Learning to Live in a Liquidity Trap

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The Starting Point

In “The Perils of Taylor Rules” (*JET*,2001), Benhabib, Schmitt-Grohé, and Uribe (BSU) show that Taylor-type interest-rate feedback rules can open the door to liquidity traps. In particular, in addition to the intended equilibrium with inflation and output near target, Taylor rules allow for the existence of a second, unintended equilibrium in which nominal rates are zero, inflation is below target, and output is low.

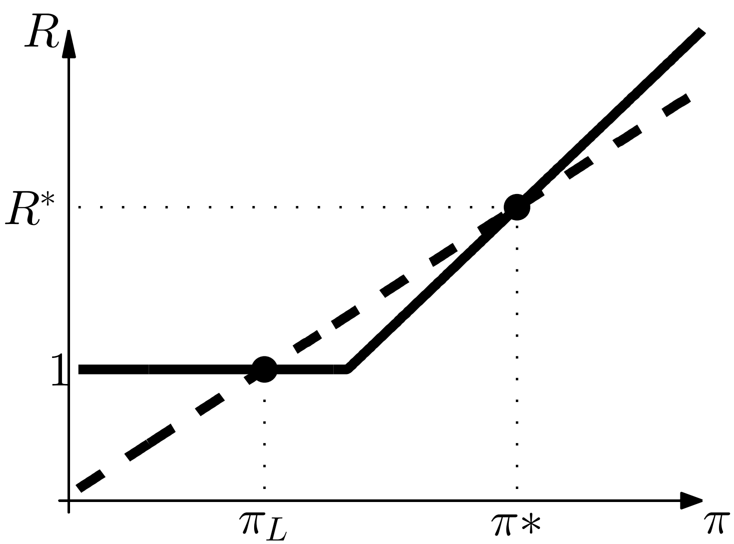
A Brief Exposition of the BSU Argument

The Taylor Rule: $R_t = \max\{1, R^* + \alpha_\pi(\pi_t - \pi^*)\}$

The Euler Equation: $U'(C_t) = \beta R_t E_t \frac{U'(C_{t+1})}{\pi_{t+1}}$

In a steady state they become, respectively,

$$R = \max\{1, R^* + \alpha_\pi(\pi - \pi^*)\} \text{ and } R = \beta^{-1}\pi$$



Solid Line: $R = \max\{1, R^* + \alpha_\pi(\pi - \pi^*)\}$

Broken Line: Euler equation $R = \beta^{-1}\pi$

Two Inflation Steady States:
The intended steady state (π^*) and the Liquidity Trap (π_L)

