Balanced-Budget Rules, Distortionary Taxes, and Aggregate Instability

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A traditional argument against a balanced-budget fiscal policy rule is that it amplifies business cycles by stimulating aggregate demand during booms via tax cuts and higher public expenditures and by reducing demand during recessions through a corresponding fiscal contraction. This paper suggests an additional source of instability that may arise from this type of fiscal policy rule. It shows that, within the standard neoclassical growth model, a balanced-budget rule can make expectations of higher tax rates self-fulfilling if the fiscal authority relies heavily on changes in labor income taxes to eliminate short-run fiscal imbalances. Calibrated versions of the model show that indeterminacy occurs for income tax rates that are empirically plausible for the U.S. economy and other Group of Seven countries.

I. Introduction

A traditional argument against a balanced-budget rule is that it amplifies business cycles by stimulating aggregate demand during booms via tax cuts and higher public expenditures and by reducing

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demand during recessions through a corresponding fiscal contraction. This argument is consistent not only with the Keynesian IS-LM model but also with the neoclassical growth model. King, Plosser, and Rebelo (1988) show that in a real business cycle model the amplitude of the business cycle increases when the government follows a balanced-budget rule and finances government spending with income taxes.

In this paper we present an additional reason why a balanced-budget rule can be destabilizing. We embed a balanced-budget rule into the neoclassical growth model and assume that the fiscal authority finances its expenditures with distortionary income taxes. We show that under this type of policy, persistent and recurring fluctuations in aggregate activity become possible in the absence of shocks to fundamentals. Specifically, under a balanced-budget rule the rational expectations equilibrium can be indeterminate and stationary sunspot equilibria may exist. In order to obtain this result, the presence of endogenous distortionary taxes is crucial: it is straightforward to show that endogenous fluctuations are impossible when the balanced-budget rule consists of fixed income tax rates and endogenous government expenditures.

To show the main result of the paper analytically, in Section II we consider a simple case in which the only source of government revenue is a labor income tax and government expenditures are constant. We also simplify the model by assuming indivisible labor, as in Hansen (1985). We show that the necessary and sufficient condition for a balanced-budget rule to generate indeterminacy of the rational expectations equilibrium is that the steady-state labor income tax rate is greater than the share of capital in output and less than the labor income tax rate that maximizes the steady-state Laffer curve.

In Section III, we extend the model to allow for capital income taxation, more general preferences, and income-elastic government spending. We calibrate the model and show numerically that indeterminacy arises within the range of capital and labor income tax rates observed for the United States and other industrialized countries. From a sensitivity analysis, we find that the rational expectations equilibrium is more likely to be indeterminate the higher the labor share and the Frisch elasticity of labor supply with respect to wage (as is also true in models in which indeterminacy is a consequence of increasing returns to scale), the less procyclical government expenditures, the more responsive labor income taxes in balancing the budget, and the larger the share of public expenditures financed by distortionary taxes. We also consider the presence of public debt and the case in which taxes are set in advance. We find that our basic result is robust to these extensions.

In Section IV, we introduce intrinsic uncertainty in the form of exogenous productivity and government purchases shocks and investigate the propagation of fundamental and sunspot shocks under various assumptions about their joint distribution. We find that when productivity and government purchases shocks are persistent, measures of comovement among endogenous variables—such as serial correlation, contemporaneous correlation with output, and standard deviation relative to output—are virtually unaffected by the assumed correlation and relative volatility of innovations in sunspot and fundamental shocks.

From a policy standpoint, our results suggest that if the proposed balanced-budget amendment to the U.S. Constitution is not to create endogenous aggregate instability, it should be combined either with restrictions on the government's ability to change tax rates in response to innovations in the state of the economy or with a reduction in the level of income tax rates currently in place. It is interesting that these are the same type of restrictions advocated by those who regard the proposed amendment primarily as a way to limit the size of the government rather than simply as a way to curb the growth of the public debt. As Milton Friedman puts it, "a balanced-budget amendment—one of the items included in the Republican Contract With America—is a means to an end. The end is holding down the growth in (or better, sharply reducing) government spending. The right kind of amendment can contribute to that end; the wrong kind can make it more difficult to achieve. The key difference is whether the amendment couples the requirement of budget balance with a limit on taxes and, hence, spending" (*Wall Street J.*, January 4, 1995, p. A12). Our paper thus adds a short-run motivation to the traditional long-run approach to tax limits. In a broader context, this paper is related to a literature suggesting that some frequently proposed policy feedback rules linking monetary and fiscal variables to the state of the economy can induce endogenous fluctuations and hence be destabilizing (Leeper 1991; Woodford 1994; Uribe 1995; Schmitt-Grohé and Uribe 1997).

II. An Economy with Labor Income Taxes

The Model

In this section we derive analytically the main result of the paper. In order to do this we analyze a simple neoclassical economy in which labor is indivisible (as in Hansen [1985]), the only source of government revenue is a labor income tax, government purchases are constant, the initial shock of public debt is zero, and the government is subject to a balanced-budget requirement. The government budget constraint is then given by $G = \tau_t w_t H_t$, where G denotes government purchases of goods, τ_t the labor income tax rate, w_t the pretax wage rate, and H_t hours worked. The representative household starts in period 0 with a positive stock of capital and chooses paths for consumption, C_t, hours, and capital, K_t, so as to maximize the present discounted value of its lifetime utility,

$$\int_0^\infty e^{-\rho t}(\log C_t - AH_t)\,dt,$$

subject to $K_t \geq 0$ and to

$$\dot{K}_t = (u_t - \delta)K_t + (1 - \tau_t)w_t H_t - C_t,$$

where u_t denotes the rental rate of capital, $\delta \in (0, 1)$ the depreciation rate, and $\rho \in (0, 1)$ the subjective discount rate. A dot above a variable denotes the time derivative, so $\dot{K}_t$ denotes net investment. The first-order conditions associated with this problem are

$$\begin{aligned} \frac{1}{C_t} &= \Lambda_t, \\ A &= \Lambda_t(1 - \tau_t)w_t, \\ \dot{\Lambda}_t &= (\rho + \delta - u_t)\Lambda_t, \end{aligned} \tag{1}$$

where Λ_t denotes the marginal utility of income. The single good is assumed to be produced with a Cobb-Douglas production technology $F(K_t, H_t)$ that uses capital and labor as inputs. Perfect competition in factor and product markets implies that factor demands are given by

$$w_t = F_H(K_t, H_t) \tag{2}$$

and

$$u_t = F_K(K_t, H_t). \tag{3}$$

Market clearing requires that aggregate demand equal aggregate supply, that is,

$$C_t + G + \dot{K}_t + \delta K_t = F(K_t, H_t). \tag{4}$$

When we solve (1) for consumption, (2) for the wage rate, and (3) for the rental rate of capital, the equilibrium conditions can be reduced to four equations:

$$A = \Lambda_t(1 - \tau_t)F_H(K_t, H_t), \tag{5}$$

$$\frac{\dot{\Lambda}_t}{\Lambda_t} = \rho + \delta - F_K(K_t, H_t), \tag{6}$$

$$\dot{K}_t = F(K_t, H_t) - \frac{1}{\Lambda_t} - G - \delta K_t, \tag{7}$$

and

$$G = \tau_t F_H(K_t, H_t) H_t. \tag{8}$$

Steady State

In the steady state there is a Laffer curve–type relationship between the tax rate and tax revenue: the number of labor tax rates that generate enough revenue to finance a given level of government purchases will in general be either zero or two. As we shall show below, the revenue-maximizing tax rate is the least upper bound of the set of tax rates for which the rational expectations equilibrium is indeterminate. Therefore, we characterize the shape of the steady-state Laffer curve in some detail.

