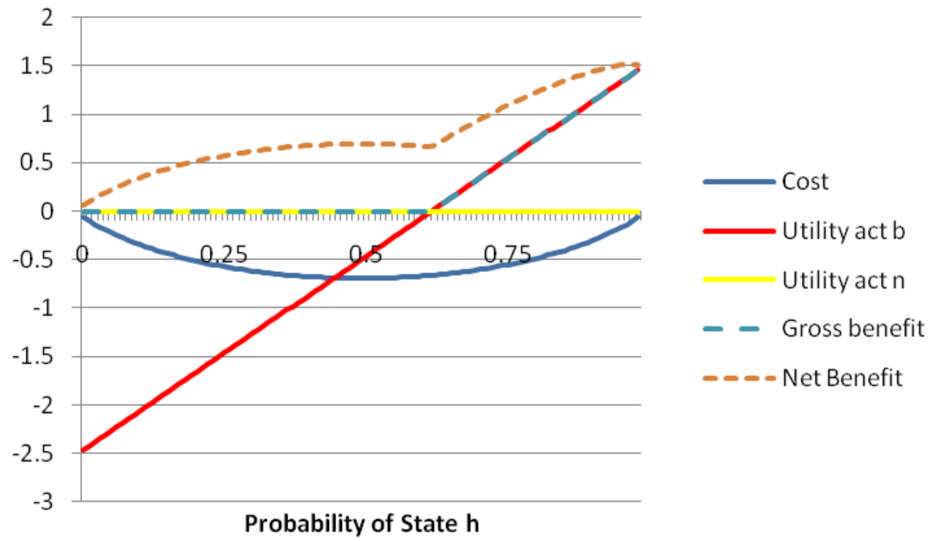
   (c) The expected utility of taking act n at each distribution

   (d) The expected utility of taking the optimal action at each distribution

   (e) The 'net utility' associated with each distribution (i.e. the expected utility of the optimal act plus entropy)

   Use the result from section two to show that the value of a particular strategy t^b, t^n must lie on the chord between the net utility function evaluated at these two points

Answer



[NOTE the second part of the question is slightl wrong. It should say that the value of a particular strategy NET OF THE ENTROPY OF THE PRIOR lies on a...]. From part 2, we know that the utility of a particular informational strategy is equal to

$$\begin{aligned} & P(t^b)[H(t^b) + U(t^b, b)] + P(t^n)[H(t^n) + U(t^n, n)] \\ = \ & P(t^b)N(t^b) + (1 - P(t^b))N(t^n) \end{aligned}$$

where $N(t)$ is the net utility function evaluated at t. this is the equation of the chord that goes between these two points

4. Show that, by Bayes' rule, once t^b and t^n have been chosen, then $P(t^b)$ and $P(t^n)$ are determined. Use this result to show that the value of a strategy is equal to the height of the chord from section 3 as it passes over the prior belief of the agent

Answer Actually, while you can use Bayes rule, an easier way is to use the fact that the

$$P(X = x) = \sum_y P(Y = y)P(X = x|Y = Y)$$

which in this case tells us that

$$\begin{aligned}\mu_h &= P(t^b)t_h^b + P(t^n)t_h^n \\ &= P(t^b)t_h^b + (1-P(t^b))t_h^n \\ &\Rightarrow P(t^b) = \frac{\mu_h - t_h^n}{t_h^b - t_h^n}\end{aligned}$$

Thus $P(t^b)$ is pinned down, and $P(t^h) = 1 - P(t^b)$

The value of a particular strategy is therefore given by

$$\begin{aligned}&\frac{\mu_h - t_h^n}{t_h^b - t_h^n}N(t^b) + \left(1 - \frac{\mu_h - t_h^n}{t_h^b - t_h^n}\right)N(t^n) \\ = \;& N(t^n) + \frac{\mu_h - t_h^n}{t_h^b - t_h^n}\Big(N(t^b) - N(t^n)\Big)\end{aligned}$$

Which is the value of the chord connecting $\big(N(t^b) - N(t^n)\big)$ as it passes over the prior.

