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Hysteresis in a simple model of currency substitution

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Abstract

A cash-in-advance model in which the cost of buying goods with a foreign currency is decreasing in the economy's accumulated experience in transacting in the foreign currency is shown to display hysteresis in money velocity; that is, a temporary increase in expected inflation can cause a permanent increase in velocity. In addition, the model implies that the domestic currency does not have to dominate the foreign currency in rate of return to induce agents to stop using the foreign currency. Finally, inflation rates that trigger currency substitution need not be associated with steady states in which the domestic currency disappears from circulation.

Keywords: Currency substitution; Dollarization; Network externalities
JEL classification: E41; F41

1. Introduction

Since the mid-1970s, many studies have documented the instability of standard econometric money demand specifications by noting that they systematically over- and under-predict actual money demand figures. Specifically, temporary changes

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in the nominal interest rate seem to have persistent effects on real money balances, which are not captured by simple money demand regressions. This phenomenon, first documented by Goldfeld (1976) using data for the United States, is present in data from both developed and developing countries [Goldfeld and Sichel (1990) provide evidence from G-7 countries and Arrau et al. (1995) from ten developing countries]. One explanation that has been advanced for these findings is that standard money demand specifications do not account for financial innovation. Among the proxies for financial innovation that have been shown to help solve the instability problem are a time trend (Lieberman, 1977), a previous peak interest rate, or ratchet variable (Goldfeld, 1976; Simpson and Porter, 1980; Cagan, 1984; Piterman, 1988), the number of electronic funds transfers (Dotsey, 1985), and a stochastic trend (Arrau et al., 1995).

In high-inflation countries, the observed difficulties with standard money demand functions have been attributed to *currency substitution*, or *dollarization*, a form of financial adaptation in which a foreign currency is used as a means of payment [see the survey by Calvo and Végh (1996), and the references therein]. Empirical studies have shown that proxies for the degree of dollarization, such as the ratio of dollar-denominated time deposits to M2, improve the fit of money demand functions (Bufman and Leiderman, 1993; Kamin and Ericsson, 1993; Rojas-Suárez, 1992; Savastano, 1992).

Despite the extensive empirical evidence documenting the role of financial innovation and financial adaptation in explaining the instability of money demand regressions, relatively little theoretical work has been devoted to this issue. Ireland (1995) extends the cash-in-advance model of Lucas and Stokey (1983, 1987) by assuming that investment in financial capital allows shoppers to buy goods on credit in markets in which money was once required. In his model, a temporary increase in the nominal interest rate generates a persistent increase in money velocity. Guidotti and Rodríguez (1992) present a model of dollarization in which agents face costs of adjusting their holdings of foreign currency. These costs result in an equilibrium inflation band within which agents choose not to change their degree of dollarization, and outside of which they choose to continuously change their degree of dollarization. In the latter case, they end up either completely dollarized or completely de-dollarized depending, respectively, on whether the inflation rate is above or below the band.[1]

The models of financial innovation and currency substitution described above ignore the possibility that such phenomena are likely to involve network effects. This paper presents a model that puts network effects at center stage by assuming that the economy's accumulated experience in using a foreign currency as a means of payment acts as an externality that reduces the private marginal cost of buying

[1] Sturzenegger (1992) develops a model of financial adaptation to study its effects on income distribution, social welfare, and seignorage revenue. Krugman (1984) presents a model of the dollar as a medium of exchange for international transactions.

goods with the foreign currency.[2] This externality produces multiple steady states and hysteresis in money velocity in an otherwise standard cash-in-advance model. Specifically, at moderate levels of inflation, the model delivers two stable steady states and one unstable steady state.[3] In one of the stable steady states, the domestic currency is the only means of payment in circulation, while in the other stable steady state both the domestic and foreign currencies are used as means of payment. If the economy starts with a high level of experience in transacting with the foreign currency, then the dollarized steady state results; otherwise, the de-dollarized steady state occurs. Therefore, a temporary increase in expected inflation during which individuals accumulate experience in using the foreign currency as a means of payment can drive the economy from the de-dollarized steady state to the dollarized steady state, causing a permanent increase in domestic money velocity.

The network externality model presented in this paper has two important additional implications. First, unlike the Guidotti–Rodríguez model, in which inflation rates that induce dollarization are associated with steady states in which the domestic currency is displaced by the foreign currency, in the network-externality model such inflation rates lead to steady states in which both currencies circulate. This implication is of particular empirical relevance because domestic currencies tend to maintain their status of media of exchange even under long periods of high inflation. Second, the network externality model implies that the domestic currency does not need to dominate the foreign currency in real rate of return in order to induce the public to stop using the foreign currency to buy goods. In particular, complete de-dollarization may occur even at domestic rates of inflation that exceed the inflation rate associated with the foreign currency.