We first show that for a given labor tax rate a steady state exists and is unique. To find such a steady state, set $\dot{\Lambda}$ in (6) equal to zero. Since $F_K(K, H)$ is monotonically decreasing in the capital/labor ratio, (6) can be solved for a unique capital/labor ratio that is independent of the tax rate. Equation (5) can then be solved for a unique and positive value of Λ. Using this value of Λ, the government budget constraint (8), and the fact that in the steady state $\dot{K} = 0$, we can write the market-clearing condition (7) as

$$\frac{(1 - \tau)F_H(K/H, 1)}{A} = K\left[F\left(1, \frac{H}{K}\right) - \delta - \tau F_H\left(\frac{K}{H}, 1\right)\left(\frac{H}{K}\right)\right], \tag{9}$$

where variables without time subscripts denote steady-state values. The left side of this expression is $C = 1/\Lambda$ and is positive and independent of K. The bracketed expression multiplying K on the right side is also independent of K. Using (6) and the linear homogeneity of the production function, we can write this bracketed expression as

$$\rho + F_H\left(\frac{K}{H}, 1\right)\left(\frac{H}{K}\right)(1 - \tau),$$

which is positive. Thus K is also positive and unique. Moreover, K is continuous in τ. Because both the capital/labor ratio and the capital stock are positive and unique in the steady state, H is also positive and unique. Finally, because the wage rate is equal to the marginal product of labor, which depends only on the capital/labor ratio, the level of government purchases given by (8) is also unique and can be written as

$$G = \left(\frac{s_h \delta}{s_i}\right) \tau K, \tag{10}$$

where K is the solution to (9), $s_h \equiv wH/F$ is the labor share in output, and $s_i \equiv \delta K/F$ is the investment share in output. Note that neither s_h nor s_i depends on τ and that the continuity of K implies that G is continuous in τ.

It follows from (9) and (10) that when $\tau = 0$, $G = 0$ because K is in this case positive and finite. When τ approaches one, K goes to zero and so does G. When τ is greater than zero but less than one, G is positive. Therefore, there must be at least one tax rate in (0, 1) that maximizes G. The critical values of τ are given by the zeros of the derivative of G with respect to τ in (10),

$$\frac{\partial G}{\partial \tau} = \frac{G}{\tau} + \frac{G}{K}\frac{\partial K}{\partial \tau},$$

where from (9) $\partial K/\partial \tau$ is given by

$$\frac{\partial K}{\partial \tau} = \frac{s_h K}{1 - s_i - \tau s_h} - \frac{K}{1 - \tau}.$$

Combining the last two expressions yields

$$\frac{\partial G}{\partial \tau} = G \frac{s_h \tau^2 - 2(1 - s_i)\tau + (1 - s_i)}{\tau(1 - \tau)(1 - \tau s_h - s_i)}.$$

For values of τ in the interval (0, 1), G and the denominator of the expression above are positive.[1] So the critical values of τ coincide with the zeros of the polynomial

$$s_h \tau^2 - 2(1 - s_i)\tau + (1 - s_i). \tag{11}$$

When $\tau = 0$, this polynomial takes the value $1 - s_i > 0$; when $\tau = 1$, it takes the value $s_h + s_i - 1 < 0$. So one of the zeros of the polynomial occurs for $\tau \in (0, 1)$ and the other for $\tau > 1$. This shows that there exists a unique maximum of the Laffer curve $\tau^* \in (0, 1)$. Moreover, since for a tax rate of .5 the polynomial is positive, the

[1] Note that $1 - \tau s_h - s_i > 1 - s_h - s_i$. From the fact that $1 - s_h = KF_K(K, H)/F(K, H)$ and the relation $\rho + \delta = F_K(K, H)$, s_i can be written as $s_i = \delta/(\rho + \delta)(1 - s_h)$, which implies that $1 - s_h - s_i > 0$.

Laffer curve is at that point upward sloping, and therefore τ^* exceeds .5.

Local Indeterminacy

Consider a log-linear approximation of the equilibrium conditions (5)–(8) around the steady state. Let λ_t, k_t, h_t, and $\hat{\tau}_t$ denote the log deviations of Λ_t, K_t, H_t, and τ_t from their respective steady states. The log-linearized equilibrium conditions then are

$$0 = \lambda_t + \frac{-\tau}{1-\tau}\hat{\tau}_t + s_k(k_t - h_t), \tag{12}$$

$$\dot{\lambda}_t = (\rho + \delta)s_h(k_t - h_t), \tag{13}$$

$$\dot{k}_t = \rho k_t + \frac{s_h\delta}{s_i}h_t + \frac{s_c\delta}{s_i}\lambda_t, \tag{14}$$

and

$$0 = \hat{\tau}_t + s_k(k_t - h_t) + h_t, \tag{15}$$

where $s_k \equiv KF_K(K, H)/F(K, H)$ and $s_c \equiv C/F(K, H)$ denote the shares of capital and consumption in output. Combining (12) and (15) yields

$$h_t = \frac{s_k}{s_k - \tau}k_t + \frac{1-\tau}{s_k - \tau}\lambda_t.$$

Using this expression to eliminate h_t in (13) and (14) results in the following system of two linear differential equations:

$$\begin{bmatrix} \dot{\lambda}_t \\ \\ \dot{k}_t \end{bmatrix} = \begin{bmatrix} -(\rho + \delta)\dfrac{s_h(1-\tau)}{s_k - \tau} & -(\rho + \delta)\dfrac{s_h\tau}{s_k - \tau} \\ \dfrac{\delta}{s_i}\left[\dfrac{s_h(1-\tau)}{s_k - \tau} + s_c\right] & (\rho + \delta)\dfrac{1-\tau}{s_k - \tau} - \delta \end{bmatrix} \begin{bmatrix} \lambda_t \\ \\ k_t \end{bmatrix}. \tag{16}$$

Let $\mathbf{J}$ be the matrix of coefficients of this linear system. Since (16) contains only one predetermined variable, k_t, the perfect-foresight equilibrium will be indeterminate if and only if both eigenvalues of $\mathbf{J}$ have negative real parts. Since the trace of $\mathbf{J}$ equals the sum of its eigenvalues and the determinant their product, requiring negative eigenvalues is equivalent to requiring that the determinant be positive and the trace negative. The trace of $\mathbf{J}$ is given by

$$\text{tr}(\mathbf{J}) = \frac{(\rho + \delta)(1-\tau)s_k}{s_k - \tau} - \delta.$$

As is easily verified,

$$\operatorname{tr}(\mathbf{J}) < 0 \quad \text{if and only if } \tau > s_k. \tag{17}$$

In the business cycle literature, the range of values used for the capital income share is fairly wide. According to Christiano (1988), estimates range from .25 to .43. Estimates of the labor income tax rate, on the other hand, range from .23 to .285 for the United States and from .27 to .47 for other Group of Seven countries (Cooley and Hansen 1992; Mendoza, Razin, and Tesar 1994). Thus, in this simple economy the trace condition for indeterminacy can be satisfied for empirically plausible parameter values.

