Slides for Chapter 3:

An Intertemporal Theory of the Current Account

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September 6, 2022

These are the slides for the textbook, "International Macroeconomics: A Modern Approach," by Stephanie Schmitt-Grohé, Martín Uribe, and Michael Woodford, Princeton University Press, 2022, ISBN: 9780691170640.

Motivation

- Build a model of an open economy to study the determinants of the trade balance and the current account.

- Study the response of the trade balance and the current account to income shocks.

- Pay special attention to how those responses depend on whether the shocks are perceived to be temporary or permanent.

A Small Open Economy

What does 'small' and 'open' mean in this context?

• An economy is small when world prices and world interest rates are independent of domestic economic conditions.

• An economy is open when it trades in goods and financial assets with the rest of the world.

• Most countries in the world are small open economies:

– Examples of developed small open economies: the Netherlands, Switzerland, Austria, New Zealand, Australia, Canada, Norway.

– Examples of emerging small open economies: Argentina, Chile, Peru, Bolivia, Greece, Portugal, Estonia, Latvia, Thailand.

– Examples of developed large open economies: United States, Japan, Germany, and the United Kingdom.

– Examples of emerging large open economies: China, India

– Examples of closed economies: North Korea, Venezuela, and to a lesser extent Cuba and Iran.

• Economic and geographic size are not necessarily related: Australia and Canada are geographically large but economically small. The UK and Japan are geographically small but economically large.

The Model Economy

- A two-period small open economy: periods 1 and 2.

- Households receive an endowment of Q_1 units of goods in period 1 and Q_2 units of goods in period 2.

- Goods are perishable.

- Initial asset holdings B_0 inherited from the past, paying the interest rate r_0 in period 1.

- In period 1, households choose consumption, C_1, and bond holdings, B_1, which pay the interest rate r_1 in period 2.

Sequential Budget Constraints of the Household

The period-1 budget constraint

$$C_1 + B_1 - B_0 = r_0 B_0 + Q_1. \tag{1}$$

The period-2 budget constraint

$$C_2 + B_2 - B_1 = r_1 B_1 + Q_2. \tag{2}$$

Because the world ends after period 2, no one is going to be around to pay or collect debts. So bond holdings must be nil at the end of period 2, that is,

$$B_2 = 0. \tag{3}$$

This expression is known as the *transversality condition*.

The Intertemporal Budget Constraint

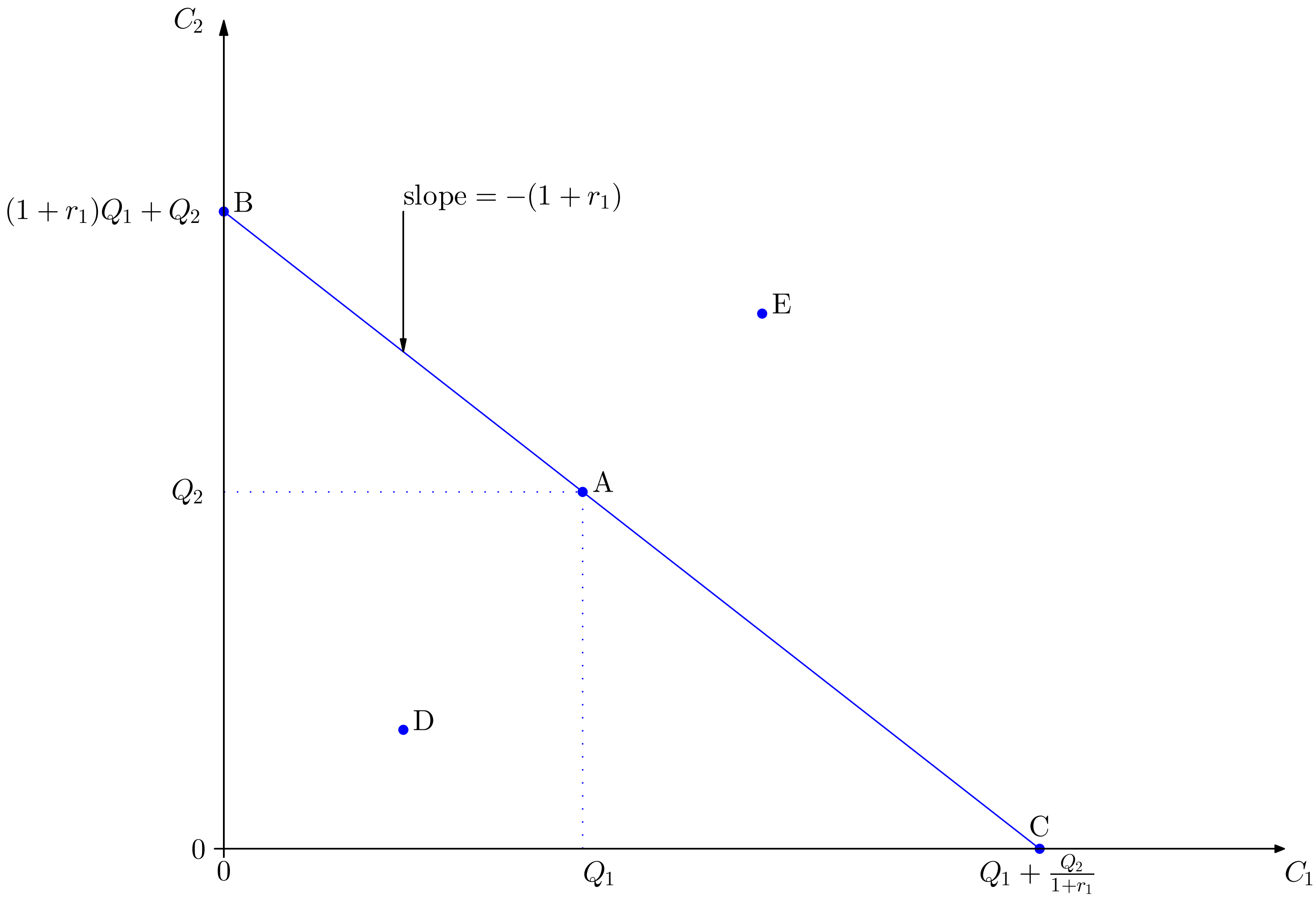
Combine (1), (2), and (3) to eliminate B_1 and B_2. This yields

$$C_1 + \frac{C_2}{1+r_1} = (1+r_0)B_0 + Q_1 + \frac{Q_2}{1+r_1}. \tag{4}$$

This expression represents the *intertemporal budget constraint*. It says that the present discounted value of the endowment plus the initial financial wealth (the right-hand side) must be equal to the present discounted value of consumption (the left-hand side).