Although the model is presented as one of currency substitution, it can also be interpreted as a one-currency, cash-in-advance model in which agents endogenously choose the set of goods to be bought on credit and in which the private cost of buying goods on credit is decreasing in the economy's accumulated experience in using credit as a means of payment. That is, the paper extends the work of Ireland (1995) by introducing network externalities in the adoption of financial innovations. Such an extension is of particular interest because in the Ireland model, money velocity, although persistent, is a stationary variable. Extending the Ireland model along the lines suggested in this paper introduces non-stationarity in money velocity, a characteristic typically found in empirical studies.

[2] Matsuyama et al. (1993) and Trejos and Wright (1995b) study two-country, two-currency extensions of the Kiyotaki and Wright (1993) and Trejos and Wright (1995a) search-theoretic models of money in which the existence of equilibria with two, one, or no international currency depends on external factors such as the frequency at which buyers and sellers meet to trade. These external factors, however, are exogenously given as part of each country's search technology. In particular, they are independent of the past inflationary experience in each country.

[3] The terms low, moderate and high inflation are defined later.

The remainder of the paper is organized as follows. Section 2 presents the model and Section 3 characterizes its steady states. Section 4 analyzes the implications of the model for the dynamics of currency substitution, money velocity, and seignorage revenue. Section 5 concludes.

2. The model

Consider a perfect-foresight, small, open economy populated by a large number of identical, infinitely lived consumers with preferences defined over sequences of consumption, $\{C_t\}_{t=0}^{\infty}$, and described by the utility function

$$\sum_{t=0}^{\infty} \beta^t U(C_t); \quad \beta \in (0,1), \tag{1}$$

where the period utility function $U(\cdot)$ is assumed to be strictly increasing, weakly concave, and continuously differentiable. Consumption is assumed to be a composite of a continuum of goods, $c_t(\theta)$, indexed by $\theta \in [0,1]$,

$$C_t = \int_0^1 u(c_t(\theta))\,\mathrm{d}\theta, \tag{2}$$

where $u(\cdot)$ is assumed to be continuously differentiable, strictly increasing, and strictly concave and to satisfy $\lim_{c\to 0} u'(c) = \infty$.

Goods can be purchased with either domestic currency (pesos) or foreign currency (dollars). Purchases of goods with dollars are subject to a transaction cost that varies across goods. Specifically, in each period $t \geq 0$, the transaction cost of buying one unit of good θ with dollars is $\phi(\theta, k_t)$ units of good θ. The variable k_t denotes the economy's accumulated experience up to period t in using the foreign currency as a means of payment and is referred to as the stock of *dollarization capital*. Households take the path of k_t as given, but (as is described later) at an aggregate level k_t is endogenously determined. The transaction cost is assumed to be strictly decreasing in k_t; this assumption is meant to capture the idea of network effects in the process of adopting an alternative means of payment. The function $\phi(\cdot,\cdot)$ satisfies the following assumption.

Assumption 1. $\phi : [0,1] \times \Re^+ \to \Re^+$ *is non-negative, twice continuously differentiable, and strictly convex and satisfies* $\phi_\theta > 0$, $\phi_k < 0$, *and* $\lim_{\theta\to 1} \phi(\theta, k) = \infty$ $\forall k \geq 0$.

The assumption that as θ approaches 1 the transaction cost becomes infinitely large is made for analytical convenience and guarantees that the demand for domestic currency is always positive.

At the beginning of each period $t \geq 0$, the government makes a transfer of T_t pesos to each household and sets the exchange rate E_t, defined as the price of one dollar in terms of pesos. The government guarantees free convertibility of the domestic currency at the rate E_t. At this point, a financial market opens which households enter with the T_t pesos from the government and with a stock of wealth W_t carried over from period $t-1$ and measured in pesos. In this market, households can buy or sell, at the price of one dollar per unit, an internationally traded, dollar-denominated bond, b_t, that pays the constant interest rate $r > 0$ in dollars in period $t+1$. They can also acquire their desired balances of foreign currency, d_t, and domestic currency, M_t. No goods can be traded in this market. Letting $w_t \equiv W_t/E_t$, $m_t \equiv M_t/E_t$, and $\tau_t \equiv T_t/E_t$, the household's budget constraint in the financial market, expressed in terms of the foreign currency, is given by

$$b_t + d_t + m_t \leq w_t + \tau_t. \tag{3}$$

After the financial market is closed, a second market opens in which goods are traded in exchange for currency and in which the central bank does not intervene. The household sends one of its members, a shopper, to this market to purchase the desired amounts of each good $\theta \in [0,1]$. At the same time, another member of the household, a firm, receives an endowment of y_t units of each good $\theta \in [0,1]$ and sells it in the goods market. All goods $\theta \in [0,1]$ are assumed to be internationally traded at the common price of one dollar per unit. Firms can sell their endowments either for pesos or for dollars. If they choose to sell goods for pesos, they cannot convert the proceeds into dollar-denominated assets before the next period's financial market opens. Consequently, in order for firms to be indifferent between receiving dollars and receiving pesos in exchange for goods, the peso price of one unit of good in period t must be E_{t+1}, the exchange rate prevailing in the financial market in period $t+1$.

