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slides

chapter 12

financial frictions

and aggregate instability

Motivation

- Emerging economies suffer from excess aggregate volatility, as documented in chapter 1.

- They also suffer from sudden stops, defined as rapid and large reversals in the current account with depressed levels of aggregate activity.

- Emerging countries are often said to overborrow, that is, to hold excessive levels of external debt.

- A central question for policymakers of individual countries and multilateral institutions is the desirability and optimal design of macroprudential policy.

- This chapter sheds light on these issues from the vantage point of models with financial frictions taking the form of collateral constraints.

This chapter analyzes questions such as:

1. Do collateral constraints amplify regular business cycles?

2. Do collateral constraints deepen large recessions?

3. Do collateral constraints open the door to nonfundamental uncertainty, exacerbating aggregate instability?

4. Do collateral constraints lead to overborrowing?

5. Can the presence of collateral constraints justify the use of countercyclical macroprudential policy?

Narrative of how collateral constraints amplify the cycle
During booms the value of collateral $\uparrow$, borrowing $\uparrow$, more borrowing further raises the value of collateral, leading to even more borrowing and hence even larger expansions of aggregate demand. Busts drive down value of collateral, the collateral constraint binds **(financial crisis)**, agents deleverage **(Fire Sale of Assets)**, $\rightarrow$ price of collateral $\downarrow$ **(Fisherian Deflation)** $\rightarrow$ further deleveraging $\rightarrow$ rapid contraction is aggregate demand for goods and services and forced current-account surpluses **(sudden stop)**.

Formulations of this idea in open economy macroeconomics

- Auernheimer and García-Saltos (2000)
- Mendoza (2002, 2010)
- Uribe (2006, 2007)
- Lorenzoni (2008)
- Jeanne and Korinek (2010)
- Bianchi (2011)
- Korinek (2011)
- Benigno, Chen, Otrok, Rebucci, and Young (2013, 2014)
- Schmitt-Grohé and Uribe (2016)

Section 12.1 Stock Collateral Constraints

Preferences:

$$\sum_{t=0}^{\infty} \beta^t \ln c_t$$

Sequential Budget Constraint:

$$c_t + d_t + q_t k_{t+1} = y_t + \frac{d_{t+1}}{1+r} + q_t k_t$$

Technology:

$$y_t = A_t k_t^{\alpha}$$

Stock Collateral Constraint:

$$d_{t+1} \leq \kappa q_t k_{t+1}; \qquad \text{with } 0 \leq \kappa < 1$$

Note: Price q_t is taken as exogenous by individual agent but is endogenous for the economy as a whole $\rightarrow$ pecuniary externality.

Capital is in fixed supply, so in equilibrium: $k_t = k > 0$ for all t.

A (bubble-free) equilibrium is a set of sequences $c_t > 0$, d_{t+1}, μ_t, and $q_t \geq 0$ satisfying

$$c_t + d_t = y_t + \frac{d_{t+1}}{1+r} \tag{1}$$

$$\frac{1}{c_t}\left[\frac{1}{1+r} - \mu_t\right] = \beta \frac{1}{c_{t+1}} \tag{2}$$

$$\frac{q_t}{c_t}\left[1 - \kappa\mu_t\right] = \frac{\beta}{c_{t+1}}\left[q_{t+1} + \alpha\frac{y_{t+1}}{k}\right] \tag{3}$$

$$\mu_t\left(\kappa q_t k - d_{t+1}\right) = 0; \quad \mu_t \geq 0; \quad d_{t+1} \leq \kappa q_t k \tag{4}$$

$$\lim_{t\to\infty}(1+r)^{-t} q_t = 0 \tag{5}$$

$$d_0 = \sum_{t=0}^{\infty} \frac{y_t - c_t}{(1+r)^t} \tag{6}$$

given $d_0 < \frac{1+r}{r} y \equiv$*natural debt limit,* A_t and $y_t \equiv A_t k^\alpha$. Instead of (6), we could have written $\lim_{t\to\infty}(1+r)^{-t} d_t = 0$. Assume that

$$\beta(1+r) = 1$$

The Steady-State Equilibrium

Assume that $A_t = A > 0 \quad \forall t$. Then $y_t = y \equiv Ak^\alpha > 0 \quad \forall t$.

A steady-state equilibrium is a set of constant sequences $c_t = c^* > 0$, $d_{t+1} = d^*$, $\mu_t = \mu^*$, and $q_t = q^* \geq 0$ for $t \geq 0$, satisfying (1)-(6). Note, 'steady-state equilibrium' and 'steady state,' are not equivalent concepts, for the steady-state equilibrium must respect the given initial condition d_0, whereas in the steady state $d_t = d^*$ for all $t \geq 0$.

By (2) $\boxed{\mu^* = 0}$, and so first two expression in (4) are also satisfied; by (3) $\boxed{q^* = \frac{\alpha y}{rk} > 0}$; and by (6) $\boxed{c^* = y - \frac{r}{1+r} d_0 > 0}$. This expression for c^* together with (1) yields $\boxed{d^* = d_0}$. Finally, the collateral constraint (last expression in (4)) is satisfied if $d_0 \leq \kappa q^* k$, or

$$\boxed{d_0 \leq \frac{\kappa \alpha y}{r}}$$

This expression yields the highest level of debt sustainable in a steady-state equilibrium. A steady-state equilibrium exists for any level of initial debt d_0 that satisfies this condition.

No Amplification of Regular Shocks

To illustrate lack of amplification of shocks of regular size, consider a negative unexpected output shock in $t = 0$

Initial condition: $t < 0$ the economy was in a steady state and $d_0 = d^*$. Then in period 0 it is learned that

$$A_t = \begin{cases} A^L & t = 0 \\ A > A^L & t \neq 0 \end{cases}$$

$$\Rightarrow y_t = \begin{cases} y^L \equiv A^L k^\alpha & t = 0 \\ y \equiv A k^\alpha > y^L & t \neq 0 \end{cases}$$

To discuss amplification we must indicate amplification relative to what. Here we mean relative to the economy without the collateral constraint.

Next we derive 2 intermediate results. One is the characterization of the steady state and the other is the characterization of the unconstrained economy. Let's begin with the steady state.

Response of the Economy Without A Collateral Constraint (NC).

$$c^{NC} = c^* - \frac{r}{1+r}\left(y - y^L\right) < c^*$$

$$d^{NC} = d_0 + (y - y^L) > d_0$$

$$ca_0^{NC} = \frac{y^L - y}{1+r} < 0 = ca^*$$

$$tb_0^{NC} = tb^* + \frac{y^L - y}{1+r} < tb^*$$

$$q^{NC} = q^* = \frac{\alpha y}{rk}$$

reduce consumption by less than output and use the current account to do so, that is, let trade balance deteriorate and borrow more

Now go back to the collateral-constraint economy (CC).

If the NC solution satisfies the equilibrium conditions of the CC economy, then we say that there is lack of amplification. Need to check if

$$d_{t+1} \leq \kappa q_t k \quad ?$$

Recall $d_1^{NC} = d_0 + (y - y_L)$; $\quad q_0^{NC} = q^*$; and $d_0 > \kappa q^* k$

It does if $d_0 + (y - y_L) \leq \kappa q^* k$, that is, if

$$\boxed{y - y^L \leq \kappa q^* k - d_0}$$

then there is no amplification.

- if shock is small, ie $y - y^L$ small.
- if not too indebted, ie d_0 small.
- if weaker constraint, ie κ large.

How often are those conditions encountered in a more realistic stochastic economy?

Quantitative Result: No amplification of regular business cycles

Mendoza (AER, 2010) makes this point in the context of a quantitative stock collateral constraint model. The model is more empirically realistic and hence much more complex than our model here. It has capital accumulation, imported inputs, labor, working capital constraints and is driven by 3 shocks, TFP, interest rate, and import price. Each shock is discretized with 2 values, so that the total number of exogenous grid points is 8.

Lack of amplification as documented in Mendoza (AER, 2010)

Stock collateral constraint: $d_{t+1} +$ working capital $\leq \kappa q_t k_{t+1}$

$\kappa = 0.20$; picked to match observed frequency of a Sudden Stop (3.3%), which is defined as a binding CC constraint **and** trade-balance-to-output ratio 2 percentage points above mean.

Variable	Std. Dev. in %	
	No CC	CC
output	3.90	3.85
consumption	4.21	3.69
investment	13.85	13.45
q_t	3.33	3.23

Source: Table 3 of Mendoza (2010).

mean debt to output is 32.6% in no CC and 10.4% in CC economy.

Prob of binding CC is 9.54%

We have thus provided a partial answer to Q1, namely, we have shown analytically and numerically that a stock collateral constraint may not amplify the cycle.

What if the contraction is large?

Large:= $y - y^L > \kappa q^* k - d_0$

We will next show that in response to a large negative output shock:

- Fisherian deflation $q_0 < q_0^{NC}$
- Amplification of contraction of demand $c_0 < c_0^{NC}$
- Fire sale/deleveraging $d_1 < d_1^{NC}$
- Less TB deterioration $tb_0 > tb_0^{NC}$
- Less CA deterioration $ca_0 > ca_0^{NC}$

... and welfare is lower than in the NC economy.

Hence, yes, financial frictions amplify busts when they trigger a binding collateral constraint.

Intuition: $A^L \downarrow$; HH want to borrow more to smooth consumption. hits CC. HH thinks if he sells 1 unit of k_{t+1} he gets $1 \times q_t$. and use proceeds to consume and to reduce debt

$$\Delta(\kappa q_t k_{t+1}) = \kappa q_t \Delta k_{t+1}$$

Must use κq_t to reduce debt. But can still use $(1-\kappa)q_t > 0$ to consume more (recall $\kappa < 1$).

However, in equilibrium selling capital cannot increase c. Capital is in fixed supply, and the fall in prices ends up being so large that the decline in the value of collateral makes debt go down and c in fact fall more than output, or the trade balance to improve.

To show this formally, combine (2) with (3) to obtain

$$q_{t+1} = \tilde{\beta}_t^{-1} q_t - r q^* \tag{7}$$

with

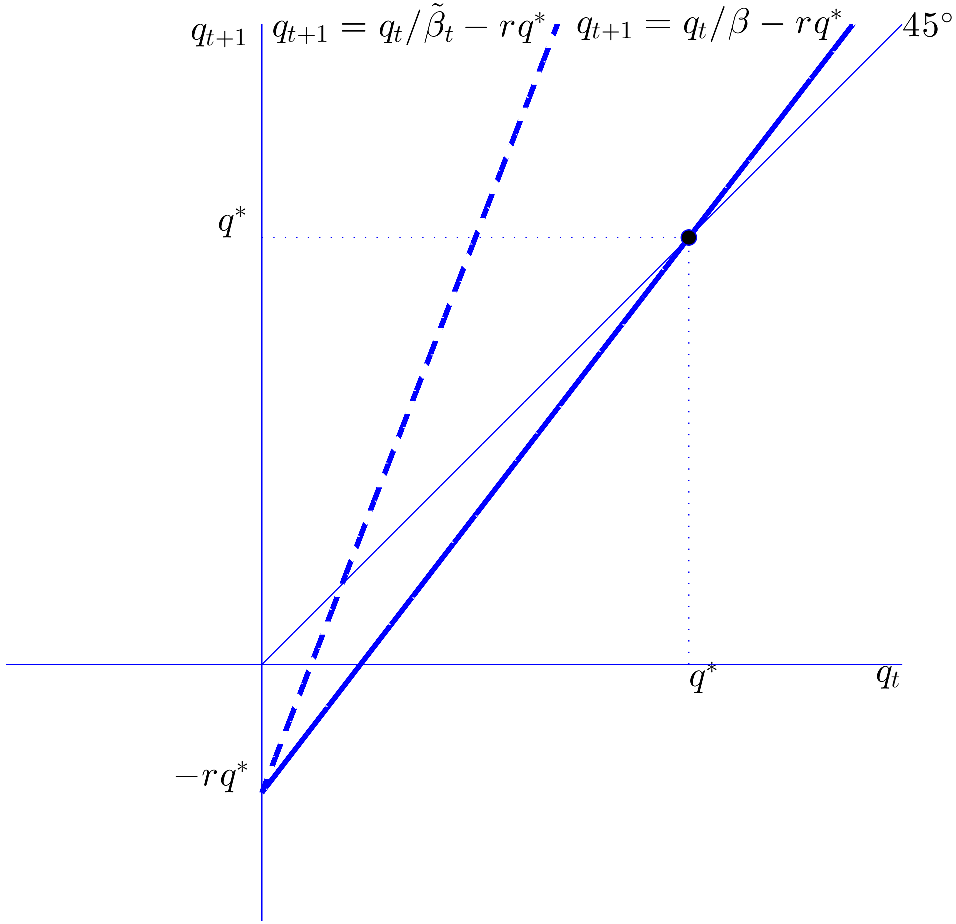
$$\tilde{\beta}_t \equiv \beta \frac{1-(1+r)\mu_t}{1-\kappa\mu_t} \tag{8}$$

Note that $\tilde{\beta}_t > 0$

$$\begin{array}{ll} \tilde{\beta}_t = \beta & \text{if } \mu_t = 0 \\ \tilde{\beta}_t < \beta & \text{if } \mu_t > 0 \end{array}$$

When constraint is binding ($\mu_t > 0$), it is as if agents are more impatient.

Figure 12.1 Phase Diagram of the Price of Capital



in any eqm: $q_t \leq q^*$
with large shock CC binds in at least 1 period
let T be the first period it binds, $\tilde{\beta}_T^{-1} > \beta^{-1} > 1$
$\Rightarrow q_T < q^*$
$\Rightarrow q_0 < q^*$

We have therefore shown that in this economy collateral constraints exacerbate the effects of large negative shocks. (that is we have addressed Q2)

Section 12.2 Stock Collateral Constraints and Self-fulfilling Financial Crises

Equilibrium multiplicity can also be a contributor excess volatility.

The existing related literature has either ignored this issue (exceptions are Mendoza 2005 and Jeanne and Korinek 2010) or added assumptions that are meant to guarantee a unique equilibrium.

We now show that self-fulfilling financial crises arise for plausible specifications in the current model. The self-fulfilling crisis coexists with an unconstrained equilibrium and features: a contraction in demand $c \downarrow$, a Fisherian deflation $q \downarrow$, and a fire sale $d \downarrow$.

Intuition: Agents become pessimistic and believe the value of collateral will be low, based on this belief they deleverage (firesale). The fire sale results in lower prices of capital, confirming the pessimistic beliefs.