The following figure provides a graphical representation of the intertemporal budget constraint.

The intertemporal budget constraint



Notes. The downward-sloping line represents the consumption paths (C_1, C_2) that satisfy the intertemporal budget constraint (4). The figure is drawn under the assumption that the household's initial asset position is zero, $B_0 = 0$.

Properties of the Intertemporal Budget Constraint

• It's downward sloping. Its slope is $-(1+r_1)$, because if you forgo one unit of consumption today and put it in the bank for one period, you get $1+r_1$ units next period.

• The set of feasible consumption paths (C_1, C_2) are those inside or at the borders of the triangle formed by the vertical axis, the horizontal axis, and the intertemporal budget constraint. Points A, B, C, and D are all feasible consumption paths.

• Point D, while feasible, violates the transversality condition because households leave money on the table at the end of period 2 ($B_2 > 0$).

• Points outside that triangle, such as point E, are infeasible. They violate the transversality condition because households leave unpaid debts at the end of period 2 ($B_2 < 0$).

• What feasible point the household will choose depends on its preferences. We turn to this issue next.

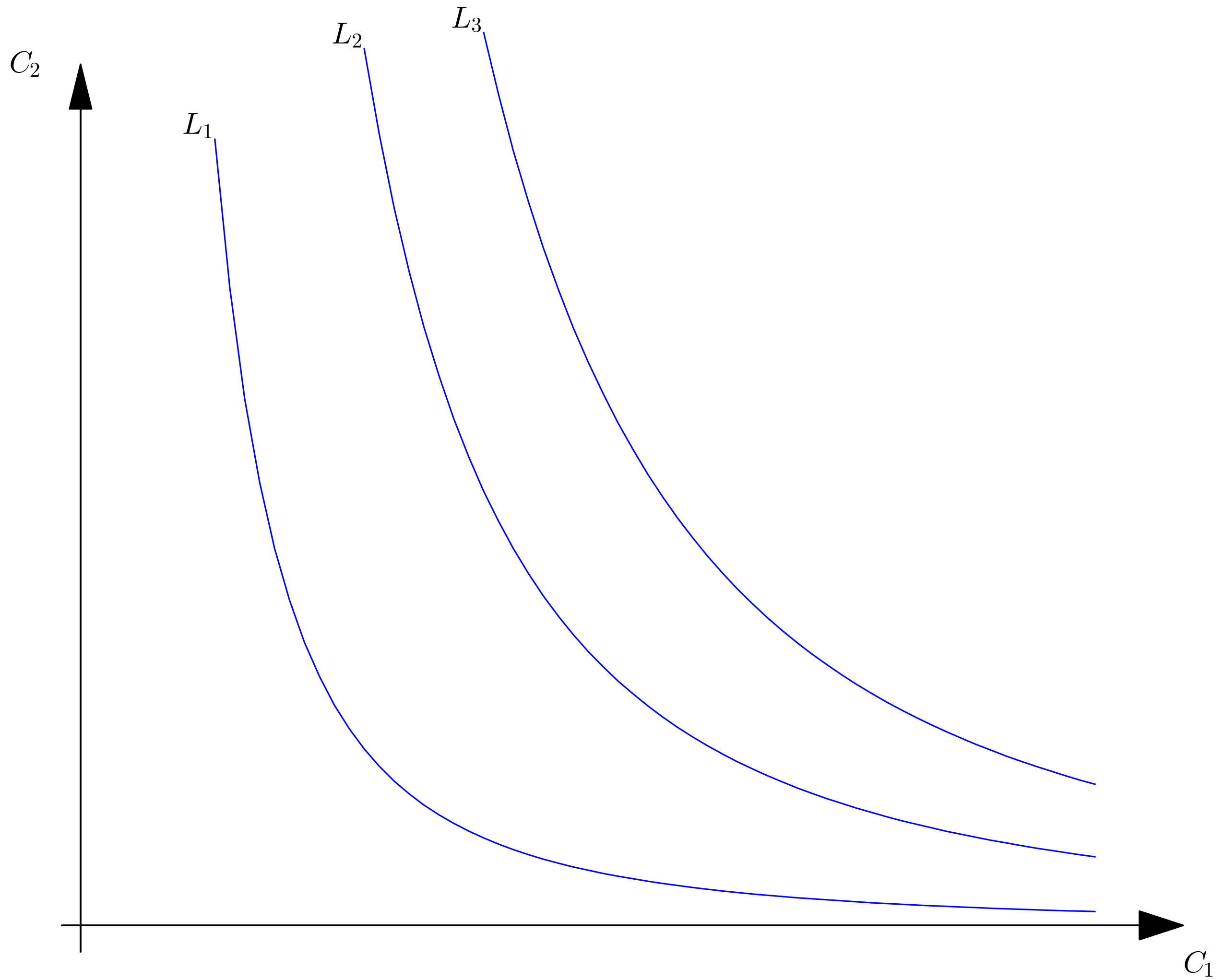
The Lifetime Utility Function

We assume that the household's happiness increases with the consumption of goods in periods 1 and 2. Preferences for consumption in periods 1 and 2 are described by the *lifetime utility function*, which is assumed to be of the form

$$U(C_1) + \beta U(C_2),$$

where $U(\cdot)$ denotes the *period utility function* and is assumed to be increasing and concave. The parameter $\beta > 0$ denotes the *subjective discount factor*. Typically it is assumed that $\beta \leq 1$, which means that in period 1 households care less about period-2 consumption than about period-1 consumption.

Indifference curves



An *indifference curve* is the set of consumption paths (C_1, C_2) that delivers the same level of welfare.

Properties of Indifference Curves

• If C_1 and C_2 are goods (i.e., objects for which more is preferred to less), indifference curves are downward sloping.

• An indifference curve located northeast of another one yields higher utility.

• One (and only one) indifference curve is associated with each point in the positive quadrant; they densely populate it.

• Indifference curves do not cross one another.