Let Θ_t^m denote the set of goods the household chooses to buy with pesos in period t and Θ_t^d the set of goods the household chooses to buy with dollars in period t. The household's wealth at the beginning of period $t+1$ is then given by

$$\begin{aligned} w_{t+1} = (1+r)b_t + \frac{m_t}{1+\pi_{t+1}} + d_t - \int_{\Theta_t^d} [1+\phi(\theta,k_t)]c_t(\theta)\,\mathrm{d}\theta \\ - \int_{\Theta_t^m} c_t(\theta)\,\mathrm{d}\theta + y_t, \end{aligned} \tag{4}$$

where $\pi_{t+1} \equiv E_{t+1}/E_t - 1$ denotes the devaluation rate in period $t+1$. To ensure that the nominal interest rate is always positive, π_{t+1} is assumed to satisfy $(1+\pi_{t+1})(1+r) > 1$ for all $t \geq 0$. Combining (3) and (4) yields the following expression for the evolution of real wealth

$$w_{t+1} \leq (1+r)(w_t+\tau_t) - \left[\frac{(1+\pi_{t+1})(1+r)-1}{1+\pi_{t+1}}\right] m_t - rd_t$$
$$- \int_{\Theta_t^d} [1+\phi(\theta,k_t)]c_t(\theta)\,\mathrm{d}\theta - \int_{\Theta_t^m} c_t(\theta)\,\mathrm{d}\theta + y_t. \tag{5}$$

The cash-in-advance constraints described above are of the form,

$$\frac{m_t}{1+\pi_{t+1}} \geq \int_{\Theta_t^m} c_t(\theta)\,\mathrm{d}\theta, \tag{6}$$

$$d_t \geq \int_{\Theta_t^d} [1+\phi(\theta,k_t)]c_t(\theta)\,\mathrm{d}\theta. \tag{7}$$

In addition, households are assumed to be subject to the following borrowing constraint that prevents them from engaging in Ponzi games

$$\lim_{t\to\infty} \frac{w_t}{(1+r)^t} \geq 0.$$

Households choose sequences $\{\{c_t(\theta)\}_{\theta\in[0,1]}, \Theta_t^m, \Theta_t^d, m_t, d_t, w_{t+1}\}_{t=0}^{\infty}$ so as to maximize (1) subject to (2), (5)–(7), and the no-Ponzi-game constraint, taking as given w_0 and the sequences $\{\pi_{t+1}, \tau_t, k_t, y_t\}_{t=0}^{\infty}$. The first-order conditions associated with the household's optimization problem are (5)–(7) holding with equality and

$$\Theta_t^d = [0, \bar{\theta}(k_t, \pi_{t+1})), \tag{8}$$

$$\Theta_t^m = [\bar{\theta}(k_t, \pi_{t+1}), 1], \tag{9}$$

$$\bar{\theta}(k,\pi) = \begin{cases} 0 & \text{if } \phi(0,k) \geq \pi \\ \theta \text{ such that } \phi(\theta,k)=\pi & \text{otherwise} \end{cases} \tag{10}$$

$$c_t(\theta) = c_t(\bar{\theta}(k_t,\pi_{t+1})) \qquad \text{for } \theta \in \Theta_t^m, \tag{11}$$

$$\frac{u'(c_t(\theta))}{1+\phi(\theta,k_t)} = \frac{u'(c_t(\bar{\theta}(k_t,\pi_{t+1})))}{1+\pi_{t+1}} \qquad \text{for } \theta \in \Theta_t^d, \tag{12}$$

$$\frac{U'(C_t)u'(c_t(\bar{\theta}(k_t,\pi_{t+1})))}{1+\pi_{t+1}} = \beta(1+r)\frac{U'(C_{t+1})u'(c_{t+1}(\bar{\theta}(k_{t+1},\pi_{t+2})))}{1+\pi_{t+2}}, \tag{13}$$