Assume that

$A_t = A$ for all $t \geq 0$

$d_0 < \kappa q^* k$ ($\Rightarrow$ the unconstrained equilibrium exists)

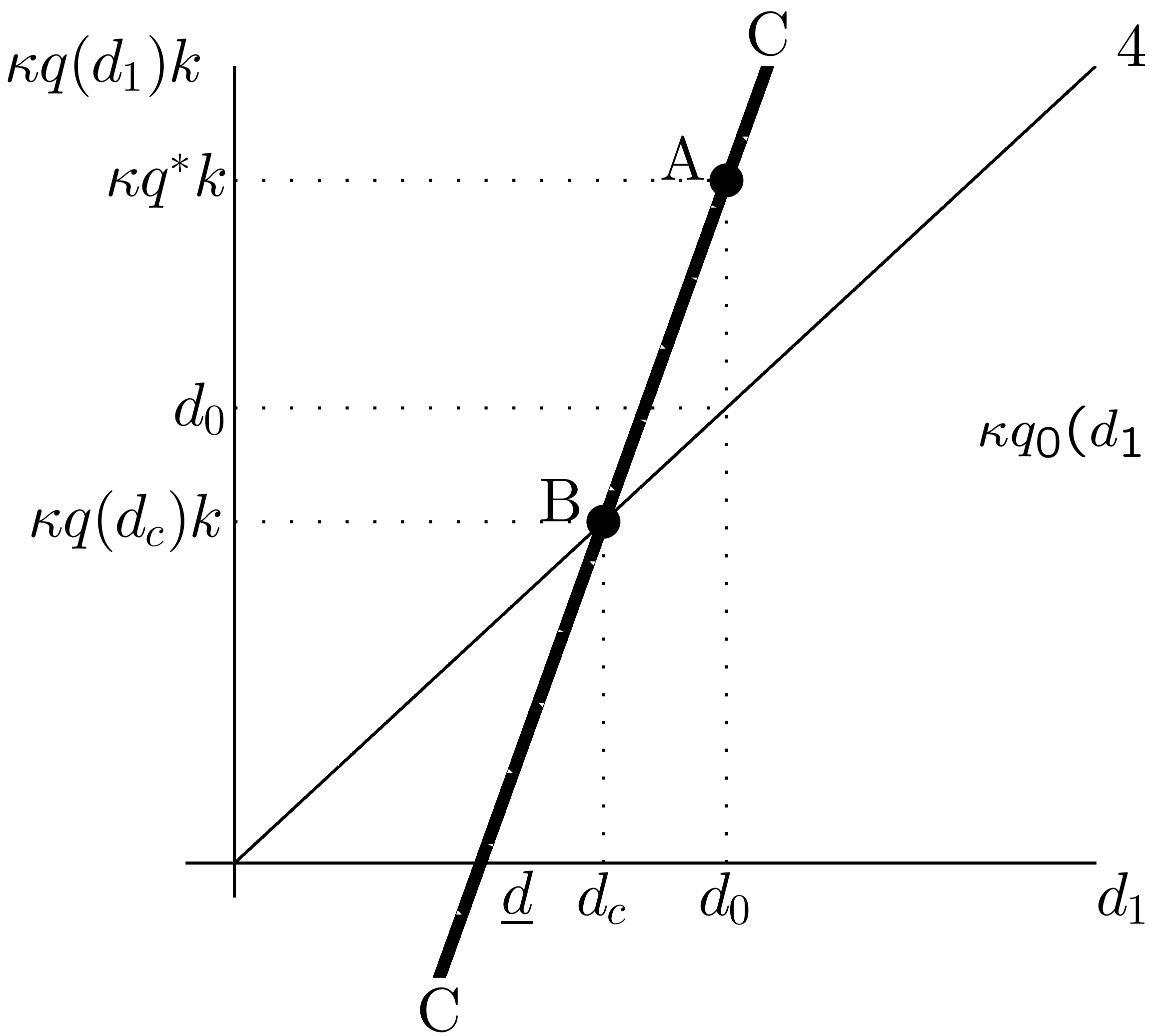
We wish to show that there exists a second equilibrium in which the collateral constraint binds in period 0 and the economy is in a steady state beginning in period 1.

$$
\begin{aligned}
d_0 &= \frac{1+r}{r}y - \frac{c_1}{r} - c_0, && (9)\\
c_1 &= y - \frac{r}{1+r}d_1, && (10)\\
\frac{1}{c_0}[1-(1+r)\mu_0] &= \frac{1}{c_1} && (11)\\
\frac{q_0}{c_0}(1-\kappa\mu_0) &= \frac{\beta}{c_1}(q^* + \alpha y/k) && (12)\\
\mu_0(\kappa q_0 k - d_1) &= 0 && (13)\\
d_1 &\leq \kappa q_0 k && (14)
\end{aligned}
$$

Now solve (9)-(12) for obtain q_0 as an increasing function of d_1

$$
\kappa q_0(d_1)k = \kappa q^* k \left[\frac{(1+r)c^* + d_1 - d_0}{(1+r)c^* + (\kappa - r)(d_1 - d_0)}\right]. \qquad (15)
$$

Figure 12.2 Stock Collateral Constraints and Self-fulfilling Financial Crisis



$$\kappa q_0(d_1)k = \kappa q^* k \left[\frac{(1+r)c^* + d_1 - d_0}{(1+r)c^* + (\kappa - r)(d_1 - d_0)}\right]$$

Sufficient condition for $\underline{d} > 0$: $d_0 > y$

Observations on the figure:
– at least 2 equilibria: Unconstrained equilibrium at point A and constrained eqm at point B
– at B: CC is binding $\rightarrow$ financial crisis; q is low, ie crisis has Fisherian deflation; $d_1 < d_0$, ie fire-sale or deleveraging.
– sufficient condition for existence of eqm B: $d_0/y > 1$. If time unit is a quarter, then debt to annual output of 25% is sufficient.
–bad economic fundamentals make the economy more vulnerable to a self-fulfilling financial crisis.

– Point B is welfare inferior to point A and at B debt is lower, therefore at B there is 'underborrowing'.
– we have either borrowing the optimal amount or less, that is, we have no overborrowing. This finding is at odds with the overborrowing result stressed in the literature.

Section 12.3 Flow Collateral Constraints

Based on Schmitt-Grohé and Uribe (NBER WP 22264, May 2016)

Households maximize

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma} - 1}{1 - \sigma}$$

subject to

$$c_t = \left[a {c_t^T}^{1-1/\xi} + (1-a){c_t^N}^{1-1/\xi}\right]^{1/(1-1/\xi)}$$

$$c_t^T + p_t c_t^N + d_t = y_t^T + p_t y^N + \frac{d_{t+1}}{1+r_t}$$

$$d_{t+1} \leq \kappa(y_t^T + p_t y^N)$$

where c_t =consumption; c_t^T, c_t^N =consumption of tradables, nontradables; d_{t+1} = debt assumed in t and maturing in $t+1$; y_t^T, y^N = endowments of tradables, nontradables; p_t =relative price of nontradables; r_t = interest rate.

Observations

(1) The last constraint is the flow collateral constraint (CC). It says that the amount of debt issued in period t, d_{t+1}, cannot exceed a fraction κ of income, $y_t^T + p_t y^N$.

(2) From the individual agent's point of view, the CC is well behaved: the larger is d_{t+1}, the closer he gets to hitting the collateral constraint. This is because he takes as exogenous all of the objects on the RHS of the collateral constraint (in particular p_t).

(3) Also, from the perspective of the individual agent, the collateral constraint defines a convex set of feasible debt levels: if d' and d'' satisfy the collateral constraint, then so does the debt level $\alpha\, d' + (1-\alpha)d''$, for any $\alpha \in [0,1]$.

(4) As we will see shortly, (2) and (3) do not hold from an aggregate perspective.

Three Equilibrium Conditions of Interest

$$d_{t+1} \leq \kappa (y_t^T + p_t y^N)$$

$$p_t = \frac{1-a}{a} \left(\frac{c_t^T}{y^N} \right)^{1/\xi}$$

$$c_t^T + d_t = y_t^T + \frac{d_{t+1}}{1+r_t}$$

These three conditions give rise to the following **equilibrium collateral constraint**

$$\boxed{d_{t+1} \leq \kappa \left[y_t^T + \left(\tfrac{1-a}{a}\right) \left(y_t^T + \tfrac{d_{t+1}}{1+r_t} - d_t \right)^{1/\xi} {y^N}^{1-1/\xi} \right]}$$

Observations

(1) d_{t+1}, appears on both the RHS and the LHS of the equilibrium CC.

(2) Because $\xi > 0$, the equilibrium value of collateral increases with the level of debt, giving rise to the possibility that the higher is d_{t+1} the less tight is the collateral constraint.

(3) Moreover, collateral (i.e., the RHS of the collateral constraint) is in general nonlinear in d_{t+1}, giving rise to the possibility that the set of debt levels that satisfy the equilibrium collateral constraint is nonconvex, that is, if d' and d'' satisfy the equilibrium collateral constraint, then $\alpha\, d' + (1-\alpha)d''$ may not for some $\alpha \in (0, 1)$.

Section 12.4 Flow Collateral Constraints and Self-Fulfilling Financial Crises

For analytical convenience assume that

(1) $r_t = r$, $y_t^T = y^T$.

(2) $\beta(1+r) = 1$.

(3) $y^N = 1$. A normalization.

(4) $\sigma = \frac{1}{\xi} = 2$.

(5) $a = 0.5$.

The equilibrium collateral constraint then becomes

$$\boxed{d_{t+1} \leq \kappa \left[y^T + \left(y^T + \frac{d_{t+1}}{1+r} - d_t \right)^2 \right]}$$

The Unconstrained Equilibrium

Let μ_t denote the multiplier associated with the collateral constraint. In the unconstrained eqm $\mu_t = 0$ and the Euler equation implies

$$\frac{c^T_{t+1}}{c^T_t} = \beta(1+r) = 1.$$

A constant consumption path, in turn, implies by the PVBC a constant debt path

$$d_t = d_0$$

for all t.

A Constrained Equilibria

With a binding collateral constraint $\mu_t > 0$, and the Euler equation becomes

$$\left(\frac{c_{t+1}^T}{c_t^T}\right)^2 = \frac{1}{1-(1+r)\mu_t} > 1$$

A binding collateral constraint introduces deviations from perfect consumption smoothing.

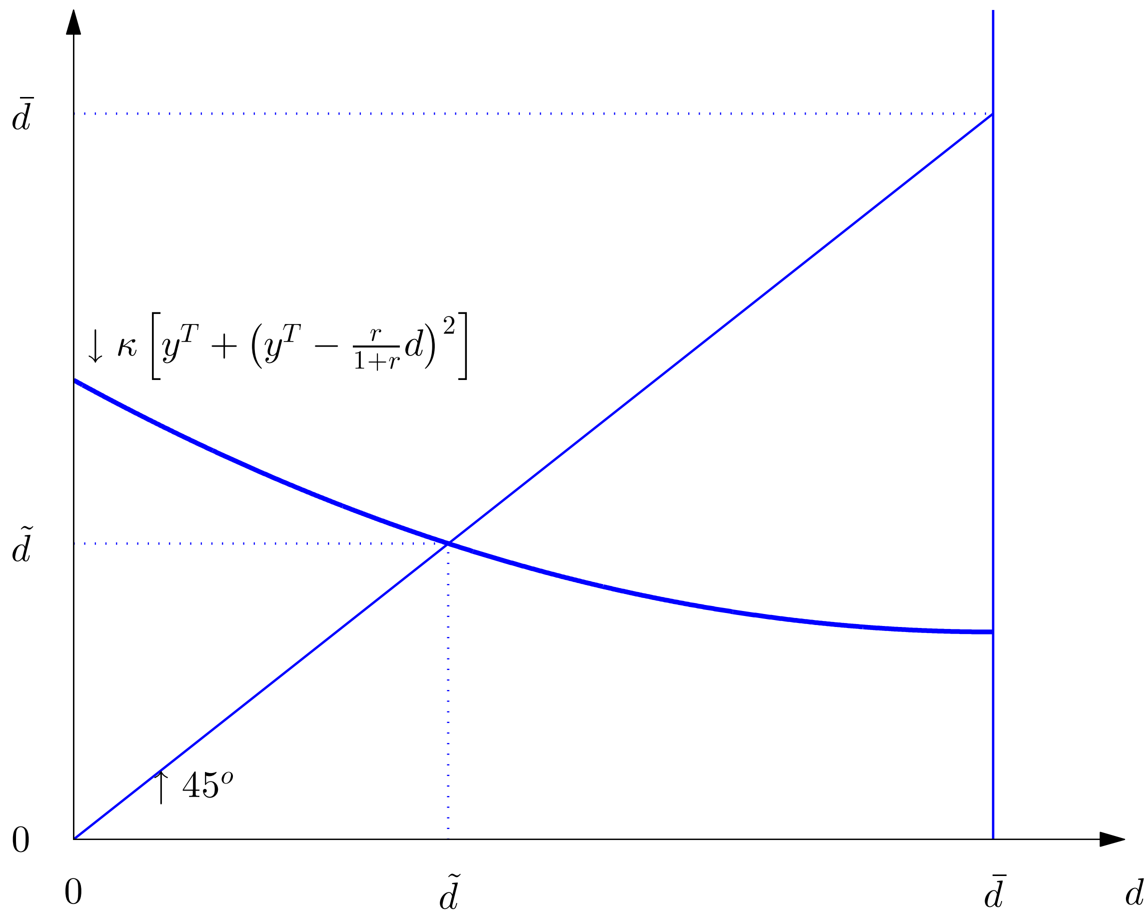
If the collateral constraint binds in period t, then

$$c_{t+1}^T > c_t^T$$

It follows that the path of consumption is non-decreasing in equilibrium.

The Steady-State Collateral Constraint

$$\boxed{d \leq \kappa \left[y^T + \left(y^T - \frac{r}{1+r} d \right)^2 \right]}$$



Observations

(1) The RHS of the long-run (LR) collateral constraint is quadratic.

(2) The expression under the power of 2 is steady state consumption, $y^T - rd/(1+r)$.

(3) The LR collateral constraint achieves a minimum when long-run consumption is 0, that is, at the natural debt limit.

(4) This means that for all relevant values of debt (i.e., all values below the natural debt limit), the LR collateral constraint is well behaved, that is, the larger is debt the tighter it gets.

(5) For any initial debt $d_0 < \tilde{d}$, an equilibrium is $d_{t+1} = d_0$ and $c_t^T = y - rd_0/(1+r)$ for all t. In these equilibria, the collateral constraint never binds.

(6) The LR collateral constraint binds at $\tilde{d}$. No steady state equilibrium is possible to the right of $\tilde{d}$.

Q: Is the steady-state equilibrium the only possible equilibrium?

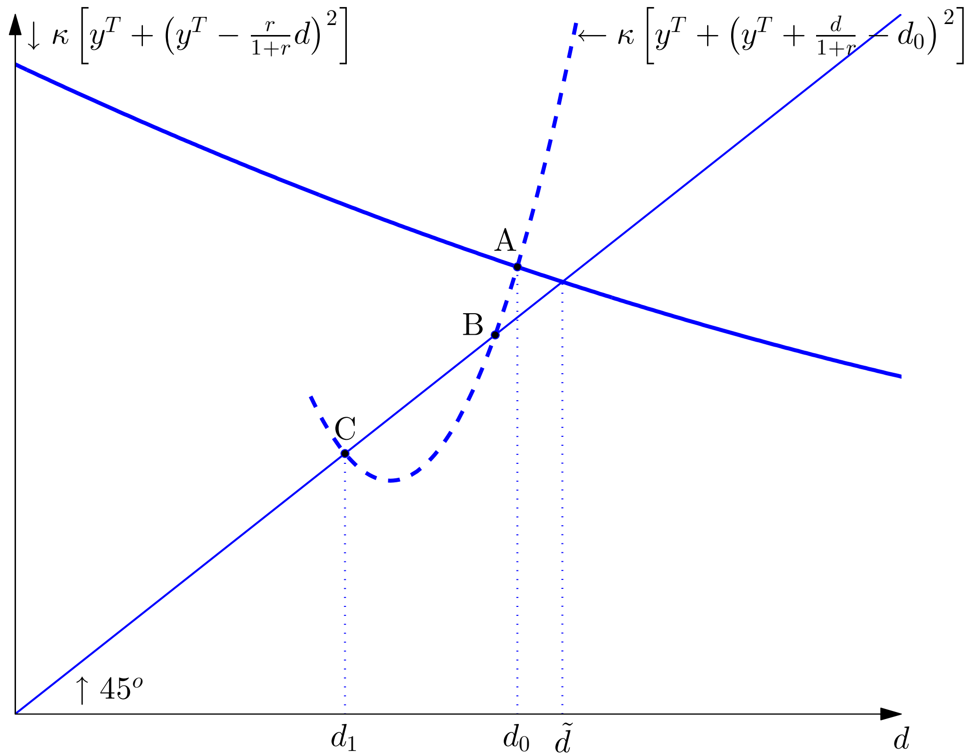
A: No.