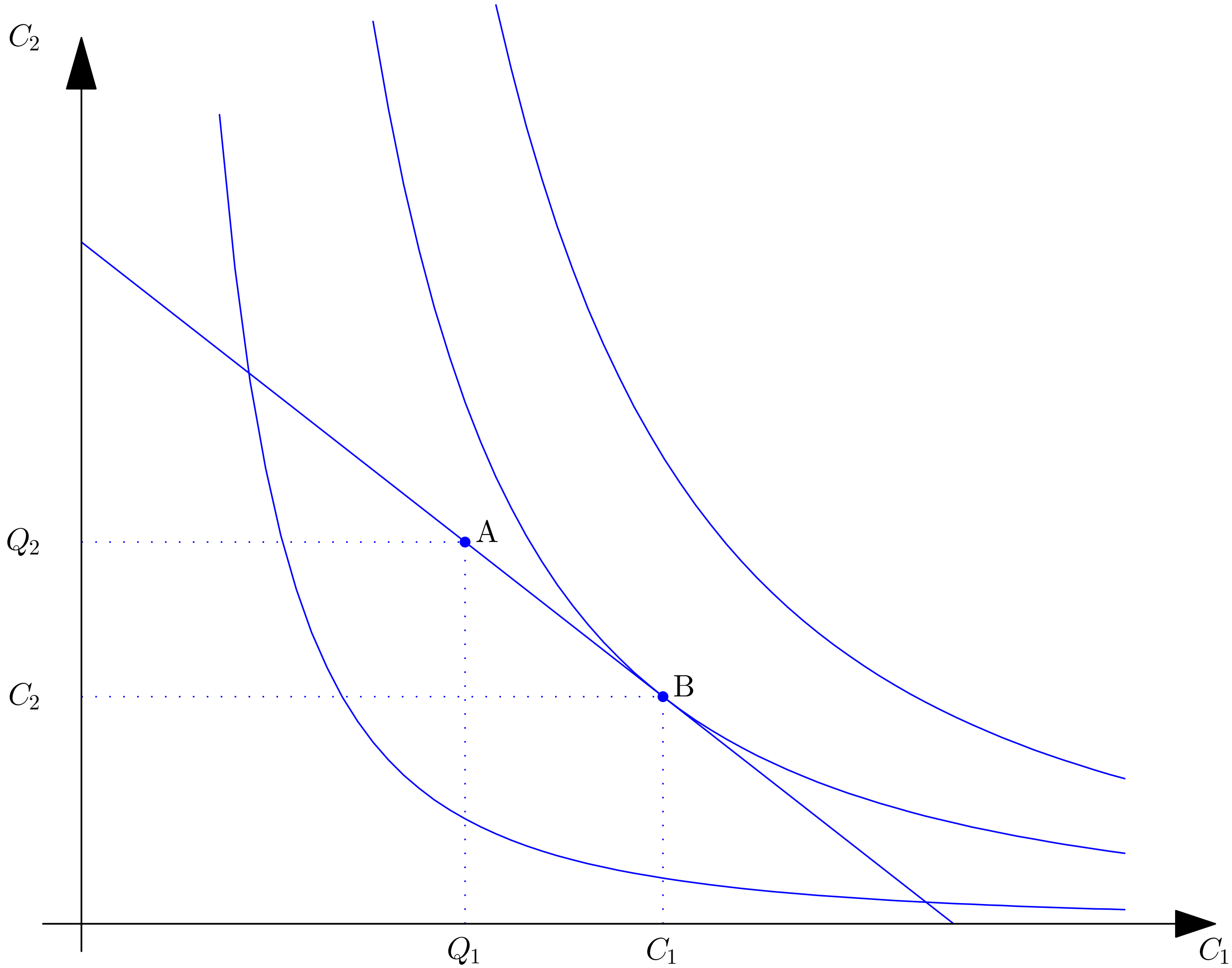
• The negative of the slope of the indifference curve is known as the *intertemporal marginal rate of substitution* of C_2 for C_1.

• The indifference curves we focus on are convex. If you are consuming a lot in period 1 and almost nothing in period 2, you are not willing to give up lots of period-2 consumption for an additional unit of period-1 consumption. But if you are consuming very little in period 1 and a lot in period 2, you are willing to give up a lot of period-2 consumption for an additional unit of period-1 consumption. This property of preferences is known as *diminishing marginal rate of substitution* of C_2 for C_1.

The Optimal Intertemporal Allocation of Consumption

- The household chooses consumption in periods 1 and 2 to maximize its lifetime utility function, subject to its intertemporal budget constraint (4).
- The next slide provides a graphical representation of how the optimal consumption path is determined. For simplicity, the graph is drawn assuming zero initial assets, $B_0 = 0$.
- The endowment point (Q_1, Q_2) is point A.
- The optimal consumption path is point B. This point is on the intertemporal budget constraint and belongs to an indifference curve that is tangent to the intertemporal budget constraint.
- The graph is drawn so that at point B, the household consumes more than its endowment. This means that it must borrow in period 1. In period 2, the household consumes less than his endowment, and uses the difference to pay back its debt including interest.
- In general, the optimal level of period-1 consumption does not need to be higher than the period-1 endowment. Whether period-1 consumption is higher, equal, or lower than the period-1 endowment depends on preferences, present and future endowments, initial wealth, and the interest rate.

The Optimal Consumption Path



Notes: The optimal consumption path (C_1, C_2) is at point B, where an indifference curve is tangent to the intertemporal budget constraint. As the figure is drawn, the household borrows in period 1 $(C_1 > Q_1)$ and pays back its debt in period 2 $(C_2 < Q_2)$. The figure is drawn under the assumption that the household's initial asset position is zero, $B_0 = 0$.

Deriving the Optimal Consumption Path

Formally, the household problem is

$$\max_{\{C_1, C_2\}} U(C_1) + \beta U(C_2)$$

subject to

$$C_1 + \frac{C_2}{1+r_1} = (1+r_0)B_0 + Q_1 + \frac{Q_2}{1+r_1}. \tag{4}$$

The household takes as given all objects on the right-hand side of the intertemporal budget constraint. Therefore, to save notation, let's call the right-hand side $\bar{Y}$:

$$\bar{Y} = (1+r_0)B_0 + Q_1 + \frac{Q_2}{1+r_1}.$$

Solve the intertemporal budget constraint for C_2 to get

$$C_2 = (1+r_1)(\bar{Y} - C_1). \tag{5}$$

Use this expression to eliminate C_2 from the lifetime utility function.

Deriving the Optimal Consumption Path (Continued)

The household maximization problem then becomes

$$\max_{\{C_1\}} U(C_1) + \beta U((1+r_1)(\bar{Y} - C_1))$$

To maximize this expression, take the derivative with respect to C_1, equate it to zero, and rearrange:

$$U'(C_1) = \beta(1+r_1)U'(C_2)$$

This optimality condition is known as the consumption *Euler equation*.