5. Solve for the optimal strategy for the case described in section 3. Draw the resulting optimal posteriors $\bar{t}^b$ and $\bar{t}^n$ on your graph. What is the relationship between the chord that connects these two posteriors and the net utility function?

Answer Using the fact that

$$\begin{aligned}\frac{u_h^b - u_h^n}{\ln t_h^b - \ln t_h^n} &= k \\ &\Rightarrow 2 = \ln\left(\frac{t_h^b}{t_h^n}\right)\end{aligned}$$

and

$$\begin{aligned}\frac{u_l^b - u_l^n}{\ln(1-t_h^b) - \ln(1-t_h^n)} &= k \\ &\Rightarrow -2 = \ln\left(\frac{1-t_h^b}{1-t_h^n}\right)\end{aligned}$$

Which solves to

$$\begin{aligned}t_h^n &= \frac{\exp(\frac{u_l^b - u_l^n}{k}) - 1}{\exp(\frac{u_l^b - u_l^n}{k}) - \exp(\frac{u_h^b - u_h^n}{k})} = \frac{\exp(-2) - 1}{\exp(-2) - \exp(2)} = 0.12 \\ t_h^b &= \exp(\frac{u_h^b - u_h^n}{k})\frac{\exp(\frac{u_l^b - u_l^n}{k}) - 1}{\exp(\frac{u_l^b - u_l^n}{k}) - \exp(\frac{u_h^b - u_h^n}{k})} = \exp(2)\frac{\exp(-2) - 1}{\exp(-2) - \exp(2)} = 0.88\end{aligned}$$

The probability that a good will be bought can be caluclated from

$$\begin{aligned}P(t^b) &= \frac{\mu_h - t_h^n}{t_h^b - t_h^n} \\ &= \frac{0.5 - 0.12}{0.88 - 0.12} \\ &= 0.5\end{aligned}$$

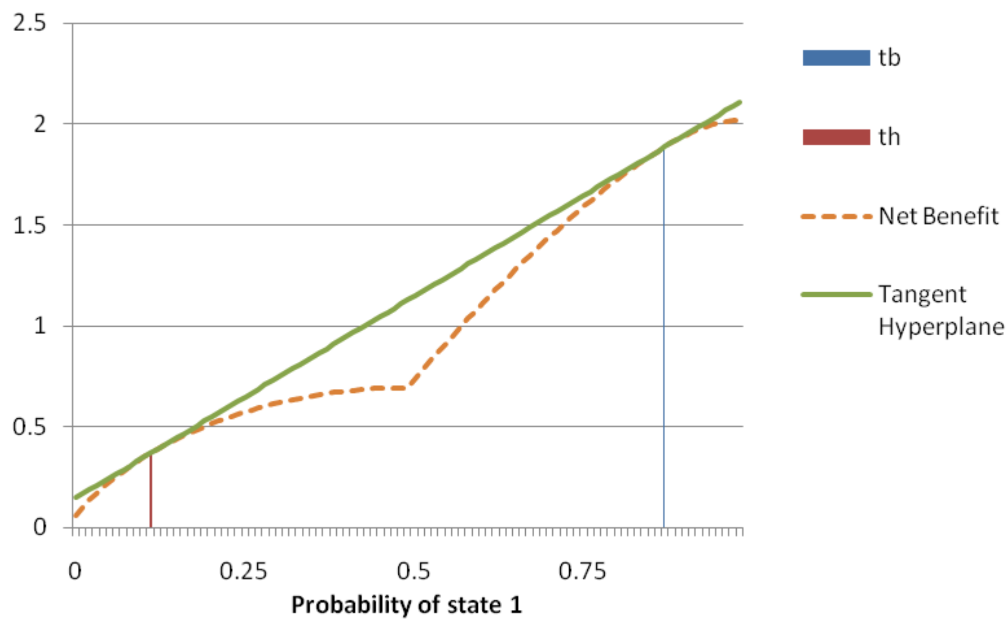
The probability that a low quality good is bought is given by

$$P(b|l) = \frac{p(l|b)p(b)}{p(l)} = \frac{(1-t_h^b)P(t^b)}{\mu_l} = 0.12$$

And the probability that a high quality good is bought is given by

$$P(b|h) = \frac{p(h|b)p(b)}{p(h)} = \frac{t_h^b P(t^b)}{\mu_l} = 0.88$$

On the graph, this looks like



The chord connecting the optimal posteriors is tangent to the net utility function at those points