We show why this is so next.

The Short-Run Equilibrium Collateral Constraint

$$\boxed{d \le \kappa \left[y^T + \left(y^T + \frac{d}{1+r} - d_0 \right)^2 \right]} \tag{12.53}$$

Figure 12.4 Multiple equilibria with flow collateral constraints



Observations

(1) The slope of the short-run (SR) CC is proportional to c_0^T. So an equilibrium must be on an upward sloping range of the SR CC.

(2) Suppose the initial debt level is d_0. The solution $d_{t+1} = d_0$ for all $t \geq 0$ does not violate the SR CC, because point A lies above the 45-degree line. Since this solution also satisfies all other equilibrium conditions, it is indeed an equilibrium. Are there more?

(3) Point C also satisfies the SR CC, since it is on the 45-degree line. But, because the slope of the SR CC is negative, c_0^T is negative. So we rule out C.

(4) Another candidate is point B. This solution satisfies the SR CC since it is on the 45-degree line. It also satisfies the LR CC. And because the slope of the SR CC is positive, c_0^T is positive. But we must check that the Euler equation is satisfied at point B for $\mu_0 \geq 0$ (next slide).

(5) At point B, the economy experiences a self-fulfilling financial crisis, caused by an arbitrary desire to deleverage.

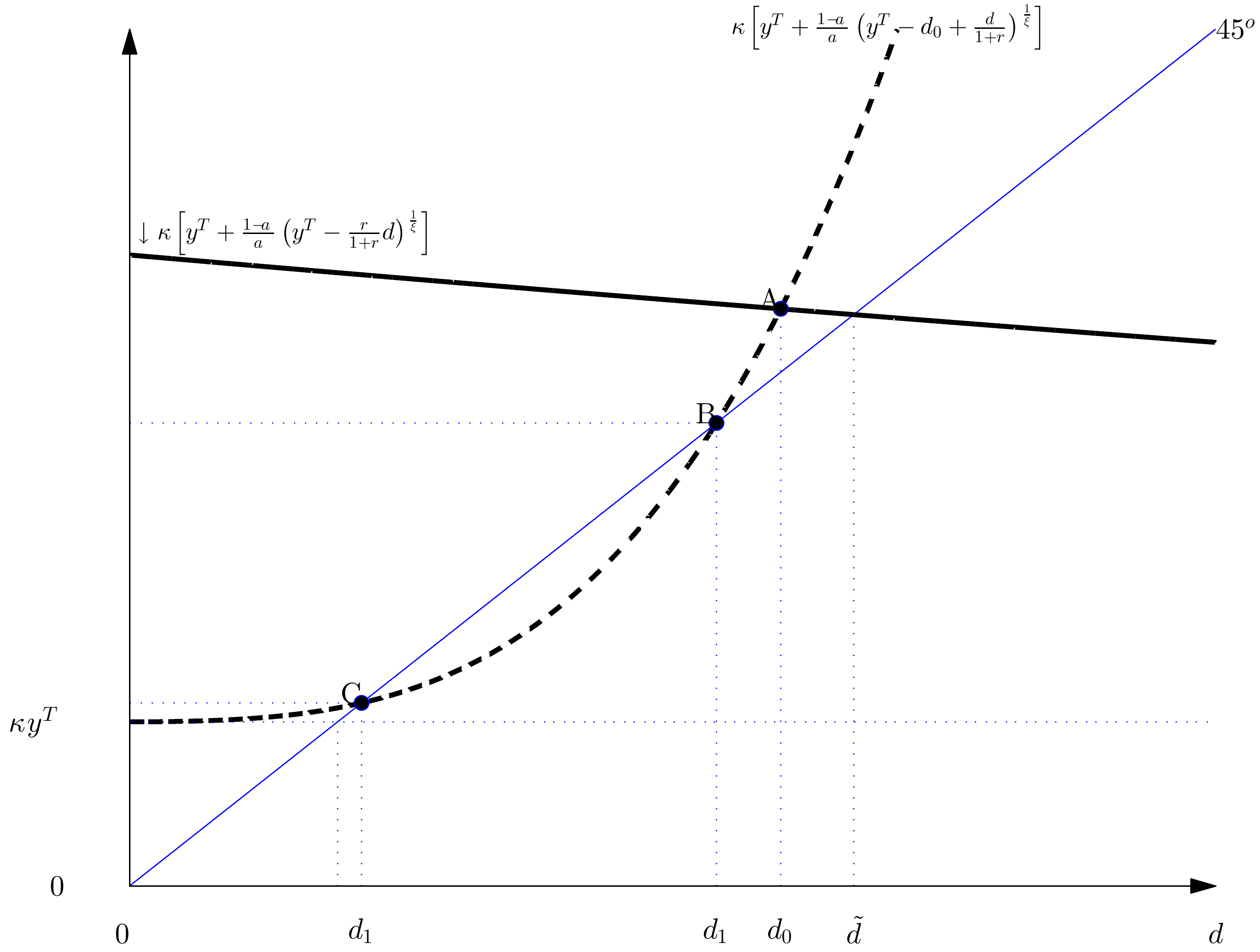
The Euler Equation in Period 0

$$\frac{c_1^T}{c_0^T} = \frac{1}{\sqrt{1-\mu_0(1+r)}},$$

where μ_0 is the multiplier associated with the collateral constraint. Is $\mu_0 \geq 0$ at point B? Yes, because at that point $c_1^T/c_0^T > 1$. This corroborates that point B on the previous graph is indeed an equilibrium.

The equilibrium at point B is costly in terms of welfare, because the initial deleveraging requires a drop in consumption, which implies a deviation from the perfect consumption smoothing induced by equilibrium A.

Figure 12.5 Third Equilibrium Under Flow collateral constraints.



Observations

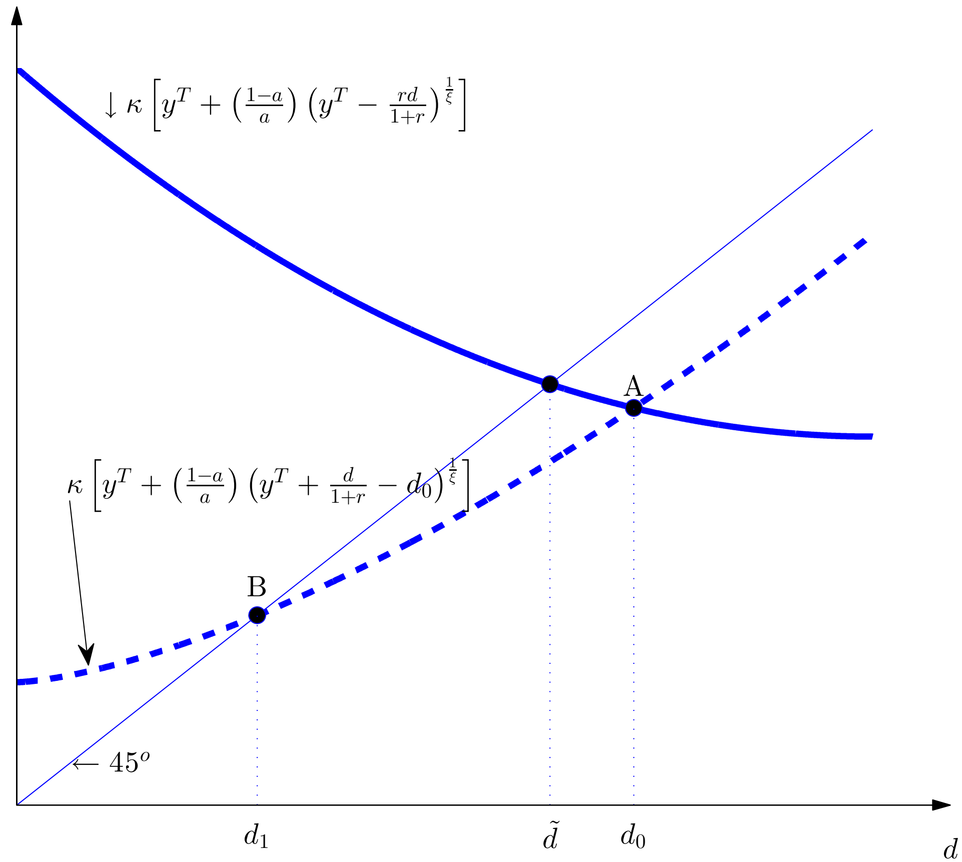
(1) Suppose the initial debt level is d_0. As before the solution $d_{t+1} = d_0$ for all $t \geq 0$ does not violate the SR CC, because point A lies above the 45-degree line. Since this solution also satisfies all other equilibrium conditions, it is indeed an equilibrium.

(2) Again another candidate is point B. This solution satisfies the SR CC since it is on the 45-degree line. It also satisfies the LR CC. And because the slope of the SR CC is positive, c_0^T is positive. And by the same argument as before, we can show that $\mu_0 > 0$. So B is an equilibrium.

(3) Point C also satisfies the SR CC, since it is on the 45-degree line. And this time the slope of the SR CC at point C is positive, so c_0^T is positive. We need to check that $\mu_0 > 0$, which is indeed the case here.

(4) Point C entails a larger drop in the value of collateral and more deleveraging than the crisis associated with point B. Thus, the contraction in aggregate demand is also larger making point C a more severe self-fulfilling financial crisis than point B.

A Unique Equilibrium



Section 12.5: Debt Dynamics in a Stochastic Economy with A Flow Collateral Constraint

Questions:

- How to solve this model numerically
- How to handle the possibility of multiplicity
- How to calibrate the economy
- What is the effect of the constraint on eqm debt dynamics
- Will the economy hit the collateral constraint

The Flow Collateral Constraint

$$d_{t+1} \leq \kappa(y_t^T + p_t y_t^N)$$

Equilibrium: $\{c_t,\ c_t^T,\ d_{t+1},\ \lambda_t,\ \mu_t,\ p_t\}$ satisfying

$$c_t^T + d_t = y_t^T + \frac{d_{t+1}}{1+r_t} \tag{16}$$

$$c_t = \left[a {c_t^T}^{1-\frac{1}{\xi}} + (1-a){c_t^N}^{1-\frac{1}{\xi}}\right]^{\frac{1}{1-\frac{1}{\xi}}} \tag{17}$$

$$\lambda_t = a c_t^{-\sigma}\left(\frac{c_t^T}{c_t}\right)^{-1/\xi} \tag{18}$$

$$\lambda_t\left[\frac{1}{1+r_t} - \mu_t\right] = \beta \mathbb{E}_t \lambda_{t+1} \tag{19}$$

$$p_t = \frac{1-a}{a}\left(\frac{c_t^T}{y^N}\right)^{1/\xi} \tag{20}$$

$$c_t^N = y_t^N \tag{21}$$

$$d_{t+1} \leq \kappa\left[y_t^T + p_t y_t^N\right], \quad \mu_t\left[\kappa\left(y_t^T + p_t y_t^N\right) - d_{t+1}\right] = 0, \quad \mu_t \geq 0 \tag{22}$$

given exogenous $\{y_t^T, y_t^N, r_t\}$ and d_0.

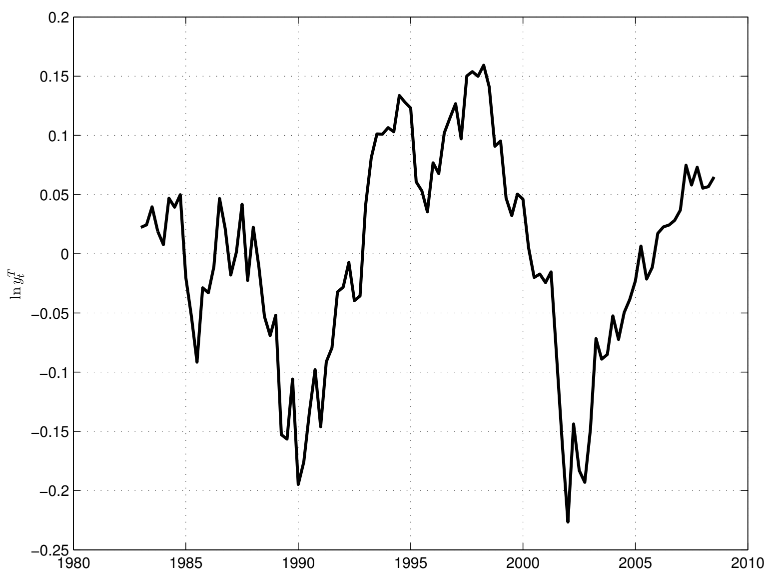
Exogenous Driving Processes:

Empirical Measure of y_t^T: sum of Argentine GDP in agriculture, manufacturing, fishing, forestry, and mining. Quadratically detrended.

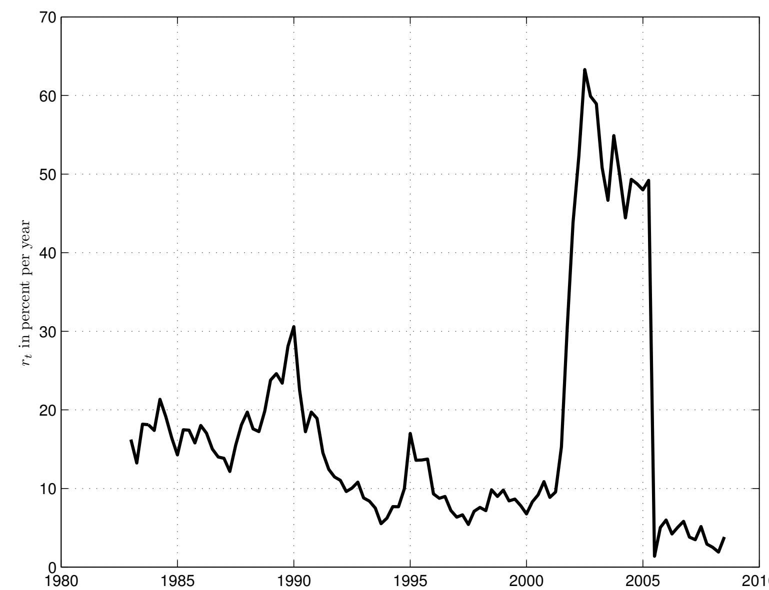
Empirical Measure of r_t: Sum of Argentine EMBI+ plus 90-day Treasury-Bill rate minus a measure of U.S. expected inflation.