After some manipulations, the determinant of $\mathbf{J}$ can be written as

$$\det(\mathbf{J}) = \frac{\delta s_h(\rho + \delta)}{s_i(\tau - s_k)} [s_h\tau^2 - 2(1 - s_i)\tau + (1 - s_i)]. \tag{18}$$

The first factor on the right side is always positive if the trace condition ($\tau > s_k$) is satisfied. The determinant condition then requires that the second factor be positive. This factor is identical to the polynomial (11), so the determinant condition requires that the labor income tax rate be below the rate τ^* that maximizes the steady-state Laffer curve. Thus the necessary and sufficient condition for the indeterminacy of the perfect-foresight equilibrium is

$$s_k < \tau < \tau^*. \tag{19}$$

A sufficient condition for the set of tax rates satisfying (19) to be nonempty is that the labor share is larger than the capital share (i.e., $s_k < .5$). To see this, recall that $\tau^* > .5$. For steady-state tax rates smaller than s_k or greater than τ^*, the determinant of $\mathbf{J}$ is negative and therefore the equilibrium is locally determinate. In particular, for tax rates on the downward-sloping side of the Laffer curve, the equilibrium is unique.[2] Figure 1 summarizes our results.

The local indeterminacy of the perfect-foresight equilibrium implies the existence of stationary sunspot equilibria (see Shigoka 1994). The intuition behind the existence of stationary sunspot equilibria in the presence of a balanced-budget rule is as follows. Suppose that agents expect future labor tax rates to be above average. This implies that, for any given capital stock, future hours worked

[2] Combining a Cagan-style money demand equation with a monetary–fiscal policy regime that fixes real seigniorage income, Sargent and Wallace (1981, 1987) show that a Laffer curve–type relationship exists between the steady-state inflation rate and steady-state seigniorage income and that the rational expectations equilibrium may be indeterminate. Contrary to our model, however, in their model indeterminacy arises for inflation tax rates on the downward-sloping side of the Laffer curve.

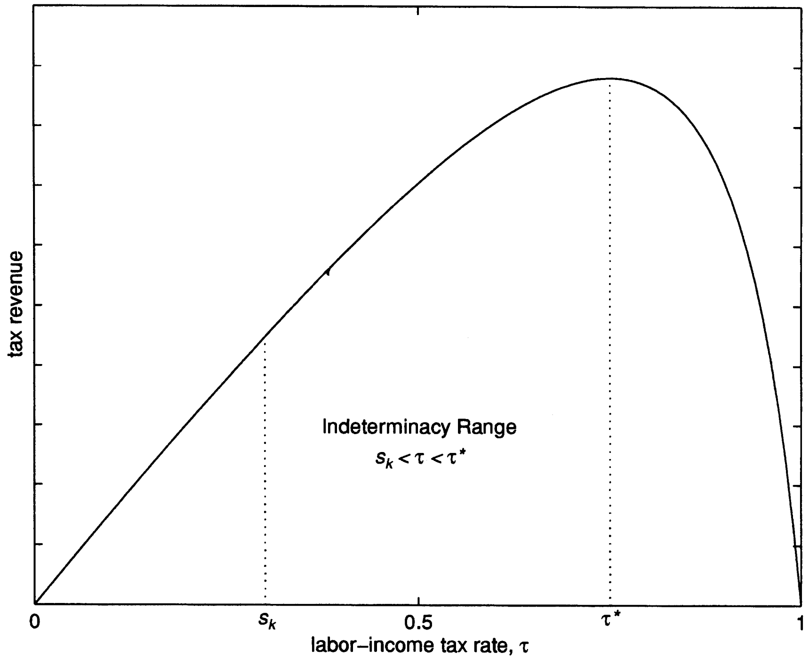

FIG. 1.—Steady-state Laffer curve

and thus the rate of return on capital will be lower (the latter is due to the fact that the marginal product of capital is decreasing in the capital/labor ratio). The decrease in the expected rate of return on capital, in turn, lowers current labor supply through its effect on the marginal utility of income, leading to a decline in current output. Since the tax base (labor income) is increasing in output, budget balance requires that the current tax rate increase. Thus expectations of above-average tax rates in the next period lead to higher current tax rates. For certain choices of the parameter values, namely those satisfying $s_k < \tau < \tau^*$, the expectation of an above-steady-state tax rate in the next period leads to an increase in tax rates today that is larger than the one expected for next period. Furthermore, for such parameter values the tax rate in period 0 is larger in absolute value than the tax rate in period t' for any $t' > 0$, so that the sequence of tax rates converges to the steady state and thus can be justified as an equilibrium outcome.

Capital accumulation plays a key role in generating expectations-driven fluctuations in this economy. In fact, one can show that in the absence of capital accumulation the equilibrium is locally unique.[3] On the other hand, the assumption that all government expenditures consist of purchases of goods is not important for our indeterminacy result. It can be shown that if all tax revenues were returned to the public in the form of lump-sum transfers, indeterminacy would still occur for steady-state tax rates greater than s_k and less than the tax rate associated with the peak of the steady-state Laffer curve. In this case, the peak of the Laffer curve is attained for a tax rate of .5.

The indeterminacy condition obtained above has a close correspondence with the one obtained in the increasing returns model of Benhabib and Farmer (1994). In both models a necessary condition for local indeterminacy is that the "equilibrium labor demand schedule" be upward sloping and steeper than the labor supply schedule. In the Benhabib-Farmer model, the equilibrium labor demand schedule is upward sloping as a consequence of sufficiently large increasing returns to scale. In our model, on the other hand, the equilibrium labor demand schedule is upward sloping because increases in aggregate employment are accompanied by decreases in the tax rate. The after-tax labor demand function can be written (in log deviations from the steady state) as

$$\tilde{w}_t = -s_k h_t - \frac{\tau}{1-\tau}\hat{\tau}_t + s_k k_t,$$

where $\tilde{w}_t \equiv w_t - [\tau/(1-\tau)]\hat{\tau}_t$ denotes the log deviation of the after-tax wage rate from the steady state. The firm's labor demand schedule is decreasing in h_t. However, using the balanced-budget equation (15) to eliminate $\hat{\tau}_t$ yields the following expression for the equilibrium labor demand schedule:

$$\tilde{w}_t = \frac{\tau - s_k}{1-\tau} h_t + \frac{s_k}{1-\tau} k_t. \tag{20}$$

If $\tau < s_k$, the equilibrium labor demand is downward sloping; if $\tau > s_k$, it is upward sloping. Since the aggregate labor supply is infinitely elastic (for a given tax rate and marginal utility of income), the labor demand schedule will be steeper than the labor supply schedule whenever $\tau > s_k$. In the Appendix, we further show that there exists a close formal correspondence between the equilibrium conditions of our model and those of two other models of endogenous business

[3] In an economy without capital, (5) becomes $A/\Lambda_t = (1-\tau_t)F_H(H_t)$, (7) becomes $1/\Lambda_t = F(H_t) - G$, and (8) becomes $\tau_t = G/[F_H(H_t)H_t]$. These three equations yield locally unique solutions for τ_t, H_t, and Λ_t.

cycles: the "implicit collusion" model of Rotemberg and Woodford (1992) and the version of Galí's (1994) "composition of aggregate demand" model analyzed in Schmitt-Grohé (1997).