Rearrange terms to write the Euler equation as

$$-\frac{U'(C_1)}{\beta U'(C_2)} = -(1+r_1),$$

which says that at the optimal consumption path, the (negative of the) *marginal rate of substitution*, $\frac{U'(C_1)}{\beta U'(C_2)}$, is equal to the (negative of the) gross interest rate $(1+r_1)$, or graphically the slope of the indifference curve is equal to the slope of the intertemporal budget constraint.

The Interest Rate Parity Condition

We assume that there is *free capital mobility*, which means that households can borrow and lend freely in the international financial market. Let r^* be the world interest rate. Then, free capital mobility guarantees that the domestic interest rate be equal to the world interest rate. That is,

$$r_1 = r^*.$$

We will refer to this condition as the *interest rate parity condition*. Any difference between r_1 and r^* would give rise to an *arbitrage opportunity* that would allow someone to make infinite profits. For instance, if $r_1 > r^*$, then one could make infinite amounts of profits by borrowing in the international market and lending in the domestic market. Similarly, if $r_1 < r^*$, unbounded profits could be obtained by borrowing domestically and lending abroad. These arbitrage opportunities disappear when $r_1 = r^*$.

Equilibrium in the Small Open Economy

We assume that all households are identical. Thus, by studying the behavior of an individual household, we are also learning about the behavior of the country as a whole. This also implies that we can interpret B_t, for $t = 0, 1, 2$, as the country's net foreign asset position or NIIP at the end of period t.

An equilibrium then is a consumption path (C_1, C_2) and an interest rate r_1 that satisfy the country's *intertemporal resource constraint*, the consumption Euler equation, and the interest rate parity condition, that is,

$$C_1 + \frac{C_2}{1 + r_1} = (1 + r_0)B_0 + Q_1 + \frac{Q_2}{1 + r_1}, \tag{6}$$

$$U'(C_1) = (1 + r_1)\beta U'(C_2), \tag{7}$$

and

$$r_1 = r^*, \tag{8}$$

given the *exogenous variables* r_0, B_0, Q_1, Q_2, and r^*.

Graphical Representation of Equilibrium

Slide 19 is a graphical representation of the three equilibrium conditions shown in slide 17 that determine C_1, C_2, and r_1.

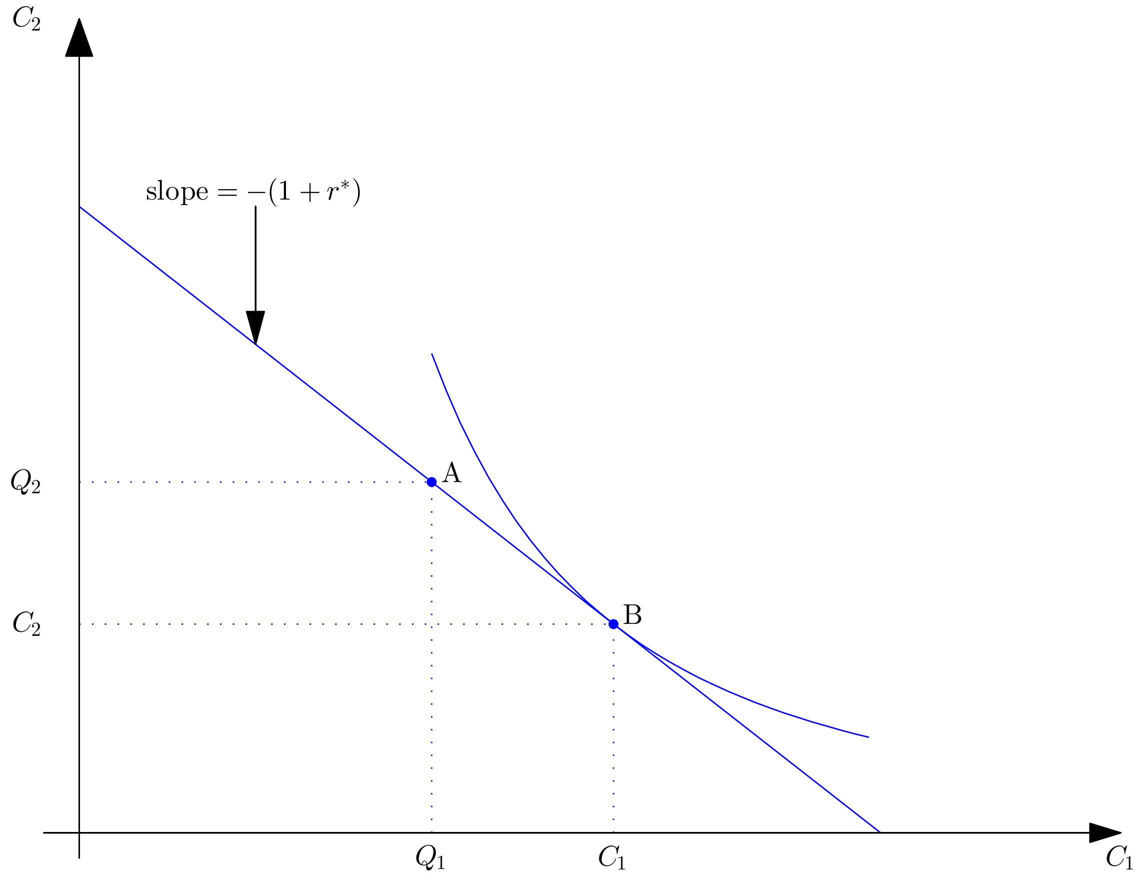
- The equilibrium is at point B.

- Point B is on the intertemporal resource constraint, as required by equilibrium condition (6).

- The indifference curve that crosses point B is tangent to the intertemporal budget constraint, whose slope is $-(1 + r_1)$. This means that equilibrium condition (7) holds.

- And the slope of the intertemporal resource constraint is $-(1+r^*)$, which means that $r^* = r_1$, as required by equilibrium condition (8).

Equilibrium in the Endowment Economy



Notes: The figure displays the equilibrium in a small open economy with free capital mobility and a zero initial net foreign asset position, $B_0 = 0$. The equilibrium is at point B, where an indifference curve is tangent to the intertemporal budget constraint. Because of free capital mobility, the domestic interest rate is equal to the world interest rate, r^*, so that the slope of the intertemporal resource constraint is $-(1+r^*)$. As the figure is drawn, the country runs trade and current account deficits in period 1, $C_1 > Q_1$.