Constant Endowment of y_t^N: $y_t^N = y^N = 1$.

(a) Traded Output, y_t^T



(b) Interest Rate, r_t



Estimate the following AR(1) system using Argentine data over the period 1983:Q1—2001:Q3:

$$\begin{bmatrix} \ln y_t^T \\ \ln \frac{1+r_t}{1+r} \end{bmatrix} = A \begin{bmatrix} \ln y_{t-1}^T \\ \ln \frac{1+r_{t-1}}{1+r} \end{bmatrix} + \epsilon_t, \qquad \epsilon_t \sim N\left(\emptyset, \Sigma_\epsilon\right)$$

OLS Estimate

$$A = \begin{bmatrix} 0.79 & -1.36 \\ -0.01 & 0.86 \end{bmatrix}; \quad \Sigma_\epsilon = \begin{bmatrix} 0.00123 & -0.00008 \\ -0.00008 & 0.00004 \end{bmatrix}; \quad r = 0.0316.$$

Some Unconditional Summary Statistics of the Driving Process

Statistic	y_t^T	r_t
Std. Dev.	12%	6%yr
Serial Corr.	0.95	0.93
Corr(y_t^T, r_t)	-0.86	
Mean	1	12%yr

Comments:

(1) High volatility of tradable y_t^T and r_t;

(2) negative correlation between y_t^T and r_t, when it rains it pours;

(3) High mean country interest rate.

How to solve the model numerically? — 3 complications

1. There is an occasionally binding constraint. $\rightarrow$ no perturbation.
2. There is an externality. $\rightarrow$ cannot be cast as value function problem.

$\Rightarrow$ These two considerations suggest using an Euler equation iteration procedure over a discretized state space.

3. There may exist multiple eqa. $\rightarrow$ impose eqm selection criterion.

Equilibrium Selection

(b) If for a given current state (y_t^T, r_t, d_t) there are one or more values of d_{t+1} for which all equilibrium conditions are satisfied pick the largest one for which the collateral constraint is binding.

(c) If for a given current state (y_t^T, r_t, d_t) there are one or more values of d_{t+1} for which all equilibrium conditions are satisfied pick the smallest one for which the collateral constraint is binding.

Criterion (b) favors equilibria like point B and criterion (c) favors equilibria like point C in figure 12.5. [One could in principle adopt other equilibrium selection criteria, including ones in which nonfundamental uncertainty (sunspot realizations) affects the real allocation.]

Discretization of the Driving Process

- How to pick the grid?

The first and last values of the grids for $\ln y_t^T$ and $\ln(1+r_t)/(1+r)$ are set to $\pm\sqrt{10}$ times the respective standard deviations (± 0.3858 and ± 0.0539, respectively).

Why $\sqrt{10}$ std(x)? Somewhat arbitrary, try to get low probability of visiting the endpoints of the grid.

$\ln y_t^T$ grid has 21 equally spaced points

$\ln \frac{1+r_t}{1+r}$ grid has 11 equally spaced points

Why 21 and 11. Again somewhat arbitrary. Tradeoff between getting the variance and serial correlation right and not having too many gridpoints.

- How to construct the transition probability matrix?

Construct the transition probability matrix of the state $(\ln y_t^T, \ln((1+r_t)/(1+r)))$ using the simulation approach, in particular the Matlab code `tpm.m` proposed in Schmitt-Grohé and Uribe (2009), which consists in simulating a time series of length 1,000,000 drawn from the AR(1) system above and associating each observation in the time series with one of the 231 possible discrete states by distance minimization.

The resulting discrete-valued time series is used to compute the probability of transitioning from a particular discrete state in one period to a particular discrete state in the next period.

The resulting transition probability matrix, stored in `tpm.mat`, captures well the covariance matrices of order 0 and 1.

Note. Some combinations of (y_i^T, r_i) are never visited. We remove those states, resulting in 145 possible pairs (y_i^T, r_i) instead of 231. Thus we have $ny = 145$ grid points for the exogenous state.

An alternative method for computing the transition probability matrix of the exogenous state is the quadrature based method proposed by Tauchen and Hussey (1991).

Functional Forms and Parameter Values

- Following the business-cycle literature we assume that the time unit is one quarter.

- $\kappa = 1.2$ ($\Rightarrow$upper limit on debt = 30 percent of annual output).

- Assume a CRRA form for preferences and a CES form for the aggregator of tradables and nontradable

$$U(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma}$$

$$A(c^T, c^N) = \left[a(c^T)^{1-\frac{1}{\xi}} + (1-a)(c^N)^{1-\frac{1}{\xi}}\right]^{\frac{1}{1-\frac{1}{\xi}}}$$

with $\sigma = 2, \xi = 1/2, a = 0.26$.

The case of Equal Intra- and Intertemporal Elasticities of Substitution

We consider the case

$$\xi = \frac{1}{\sigma}$$

Why this case is of interest:

- It is empirically plausible, see Akinci (2011).

- Preferences become separable in c_t^T and c_t^N, which in some applications facilitates the characterization of equilibrium (although not in the application we are interested in here.)

$$U(A(c_t^T, c_t^N)) = \frac{a {c_t^T}^{1-\sigma} + (1-a) {c_t^N}^{1-\sigma} - 1}{1-\sigma}$$

We need to make the model stationary — how?

Make households impatient

$$\beta(1+r) < 1.$$

Specifically, pick $\beta = 0.9635$ to match the average external-debt-to-output ratio (in the model without the collateral constraint) of 23 percent per year observed in Argentina over the period 1983-2001 (Lane and Milesi-Ferretti, 2007). This value implies that $\beta(1+r) = 0.9939 < 1$, which means that the subjective discount rate, $\beta^{-1} - 1$, is 63 basis points per quarter below the pecuniary discount rate, r.

Note that here, contrary to the analysis in section 4.10.8 of Chapter 4, we are able to match the desired debt to output ratio, with a relatively small amount of impatience. [Suggestion for replication: use a different stationarity inducing device, say EDEIR, and analyze the sensitivity of the results to this modification.]

How to pick the grid for debt, d_t

Use $nd = 501$ equally spaced points for d_t in the interval $[\underline{d}, \overline{d}]$

How to pick the first and last points of the grid?

First, try $\underline{d} = 0$ and $\overline{d} = d^n$, where d^n is equal to the natural debt limit,

$$d^n \equiv \underline{y}^T \frac{1+\overline{r}}{\overline{r}} = 8.3416.$$

It turns out that d^n is never visited in eqm. To have a more efficient grid, we thus set $[\underline{d}, \overline{d}] = [0, 3.5]$.

Overall grid size: $n = ny \times nd = 145 \times 501 = 72{,}645$ points.

Other issues complicating the numerical algorithm:

• for some current states (y_t^T, r_t, d_t) there exists no value of d_{t+1} that ensures both the satisfaction of the collateral constraint and positive consumption of tradables.

• some debt choices lead with positive probability to areas of the state space for which either consumption is non-positive or the aggregate collateral constraint is violated in the next period.

$\Rightarrow$ to address those issues we introduce a path-finder refinement of the solution algorithm that avoids such debt choices.

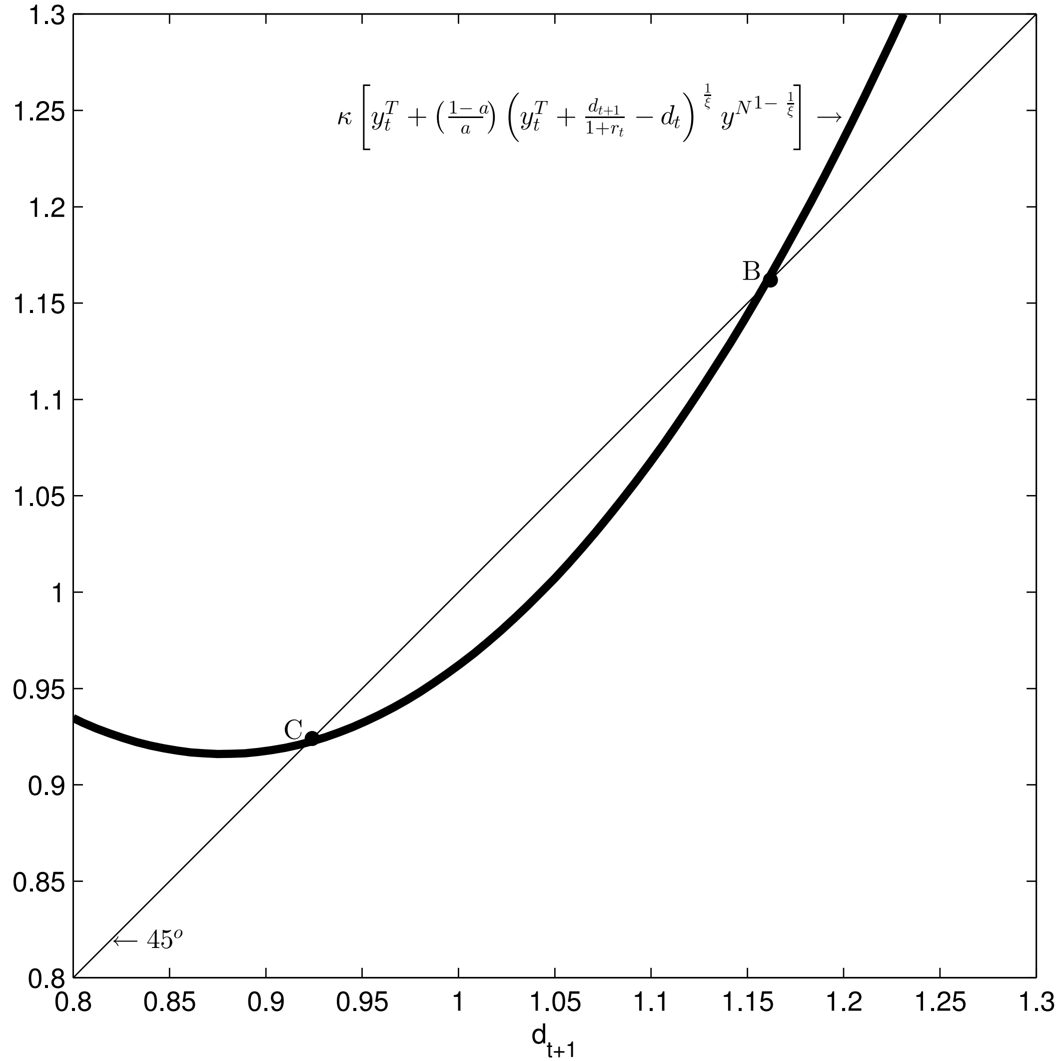
The Matlab program `constrained.m` computes the equilibrium policy function. It uses the refinement `pathfinder.m`. And `simu.m` produces simulated time series of variables of interest.

Summary of the Calibration of the Flow-Collateral-Constraint Economy

Parameter	Value	Description
κ	0.3×4	Parameter of collateral constraint
σ	2	Inverse of intertemporal elasticity of consumption
β	0.9635	Quarterly subjective discount factor
r	0.0316	Steady state quarterly country interest rate
ξ	0.5	Elasticity of substitution between tradables and nontradables
a	0.26	Share of tradables in CES aggregator
y^N	1	Nontradable output
y^T	1	Steady-state tradable output
	Discretization of State Space	
n_{y^T}	21	Number of grid points for $\ln y_t^T$, equally spaced
n_r	11	Number of grid points for $\ln[(1+r_t)/(1+r)]$, equally spaced
n_d	501	Number of grid points for d_t, equally spaced
$\left[\ln \underline{y}^T, \ln \overline{y}^T\right]$	[-0.3858,0.3858]	Range for tradable output
$\left[\ln\left(\frac{1+\underline{r}}{1+r}\right), \ln\left(\frac{1+\overline{r}}{1+r}\right)\right]$	[-0.0539,0.0539]	Range for interest rate
$[\underline{d}, \overline{d}]$	[0,3.5]	Range for debt

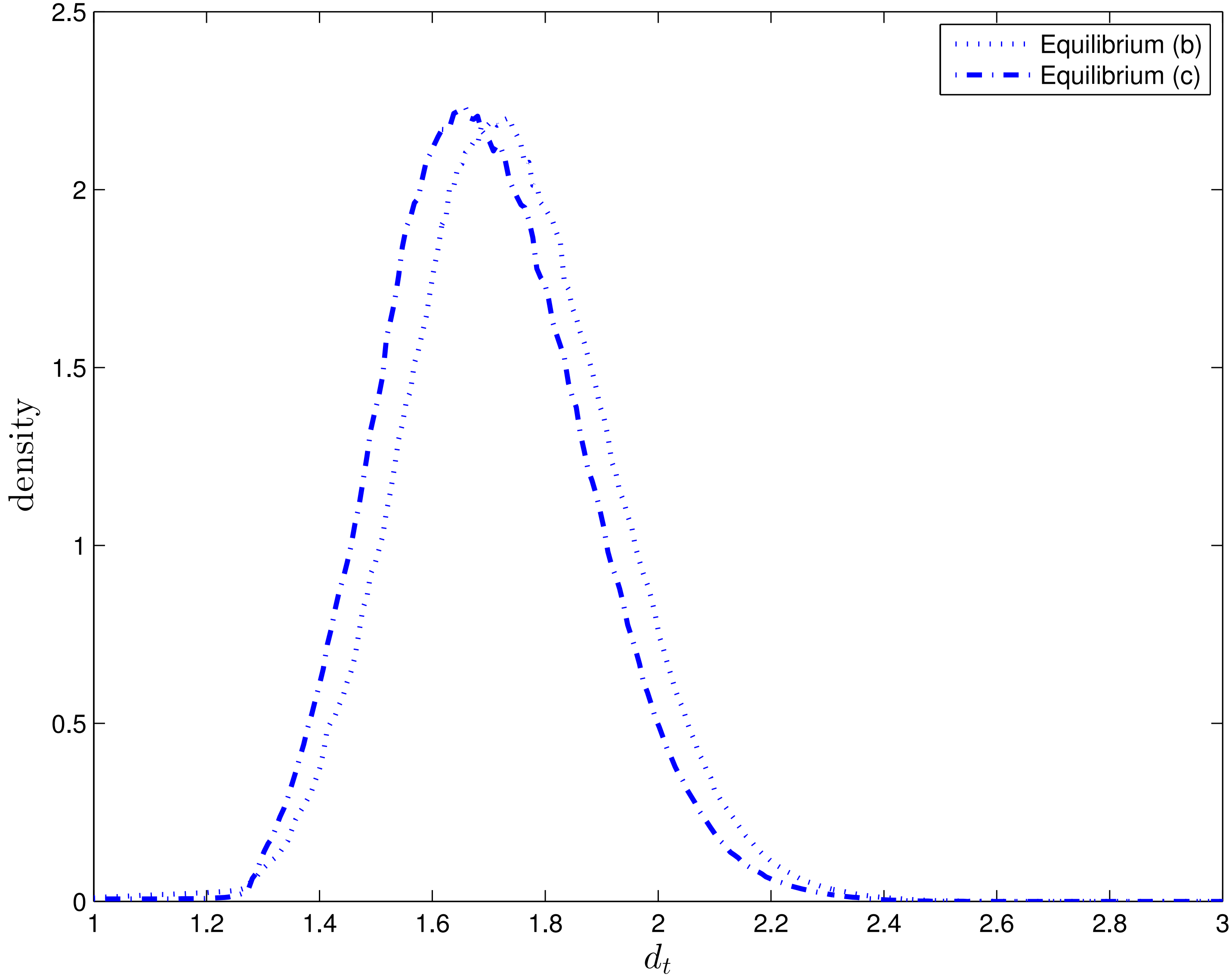
Note. The time unit is one quarter.