6. What would be the optimal behavior of the consumer if they had a prior belief about the probability of a high quality firm that was different from 0.5 but between $\bar{t}^b$ and $\bar{t}^n$? (Hint, think about the result in section 4). What about if their prior belief was above $\bar{t}^b$? Or below $\bar{t}^n$?

Answer The result from section 4 tells us that the optimal posteriors are the ones that support the highest chord as it goes over the prior. The posteriors $\bar{t}^b$ and $\bar{t}^n$ support the highest chord going over the posterior $\mu_h = 0.5$, but also the highest chord going over any posterior in the range $[\bar{t}^n, \bar{t}^b]$, thus, the optimal strategy for all prior beliefs in this range uses the same posteriors, - though the probabilites $P(t^b)$ and $P(t^n)$ will change Outside this range, as is obvious from the graph, the optimal strategy is innatention.

7. Consider the effect of an exogenous price rise to 2.5. (Note at this price, a fully informed consumer would still like to buy a high quality good and not buy a low quality good). What will happen to the demand for high and low quality goods? What will happen to the profits of these firms?

   Answer Plugging the new values into the equations from part 5 gives

$$
\begin{aligned}
t_h^n &= \frac{\exp(\frac{u_l^b - u_l^n}{k}) - 1}{\exp(\frac{u_l^b - u_l^n}{k}) - \exp(\frac{u_h^b - u_h^n}{k})} = \frac{\exp(-2.5) - 1}{\exp(-2.5) - \exp(1.5)} = 0.21 \\
t_h^b &= \exp(\frac{u_h^b - u_h^n}{k}) \frac{\exp(\frac{u_l^b - u_l^n}{k}) - 1}{\exp(\frac{u_l^b - u_l^n}{k}) - \exp(\frac{u_h^b - u_h^n}{k})} = \exp(1.5) \frac{\exp(-2.5) - 1}{\exp(-2.5) - \exp(1.5)} = 0.94
\end{aligned}
$$

Thus the price rise means that the decision maker is *less* informed when they choose the low quality good (a the probability the good is high quality and when the subject does not buy rises from 0.12 to 0.21), but is more informed when they do buy (probability of the good being high quality when the subject buys rises from 0.88 to 0.94). The unconditional probability that the good will be bought is given by

$$
\begin{aligned}
P(t^b) &= \frac{\mu_h - t_h^n}{t_h^b - t_h^n} \\
&= \frac{0.5 - 0.21}{0.94 - 0.21} \\
&= 0.4
\end{aligned}
$$

Unsurprisingly, the price rise means that the good is less likely to be bought

The probability that a low quality good is bought is given by

$$
\begin{aligned}
P(b|l) &= \frac{p(l|b)p(b)}{p(l)} \\
&= \frac{(1 - t_h^b)P(t^b)}{\mu_l} \\
&= 0.05
\end{aligned}
$$

The probability that a high quality good is sold is given by

$$\begin{aligned} P(b|h) &= \frac{p(h|b)p(b)}{p(h)} \\ &= \frac{t_h^b P(t^b)}{\mu_l} \\ &= 0.75 \end{aligned}$$

Both high quality firms and low quality firms sell less, and while the proportional fall in the likelihood of a sale is much bigger for low quality firms (60% vs 15%), the absolute fall is actually bigger for high quality firms (13pp vs 7pp). For the low quality firms, gross expected revenue falls from 0.24 to 0.12. For the high quality firms, gross expected revenue rises from 1.76 to 1.88.

8. How does the price elasticity of demand depend on the cost of information k?