The Trade Balance and the Current Account

We can now answer the question posed at the beginning: What determines the trade balance and the current account? The trade balance is the difference between output and consumption,

$$TB_1 = Q_1 - C_1$$

$$TB_2 = Q_2 - C_2$$

The current account equals the trade balance plus net investment income (NII)

$$CA_1 = TB_1 + r_0 B_0$$

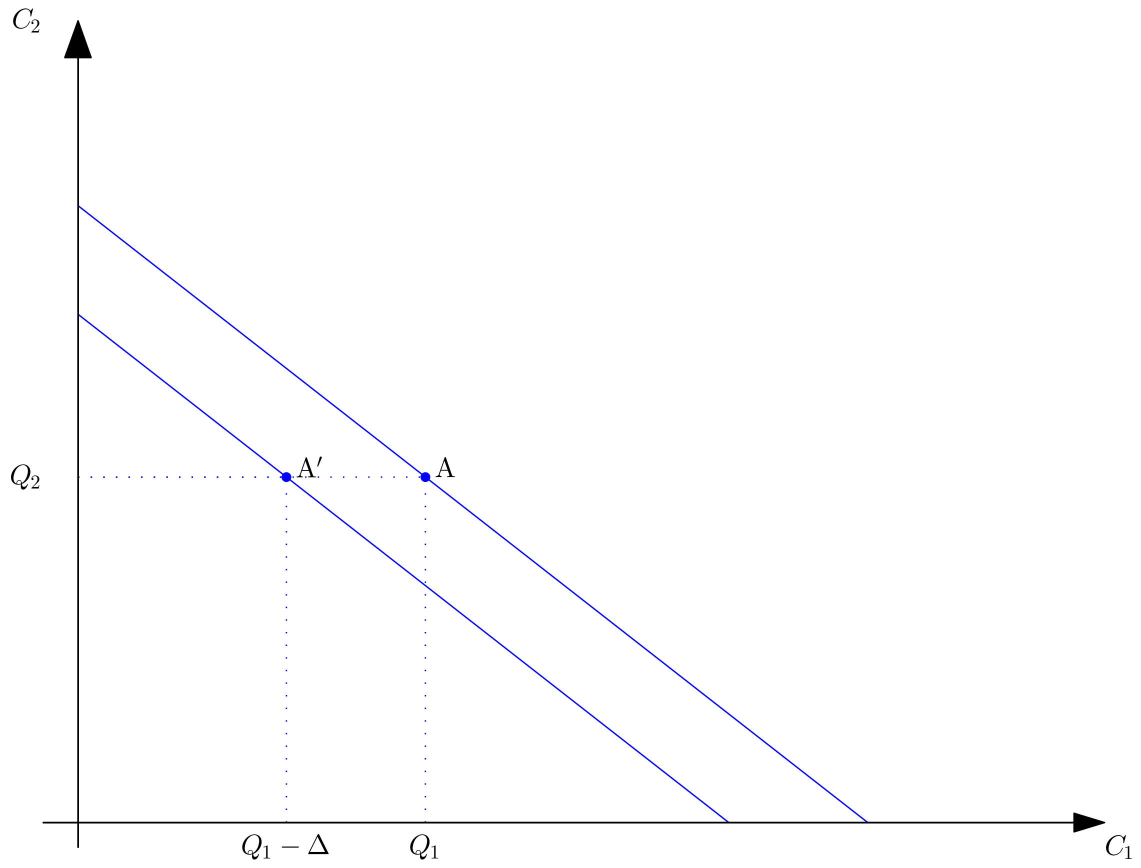
$$CA_2 = TB_2 + r_1 B_1$$

We have shown that the *endogenous variables*, C_1 and C_2 and r_1, depend on the *exogenous variables* Q_1, Q_2, r_0, B_0, and r^*. Therefore, we have that preferences, endowments, the world interest rate, and the initial net international investment position of a country are the determinants of the trade balance and the current account.

Adjustment to Temporary Output Shocks

Assume that output in period 1 falls from Q_1 to $Q_1 - \Delta < Q_1$ and output in period 2, Q_2 is unchanged. How does this temporary output decline affect the intertemporal budget constraint?

A Temporary Decline in Output and the Intertemporal Budget Constraint



Notes: In response to a decline in Q_1 equal to Δ, the intertemporal budget constraint shifts to the left by Δ. Point A indicates the endowment point prior to the output decline and point A′ indicates the endowment point after it. The figure is drawn under the assumption that the household's initial asset position is zero, $B_0 = 0$.

