Single Variable Calculus Homework[1]

Differentiate the following:

1. $\frac{d}{dx}f(x) = \frac{d}{dx}(x^2 + 3x - 4) = 2x + 3$
2. $\frac{d}{dx}y = \frac{d}{dx}x^{-\frac{2}{3}} = -\frac{2}{3}x^{-\frac{5}{3}}$
3. $\frac{d}{dx}g(x) = \frac{d}{dx}x^2 + x^{-3} = 2x - 3x^{-4}$
4. $\frac{d}{dx}y = \frac{d}{dx}\frac{x^2+4x+3}{\sqrt{x}} = \frac{3}{2}x^{\frac{1}{2}} + 2x^{-\frac{1}{2}} - \frac{3}{2}x^{-\frac{3}{2}}$
5. $\frac{d}{dx}f(x) = \frac{d}{dx}x^2e^x = 2xe^x + x^2e^x$
6. $\frac{d}{dx}V(x) = \frac{d}{dx}\left(2x+3\right)\left(x^4 - 2x\right) = 2\left(x^4 - 2x\right) + \left(2x+3\right)\left(4x^3 - 2\right)$
7. $\frac{d}{dx}f(x) = \frac{d}{dx}\frac{x}{x+\frac{c}{x}} = \frac{2\frac{c}{x}}{\left(x+\frac{c}{x}\right)^2}$
8. $\frac{d}{dx}F(y) = \frac{d}{dx}\left(\frac{1}{y^2} - \frac{3}{y^4}\right)\left(y + 5y^3\right) = \left(-\frac{2}{y^3} + \frac{12}{y^5}\right)\left(y + 5y^3\right) + \left(\frac{1}{y^2} - \frac{3}{y^4}\right)\left(1 + 15y^2\right) = \frac{9}{y^4} + \frac{14}{y^2} + 5$
9. $\frac{d}{dx}y = \frac{d}{dx}e^{\sqrt{x}} = e^{\sqrt{x}}\frac{1}{2\sqrt{x}}$
10. $\frac{d}{dx}y = \frac{d}{dx}e^{ax^2+bx+c} = e^{ax^2+bx+c}(2ax + b)$
11. $\frac{d}{dt}y = \frac{d}{dx}\ln(t+9) = \frac{1}{t+9}$
12. $\frac{d}{dx}y = \frac{d}{dx}(\ln(x) - \ln(1+x)) = \frac{1}{x} - \frac{1}{1+x}$

Find the following:

1. $\int 8x^{-5}dx,\ x \neq 0.$

$$\begin{aligned} \int 8x^{-5}dx &= -\frac{8}{4}x^{-4} + C \\ &= -2x^{-4} + C \end{aligned}$$

2. $\int (7e^x + 3)dx$

$$\int (7e^x + 3)dx = 7e^x + 3x + C$$

3. $\int \frac{6x}{x^2+13}dx$

Using $f(x) = x^2 + 13 \Leftarrow f'(x) = 2x$

$$\begin{aligned} \int \frac{6x}{x^2+13}dx &= 3\int \frac{2x}{x^2+13}dx \\ &= 3\int \frac{f'(x)}{f(x)}dx \\ &= 3\ln(f(x)) + C \\ &= 3\ln(x^2+13) + C \end{aligned}$$

[1] If you find any typo please email me: Maria_Jose_Boccardi@Brown.edu

4. $\int (x+3)(x+1)^{\frac{1}{2}} dx$

Using integration by parts where

$$f(x) = x + 3$$

and

$$g'(x) = (x+1)^{\frac{1}{2}} \Leftarrow g(x) = \frac{2}{3}(x+1)^{\frac{3}{2}}$$

$$\begin{aligned} \int (x+3)(x+1)^{\frac{1}{2}} dx &= \frac{2}{3}(x+3)(x+1)^{\frac{3}{2}} - \int \frac{2}{3}(x+1)^{\frac{3}{2}} dx + C \\ &= \frac{2}{3}(x+3)(x+1)^{\frac{3}{2}} - \frac{2}{5}(x+1)^{\frac{5}{2}} dx + C \end{aligned}$$

5. $\int xe^x dx$

Using integration by parts where

$$f(x) = x \Leftarrow f'(x) = 1$$

and

$$g'(x) = e^x \Leftarrow g(x) = e^x$$

$$\begin{aligned} \int xe^x dx &= xe^x - \int e^x dx \\ &= xe^x - e^x + C \end{aligned}$$

6. $\int x^3\sqrt{1+x^2} dx$

We can rewrite

$$\int x^3\sqrt{1+x^2} dx = \int \frac{1}{2}x^2\sqrt{1+x^2} 2x dx$$

Use substitution where

$$u = g(x) = 1 + x^2, \text{ therefore } du = 2x dx$$

and we have that

$$f(x) = \frac{1}{2}(u-1)\sqrt{u} = \frac{1}{2}(u-1)u^{\frac{1}{2}}$$

We also use parts where

$$f(u) = u - 1 \Leftarrow f'(u) = 1$$

and

$$g'(u) = \frac{1}{2}u^{\frac{1}{2}} \Leftarrow g(u) = \frac{1}{3}u^{\frac{3}{2}}$$

and therefore we have that

$$\begin{aligned} \int x^3\sqrt{1+x^2} dx &= \int \frac{1}{2}x^2\sqrt{1+x^2} 2x dx \\ &= \int \frac{1}{2}(u-1)u^{\frac{1}{2}} du \\ &= (u-1)\left(\frac{1}{3}u^{\frac{3}{2}}\right) - \int \frac{1}{3}u^{\frac{3}{2}} du \\ &= \frac{1}{3}\left[(u-1)u^{\frac{3}{2}} - \frac{2}{5}u^{\frac{5}{2}} + C\right] \end{aligned}$$

Evaluate the following:

1. $\int_0^1 x(x^2+6)dx$

$$\int_0^1 x(x^2+6)dx = \int_0^1 x^3 + 6xdx = \left[\frac{1}{4}x^4 + 3x^2\right]_0^1$$

2. $\int_{-1}^1 (ax^2+bx+c)dx$

$$\int_{-1}^1 (ax^2+bx+c)dx = \left[\frac{a}{3}x^3 + \frac{b}{2}x^2 + cx\right]_{-1}^1 = \frac{2}{3}a + 2c$$

3. $\int_1^2 e^{-2x}dx$

$$\int_1^2 e^{-2x}dx = -\frac{1}{2}\int_1^2 e^{-2x}(-2)dx$$

Using substitution we have that

$$g(x) = -2x = u \Leftarrow g(1) = -2, g(2) = -4$$
$$g'(x) = -2 \Leftarrow du = -2dx$$
$$f(x) = e^x$$

we have that

$$\begin{aligned} -\frac{1}{2}\int_1^2 e^{-2x}(-2)dx &= -\frac{1}{2}\int_{-2}^{-4} e^u du \\ &= \frac{1}{2}\int_{-4}^{-2} e^u du \\ &= \frac{1}{2}e^u|_{-4}^{-2} \\ &= \frac{1}{2}\left[e^{-2} - e^{-4}\right] \end{aligned}$$