Dynamics in a Flexible-Price Endowment Economy

The Taylor Rule

$$R_t = \max\{1, R^* + \alpha_\pi (\pi_t - \pi^*)\}$$

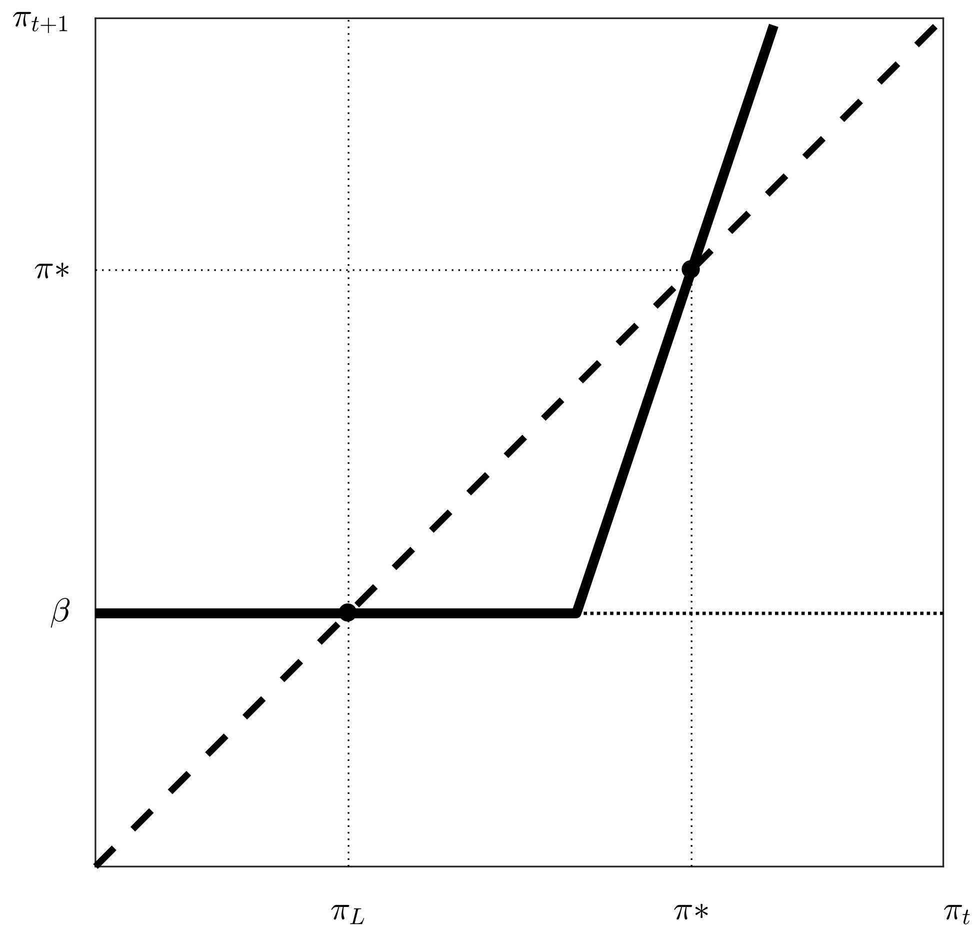
The Euler Equation

$$\pi_{t+1} = \beta R_t$$

Combining yields the equilibrium dynamics of inflation

$$\pi_{t+1} = \max\{\beta, \beta R^* + \beta\alpha_\pi (\pi_t - \pi^*)\}$$

The next slide shows the phase diagram implied by this difference equation.



Solid line: $\pi_{t+1} = \max \{\beta, \beta R^* + \beta\alpha_\pi (\pi_t - \pi^*)\}$
Broken line: 45-degree line

Comment: Similar results obtain in sticky-price/wage economies.

Are the “Perils of Taylor Rules” Theoretically Relevant?

• **Main Criticism:** The liquidity trap equilibrium is not stable under some alternatives to rational expectations, in particular various forms of least-square learning of the type pioneered by Marcet and Sargent (JET, 1989).

Examples

• Bullard and Mitra (JME, 2002), Evans and Honkapohja (RES, 2003), Evans and McGough (JMCB forthcoming) show that pure interest rate pegs are unstable under least square learning.

• Evans, Guse, Honkapohja (EER, 2008) and others show that the Perils of Taylor rules are not *E-stable.*

This paper

- shows that the Liquidity-Trap equilibrium is stable under social learning.

- The social learning mechanism includes three elements:

– **Mutation:** People learn from mistakes and try new ideas.
– **Crossover:** People imitate other people.
– **Tournaments:** Successful people pass their views to others.

- **Main Result:** Agents can learn to have pessimistic sentiments about the central bank's ability to generate price growth, giving rise to a stochastically stable environment characterized by zero nominal rates, deflation, and stagnation.

- This paper extends the work of Arifovic, Bullard, and Kostyshyna (EJ, 2012) showing that equilibria under passive interest-rate rules are stable under social learning.

The Environment: New-Keynesian Model with a Taylor Rule

$$y_t = y^e_{t+1} - \sigma^{-1}(i_t - \pi^e_{t+1}) + \sigma^{-1} r^n_t \tag{1}$$

$$\pi_t = \kappa y_t + \beta \pi^e_{t+1} \tag{2}$$

$$i_t = \max\{-i^*, \phi_\pi \pi_t + \phi_y y_t\} \tag{3}$$

y_t = the output gap
y^e_{t+1} = period-t expected value of y_{t+1}
π_t = deviation of the inflation rate from target
π^e_{t+1} = period-t expected value of π_{t+1}
i_t = deviation of the nominal interest rate from target i^*
r^n_t = exogenous natural rate shock, follows 2-state Markov process
$r^n_t \in \{r^n_H, r^n_L\}$ with transition probability matrix $\begin{bmatrix} \rho_H & 1-\rho_H \\ 1-\rho_L & \rho_L \end{bmatrix}$

Minimum-State-Variable Rational Expectations Equilibria (MSV-REE): A MSV-REE is a 4-tuple

$$z = \begin{bmatrix} y_H \\ \pi_H \\ y_L \\ \pi_L \end{bmatrix}$$

satisfying (1)-(3).