Multiple Binding Debt Levels In the Stochastic Economy



Note. The value of collateral is evaluated at the state $(y_t^T, r_t, d_t) = (0.7633, 0.0541, 1.5960)$. The state space has 26,024 states (or 36 percent of all states) with multiple binding debt levels.

Figure 12.6: External Debt Densities



Note. Replication Matlab program `plotdu.m`.

Observations on the figure

–Equilibrium selection criterion (b) gives rise to a different debt density than equilibrium selection criterion (c). Difference in densities suggests that there is also equilibrium multiplicity in the stochastic economy.

–more pessimistic criterion (c) yields mean debt, 12% of output.

–criterion (b) yields slightly higher mean debt of 12.4% of output.

–ignoring eqm selection might cause non-convergence of the Euler-equation iteration procedure.

The Unconstrained Economy

(no externality, eqm can be expressed as solution to a value function problem)

$$v(y^T, r, d) = \max_{c^T, d'} \left\{ U(A(c^T, y^N)) + \beta \mathbb{E} \left[v(y^{T\prime}, r', d') \,\Big|\, y^T, r \right] \right\}$$

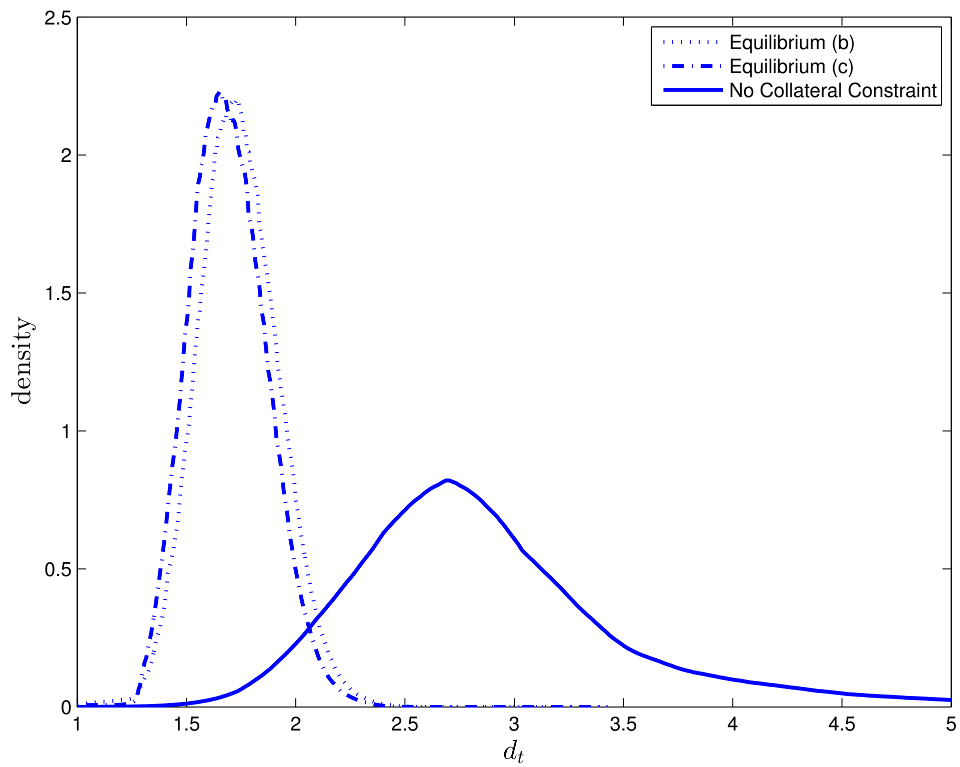
subject to

$$c^T + d = y^T + \frac{d'}{1+r},$$

where a prime superscript denotes next-period values.

Figure 12.7: External Debt Densities

With And Without a Collateral Constraint



Replication program `plotdu.m`.

Observations

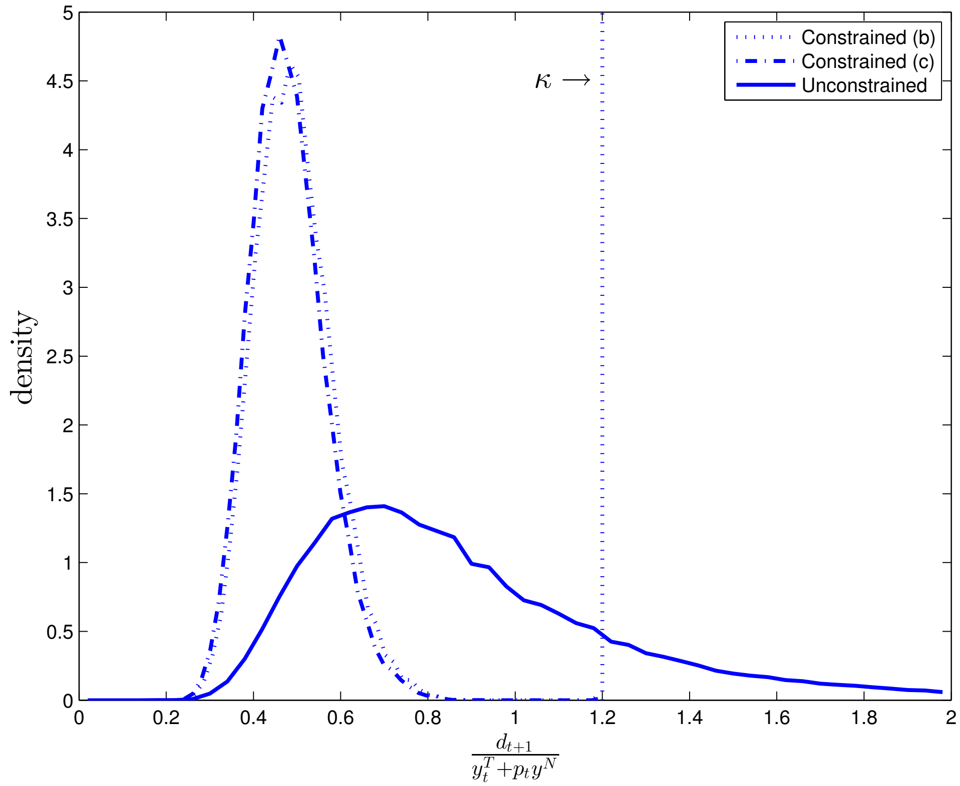
- mean debt is larger without constraint
- debt to output is 23% versus 12% with constraint
- constraint compresses debt distribution

	(b)	(c)	NoCC
$E(d_t)$	1.74	1.69	2.89
$E\left(\frac{d_{t+1}}{4y_t}\right)$	12.4	12.0	23.3
std(d_t)	0.18	0.17	0.67
std$\left(\frac{d_{t+1}}{4y_t}\right)$	0.022	0.023	0.121
corr(d_t, d_{t-1})	0.983	0.984	0.999

Yet, the constraint almost never binds ...

Figure 12.8

The Equilibrium Distribution of Leverage



Replication program `plot_leverage.m`.

$$\text{leverage} = \frac{d_{t+1}}{y_t^T + p_t y^N}$$

- households stay well clear of upper bound on leverage, κ.

- this precautionary savings arises because, as we will see shortly, hitting the constraint is painful.

- In 1 million quarters the constraint binds
  – 287 times in economy (c) and
  – 1,113 times in economy (b)

- The prob that leverage exceeds κ in the unconstrained economy is 18.6%

[Suggestion for an exercise: Show that the frequency of a binding constraint rises when agents are made more impatient.]

Section 12.6 Financial Amplification

Even if the constraint binds so rarely, are regular business cycle fluctuations different in the collateral constrained economy than in the economy without the collateral constraint?

Table 12.2: No Amplification of Regular Business Cycles

Indicator	Std. Dev.			Serial Corr.			Corr. w. Output		
	CC	No CC	R	CC	No CC	R	CC	No CC	R
Tradable Output, y_t^T	0.12	0.12	0.12	0.94	0.94	0.94	0.97	0.95	0.96
Interest Rate, r_t	0.02	0.02	0.02	0.90	0.90	0.90	-0.91	-0.92	-0.93
Output, $y_t^T + p_t y^n$	0.96	0.99	0.95	0.93	0.95	0.94	1.00	1.00	1.00
Consumption of Tradables, c_t^T	0.15	0.16	0.15	0.92	0.95	0.94	0.99	0.99	0.99
Relative Price of Nontradables, p_t	0.85	0.87	0.83	0.92	0.95	0.94	1.00	1.00	1.00
Trade Balance, tb_t	0.05	0.06	0.05	0.61	0.79	0.67	-0.65	-0.71	-0.63
Current Account, ca_t	0.04	0.04	0.04	0.30	0.30	0.32	-0.27	0.13	-0.13
Capital Control Tax, τ_t in percent			0.96			0.10			-0.11

Note. All moments are unconditional. CC stands for the collateral-constraint economy under equilibrium selection criterion (c), No CC for the economy without a collateral constraint, and R for the economy with Ramsey optimal capital control policy.

Observations on the table

• Amplitude of business cycle (std.) not increased in CC economy. (Mendoza AER 2010 first pointed this out in context of a stock collateral constraint economy). Why? Because of precautionary savings main effect of CC is to shift the debt density to the left.

• Serial Correlations also little affected by the presence of the CC.

• These findings are important because they suggest that it is unlikely that emerging economies are more volatile than rich countries because they face more severe borrowing limits of the type studied here.

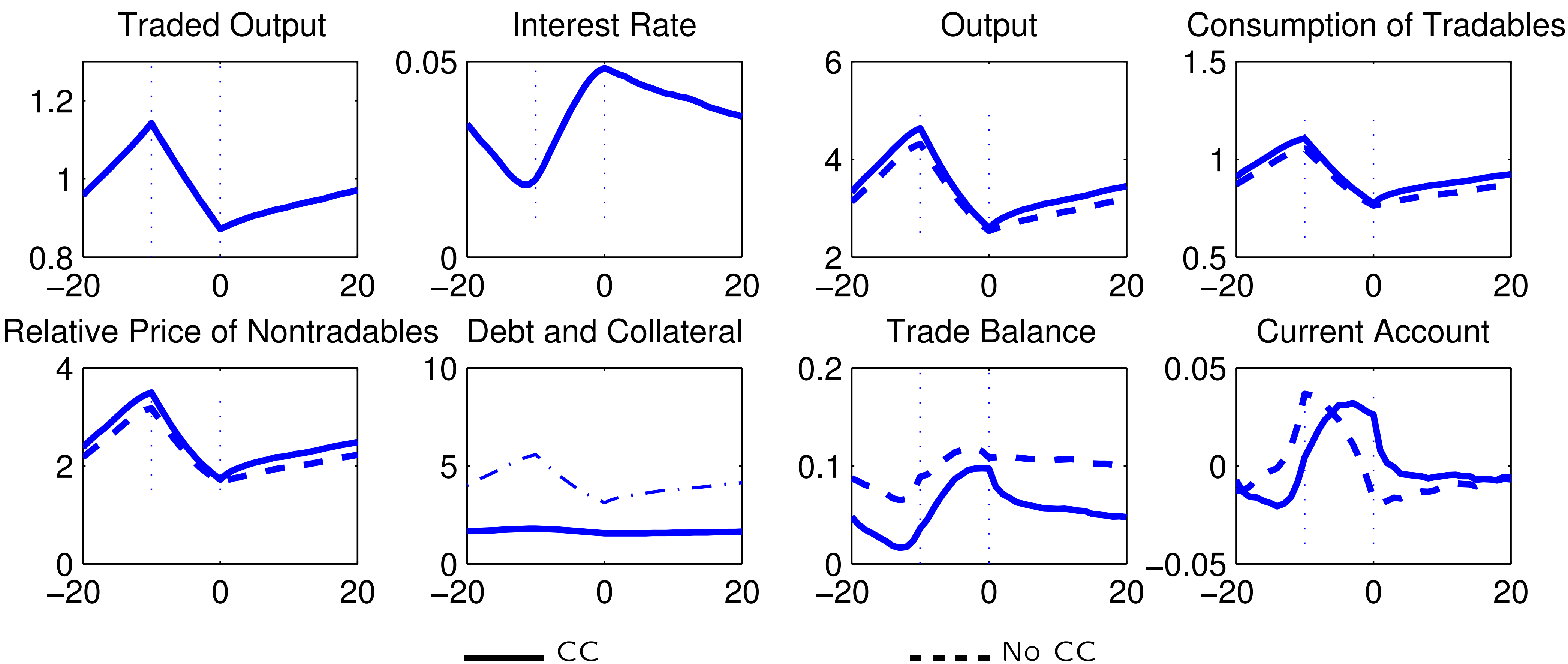
What about amplification of more dramatic events, like a boom bust episode.

Definition of a boom bust episode:

In quarter $t = -20$, y_t^T is below the mean, in quarter $t = -10$, y_t^T is at least 1 std above its mean (the boom). Then in the course of 2.5 years, y^T falls to 1 standard deviations below the mean, that is, in $t = 0$ y_t^T is one standard deviation below the mean (the bust).

The assumed driving process implies that this type of boom bust episode occurs once every 130 years. In this sense it is a rare event.

Figure 12.9: No Amplification of Boom-Bust Episodes



Note. CC indicates the economy with the collateral constraint under equilibrium selection criterion (c). Replication program `boom_bust.m`.

Observations on the figure:

- dynamics in CC and no-CC economies are not very different.

- we can see that the value of collateral displays a clear boom-bust pattern. Yet, these variations in the value of collateral do not lead to a boom bust pattern in external debt. Debt is flat over the entire boom-bust episode. Thus, this model does not have the feature that expansions in the value of collateral during booms lead to more debt. Similarly, the collapse of the value of collateral during the bust, leaves debt unaffected.