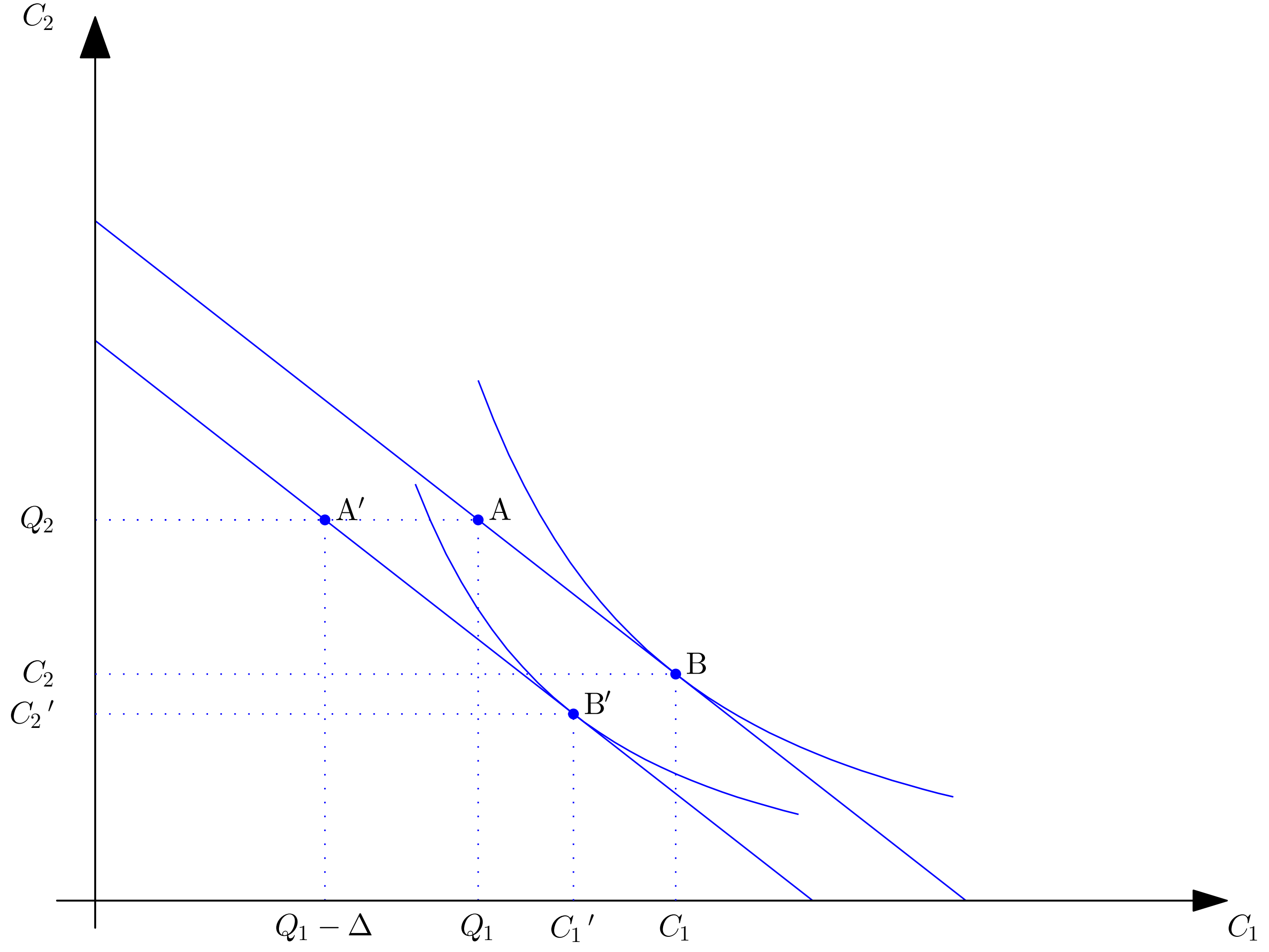
How will the household adjust to the temporary negative output shock?

Assume that both C_1 and C_2 are *normal goods* (i.e., goods whose consumption increases with income)

Then the household would want to cut both C_1 and C_2. By also cutting C_2, the household does not need to cut C_1 by Δ but by less.

The next figure illustrates the adjustment.

Adjustment to a Temporary Decline in Output



Notes: The figure depicts the adjustment of the economy to a decline in the period 1 endowment equal to Δ. The endowment point shifts left from point A to point A′ and the optimal consumption path shifts from point B to point B′. Period 1 consumption declines by less than Δ. The period 1 trade balance becomes more negative, $Q_1 - \Delta - C_1' < Q_1 - C_1$. The figure is drawn under the assumption that the household's initial asset position is zero, $B_0 = 0$.

In smoothing consumption over time, the country runs a larger trade deficit in period 1 (recall that it was running a trade deficit even in the absence of the shock) and finances it by acquiring additional foreign debt. Thus, the current account deteriorates. In period 2, the country must generate a larger trade surplus than the one it would have produced in the absence of the shock in order to pay back the additional debt acquired in period 1.

The important principle to take away from this example is that **temporary negative income shocks are smoothed out by borrowing from the rest of the world rather than by fully adjusting current consumption by the size of the shock.**

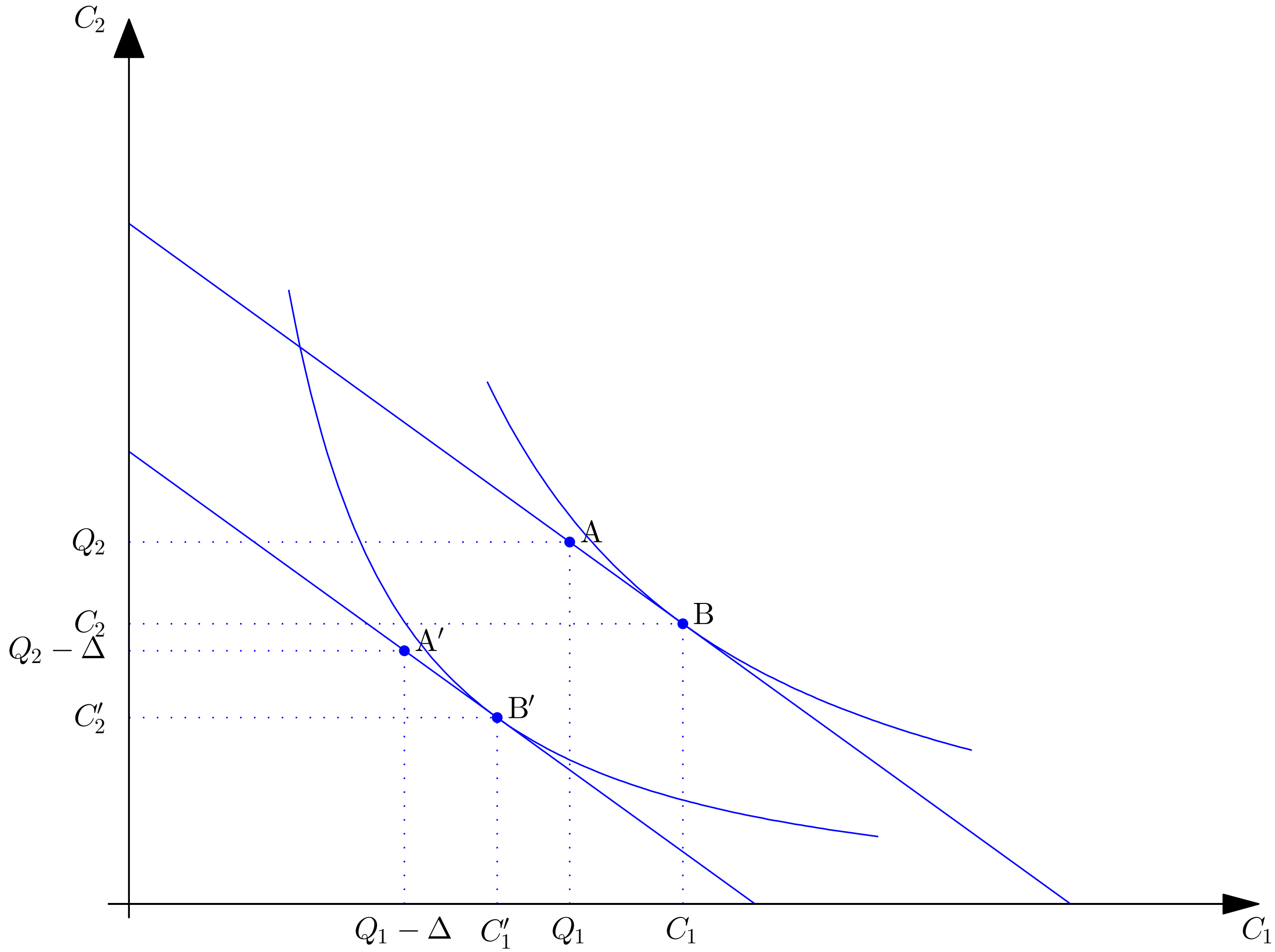
Adjustment to Permanent Output Shocks

Suppose Q_1 and Q_2 both fall by Δ.