Under the most common calibration of the canonical NK model (which we adopt), there exist the following two MSV-REE:

• **REE-NB:** The zero lower bound never binds. This is the intended equilibrium, because output and inflation fluctuate around their respective targets.

• **REE-AB:** The zero lower bound always binds. This is the liquidity-trap equilibrium. It is an unintended equilibrium because the economy suffers chronic deflation and unemployment.

Perceived and Actual Laws of Motion (PLM and ALM)

There are N agents. The **PLM** of agent i is:

$$z_{iH} = \left[\begin{array}{cc} y_{iH} & \pi_{iH} \end{array} \right]' \text{ and } z_{iL} = \left[\begin{array}{cc} y_{iL} & \pi_{iL} \end{array} \right]'$$

• **Expectations of agent i** about next period's output and inflation conditional on the current state being H or L:

$$z_{iH}^e = \rho_H z_{iH} + (1 - \rho_H) z_{iL}; \quad \text{and} \quad z_{iL}^e = (1 - \rho_L) z_{iH} + \rho_L z_{iL}$$

• **Aggregate expectations**

$$z_H^e = \frac{1}{N} \sum_{i=1}^{N} z_{iH}^e \quad \text{and} \quad z_L^e = \frac{1}{N} \sum_{i=1}^{N} z_{iL}^e$$

ALM: Plug the aggregate expectations into the equations of the NK model and solve the resulting static system for output and inflation.

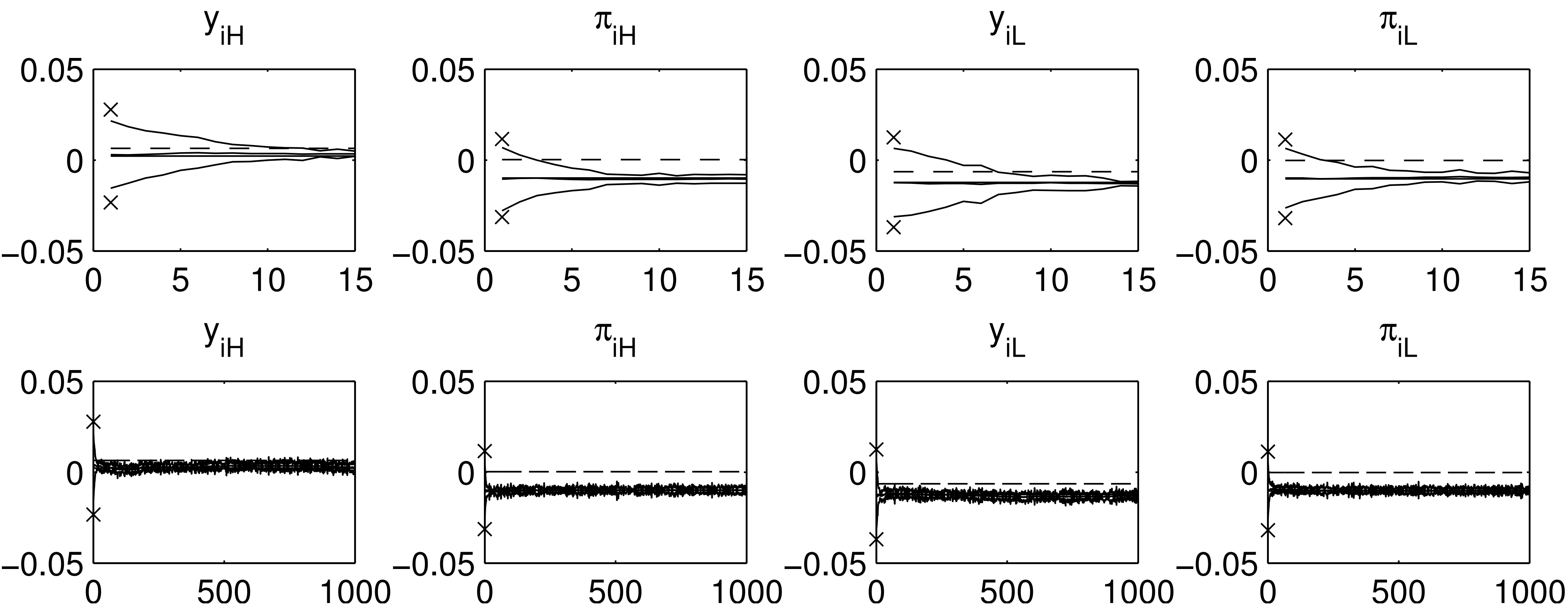
Evaluating the Stability of the Liquidity-Trap Equilibrium Under Social Learning

(1) Assume that the economy has been at the liquidity trap for some time (all individuals have the same PLMs).