III. An Economy with Labor and Capital Income Taxes

In this section we introduce capital income taxes and income-elastic government expenditures to make the model's tax and expenditure structure closer to the one observed in the United States. The government budget constraint is then given by

$$G_t = \tau_t^h w_t H_t + \tau_t^k (u_t - \delta) K_t, \tag{21}$$

where τ_t^h and τ_t^k denote the labor and capital income tax rates, respectively. The term $-\tau_t^k \delta K_t$ represents a depreciation allowance. We investigate the possibility of indeterminacy under four fiscal policies within the class of balanced-budget rules: (1) endogenous capital and labor income tax rates with exogenous government purchases, (2) endogenous capital income and fixed labor income tax rates with exogenous government purchases, (3) fixed capital income and endogenous labor income tax rates with exogenous government purchases, and (4) endogenous capital and labor income tax rates with income-elastic government purchases. We also consider more general preferences than the ones used in the previous section by allowing for less than perfectly elastic aggregate labor supply. The class of period utility functions considered is

$$U(C_t, H_t) = \log C_t - \frac{H_t^{1+\gamma}}{1+\gamma}, \quad \gamma \geq 0. \tag{22}$$

These preferences imply that the Frisch elasticity of labor supply with respect to wage, ϵ_{hw}, is equal to $1/\gamma$.[4] We calibrate the model using a baseline parameterization that reflects typical values found in the real business cycle literature. The values used are .7 for the labor share (s_h), .04 for the annual real interest rate (ρ), .1 for the annual depreciation rate (δ), and infinity for the Frisch labor supply elasticity ($\gamma = 0$).

Figure 2 shows with dots the steady-state values of τ^h and τ^k for which the equilibrium is indeterminate in the case of policy 1; that

[4] The assumption that labor is indivisible implies that the aggregate elasticity of labor supply (with tax rates and the marginal utility of income held constant) is infinite; however, it does not imply that an individual's labor supply elasticity is also infinite. In fact, it is perfectly consistent with low individual Frisch labor supply elasticities.

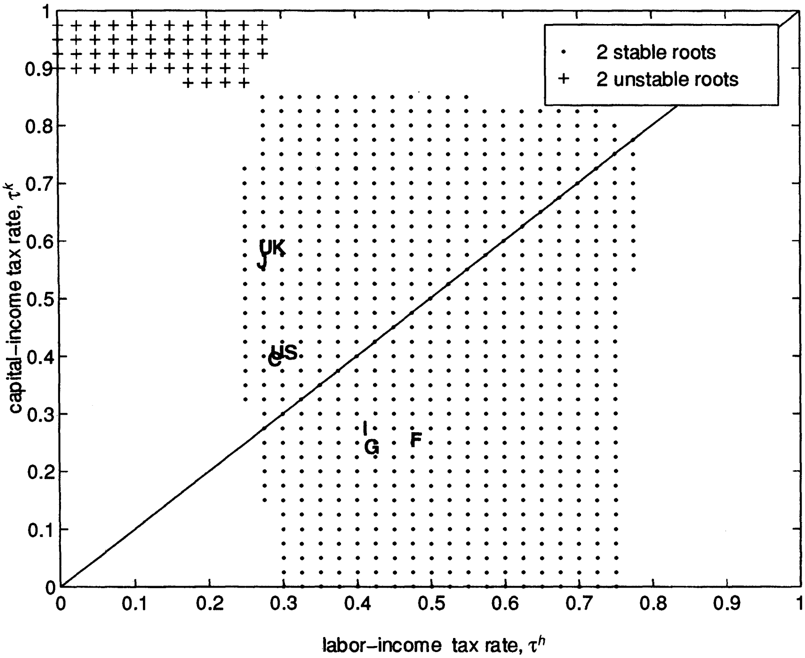

FIG. 2.—Endogenous capital and labor income tax rates, policy 1

is, government purchases are fixed and τ^h and τ^k vary in the same proportion to balance the budget. Note first that if $\tau^k = 0$, the model collapses to the one analyzed in the previous section, so indeterminacy arises for $\tau^h > s_k = .3$. The pairs (τ^k, τ^h) for which $\tau^h = \tau^k$ (the solid line) correspond to the case of an income tax regime with depreciation allowance. Indeterminacy in this case arises for income tax rates above 27 percent. The figure also includes the labor and capital income tax rates estimated by Mendoza et al. (1994) for the United States (US), the United Kingdom (UK), France (F), Germany (G), Italy (I), Canada (C), and Japan (J).[5] The tax rate pairs for all seven countries fall inside the range of values for which the equilibrium is indeterminate.[6]

[5] Almost all the estimated tax rate time series show an upward trend. We use the most recent value reported, which is 1988.

[6] Two issues should, however, be pointed out. First, the calibration reflects mostly U.S. long-run data restrictions, and second, consumption taxes, which are a significant source of tax revenue in the European countries, are ignored. In fact, it can be shown that if the only source of government revenues is a tax on consumption, then the rational expectations equilibrium under a balanced-budget rule implemented through an endogenous consumption tax rate is locally determinate.

Next we consider policy 2, in which the labor income tax rate and government purchases are constant and the capital income tax rate is endogenous. If the labor income tax rate is zero, one can show analytically (see Schmitt-Grohé and Uribe 1996) that stationary sunspot equilibria are altogether impossible. The intuition for this result is as follows. Expectations of future capital income tax increases lower the interest rate and thus reduce current labor supply. The current tax base, however, declines only marginally since the stock of capital is predetermined. Thus a small increase in the current capital income tax rate will suffice to balance the budget. As a result, a higher current tax rate is associated with an expectation of an even higher tax rate next period. Such a path is explosive and therefore is not consistent with a stationary sunspot equilibrium. If, on the other hand, the labor income tax rate is strictly positive, tax revenues fall by more in response to a given decline in employment, and hence a larger increase in the current capital income tax rate is necessary to bring about budget balance. Consequently, stationary sunspot equilibria become possible. However, one can show that for the calibration assumed here the equilibrium is indeterminate only for steady-state capital income tax rates above 80 percent (see Schmitt-Grohé and Uribe 1996).