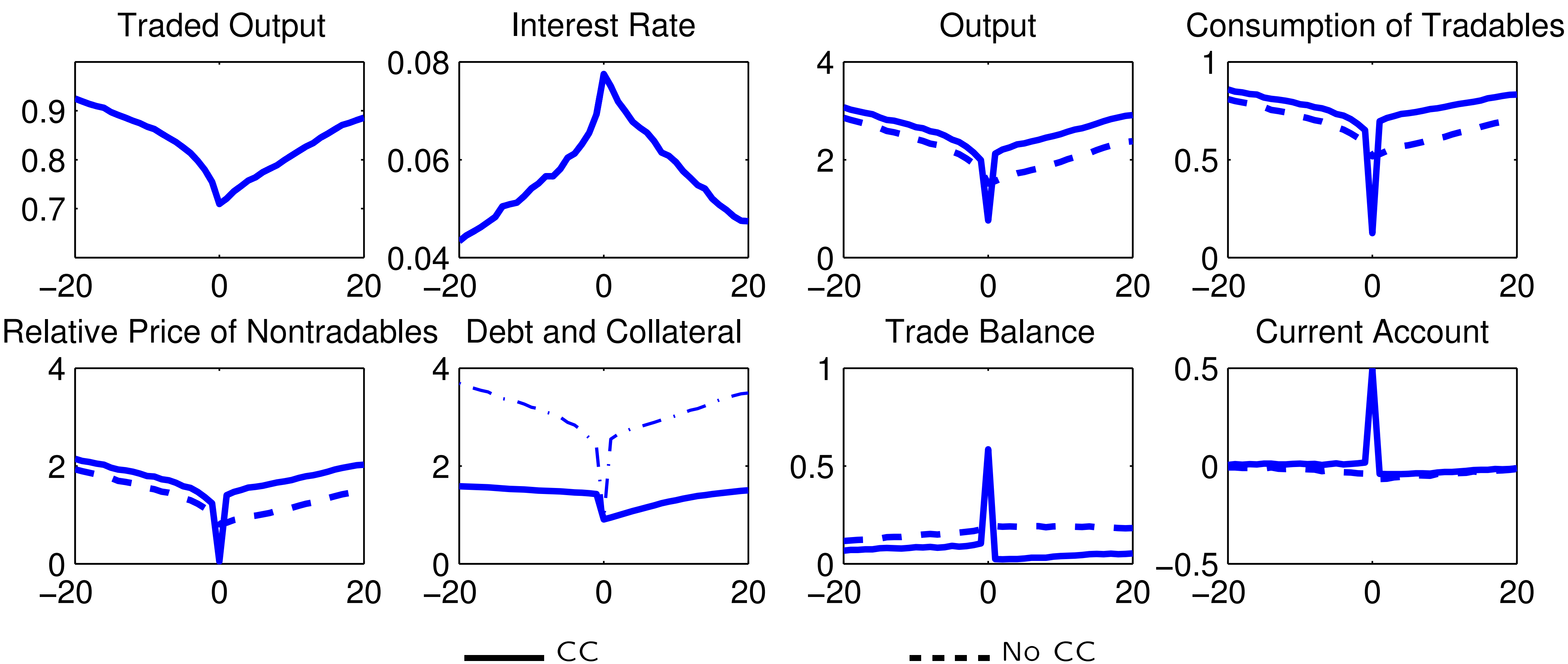
Finally, let's look at the dynamics when the collateral constraint actually binds.

As explained earlier this happens rarely, for equilibrium selection criterion (c), this happens once every 870 years.

Again, we simulated the economy for 1e6 periods, and then averaged over all windows in which the collateral constraint binds. The window starts 20 quarters before the collateral constraint binds ($t = 0$) and ends 20 quarters after the constraint binds.

For comparison, we also show the behavior of the unconstrained economy during the same periods, i.e., having experienced the identical paths of y_t^T and r_t

Figure 12.10: Amplification During Financial Crises



Note. Replication program `typical_crisis.m`.

Observations:

- when does the constraint bind? after the economy got one big negative surprise after another and each negative shock larger than the previous one. After 5 years of such bad luck, finally, the constraint binds.

- the financial crisis is not preceded by a build-up in debt.

- the collateral constraint becomes binding because the value of collateral falls, not because debt rises.

- once that happens the entire decline in the value of collateral (between periods $t = -1$ to period $t = 0$) must be accommodated by a reduction in debt, $\rightarrow$ deleveraging.

- at that point c^T falls, and hence p falls, setting off a Fisherian debt deflation and a firesale.

- the crisis is short lived. Once the stock of debt was reduced, the economy returns immediately to ‘normal’.

Section 12.7 Optimal Capital Control Policy

Models with endogenous collateral constraints (stock or flow) display a pecuniary externality.

The existing related literature has stressed that the pecuniary externality induces **overborrowing** in the sense that a planner who internalizes the pecuniary externality would borrow less.

This is the topic of this section.

Assumptions:

– the government has commitment
– the government is benevolent
– the government has access to state contingent capital control taxes, τ_t, and lump sum taxes.

We will show:
– capital control tax results in full internalization of pecuniary externality.

– one calibration will yield underborrowing and another overborrowing.

τ_t = proportional tax on debt assumed in period t; $\tau_t > 0$ capital control tax , $\tau_t < 0$ borrowing subsidy

Financed with lump sum taxes: ℓ_t = lump-sum taxes in period t

Tax revenue: $\tau_t \frac{d_{t+1}}{1+r_t}$

Government budget constraint in period t: $\tau_t \frac{d_{t+1}}{1+r_t} = \ell_t$

The household budget constraint:

$$c_t^T + p_t c_t^N + d_t = y_t^T + p_t y_t^N + (1 - \tau_t) \frac{d_{t+1}}{1 + r_t} + \ell_t$$

Interest rate on foreign borrowing absent the capital control tax is$(1 + r_t)$. With the tax it becomes:

$$\frac{1 + r_t}{1 - \tau_t} > 1 + r_t, \quad \text{if } \tau_t > 0$$

Competitive equilibrium in the economy with capital control taxes are processes c_t^T, d_{t+1}, λ_t, μ_t, and p_t satisfying

$$
\begin{aligned}
c_t^T + d_t &= y_t^T + \frac{d_{t+1}}{1+r_t} && (23)\\
\lambda_t &= U'(A(c_t^T, y_t^N))A_1(c_t^T, y_t^N) && (24)\\
\left(\frac{1-\tau_t}{1+r_t} - \mu_t\right)\lambda_t &= \beta \mathbb{E}_t \lambda_{t+1} && (25)\\
p_t &= \frac{A_2(c_t^T, y_t^N)}{A_1(c_t^T, y_t^N)} && (26)\\
d_{t+1} &\leq \kappa \left[y_t^T + p_t y_t^N\right] && (27)\\
\mu_t[\kappa(y_t^T + p_t y_t^N) - d_{t+1}] &= 0 && (28)\\
\mu_t &\geq 0 && (29)
\end{aligned}
$$

given $\{\tau_t\}$, $\{y_t^T\}$ and $\{r_t\}$, and d_0.

How to pick τ_t? To maximize

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(A(c_t^T, y_t^N))$$

subject to (23)-(29).

<u>Claim</u>: $\{c_t^T\}$ and $\{d_{t+1}\}$ satisfy (23)-(29) if and only if they satisfy (23) and

$$d_{t+1} \leq \kappa \left[y_t^T + \frac{A_2(c_t^T, y_t^N)}{A_1(c_t^T, y_t^N)} y_t^N \right]. \tag{30}$$

<u>Proof:</u> Suppose $\{c_t^T\}$ and $\{d_{t+1}\}$ satisfy (23) and (30). Show that they also satisfy (23)-(29). (To show the reverse is also needed, but as it is trivial not shown here.)

Pick $\{p_t\}$ to satisfy (26). Then by (30), (27) holds.
Pick $\{\mu_t\} = 0\ \forall t$, then (28) and (29) hold
Pick λ_t to satisfy(24).
Pick τ_t to satisfy (25) ■

[Can you show that τ_t is not unique? Ie, $\exists$ other picks for μ_t and τ_t that are consistent with the same allocation?]

The Ramsey Optimal Capital Control Tax Problem

$$\max_{\{c_t^T, d_{t+1}\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(A(c_t^T, y_t^N)) \tag{31}$$

subject to

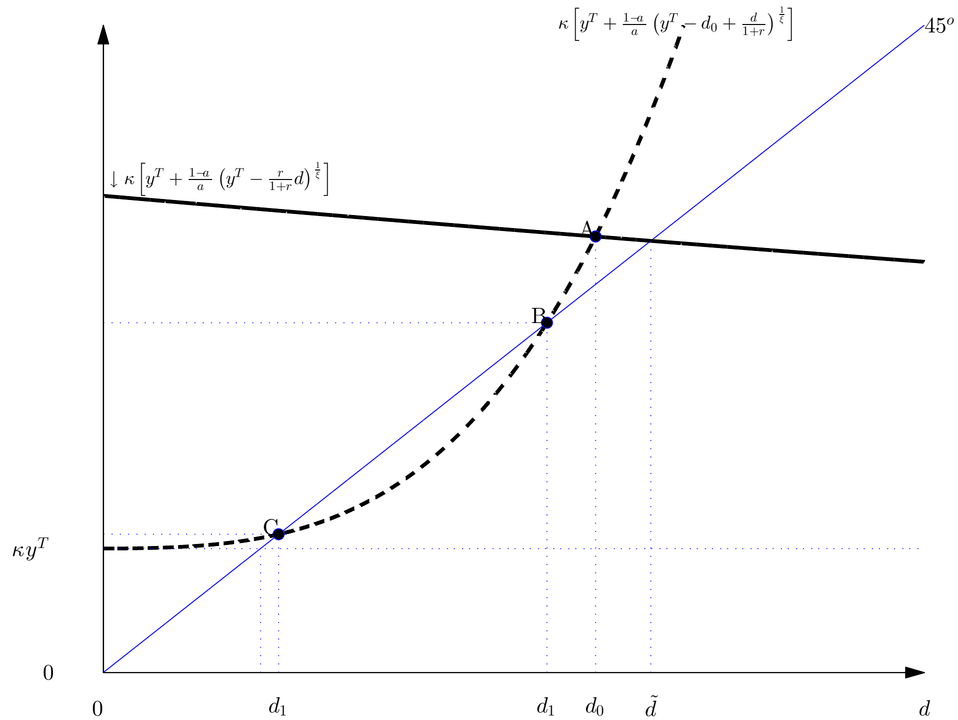
$$c_t^T + d_t = y_t^T + \frac{d_{t+1}}{1+r_t} \tag{23}$$

$$d_{t+1} \leq \kappa \left[y_t^T + \frac{A_2(c_t^T, y_t^N)}{A_1(c_t^T, y_t^N)} y_t^N \right] \tag{30}$$

We have thus shown that with a capital control tax the Ramsey planner fully internalizes the pecuniary externality.

Section 12.7.1 Overborrowing or Underborrowing in the Analytical Example of Section 12.4

Underborrowing!!



The competitive equilibrium at point A can be supported with $\tau_t = 0$ for all t. It is also the first-best allocation, so it must be Ramsey optimal.

Thus, if agents coordinate on equilibrium A, there is neither overborrowing nor underborrowing.

But if agents coordinate on equilibrium B or C, the economy suffers underborrowing.

Implementation of Ramsey Optimal Equilibrium

The Ramsey optimal tax rate in this economy is $\tau_t = 0$ at all times.

However, announcing the policy $\tau_t = 0$ for all t does not guarantee that the Ramsey optimal equilibrium will emerge. Indeed, this tax policy also supports the deleveraging equilibria B or C.

What capital control policy can induce the Ramsey-optimal equilibrium? Consider a debt-dependent feedback rule for τ_t:

$$\tau_t = \tau(d_{t+1}, d_t)$$

satisfying $\tau(d, d) = 0$ and $\tau > 0$ if $d_{t+1} < d_t$.

Implementation (continued)

Under this tax-policy rule, the Euler equation in period 0 becomes:

$$\frac{c_1^T}{c_0^T} = \frac{1}{\sqrt{1 - \tau(d_1, d_0) - (1+r)\mu_0}}$$

(1) In the intended (Ramsey) equilibrium, $c_1^T/c_0^T = 1$, $d_1 = d_0$, and $\mu_0 = 0$, so the Euler equation holds and $\tau(d_1, d_0) = 0$.

(2) In the unintended equilibrium (points B or C), $c_1^T/c_0^T > 1$, and $d_1 < d_0$. Make $\tau(d_1, d_0) > 0$ and so large that μ_0 has to be negative for the Euler equation to hold. Since μ_0 must be nonnegative, this capital-control policy rules out the unintended equilibrium.

(3) $\Rightarrow$ The thread of imposing capital controls in response to outflows eliminates self-fulfilling crises.

Section 12.8 Overborrowing and Underborrowing in the Stochastic Economy

Computation: The Ramsey optimal allocation is relatively easy to compute because the Ramsey problem can be cast in the form of a Bellman equation problem.

The recursive version of the Ramsey problem of maximizing (31) subject to (23) and (30) is given by

$$v(y^T, r, d) = \max_{c^T, d'} \left\{ U(A(c^T, y^N)) + \beta \mathbb{E} \left[v(y^{T\prime}, r', d') \, \middle| y^T, r \right] \right\}$$

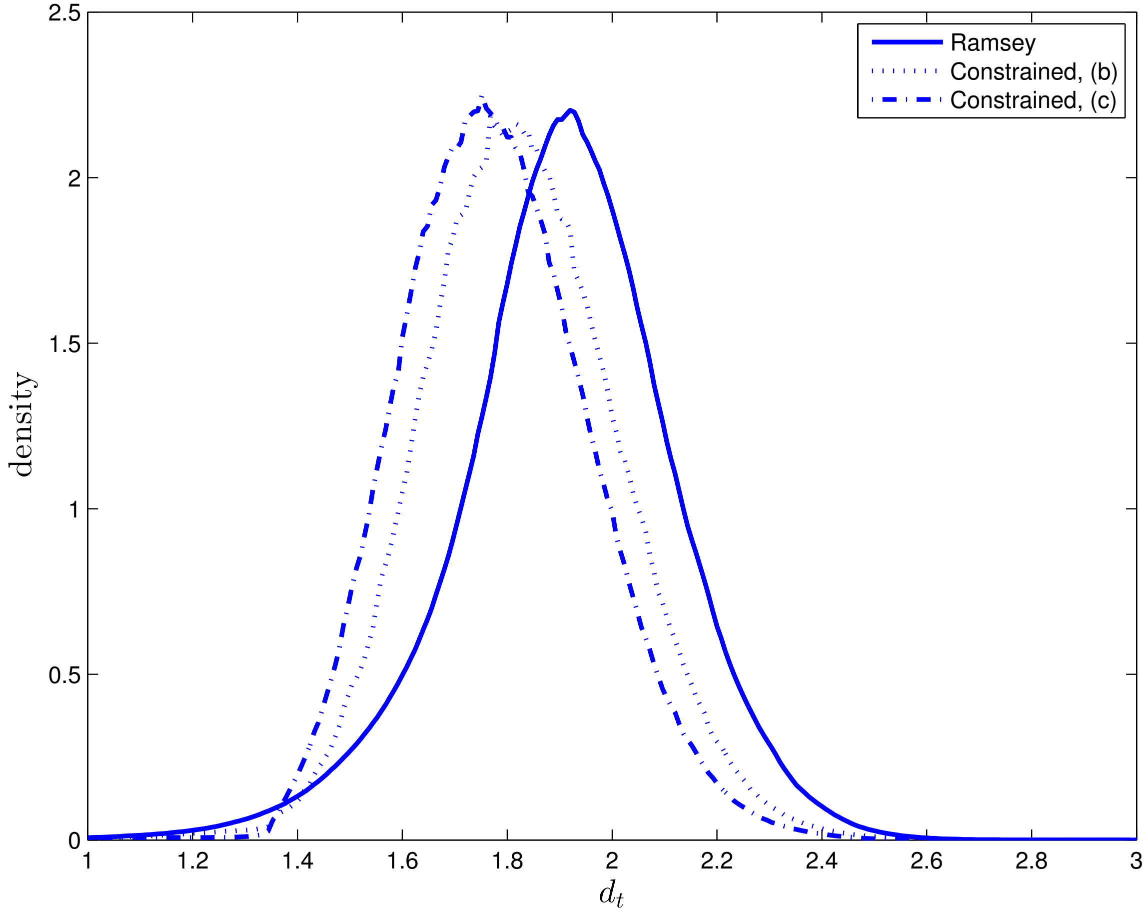
subject to

$$\begin{aligned} c^T + d &= y^T + \frac{d'}{1+r} \\ d' &\leq \kappa \left[y^T + \frac{1-a}{a} \left(\frac{c^T}{y^N} \right)^{\frac{1}{\xi}} y^N \right] \end{aligned}$$

where a prime superscript denotes next-period values.