$$\lim_{t\to\infty} \frac{w_t}{(1+r)^t} = 0.$$

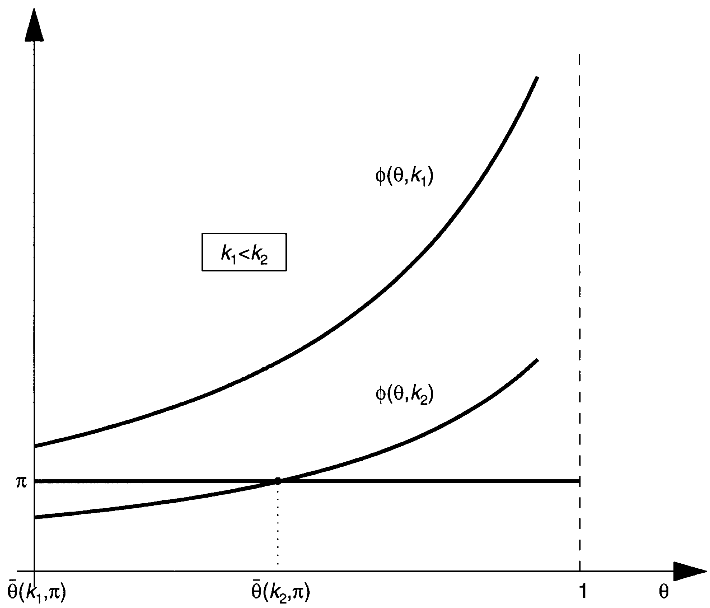

Fig. 1. The degree of dollarization.

Eqs. (8) and (9) say that in each period $t \geq 0$ there exists a cut-off good $\bar{\theta}(k_t, \pi_{t+1}) \in [0, 1]$ such that goods with index $\theta < \bar{\theta}(k_t, \pi_{t+1})$ are purchased with dollars and goods with index $\theta \geq \bar{\theta}(k_t, \pi_{t+1})$ are purchased with pesos (Fig. 1). Accordingly, $\bar{\theta}(k_t, \pi_{t+1})$ is referred to as the *degree of dollarization* in period t. Eq. (10) says that when the degree of dollarization is positive, it is given by the good whose cost is the same whether it is bought with dollars, $\phi(\theta, k_t)$, or with pesos, π_{t+1}. Assumption 1 implies that $\bar{\theta}(\cdot, \cdot)$ is continuous (see the appendix) and that for any pair (k, π) such that $\bar{\theta}(k, \pi) > 0$, $\bar{\theta}(\cdot, \cdot)$ is strictly increasing in both arguments, continuously differentiable, and strictly concave in k (Fig. 2). The assumption that $u'(0) = \infty$ implies that every type of good is always consumed in positive quantities, that is, $c_t(\theta) > 0$, $\forall \theta$, t; Eqs. (11) and (12) say that $c_t(\theta)$ is continuous in θ, strictly decreasing in θ for $\theta \in \Theta_t^d$, and constant for $\theta \in \Theta_t^m$; the assumption $\lim_{\theta \to 1} \phi(\theta, k) = \infty$ implies that households always choose to buy some goods with pesos (that is, $\bar{\theta}(k, \pi) < 1$); as a result, by the cash-in-advance constraint (6), the demand for domestic currency is always positive ($m_t > 0$, $\forall t$).

I postulate the following law of motion for the stock of dollarization capital k_t:

$$k_{t+1} = (1 - \delta)k_t + F(\bar{\theta}(k_t, \pi_{t+1})) \equiv G(k_t, \pi_{t+1}), \tag{14}$$

where $F(\cdot)$ satisfies

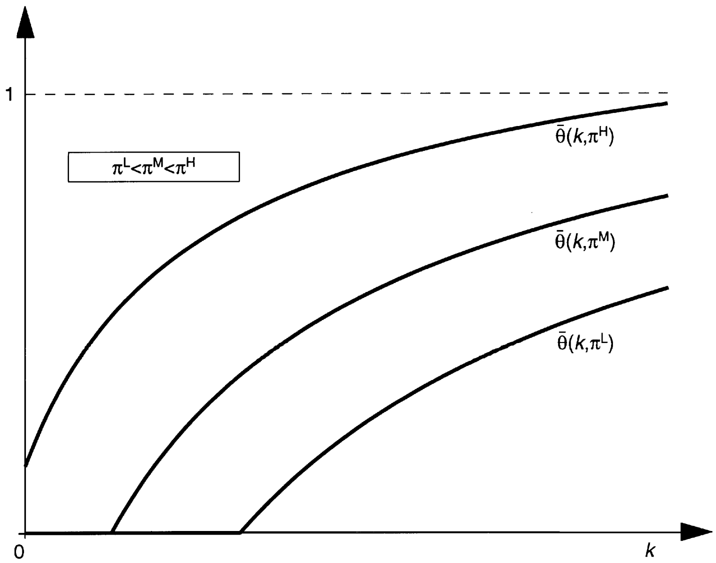

Fig. 2. The function $\bar{\theta}(k,\pi)$.

Assumption 2. $F:[0,1]\rightarrow\Re^{+}$ *is continuously differentiable, strictly increasing, and strictly concave, with* $F(0)=0$.