Under a balanced-budget rule in which the labor income tax rate is endogenous and the capital income tax rate and government purchases are exogenous (policy 3), indeterminacy arises for roughly the same steady-state capital and labor income tax rates as under policy 1. One difference is that as τ^k increases, indeterminacy arises for slightly smaller values of τ^h under policy 3 than under policy 1. The reason for this difference is that as τ^k increases, the share of labor tax revenues in total tax revenues becomes smaller and, because τ^k is constant, the change in τ^h necessary to balance the budget in response to changes in output becomes larger. Comparing the implications of policies 1, 2, and 3, we conclude that endogenous labor income tax rates are essential for the existence of stationary sunspot equilibria.

We now turn to the analysis of policy 4, where government expenditures are income elastic and tax rates adjust in equal proportions to bring about budget balance. Panel *a* of figure 3 shows the smallest value of τ^h for which indeterminacy occurs for each value of τ^k and for a given value of the income elasticity of government expenditures, ϵ_{GY}. We consider three cases: procyclical government expenditures ($\epsilon_{GY} = .5$), acyclical government expenditures ($\epsilon_{GY} = 0$), and countercyclical government expenditures ($\epsilon_{GY} = -.5$). For a capital income tax rate of 40.7 percent (the U.S. value), the smallest labor income tax rate that makes indeterminacy possible increases from

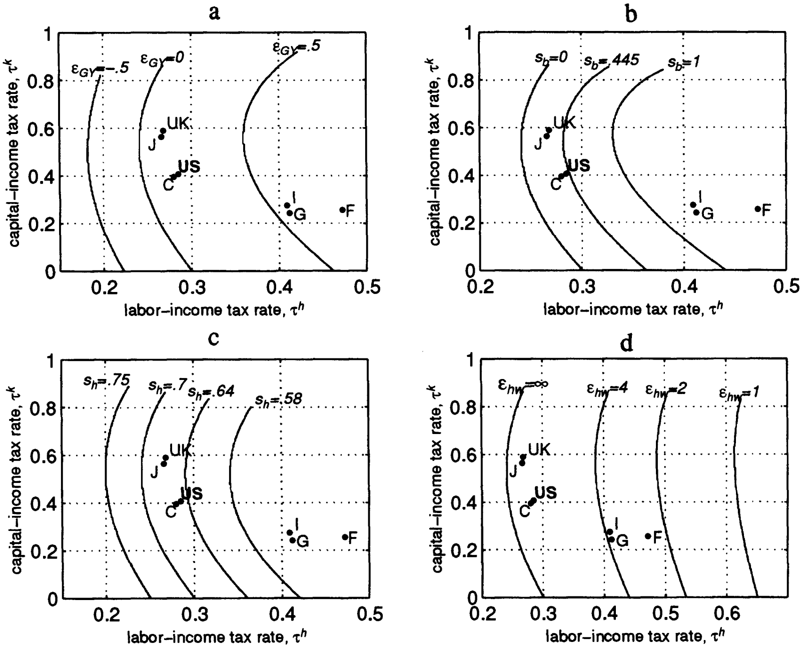

FIG. 3.—Sensitivity analysis. Each line shows, for a given value of (a) the income elasticity of government purchases, ϵ_{GY}, (b) the debt to GDP ratio, s_b, (c) the labor share, s_h, or (d) the Frisch wage elasticity of labor supply, ϵ_{hw}, and for every value of the capital income tax rate, τ^k, the minimum value of the labor income tax rate, τ^h, for which the perfect-foresight equilibrium is indeterminate.

28 to 38 percent as the income elasticity of government expenditures increases from zero to .5; that is, the more procyclical government expenditures are, the less likely it is that the equilibrium is indeterminate. The reason for this result is that the more procyclical government expenditures are, the smaller the required change in tax rates necessary to balance the budget for a given change in output. It is straightforward to show that in the particular case in which government purchases are proportional to output ($\epsilon_{GY} = 1$) and are financed by an income tax without a depreciation allowance, the balanced-budget rule implies a constant income tax rate and hence the equilibrium is determinate. An example of a procyclical component of government expenditures in the presence of a balanced-budget requirement is interest payments on the public debt (provided that the interest rate is itself procyclical). We shall return to this point later. If, on the other hand, government expenditures are countercyclical, for a capital income tax rate of 40.7 percent the smallest labor income tax rate that makes indeterminacy possible decreases from

28 to 19 percent as ϵ_{GY} decreases from zero to $-.5$; that is, the more countercyclical government expenditures are, the more likely it is that the equilibrium is indeterminate. An example of a countercyclical component of government expenditures is unemployment compensation.

Public Debt

In our model, the presence of public debt introduces a procyclical component into government expenditures: when the only source of uncertainty is revisions in expectations about the future path of the economy, the real interest rate turns out to be procyclical; since by the balanced-budget rule the stock of public debt is constant, interest payments on the public debt are also procyclical. It follows from our analysis of cyclical government expenditures that the larger the steady-state share of interest payments in total government outlays, the less likely it is that the rational expectations equilibrium becomes indeterminate as a result of the implementation of a balanced-budget rule. We now assess the quantitative implications of the presence of public debt for the determinacy of the equilibrium. In this case the balanced-budget rule (21) becomes

$$G + (1 - \tau_t^k) r_t B = \tau_t^h w_t H_t + \tau_t^k (u_t - \delta) K_t, \tag{23}$$

where the new term on the left side, $(1 - \tau_t^k) r_t B$, denotes interest payments on the public debt; $B > 0$ denotes the stock of public debt, which is constant by the balanced-budget rule; and r_t denotes the rate of interest paid on the debt, which in equilibrium must be equal to $u_t - \delta$. We calibrate the model using the same parameter values used to calibrate the model without debt and assume that the capital and labor income tax rates adjust in equal proportions to bring about budget balance. Panel *b* of figure 3 shows for every value of the capital income tax rate, τ^k, and for three values of the ratio of steady-state debt to gross domestic product ($s_b = 0$, .445, and 1) the minimum value of the labor income tax rate that renders the perfect-foresight equilibrium indeterminate. A value of 44.5 percent for the debt to GDP ratio corresponds to the debt to GDP ratio observed in the United States in 1995 (1997 *Economic Report of the President*, table B-79). The graph shows that, as expected, the higher the debt to GDP ratio, the higher the minimum labor income tax rate for which the equilibrium is indeterminate (see also Stockman 1996). For a capital income tax rate of 40.7 percent, the critical labor income tax rate increases from 24.4 to 34.1 percent as the debt to GDP ratio increases from zero to 100 percent. For a debt to GDP ratio of 44.5 percent and a capital income tax rate of 40.7 percent, the critical labor income tax rate is 28.8 percent, which is just above the value reported in Mendoza et al. (1994) for the U.S. labor income tax rate of 28.5 percent.