In general the decline in consumption should be expected to be close to Δ, implying that a permanent output shock has little consequences for the trade balance and the current account.

The figure on the next slide illustrates this point.

Adjustment to a Permanent Decline in Output



Notes: The figure depicts the adjustment to a decline in Q_1 and Q_2 equal to Δ. The endowment point A shifts down and to the left to point A′. The intertemporal budget constraint shifts down in a parallel fashion. The optimal consumption path (C_1, C_2) shifts from point B to point B′. The figure is drawn for the case $B_0 = 0$. The period 1 trade balance is little changed.

Comparing the effects of temporary and permanent output shocks on the current account, the following general principle emerges:

Economies tend to finance temporary shocks (by borrowing or lending on international capital markets) and adjust to permanent ones (by varying consumption in both periods up or down).

Thus, temporary shocks tend to produce large movements in the current account while permanent shocks tend to leave the current account largely unchanged.

Anticipated Income Shocks

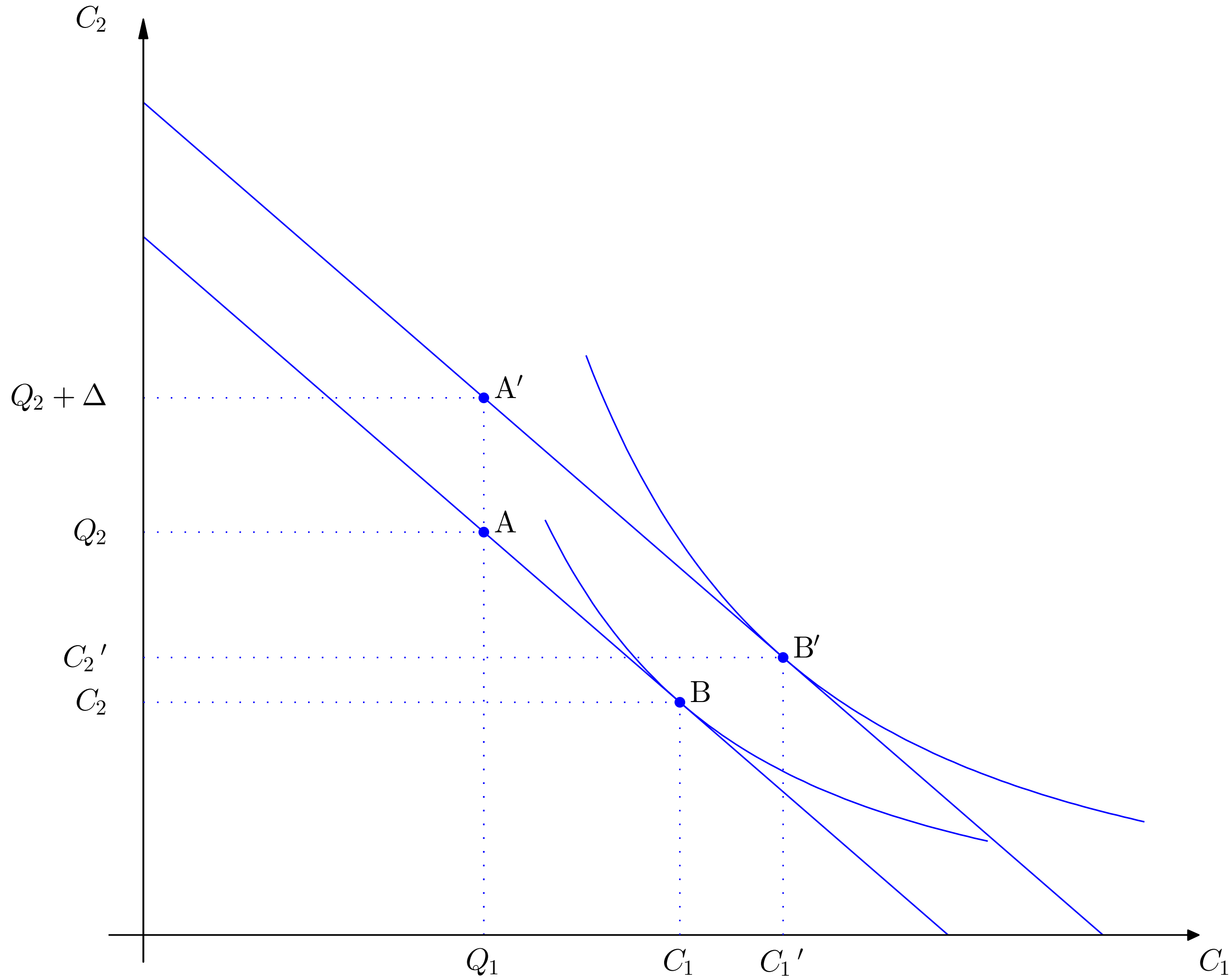
Consider now the case that in period 1 households learn that their endowment, Q_2, will be higher in period 2.

What will be the effect of this news on: consumption, the domestic interest rate, the trade balance, and the current account?

The figure on the next slide depicts the adjustment to an anticipated increase in Q_2 equal to $\Delta > 0$. The intertemporal budget constraint shifts up by Δ. The increase in the period-2 endowment causes an increase in period-1 consumption from C_1 to C_1'. Because the endowment in period 1 is unchanged, the period-1 trade balance and current account deteriorate.

Thus, good news about the future lead to a deterioration of the current account. This shows that current account deficits are not necessarily an indication of a weak economy.