The assumption that F is strictly increasing states that the stock of dollarization capital in period $t+1$ is strictly increasing in the degree of dollarization of the economy in period t. That is, there is social learning by doing in the process of adopting a foreign currency as a means of exchange. The assumption that F is strictly concave is made to rule out multiple steady states due to increasing returns to scale in the learning-by-doing technology. The assumption that the stock of dollarization capital depreciates at a constant rate $\delta\in(0,1]$, together with the assumption that $F(0)=0$, means that when the foreign currency is not used as a means of payment (i.e., when $\bar{\theta}(k_t,\pi_{t+1})=0$), the stock of dollarization capital gradually decreases, that is, the economy as a whole forgets how to use the foreign currency to purchase goods.

Assumptions 1 and 2 imply that the function $G(\cdot,\cdot)$ is continuous, strictly increasing in k, non-decreasing in π, and bounded above by $(1-\delta)k+F(1)$, and that for any pair (k,π) such that $\bar{\theta}(k,\pi)>0$, $G_{\pi}(k,\pi)>0$, $G_k(k,\pi)>1-\delta$, and $G_{kk}(k,\pi)<0$. Because the initial condition k_0 and the time path of the devaluation rate $\{\pi_{t+1}\}_{t=0}^{\infty}$ are exogenously given, Eq. (14) is a scalar system that can be solved for $\{k_{t+1}\}_{t=0}^{\infty}$ independently of the rest of the equilibrium conditions.

3. Characterization of steady states

The phase diagram corresponding to the difference equation (14) looks qualitatively identical to the graph of $\bar{\theta}(k_t, \pi)$ mounted on the ray $(1-\delta)k_t$. Fig. 3 displays the phase diagram of (14) for three different inflation rates: π^{L}, π^{M}, and π^{H}, where $\pi^{\mathrm{L}} < \pi^{\mathrm{M}} < \pi^{\mathrm{H}}$. At low inflation rates such as π^{L}, the degree of dollarization is too low to generate enough learning, $F(\bar{\theta}(k,\pi))$, to offset the loss of dollarization capital due to depreciation; consequently, regardless of the initial value of k, the economy converges to a steady state in which $k=0$, and domestic currency is the only means of exchange. At high inflation rates such as π^{H}, agents have incentives to buy some goods with dollars even when the initial stock of dollarization capital is zero; therefore, the economy converges to a steady state in which $k>0$, and both the domestic and the foreign currencies are used as means of payment. At intermediate levels of inflation such as π^{M}, the economy converges to the de-dollarized steady state for low initial levels of dollarization capital and to a steady state in which some markets are dollarized for high initial levels of dollarization capital. In the remainder of this section, I formally characterize the steady states associated with Eq. (14).

I begin by characterizing the set of inflation rates for which there exists a unique, dollarized steady state. Let $\bar{\pi} \equiv \phi(0,0)$. By assumption 1, $\bar{\pi}$ is positive. For inflation rates $\pi \leq \bar{\pi}$, $k=0$ is a steady state of (14) because if $k_t = 0$, then from (10) we have $\bar{\theta}(k_t, \pi) = 0$, which implies that $k_{t+1} = F(0) = 0$. If, on

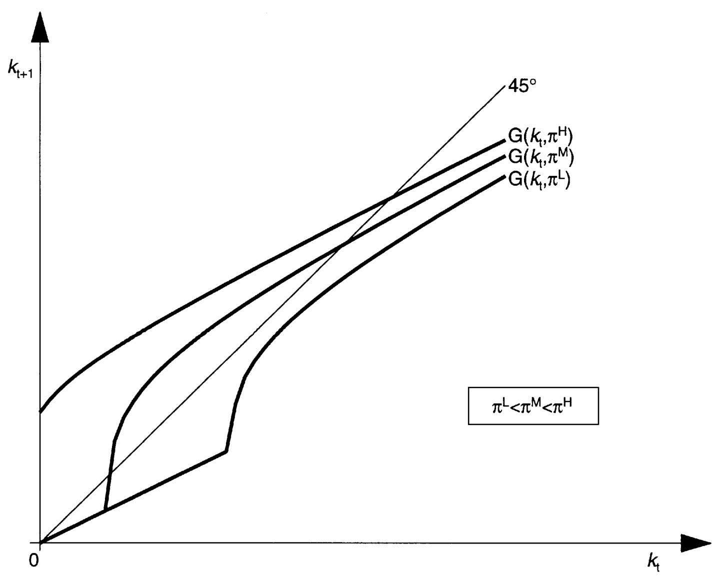

Fig. 3. Phase diagram.