Our specification of the budget constraint (23) overstates the procyclical component that debt introduces into government expenditures. The reason is that our specification implicitly assumes that the entire stock of debt is refinanced instantaneously at the current interest rate. In reality, however, a substantial fraction of the U.S. public debt is in the form of medium- and long-term fixed-income securities. For example, in 1995 the average maturity of U.S. Treasury securities held by the public was 5 years and 4 months (1997 *Economic Report of the President,* table B-86). Consequently, interest payments on a significant fraction of the stock of debt are not sensitive to changes in the current interest rate, and thus the frontiers depicted in panel *b* of figure 3 should be regarded as upper bounds for the minimum labor income tax rates that render the equilibrium indeterminate. That is, for the debt to GDP ratio, the level of tax rates, and the maturity structure of the public debt observed in the United States, the presence of debt does not eliminate the possibility of policy-induced sunspot equilibria.

Sensitivity Analysis

We now turn to a sensitivity analysis with respect to two critical parameters governing the determinacy of equilibrium: the labor share and the Frisch elasticity of the labor supply with respect to the wage rate. These two parameters have been shown to be critical for the determinacy of equilibrium in other endogenous business cycle models. For example, in models in which indeterminacy arises as a consequence of increasing returns to scale, indeterminacy is more likely the higher the Frisch labor supply elasticity and the larger the labor share (see Schmitt-Grohé 1997). A similar result emerges in the present model.

In performing the sensitivity analysis, we assume that the balanced-budget rule is implemented via policy 1; that is, tax rates are endogenous and government purchases are constant. Each locus in panel *c* of figure 3 shows, for a given capital income tax rate and labor share, the smallest labor income tax rate that generates indeterminacy. We consider four values of the labor share commonly used in calibrating U.S. business cycle models: .58, .64, .7, and .75.[7]

[7] These values are taken from King et al. (1988), Kydland and Prescott (1982), Farmer and Guo (1994), and Rotemberg and Woodford (1992), respectively. The main differences among these measures of the labor share are due to the treatment given to proprietors' income, indirect business taxes, and imputed services from consumer durables.

As s_h decreases, the minimum labor income tax rate that generates indeterminacy increases. Some intuition for this finding can be gained by considering equilibrium in the labor market. Suppose that expectations of a future tax increase shift the labor supply schedule up. Then because the slope of the labor demand schedule is equal to $-s_k$, the smaller s_k is, the larger the decline in employment. Consequently, the increase in the tax rate required to bring about budget balance is larger the smaller s_k is or the larger s_h is, and hence stationary sunspot equilibria become more likely the larger s_h is.

Each locus in panel *d* of figure 3 represents, for a given value of the capital income tax rate, τ^k, and of the Frisch wage elasticity of labor supply, ϵ_{hw}, the smallest labor income tax rate that generates indeterminacy. The graph shows that the smallest labor income tax rate that generates indeterminacy is decreasing in the labor supply elasticity. Again, this relationship can be understood by considering equilibrium in the labor market. The slope of the labor supply schedule is equal to the inverse of ϵ_{hw}, so employment falls by more the larger ϵ_{hw} is when expectations of a future tax increase shift the labor supply schedule up. Hence, the increase in the tax rate required to bring about budget balance is greater, the greater ϵ_{hw} is. Consequently, stationary sunspot equilibria become more likely as ϵ_{hw} increases. Recall also that the necessary condition for indeterminacy could be interpreted as requiring that the "equilibrium labor demand" schedule be steeper than the labor supply schedule. From equation (20) it follows that the slope of the equilibrium labor demand schedule is increasing in τ^h. As ϵ_{hw} decreases, the slope of the labor supply schedule increases; therefore, as ϵ_{hw} decreases, τ^h has to increase in order for the labor demand function to still be steeper than the labor supply schedule.

Panel *d* reveals that if the aggregate labor supply elasticity is below four, the smallest labor income tax rate that makes indeterminacy possible exceeds the U.S. labor income tax rate; if the labor supply elasticity falls below two, it exceeds the labor income tax rate estimates of all Group of Seven countries. This sensitivity analysis shows that for empirically realistic values of the U.S. income tax rates, a balanced-budget policy induces indeterminacy only if the Frisch elasticity of labor supply with respect to wage is relatively high. In Schmitt-Grohé and Uribe (1996), we show that once the model is augmented to include a nontaxed sector (such as home production), a balanced-budget rule induces indeterminacy for empirically plausible values of the preference parameter γ. In particular, we show that for the estimates of γ presented in the paper by Rupert, Rogerson, and Wright (1996), who take household production explicitly into account in their estimation of γ, stationary sunspot equilibria exist for values of the labor and capital income tax rates observed in the U.S. economy.

Predetermined Tax Rates

Finally, throughout the paper we have ignored the fact that in practice tax rates are typically set in advance. Although clearly unrealistic, our assumption that tax rates and the tax base are determined simultaneously turns out not to be essential in obtaining the basic result of the paper that under a balanced-budget rule the rational expectations equilibrium is indeterminate for empirically plausible parameterizations. Specifically, suppose that time is discrete and that tax rates are set $k \geq 1$ periods in advance, so that in each period $t \geq 0$, the tax rates for periods $t, \ldots, t + k - 1$ are predetermined. Suppose also that the balanced-budget requirement stipulates that the period $t + k$ tax rate τ_{t+k} must be set in period t in such a way that the expected value of the stock of public debt in period $t + k$, given information available in period t, is equal to the initial stock of debt (which we assume to be zero). It can be shown numerically that the minimum steady-state labor income tax rate for which the equilibrium is indeterminate does not depend on the number of tax lags k. This result holds for implementations of the balanced-budget rule in which government purchases are constant and either capital and labor income tax rates adjust in equal proportions or capital income tax rates are constant and only labor income tax rates adjust. The intuition for this result is as follows. Consider the indeterminacy of the perfect-foresight equilibrium when taxes are set k periods in advance. From the perspective of period t, the economy looks as though taxes were not set in advance from period $t + k$ on. Therefore, if the equilibrium is indeterminate when taxes are determined simultaneously with the tax base, then when taxes are set k periods in advance, the tax rate in period $t + k$ must also be indeterminate in period t.

IV. Indeterminacy and Propagation

In this section we investigate the implications of a balanced-budget rule for the propagation of sunspot and fundamental shocks. We analyze the solution to a log-linear approximation of the equilibrium conditions of a discrete-time version of our model that, in addition to sunspot shocks, is subject to technology and government purchases shocks. We assume that the balanced-budget rule is implemented by adjusting capital and labor income tax rates in equal proportions, policy 1, so that $\hat{\tau}_t^k = \hat{\tau}_t^h \equiv \hat{\tau}_t$. If the equilibrium is indeterminate, the equilibrium conditions can be reduced to the following first-order vector stochastic linear difference equation:

$$\begin{bmatrix} \hat{\tau}_{t+1} \\ k_{t+1} \\ a_{t+1} \\ g_{t+1} \end{bmatrix} = \begin{bmatrix} \mathbf{M} & \tilde{\mathbf{M}} \\ \varnothing & \mathbf{\Theta} \end{bmatrix} \begin{bmatrix} \hat{\tau}_{t} \\ k_{t} \\ a_{t} \\ g_{t} \end{bmatrix} + \begin{bmatrix} \epsilon^s_{t+1} \\ 0 \\ \epsilon^a_{t+1} \\ \epsilon^g_{t+1} \end{bmatrix},$$

where $\mathbf{M}$ is a 2×2 matrix with both eigenvalues inside the unit circle and $\mathbf{\Theta}$ is a 2×2 diagonal matrix, with diagonal (θ^a, θ^g) denoting the serial correlation of the exogenous processes for productivity, a_t, and government purchases, g_t, respectively. The sunspot innovation ϵ^s_t is assumed to have mean zero and to be serially uncorrelated but potentially contemporaneously correlated with the productivity or the government purchases innovations ϵ^a_t and ϵ^g_t, which in turn are assumed to have mean zero and to be serially uncorrelated and orthogonal to each other.