Observation: Although the constraints of this control problem may not represent a convex set in tradable consumption and debt, the fact that the Ramsey allocation is the result of a utility maximization problem, implies that its solution is generically unique.

Debt Densities With Optimal Capital Controls: A Case of Underborrowing



Observations on the figure:

Equilibrium selection criterion (c) $E(D/Y^{annual}) = 12.0$ percent

Equilibrium selection criterion (b) $E(D/Y^{annual}) = 12.4$ percent

Ramsey optimal capital control policy: $E(D/Y^{annual}) = 13.1$ percent

Therefore there is **underborrowing** in the unregulated economy. (between 0.7 and 1.1 percent of output)

Agents engage in excessive precautionary savings.

How often is there a crisis under the Ramsey policy? 179 times in one million periods. [and 287 and 1,133 times, respectively, under selection criteria (c) and (b).]

For comparison let's look now at a different calibration of the model, namely, that studied in Bianchi (AER 2011)—the central reference for the overborrowing result in the quantitative flow collateral constraint literature.

Bianchi has a different driving force, y_t^T and y_t^N are stochastic but r_t is not.

The Driving Process of Bianchi 2011

The natural logarithms of the traded and nontraded endowments follow a bivariate AR(1), which is estimated on annual HP-filtered Argentine data spanning the period 1965 to 2007. Traded GDP: Manufacturing and primary products. Nontraded GDP: remaining components.

$$\begin{bmatrix} \ln y_t^T \\ \ln y_t^N \end{bmatrix} = \begin{bmatrix} 0.901 & -0.453 \\ 0.495 & 0.225 \end{bmatrix} \begin{bmatrix} \ln y_{t-1}^T \\ \ln y_{t-1}^N \end{bmatrix} + \epsilon_t, \tag{32}$$

where $\epsilon_t \sim N(\emptyset, \Omega_\epsilon)$, with $\Omega_\epsilon = \begin{bmatrix} 0.00219 & 0.00162 \\ 0.00162 & 0.00167 \end{bmatrix}$.

Some Unconditional Summary Statistics of the Bianchi (2011) Driving Process

Statistic	$\ln y^T$	$\ln y^N$
Std. Dev.	6%	6%
Serial Corr.	0.53	0.62
Corr($\ln y_t^T, \ln y_t^N$)	0.83	

Observations:

(1) High volatility of tradable and nontradable endowments.

(2) Strong positive correlation between y_t^T and y_t^N.

Discretization of the State Space

There are 4 distinct grid points for $ln(y^T)$,

$$\begin{bmatrix} -0.1093 \\ -0.0347 \\ 0.0347 \\ 0.1093 \end{bmatrix}$$

and 16 distinct pairs (y^T, y^N).

There are 800 grid points for d_t.

The total grid has $16 \times 800 = 12,800$ points.

Summary of the Calibration

Time unit is one year.

Parameter	Value	Description
κ	0.33	Parameter of collateral constraint
σ	2	Inverse of intertemporal elasticity of consumption
β	0.91	Subjective discount factor
r	0.04	Interest rate (annual)
ξ	0.83	Elasticity of substitution between tradables and nontradables
a	0.31	Weight on tradables in CES aggregator
y^N	1	Steady-state nontradable output
y^T	1	Steady-state tradable output
n_y	16	Number of grid points for $(\ln y_t^T, \ln y_t^N)$
n_d	800	Number of grid points for d_t, equally spaced
$\left[\ln \underline{y}^T, \ln \overline{y}^T\right]$	[-0.1093,0.1093]	Range for tradable output
$\left[\ln \underline{y}^N, \ln \overline{y}^N\right]$	[-0.1328,0.1328]	Range for nontradable output
$[\underline{d}/(1+r), \overline{d}/(1+r)]$	[0.4 1.02]	Range for debt

Comment on Impatience

The calibration implies that consumers are impatient:

$r = 0.04;$

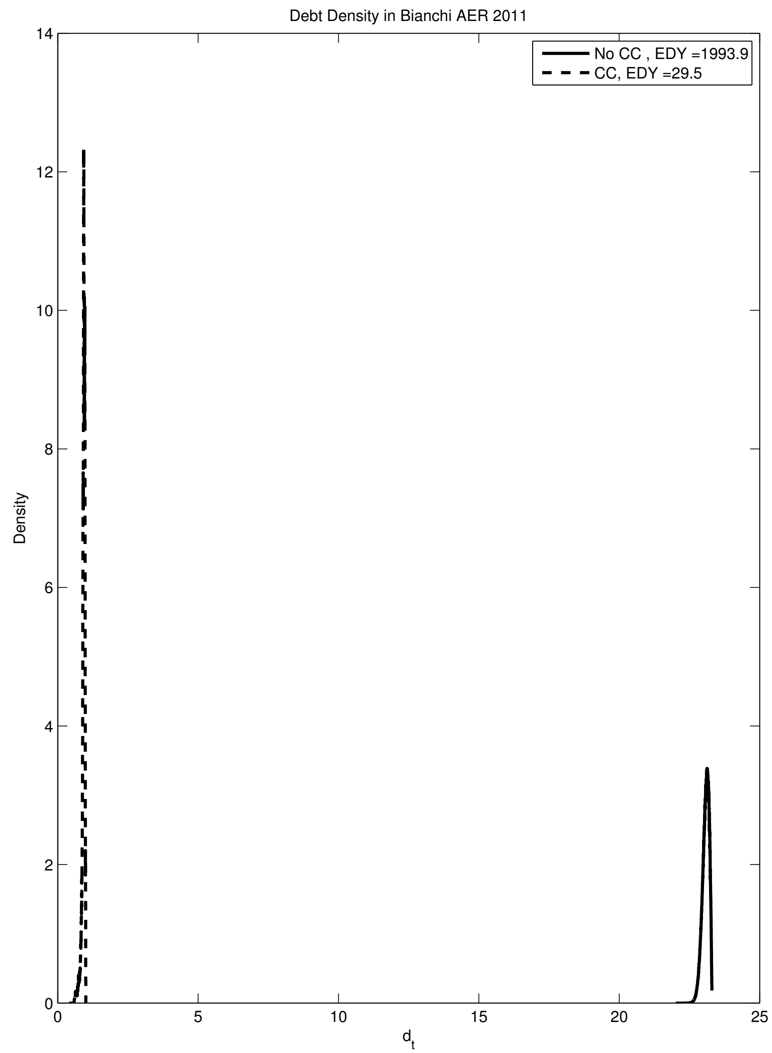
$\beta = 0.91;$

$\beta(1 + r) = 0.95$

The high degree of impatience influences the role of optimal capital controls. The Ramsey planner is constantly negotiating a trade off between, on the one hand, allowing impatient consumers to front-load consumption and, on the other hand, preventing the economy from hitting the collateral constraint.

The high degree of impatience determines how far apart the debt densities are between the CC and the no CC economy. See the next figure

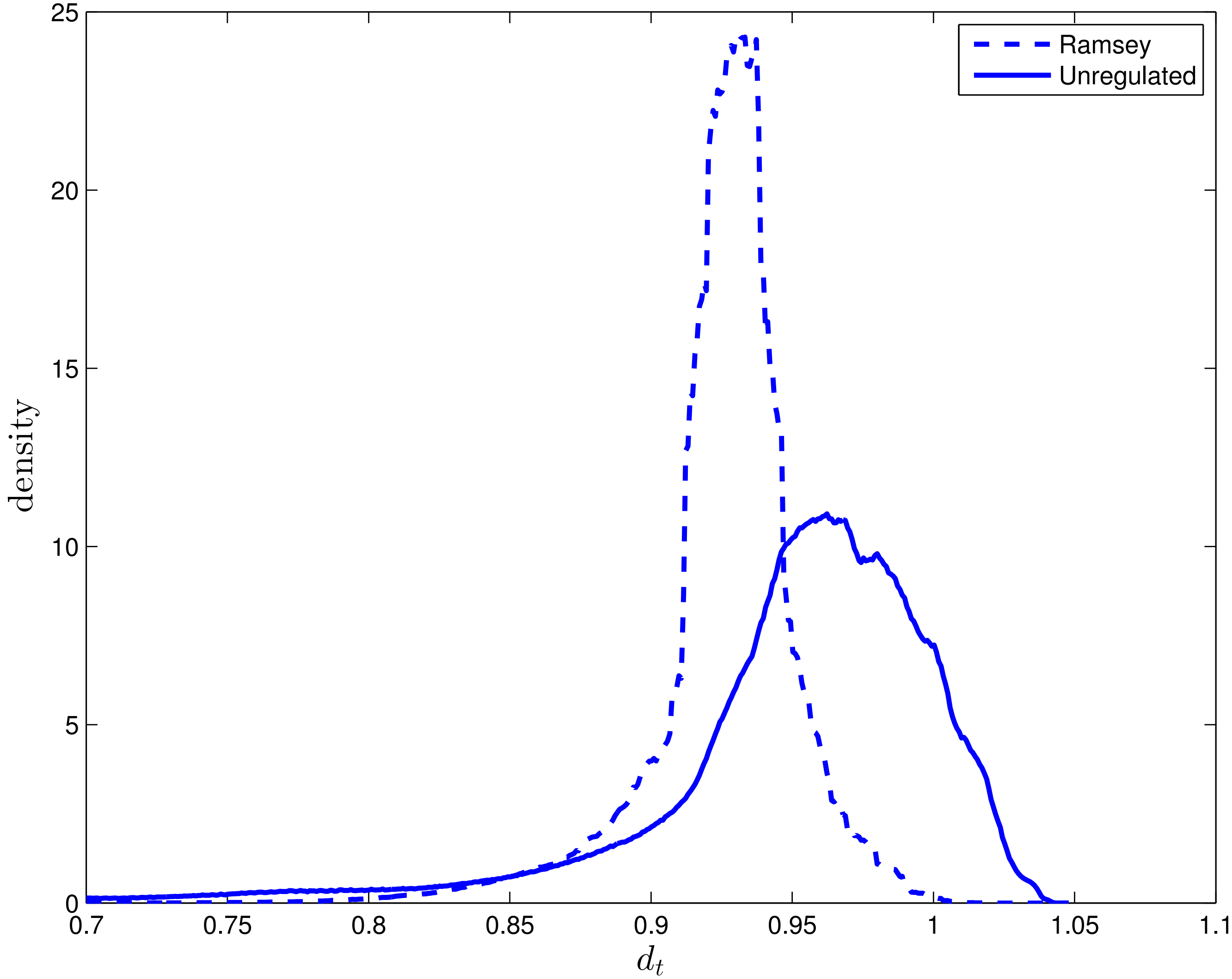
Debt Densities under the Bianchi Calibration



Let's next turn ask how much overborrowing there is in this calibrated economy. Recall overborrowing is defined as the amount of borrowing in the economy in which agents fail to internalize the pecuniary externality compared to the economy in which they do, or in which they do not but face optimally set capital control taxes that makes households behave as if they internalized the pecuniary externality.

How much overborrowing is there?

Modest Amount of Overborrowing under the Bianchi Calibration



Under Ramsey Optimal Capital Controls:

$E\left(\frac{d_{t+1}}{(1+r)y_t}\right) = 28.5\%$

With collateral constraint:

$E\left(\frac{d_{t+1}}{(1+r)y_t}\right) = 29.2\%$

$\Rightarrow$ Pecuniary externality leads to overborrowing of 0.7 percentage points of output

(These results are our replication of those reported in Bianchi. He reports, 28.6% and 29.2%, respectively.)

This suggests that the main role of optimal capital control taxes is not a sizable reduction in the amount of debt.

What do the taxes affect?

Frequency of crisis (defined as a binding collateral constraint) falls from once every **12 years** in the unregulated economy to once every **26 years** in the Ramsey economy.

This suggests that the main role of optimal capital controls is to avoid a binding collateral constraint.

We conclude, depending on the calibration the pecuniary externality can lead either to overborrowing or to underborrowing vis-à-vis the allocation under Ramsey optimal capital control taxes.

Note on the literature:

An exception to the standard overborrowing result is Benigno, Chen, Otrok, Rebucci, and Young (2013) who obtain underborrowing by replacing the assumption of an endowment economy maintained in Bianchi (2011) with the assumption that output is produced with labor. In their production economy, the social planner sustains more debt than in the unregulated economy by engineering sectoral employment allocations conducive to elevated values of the collateral in terms of tradable goods. The underborrowing result obtained in this section is complementary but different from that of Benigno et al. Here, underborrowing arises even in the context of an endowment economy and is due to the possibility of self-fulfilling crises.

Section 12.9 Is Optimal Capital Control Policy Macroprudential?

Macroprudential can mean two slightly different things.

Meaning 1: On average there is a tax on capital inflows, ie, $\mathbb{E}\tau_t > 0$

Meaning 2: Taxes on capital flows are cyclical, in particular, high during good times and low during bad times, ie, capital controls are countercylical, $corr(\tau_t, y_t) > 0$.

Calibration 1: USG, Chapter 12. mean(τ_t) = 0

Calibration 2: Bianchi, AER 2011. mean(τ_t) = 4.2% ; median(τ_t) = 2.5%

According to meaning 1 optimal capital controls fail to be macroprudential in the USG calibration but are indeed macroprudential in the Bianchi calibration.

However, neither calibration implies that the optimal capital control tax is raised during booms and then lowered during the bust, and hence the optimal capital control tax fails to be macroprudential in this precise sense.

Figure 12.13 Are Optimal Capital Controls Macroprudential?