the other hand, $\pi > \bar{\pi}$, then $k = 0$ is no longer a steady state of (14) because in this case $\bar{\theta}(0,\pi) > 0$ and therefore, if $k_t = 0$, then $k_{t+1} = F(\bar{\theta}(0,\pi)) > 0$. Further, because for any pair (k,π) such that $\bar{\theta}(k,\pi) > 0$, $G(k,\pi)$ is strictly increasing and strictly concave in k and is bounded above by $(1-\delta)k + F(1)$, it follows that if $\pi > \bar{\pi}$, Eq. (14) has a unique, positive steady state. Accordingly, I define the set of *high* inflation rates, Π^{H}, as the collection of inflation rates such that regardless of the initial value of k, the economy converges to a steady state in which the degree of dollarization is positive, that is,

$$\Pi^{\mathrm{H}} = \{\pi : \pi > \bar{\pi} \equiv \phi(0,0)\}.$$

Similarly, I define the set of *low* inflation rates, Π^{L}, as the collection of inflation rates such that regardless of the initial value of k, the economy converges to a completely de-dollarized steady state, that is,

$$\Pi^{\mathrm{L}} \equiv \{\pi : (1+r)(1+\pi) > 1 \quad \text{and} \quad G(k,\pi) < k \ \ \forall k > 0\}.$$

This set is not empty. Consider, for instance, the inflation rate $\pi = 0$. By assumption, $\phi(\theta,k) \geq 0$, so from (10) it follows that if $\pi = 0$, then $\bar{\theta}(k,\pi) = 0 \ \forall k$. Hence $G(k,\pi) = (1-\delta)k < k \ \forall k > 0$, so $0 \in \Pi^{\mathrm{L}}$. Moreover, because $G(k,\pi)$ is non-decreasing in π, it follows that if π belongs to Π^{L}, then all inflation rates in the interval $((1+r)^{-1} - 1, \pi]$ belong to Π^{L}.

Let $\underline{\pi}$ be the least upper bound of Π^{L}. Since Π^{L} is bounded above by $\bar{\pi}$, $\underline{\pi}$ is well defined. Define the set of moderate inflation rates, Π^{M}, as the collection of inflation rates that are less than or equal to any high inflation rate and greater than or equal to any low inflation rate. That is,

$$\Pi^{\mathrm{M}} = \{\pi : \underline{\pi} \leq \pi \leq \bar{\pi}\}.$$

For any inflation rate in the interior of Π^{M}, Eq. (14) has three steady states: two stable steady states and one unstable steady state. To see this, let $\pi \in (\underline{\pi}, \bar{\pi})$. First, recall that since $\pi < \bar{\pi}$, $k = 0$ is a steady state of (14). Moreover, $k = 0$ is a locally stable steady state. This can be seen by noting that since $\phi(0,0) = \bar{\pi} > \pi$, by continuity there exists a value $k' > 0$ such that $\phi(0,k') > \pi$. Consequently, for any $k \leq k'$, (10) implies that $\bar{\theta}(k,\pi) = 0$ and so $G(k,\pi) = (1-\delta)k < k$, which implies that $k = 0$ is a locally stable steady state. Next, recall that $G(\cdot,\cdot)$ is continuous and that for any k such that $G(k,\pi) > (1-\delta)k$, the function $G(k,\pi)$ satisfies $G_k(k,\pi) > 1-\delta$ and $G_{kk}(k,\pi) < 0$. Therefore, Eq. (14) can have at most one positive stable steady state and one positive unstable steady state. Now note that because $\underline{\pi}$ is the least upper bound of Π^{L}, for any $\pi > \underline{\pi}$ there exists a $k > 0$ such that $G(k,\pi) > k$. Thus, since $G(k,\pi)$ is continuous and bounded above by $(1-\delta)k + F(1)$, it follows that for any $\pi \in (\underline{\pi}, \bar{\pi})$, Eq. (14) has a positive, stable steady state. Also, the facts that $G(\cdot,\cdot)$ is continuous and that for any $\pi \in (\underline{\pi}, \bar{\pi})$

Eq. (14) has two stable steady states imply that for any $\pi \in (\underline{\pi}, \bar{\pi})$, Eq. (14) has a positive, unstable steady state located between the two stable ones.

I now show that the necessary and sufficient condition for the interior of Π^{M} to be non-empty is

$$G_k(0, \bar{\pi}) > 1. \tag{15}$$

To see that condition (15) is necessary, assume that $G_k(0, \bar{\pi}) \leq 1$. Since $\bar{\theta}(k, \bar{\pi}) = 0$ for $k = 0$ and is strictly positive for $k > 0$, it follows that $G(0, \bar{\pi}) = 0$ and $G_{kk}(k, \bar{\pi}) < 0$ for $k > 0$. Therefore, $G(k, \bar{\pi}) < k$ $\forall k > 0$ and $\bar{\pi} \in \Pi^{\mathrm{L}}$. Consequently, $\underline{\pi} = \bar{\pi}$, that is, the interior of Π^{M} is empty. To see that condition (15) is sufficient for Π^{M} to be non-empty, assume that (15) is satisfied, and consider the inflation rate $\pi = \bar{\pi}$. Since $G(0, \bar{\pi}) = 0$ and $G_k(0, \bar{\pi}) > 1$, there exists a positive value of k for which $G(k, \bar{\pi}) > k$. This implies that $\bar{\pi} > \underline{\pi}$, that is, that the interior of Π^{M} is non-empty.