Figure 4 displays the impulse responses of the tax rate, output, hours, and consumption to sunspot, technology, and government purchases shocks. The impulse responses are computed assuming that the time unit is a quarter, the steady-state labor income tax rate is 35 percent, the steady-state capital income tax rate is 40 percent, and the serial correlation of both fundamental shocks is .9. The remaining parameters take the same values as before. For this calibration of the model the rational expectations equilibrium is indeterminate. All variables are expressed as percentage deviations from the steady state, so that, for example, a unit innovation in the labor income tax rate corresponds to an increase in the tax rate from 35 to 35.35 percent. Panel *a* of figure 4 shows the impulse responses to a unit innovation in the tax rate under the assumption that ϵ^s_t is uncorrelated with any fundamental shock. Note that the path of the tax rate is endogenously determined up to the initial value $\hat{\tau}_0$. The initial increase in the tax rate triggers a highly persistent, hump-shaped response in aggregate activity and tax rates.

As is well known, when the equilibrium is determinate, the presence of a balanced-budget rule with endogenous tax rates amplifies the response of endogenous variables to fundamental shocks. For example, the response of output to a positive productivity shock is amplified because by the balanced-budget rule the expansion in aggregate activity leads to a decline in tax rates. In addition, when the equilibrium is determinate, the responses of tax rates, output, and hours are typically monotone. By contrast, when the equilibrium is indeterminate, the initial response of endogenous variables to fun-

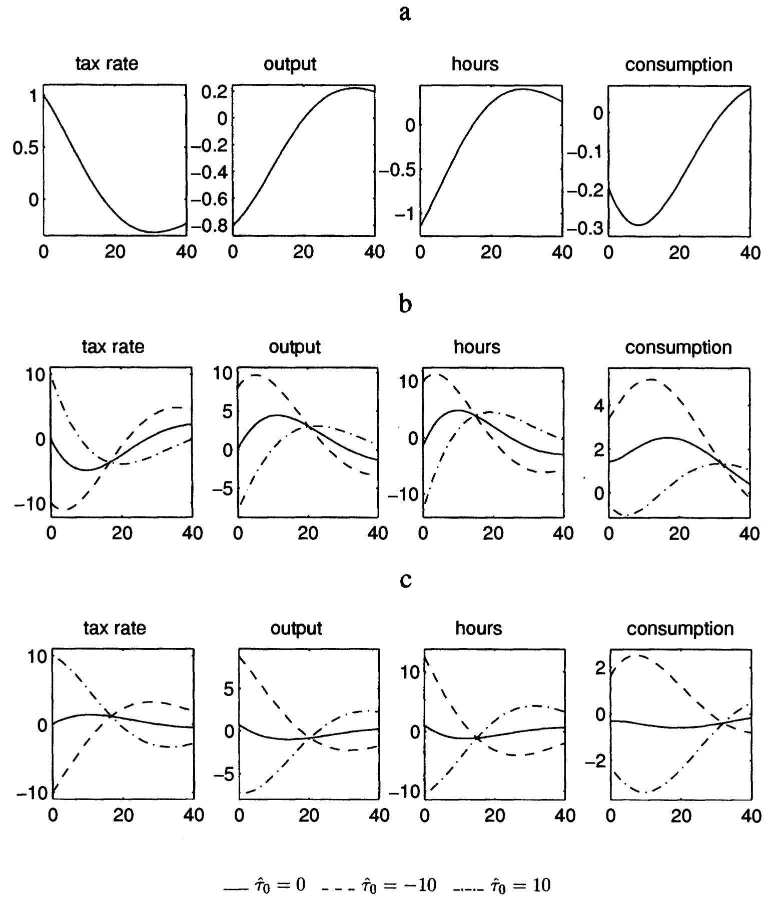

FIG. 4.—Impulse responses. *a*, Sunspot shock, $\hat{\tau}_0 = 1$. *b*, Technology shock, $a_0 = 1$. *c*, Government purchases shock, $g_0 = 1$. The technology and government purchases shocks are assumed to follow univariate AR(1) processes with a serial correlation of .9. The steady-state tax rates are $\tau^h = .35$ and $\tau^k = .40$, which imply that the rational expectations equilibrium is indeterminate. All variables are expressed in percentage deviations from the steady state.

damental shocks is not necessarily magnified. Panels *b* and *c* of figure 4 show, respectively, the impulse responses to technology and government purchases shocks under three alternative assumptions about the initial percentage increase in the tax rate: −10, 0, and 10. As shown in panel *b*, for example, a positive innovation in the technology shock can lead to a contraction in output if the initial response of the tax rate is sufficiently above the steady state. Also, contrary to the determinacy case, under indeterminacy the impulse responses of tax rates, output, and hours are hump-shaped regardless of the initial value of the tax rate.