(2) In period 1, individual PLMs receive an exogenous perturbation.

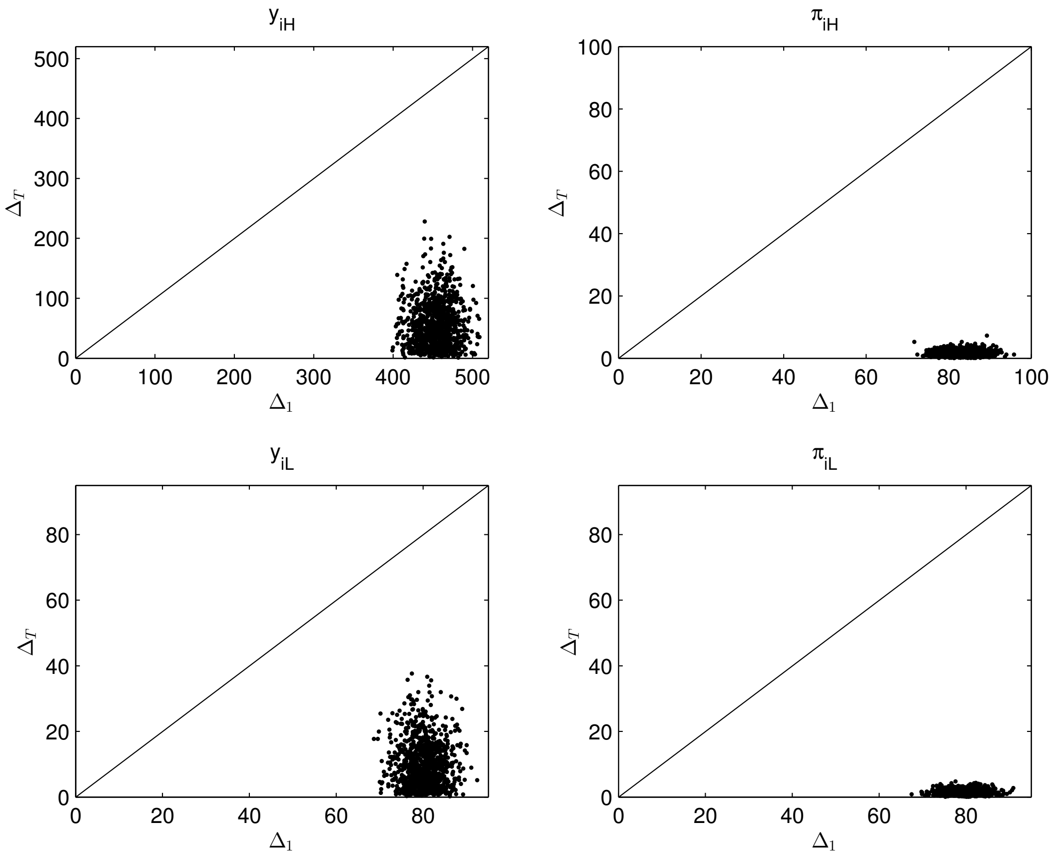
(3) Check whether, through social learning (crossovers, mutations, and tournaments), the economy returns to the liquidity trap.

Learning the Liquidity Trap: Evolution of the Cross-Sectional Distribution of PLMs



Notes. The first row displays the first 15 periods of the simulation and the second row the first 1000 periods. Mean and two-standard deviation band around the mean of the cross-sectional distribution of PLMs. The solid horizontal line is the PLM of the liquidity-trap equilibrium, and the broken horizontal line is the PLM of the intended equilibrium. Crosses mark the two-standard-deviation band of the cross sectional distribution of PLMs immediately after the perturbation.