It is useful to express the necessary and sufficient condition for the interior of Π^{M} to be non-empty in terms of the structural functions $\phi(\cdot\,,\cdot)$ and $F(\cdot)$. Using (10) and (14), condition (15) can be written as

$$1 - \delta - F'(0)\frac{\phi_k(0,0)}{\phi_\theta(0,0)} > 1. \tag{16}$$

This condition says that multiple steady states are more likely to exist when network externalities are important ($|\phi_k(0,0)|$ large), when social learning by doing is important ($F'(0)$ large), when the transaction cost of buying goods with foreign currency is relatively flat across low-θ goods ($\phi_\theta(0,0)$ small), and when the stock of experience in using the foreign currency as a means of payment depreciates slowly (δ small).[4]

Fig. 4 depicts the steady-state values of k corresponding to each level of inflation. Stable steady states are shown with solid lines, and unstable steady states are shown with broken lines. If condition (15) is satisfied, then Eq. (14) has two bifurcation points: $\bar{\pi}$ and $\underline{\pi}$. For inflation rates $\pi > \bar{\pi}$, (14) has a unique, stable, positive steady state. For inflation rates $\pi \in ((1+r)^{-1} - 1, \underline{\pi})$, Eq. (14) has a unique stable steady state at $k = 0$, and for inflation rates $\pi \in (\underline{\pi}, \bar{\pi})$, Eq. (14)

[4] The characterization of steady states is based on the assumption that the depreciation rate is strictly positive. When $\delta = 0$, the dynamic structure of the model is substantially different. Specifically, it can be shown that if $\delta = 0$, then for $(1+r)^{-1} - 1 < \pi \leq \bar{\pi}$, Eq. (14) has a continuum of steady states $[0, k^*(\pi)]$, where $k^*(\pi) > 0$ is given by the value of k that solves $\phi(0, k) = \pi$, if such value exists, or else by $+\infty$. In any such steady state, the domestic currency is the only medium of exchange in circulation. Also, all steady states are unstable; in particular, if $k_0 > k^*(\pi)$, then $k_t \to +\infty$ and the domestic currency is eventually driven out of circulation by the foreign currency. For inflation rates $\pi > \bar{\pi}$, k_t converges to infinity independently of k_0, and the domestic currency is gradually displaced by the foreign currency.

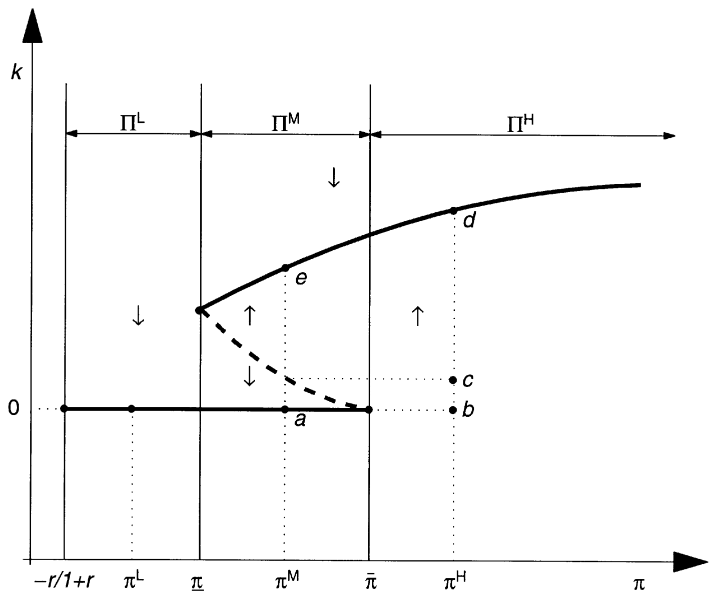

Fig. 4. Steady states.

has three steady states: one stable steady state at $k=0$, one positive stable steady state, and one unstable steady state located between the two stable ones.[5]

4. Hysteresis

The main prediction of the network externality model developed above is that the presence of network effects in the adoption of alternative means of exchange causes hysteresis in money demand; specifically, temporary changes in expected inflation may permanently affect the degree of dollarization and money velocity. Fig. 4 illustrates this effect. Suppose that the economy starts at a steady state with no dollarization capital ($k=0$) and a moderate level of inflation $\pi^{\mathrm{M}} \in (\underline{\pi}, \bar{\pi})$, as in point a in Fig. 4. Suppose now that the government decides to raise the inflation rate to a high level $\pi^{\mathrm{H}} > \bar{\pi}$. In response to this policy, the economy will gradually move from point b to point d, a steady state in which $k>0$. Further, suppose that the government decides to stabilize by setting the inflation rate back to π^{M}