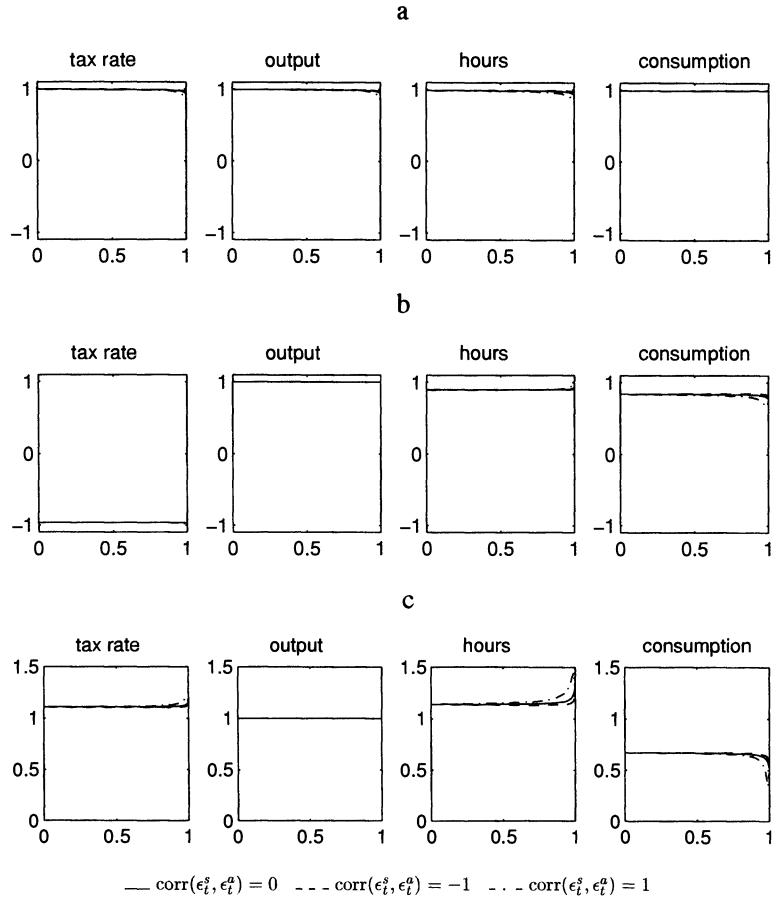

FIG. 5.—Comovements. Each plot shows either the serial correlation (panel *a*), the contemporaneous correlation with output (panel *b*), or the standard deviation relative to output (panel *c*) as a function of the variance of the sunspot shock, $\sigma^2_{\epsilon^s}$, for three different values of the correlation between the sunspot and the technology shocks: −1, 0, and 1. The variance of the sunspot shock, $\sigma^2_{\epsilon^s}$, is shown on the horizontal axis; it takes values between zero and one. The variance of innovation in the technology shock, $\sigma^2_{\epsilon^a}$, is set so that the sum of the variances of the innovations in the technology and sunspot shocks is equal to one, $\sigma^2_{\epsilon^a} + \sigma^2_{\epsilon^s} = 1$.

When the economy is subject to fundamental and sunspot shocks, the comovements of output, taxes, hours, and consumption depend in principle on the assumed correlation of the sunspot shock with the fundamental shock and on the relative importance of each source of uncertainty. Figure 5 summarizes this relationship when the only sources of uncertainty are sunspot shocks and persistent productivity shocks ($\theta^a = .9$). It shows the first-order serial correlation (panel *a*), the contemporaneous correlation with output (panel *b*), and the standard deviation relative to output (panel *c*) of the tax rate, output, hours, and consumption as a function of the variance of the sunspot shock, $\sigma^2_{\epsilon^s}$. The variance of the sunspot shock, shown on the horizontal axis, is assumed to take values between zero and one. The sum of the variances of the sunspot and technology innovations is held constant at one, that is, $\sigma^2_{\epsilon^s} + \sigma^2_{\epsilon^a} = 1$. Thus figure 5 encompasses two extreme cases: one in which the economy is hit only by sunspot shocks ($\sigma^2_{\epsilon^s} = 1$, $\sigma^2_{\epsilon^a} = 0$) and another in which the economy is hit only by technology shocks ($\sigma^2_{\epsilon^s} = 0$, $\sigma^2_{\epsilon^a} = 1$). Each plot displays three lines: a solid line corresponds to the case in which the sunspot and the technology innovations are uncorrelated ($\mathrm{corr}(\epsilon^s_t, \epsilon^a_t) = 0$), a broken line corresponds to the case in which the correlation between the sunspot and the technology innovations is equal to minus one ($\mathrm{corr}(\epsilon^s_t, \epsilon^a_t) = -1$), and a chain-dotted line corresponds to the case in which the correlation between the sunspot and the technology innovations is equal to one ($\mathrm{corr}(\epsilon^s_t, \epsilon^a_t) = 1$).

The main implication of figure 5 is that neither the first-order serial correlations, the contemporaneous correlations with output, nor the standard deviation relative to output of taxes, output, hours, and consumption is affected by the relative volatility of the sunspot shock or its correlation with the technology shock. This is reflected in the fact that in most cases the three lines are indistinguishable from each other and perfectly flat. This demonstrates that in our model indeterminacy of the rational expectations equilibrium does not necessarily imply that any arbitrary pattern of comovement in endogenous variables can be supported as an equilibrium outcome by an appropriate choice for the joint distribution of sunspot and fundamental shocks.[8]

V. Concluding Remarks

This paper embeds a balanced-budget rule in a standard neoclassical growth model and shows that if the fiscal authority relies heavily on changes in labor income tax rates to achieve budget balance, such a rule may be destabilizing because persistent and recurring fluctuations in aggregate activity become possible in response to arbitrary changes in expectations. Calibrated versions of the model suggest that this source of instability is not just a theoretical possibility but occurs for empirically realistic parameter values. The results give

[8] A similar result obtains if the economy is subject to persistent government purchases shocks and sunspot shocks.

additional support to the policy recommendation that balanced-budget rules should be coupled with restrictions on the fiscal authority's ability to change tax rates. In the context of our model, such a restriction would eliminate the possibility of expectations-driven fluctuations.

Appendix

In this Appendix we show that there exists a close formal correspondence between the equilibrium conditions of the model with a balanced-budget rule, distortionary income taxes, and constant government purchases presented in this paper and those of two other models of endogenous business cycles: the "implicit collusion" model of Rotemberg and Woodford (1992) and the version of Galí's (1994) "composition of aggregate demand" (CAD) model analyzed in Schmitt-Grohé (1997). Consider the case of an income tax ($\tau_t^h = \tau_t^k = \tau_t$) without depreciation allowance. The balanced-budget rule is then given by

$$G = \tau_t F(K_t, H_t).$$

To facilitate comparison, we assume that time is discrete. The following equilibrium conditions hold for all three models:

$$U_c(C_t, H_t) = \lambda_t,$$

$$-U_H(C_t, H_t) = \lambda_t w_t,$$

$$Y_t = C_t + K_{t+1} - (1 - \delta)K_t,$$

$$1 = E_t\left[\left(\frac{\beta\lambda_{t+1}}{\lambda_t}\right)(u_{t+1} + 1 - \delta)\right].$$

In the balanced-budget model, disposable income, Y_t, is given by

$$Y_t = F(K_t, H_t) - G.$$

The equilibrium conditions of the CAD and implicit collusion models include an identical expression, in which G represents a fixed cost that ensures that imperfectly competitive firms do not make pure profits in the long run. In all three models, the (after-tax) wage rate, w_t, and the (after-tax) rental rate of capital, u_t, are given by

$$F_H(K_t, H_t) = \mu_t w_t$$

and

$$F_K(K_t, H_t) = \mu_t u_t.$$

In the balanced-budget model, μ_t represents the wedge between marginal product and after-tax factor prices introduced by distortionary income taxation. Specifically,

$$\mu_t \equiv \frac{1}{1-\tau_t} = 1 + \frac{G}{Y_t}. \tag{A1}$$

The last equality follows from the balanced-budget constraint. In the CAD and implicit collusion models, on the other hand, μ_t represents a markup of prices over marginal cost charged by imperfectly competitive producers. In the CAD model, the markup is a function of the investment share,

$$\mu_t = \mu\left(\frac{K_{t+1} - (1-\delta)K_t}{Y_t}\right),$$

whereas in the implicit collusion model, μ_t is a function of the ratio of the present discounted value of future profits, X_t, to current output, Y_t:

$$\mu_t = \mu\left(\frac{X_t}{Y_t}\right).$$

These equations describe the complete set of equilibrium conditions of the balanced-budget and CAD models. The complete set of equilibrium conditions of the implicit collusion model includes one additional equation specifying the evolution of X_t over time.

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