USG calibration, boom bust

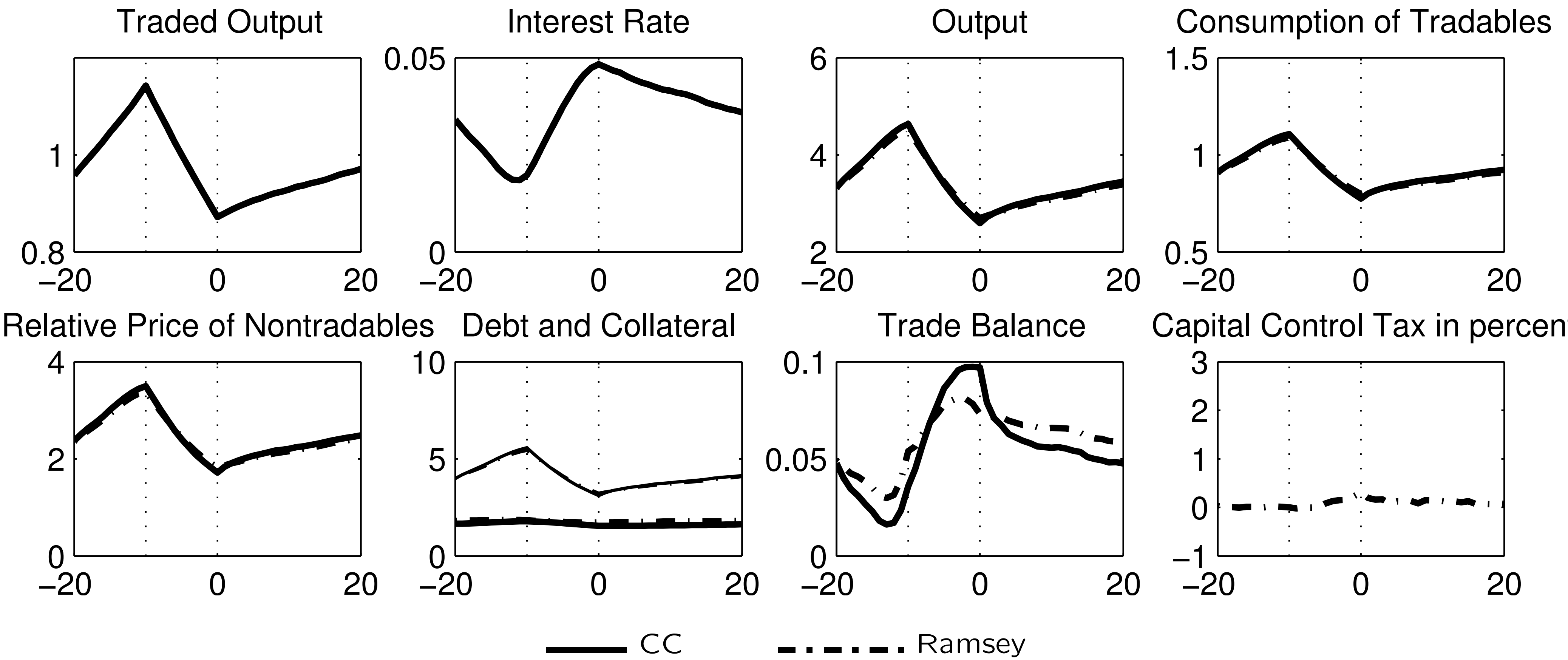
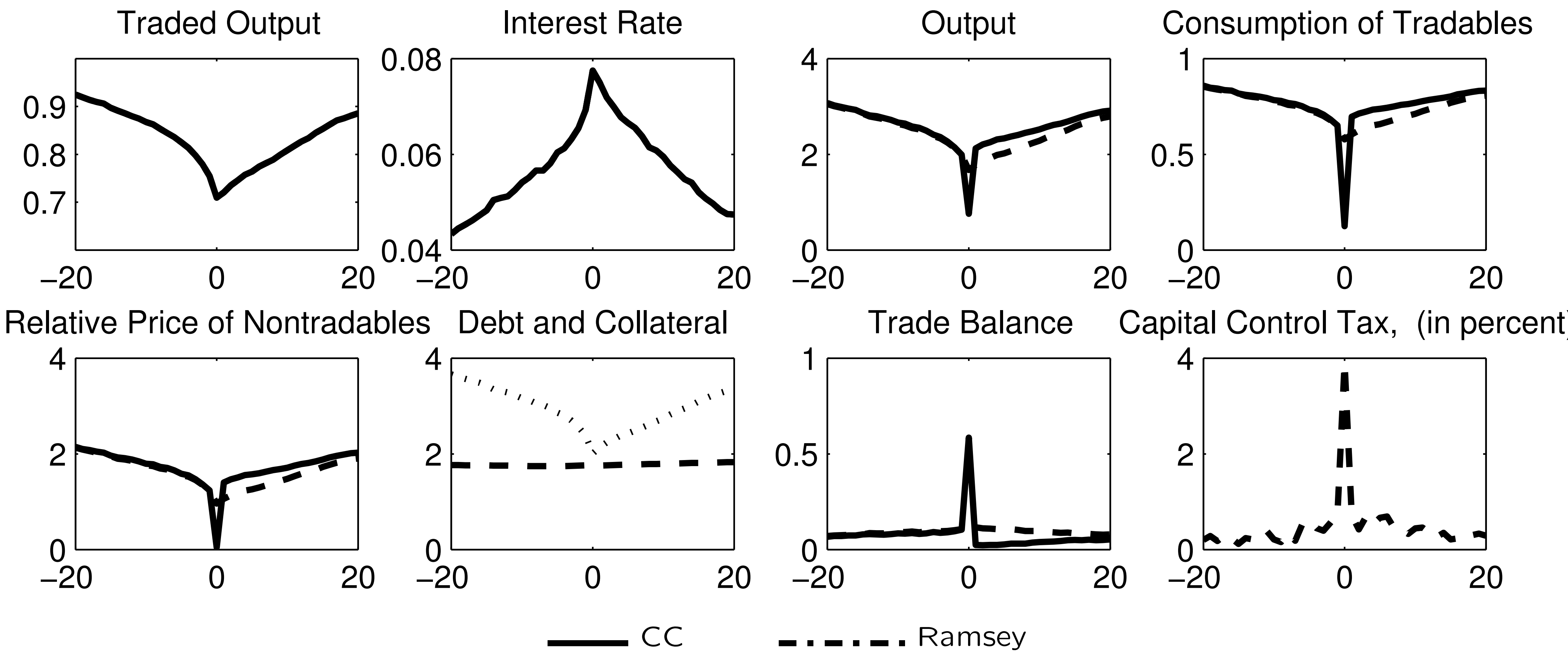
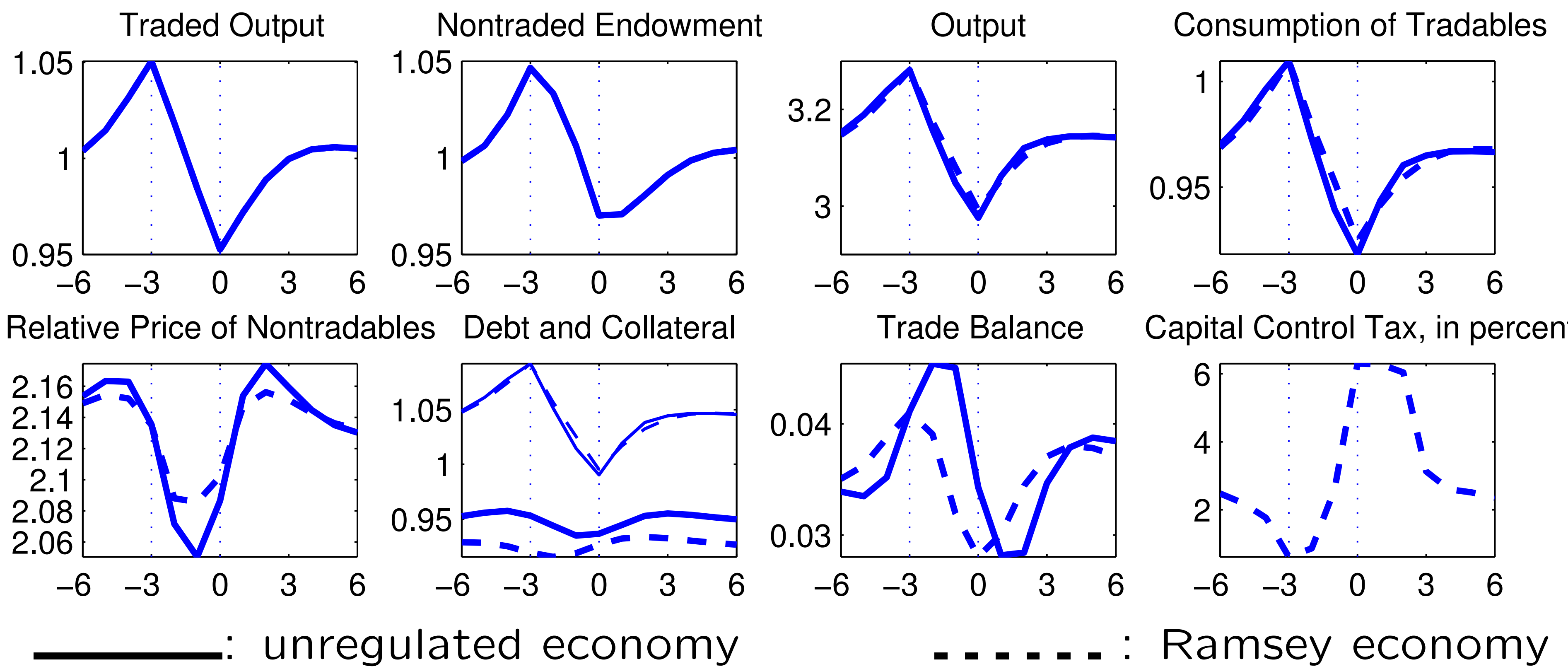


Figure 12.12 Macroprudential Capital Controls in a Financial Crisis.

USG calibration



Are Optimal Capital Controls Cyclical? Bianchi calibration



Source: Schmitt-Grohé and Uribe (IMF ER, 2017). Definition of boom-bust episode: $y^T_{-3} > 1$ and $y^T_0 < 1$; given grid this implies that during a typical boom bust episode output falls from 5% above mean to 5% below mean over 3 years. Frequency, 12.3%. Each line is the mean across all windows containing a boom-bust cycle in a time series of 1 million years. For the capital-control tax rate, the figure displays the median instead of the mean across windows because this variable is skewed, with an unconditional mean of 4.2 percent and an unconditional median of 2.5 percent. Because the capital control tax rate is indeterminate when the collateral constraint binds under the Ramsey policy, this variable is given a number only if the collateral constraint is slack under the Ramsey policy. Replication file typical_boom_bust.m in sgu_endowment_shocks.zip.

Observation:

Over the typical boom bust cycle the optimal capital control tax is not countercyclical. It is lowered during booms and raised during recessions.

What is the role of Ramsey optimal capital control taxes? — To avoid a binding constraint.

• Capital Control Taxes are positive on average Median(τ_t) = 0.0258

this should lower debt. Mean debt in the Ramsey economy is 0.926 (or 28.5% of output) as opposed to 0.9483 (or 29.2% of output) in the unregulated economy.

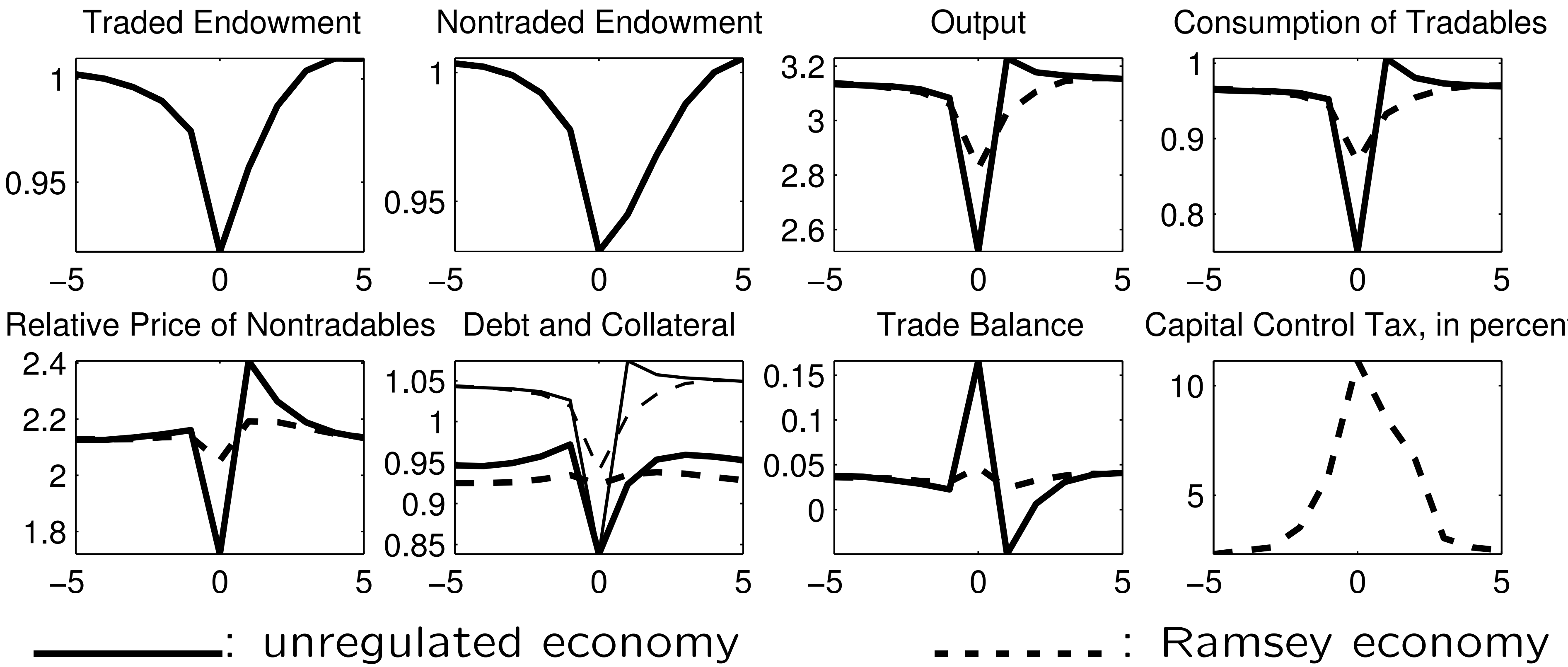
Positive taxes help the economy stay clear of a binding constraint.

• Capital control taxes are quite volatile, std(τ_t) is 4.2%. They are moved around to avoid a binding collateral constraint.

• Frequency of binding constraint in Ramsey 3.9% (or once every 26 years) and in unregulated economy 8.5% (or once every 12 years).

Why is it so important to avoid a binding constraint? Because it leads to a deep (albeit short) contraction:

The Typical Financial Crisis in the Bianchi Economy



Source: Schmitt-Grohé and Uribe (IMF ER 2017). Note. Each line is the mean across all 11-year windows containing a binding collateral constraint in the center in a one-million-year time series from the unregulated economy. For the capital-control tax rate, the figure displays the median instead of the mean across windows because this variable is skewed, with an unconditional mean of 4.2 percent and an unconditional median of 2.5 percent. Because the capital control tax rate is indeterminate when the collateral constraint binds under the Ramsey policy, this variable is given a number only if the collateral constraint is slack under the Ramsey policy. Replication file typical_crisis.m in sgu_endowment_shocks.zip.

Observation:

Ramsey planner **raises** capital control taxes in run up to crisis and **lowers** them once crisis is over. $\Rightarrow$ optimal capital controls are **not** countercyclical. And not macroprudential in that sense.

What about unconditionally?

corr($\tau_t, \ln y_t$) = **-0.84**,

corr($\tau_t, \ln c_t^T$) = **-0.88**

$\Rightarrow$ optimal capital controls are **not** countercyclical in that sense either.

Summary of Findings on the Cyclicality of Optimal Capital Control Taxes

- Ramsey optimal capital control taxes make households fully internalize the collateral constraint induced pecuniary externality.

- Ramsey optimal capital control taxes are found to be **procyclical**, they are raised during recessions and lowered during booms. Therefore, the pecuniary externality does not support adoption of cyclical macroprudential policy.

- What drives the result?— Ramsey planner navigates a tradeoff between allowing agents to frontload consumption as much as possible and avoiding a binding collateral constraint.