Cross-Sectional Dispersion of Beliefs at the Beginning and End of the Simulation Period for One Thousand Simulations



Note. Δ_1 and Δ_T stand for the cross-sectional average percent absolute deviation of PLMs from the liquidity-trap RE equilibrium in periods 1 and $T = 1000$, respectively. Each dot is one simulation, and there are 1000 simulations.

Conclusion

• The Taylor rule in combination with the zero lower bound on nominal rates has been shown to create an unintended liquidity-trap equilibrium.

• The relevance of this equilibrium has been challenged on the basis that it is not stable under least-square learning.

• In this paper, we show that the liquidity-trap equilibrium is stable under social learning.

• The learning mechanism we employ includes three realistic elements: mutation, crossover, and tournaments.

• We show that agents can learn to have pessimistic sentiments about the central bank's ability to generate price growth, giving rise to a stochastically stable environment characterized by deflation and stagnation.

EXTRAS

1. Crossover

Agents are randomly matched into pairs without replacement. With probability 0.5 an element of the PLM of agent i_1, $\{y_{i_1 H}, \pi_{i_1 H}, y_{i_1 L}, \pi_{i_1 L}\}$, will be exchanged for the corresponding element of the PLM of agent i_2.

Not all crossovers will actually be adopted. Only a fraction $pc = 0.1$ of crossovers will be.

old PLM of agent i_1: $\{y_{i_1 H}, \pi_{i_1 H}, y_{i_1 L}, \pi_{i_1 L}\}$
new PLM of agent i_1: $\{y_{i_2 H}, \pi_{i_1 H}, y_{i_1 L}, \pi_{i_1 L}\}$

old PLM of agent i_2: $\{y_{i_2 H}, \pi_{i_2 H}, y_{i_2 L}, \pi_{i_2 L}\}$
new PLM of agent i_2: $\{y_{i_1 H}, \pi_{i_2 H}, y_{i_2 L}, \pi_{i_2 L}\}$

Need to pick: pc.

2. Mutation

PLM prior to mutation: $z_i = [y_{iH}\, \pi_{iH}\, y_{iL}\, \pi_{iL}]'$

PLM after mutation: $z_i' = z_i + \Sigma_m \epsilon$

The probability that an element of an individual's PLM experiences a mutation is given by $pm = 0.1$.

Need to pick: Σ_m and pm

3. Tournament Selection

In each period N matches of two individuals are formed with replacement. For each match the PLMs for inflation and output, separately, that produced the better *fitness measure* will be adopted.

$$z_{i,k}^f = \begin{cases} \Gamma_H z_i, & \text{if } r_{k-1}^n = r_H^n \\ \Gamma_L z_i, & \text{if } r_{k-1}^n = r_L^n \end{cases}$$

Then the fitness of the period-t PLM of agent i for output and inflation is defined as