Adjustment to an anticipated increase in output



An Economy with Logarithmic Preferences

Assume that preferences are logarithmic and that there is no discounting ($\beta = 1$). Then the lifetime utility is given by

$$U(C_1) + \beta U(C_2) = \ln C_1 + \ln C_2$$

The intertemporal budget constraint is

$$C_1 + \frac{C_2}{1 + r_1} = \bar{Y}$$

Solving the intertemporal budget constraint for C_2 and using the result to eliminate C_2 from the lifetime utility function, the household's optimization problem reduces to choosing C_1 to maximize

$$\max_{\{C_1\}} \quad \ln(C_1) + \ln((1 + r_1)(\bar{Y} - C_1)).$$

The first-order condition associated with this problem is

$$\frac{1}{C_1} - \frac{1}{\bar{Y} - C_1} = 0.$$

Solving for C_1 yields

$$C_1 = \frac{1}{2}\bar{Y} \tag{9}$$

This result says that households find it optimal to consume half of their lifetime wealth in the first half of their lives.

Combining (5) and (9) yields

$$C_2 = \frac{1}{2}\bar{Y}(1 + r_1) \tag{10}$$

This is also intuitive. The household consumes half of $\bar{Y}$ in period 1 and puts the other half in the bank, receiving $\frac{1}{2}\bar{Y}(1 + r_1)$ for consumption in period 2.

Deriving the Optimal Consumption Path (Continued)

Now recall that $\bar{Y} = (1 + r_0)B_0 + Q_1 + \frac{Q_2}{1+r_1}$ and that in equilibrium $r_1 = r^*$, to write (9), as

$$C_1 = \frac{1}{2}\left[(1 + r_0)B_0 + Q_1 + \frac{Q_2}{1 + r^*}\right] \quad (11)$$

According to this expression, period-1 consumption is increasing in Q_1, Q_2, and $(1 + r_0)B_0$, and decreasing in the interest rate r^*.

The response of C_1 to an increase in Q_1 depends crucially on what households expect to happen with Q_2. If the increase in output is temporary, so that Q_2 is not expected to change, then C_1 increases by 1/2 times the change in period-1 output. Households leave the other half for future consumption.

$$\Delta C_1 = \frac{1}{2}\Delta Q_1 \quad (12)$$

But if the increase in Q_1 is expected to be associated with an equal increase in Q_2, households consume most of the current output increase in the current period because there is no need to save much for next period when output will also be higher,

$$\Delta C_1 = \frac{1}{2}\left(\Delta Q_1 + \frac{1}{1+r^*}\Delta Q_2\right) \tag{13}$$

We do not observe Q_2 in period 1, so the reaction of C_1, which we do observe, allows us to infer in period 1 whether the change in Q_1 is expected to be temporary or permanent.

Effect of a Temporary Output Shock on the Current Account

Suppose that output increases in period 1 but is expected not to change in period 2. That is, assume that

$$\Delta Q_1 > 0 \text{ and } \Delta Q_2 = 0$$

Recall that

$$CA_1 = TB_1 + r_0 B_0 = Q_1 - C_1 + r_0 B_0 \tag{14}$$

Differentiating this expression and recalling from (12) that $\Delta C_1 = \frac{1}{2}\Delta Q_1$, we get

$$\Delta CA_1 = \Delta Q_1 - \Delta C_1 = \Delta Q_1 - \frac{1}{2}\Delta Q_1 = \frac{1}{2}\Delta Q_1$$

The current account improves by half the increase in output. Households know the output increase is temporary. So, because they like to smooth consumption over time, they save half of it for consumption next period.

Effect of a Permanent Output Shock on the Current Account

Suppose that output increases by the same amount in both periods 1 and 2. That is, assume that

$$\Delta Q_1 = \Delta Q_2 > 0$$

Differentiating the expression for the current account (14), we have

$$\Delta CA_1 = \Delta Q_1 - \Delta C_1$$

and from (13)

$$\Delta C_1 = \frac{1}{2}\left[\Delta Q_1 + \frac{\Delta Q_2}{1+r^*}\right]$$

Since $\Delta Q_1 = \Delta Q_2$, we can write

$$\Delta CA_1 = \frac{1}{2}\frac{r^*}{1+r^*}\Delta Q_1$$

The increase in the current account is now only a fraction $\frac{1}{2}\frac{r^*}{1+r^*}$ of the change in output, much smaller than in the case of a temporary shock. This makes sense: Why save a large part of the increase in current output if output is also expected to increase next period?

Intuition

If you lose your lunch money one day, it's not a big problem. You simply borrow from a friend. Next time, you pay his lunch. However, if your father cuts your monthly allowance, you will have to make plans to reduce your spending accordingly. We have seen that a similar principle is at work with the current account. We summarize this principle as follows:

A General Principle

Finance temporary output shocks (by running current account deficits or surpluses without much change in spending) and adjust to permanent output shocks (by changing spending, without much change in the current account).

An Anticipated Increase in Future Output

Suppose Q_2 increases, $\Delta Q_2 > 0$, while Q_1 is unchanged, $\Delta Q_1 = 0$.

By equation (11),

$$\Delta C_1 = \frac{1}{2(1+r^*)}\Delta Q_2 > 0$$

From the definition of the trade balance, we have

$$\Delta TB_1 = \Delta Q_1 - \Delta C_1 = 0 - \frac{1}{2(1+r^*)}\Delta Q_2 < 0$$

And by (14),

$$\Delta CA_1 = -\frac{1}{2(1+r^*)}\Delta Q_2 < 0$$

Intuition: The increase in Q_2 makes households richer, inducing them to increase C_1 and C_2. With Q_1 unchanged, the increase in C_1 causes a fall in TB_1, which must be financed by external borrowing, that is, by a fall in CA_1. Thus, good news about the future causes a deterioration of the current account. This shows that current account deficits are not necessarily an indication of a weak economy.

Summing Up

This chapter presents an intertemporal model of the current account with 3 building blocks:

- Households face an intertemporal budget constraint.
- Households have preferences over present and future consumption.
- Households choose a consumption path that maximizes lifetime utility subject to the intertemporal budget constraint.
- Free capital mobility equalizes the domestic and world interest rates.

The model delivers the following key insights:

- In response to temporary income shocks, countries use the current account to smooth consumption over time. Positive temporary shocks cause an improvement in the current account and negative temporary shocks cause a deterioration.
- In response to permanent income shocks, countries adjust consumption without much movement in the current account.
- In response to an anticipated increase in future income, the trade balance and the current account deteriorate.