$$F_{i,t}^y = -\frac{1}{t}\sum_{k=2}^{t}\left(y_k - y_{i,k}^f\right)^2$$

and

$$F_{i,t}^\pi = -\frac{1}{t}\sum_{k=2}^{t}\left(\pi_k - \pi_{i,k}^f\right)^2$$

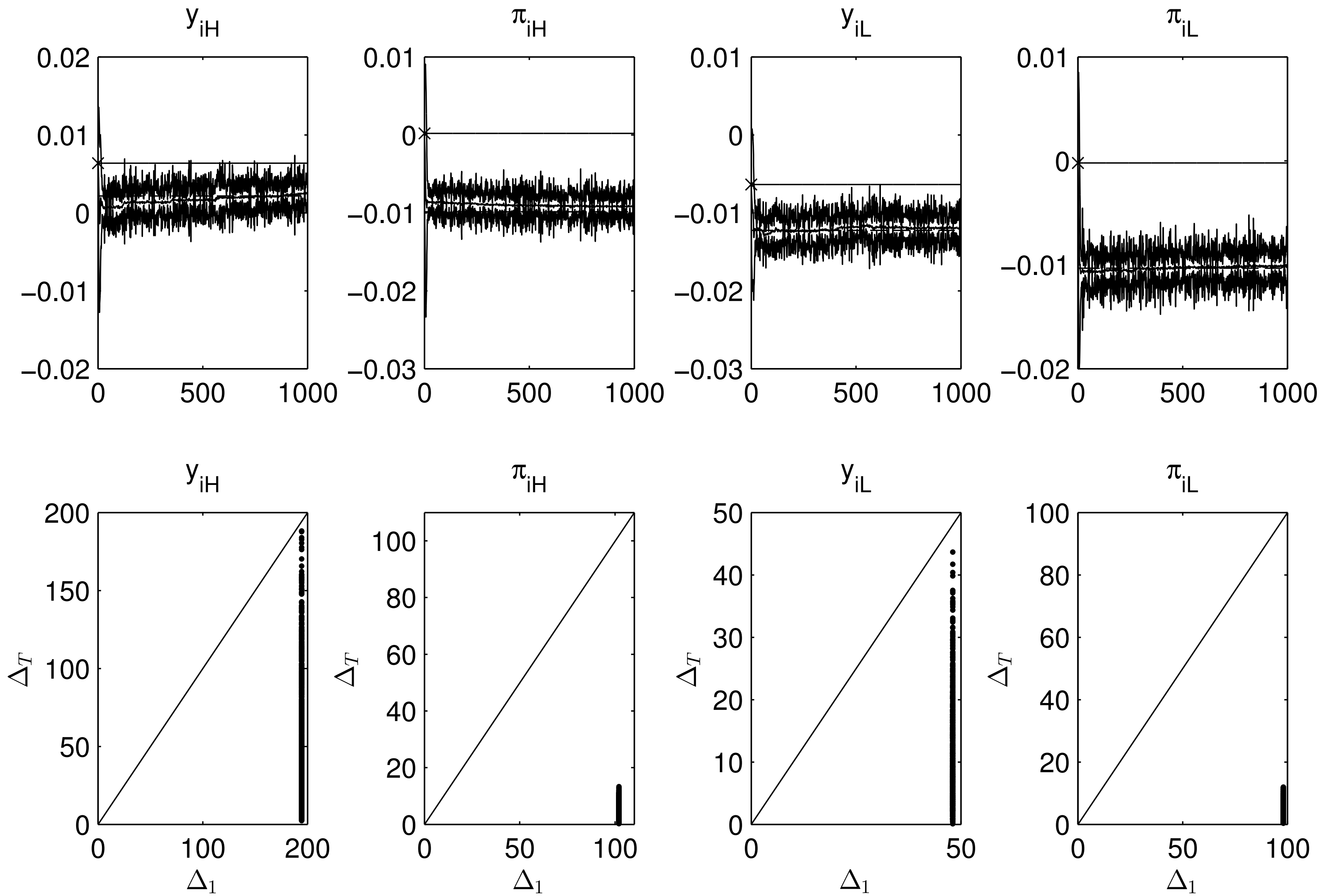
This fitness measure has the interpretation of the squared-sum of one-period-ahead forecast errors. The purpose of tournament selection is to promote the PLMs with higher forecasting accuracy.

Calibration

Symbol	Value	Description
		Structural Parameters
β	0.99	Subjective discount factor
σ	2	Reciprocal of Intertemp. elast. of substit.
κ	0.02	Output coefficient of the Phillips curve
i^*	0.0101	Nominal interest rate target ($= \beta^{-1} - 1$)
ϕ_π	1.5	Inflation coefficient of the Taylor rule
ϕ_y	0.125	Output coefficient of the Taylor rule
		r_t^n Process
r_H^n	0.0093	Deviation of r_t^n from steady state in state H
r_L^n	-0.0093	Deviation of r_t^n from steady state in state L
ρ_H	0.675	Prob$(r_{t+1}^n = r_H^n \| r_t^n = r_H^n)$
ρ_L	0.675	Prob$(r_{t+1}^n = r_L^n \| r_t^n = r_L^n)$ ($= \rho_H$)
		Social Learning Hyperparameters
T_{IH}	100	Length of initial history
T	1000	Length of simulation period
N	300	Number of Agents
pc	0.1	Probability of crossover
pm	0.1	Probability of mutation

Notes. Quarterly calibration. $\Sigma = \Sigma_m$ are diagonal, with diagonal $[\ 0.0123 \quad 0.0103 \quad 0.0123 \quad 0.0103\ ]'$.

Convergence from the Intended Equilibrium to the Liquidity Trap



Note. See notes to the previous figure.