[5] It is straight-forward to show that if (15) is satisfied, then for $\pi = \bar{\pi}$ the difference Eq. (14) has two steady states: one unstable at $k=0$ and one stable at some $k>0$. For $\pi = \underline{\pi}$, (14) also has two steady states: one stable at $k=0$ and one unstable at some $k>0$. Bifurcation points such as $\underline{\pi}$ and $\bar{\pi}$ are referred to as points of catastrophe because small deviations in π away from them change the qualitative nature of the system's orbit structure (see, for example, Azariadis, 1993).

at a moment in which k is beyond point c. As a consequence, the economy will not return to the initial steady state a, but will be attracted to point e, a steady state in which $k > 0$.

On the other hand, if the economy starts at a steady state such as e in which both pesos and dollars are circulating as means of exchange, a policy aimed at achieving complete de-dollarization on a permanent basis must consist of a stabilization program that sets the inflation rate at a low level such as $\pi^{\mathrm{L}} < \underline{\pi}$ for a period of time long enough to allow the stock of dollarization capital to fall, via depreciation, below the level corresponding to point c. At that moment, an increase in the inflation rate back to π^{M} will not interrupt the ongoing process of de-dollarization, which will continue until the economy reaches the steady state a in which the domestic currency is the only medium of exchange in circulation. Note that the reduction in expected inflation required to set in motion the process of de-dollarization does not need to be so ‘aggressive’ as to make the real rate of return on the domestic currency $(-\pi)$ higher than the real rate of return on the foreign currency (zero). In other words, the government does not need to set the domestic inflation rate below the inflation rate associated with the foreign currency in order to induce people to de-dollarize. This is because the relevant real rate of return on the foreign currency, given by the difference between its pecuniary rate of return (zero) and the transactions cost $(\phi(\theta,k) > 0)$, is always negative.

Now I turn to the implications of the model for the dynamics of money velocity. In each period $t \geq 0$, the expenditure-based velocity of circulation of the domestic currency, v_t, is given by the ratio of aggregate domestic spending in period t (including transactions costs) to domestic real balances in period t[6]

$$v_t = \frac{\int_{\Theta_t^m} c_t(\theta)\,\mathrm{d}\theta + \int_{\Theta_t^d} [1 + \phi(\theta, k_t)] c_t(\theta)\,\mathrm{d}\theta}{m_t/(1+\pi_{t+1})}.$$

A simple expression for v_t results from assuming a unitary intra-temporal elasticity of substitution in consumption, that is, $u(c) = \ln(c)$ in Eq. (2). This assumption implies a unitary elasticity of the demand for money with respect to aggregate domestic spending. Specifically, using conditions (8), (9), (11) and (12) and the cash-in-advance constraint (6), which must hold with equality given the maintained assumption that $(1+r)(1+\pi_{t+1}) > 1$ for all $t \geq 0$, v_t takes the form

$$v_t = \frac{1 + \pi_{t+1}\bar{\theta}(k_t, \pi_{t+1})}{1 - \bar{\theta}(k_t, \pi_{t+1})}. \tag{17}$$

[6] Recall that $m_t \equiv M_t/E_t$, where E_t is the exchange rate prevailing in the financial market of period t, and that the price prevailing in the goods market of period t is E_{t+1}; so $M_t/E_{t+1} = m_t/(1+\pi_{t+1})$.

This expression implies that when the only medium of exchange is the domestic currency ($\bar{\theta}(k_t, \pi_{t+1}) = 0$), money velocity is equal to one and the model collapses to the standard cash-in-advance model without credit goods (Lucas, 1980). In this case, small disturbances in expected inflation have no effect on money velocity. In contrast, as shown above, temporary but large changes in expected inflation can drive the economy away from the single-currency steady state to one in which both the domestic and foreign currencies circulate (i.e., $\bar{\theta}(k, \pi) > 0$), causing a permanent increase in money velocity. Unlike the single-currency steady state, in which money velocity is constant, in dollarized steady states money velocity is an increasing function of both expected inflation and the stock of dollarization capital; as a result, small, temporary innovations in expected inflation have not only direct, temporary effects on money velocity but also indirect, persistent effects through the stock of dollarization capital.

Consider now the results of estimating a money demand function like (17) by simply regressing money velocity onto expected inflation (or the nominal interest rate). Clearly, omitting k_t as a regressor would lead to problems of 'missing money' like the one first documented by Goldfeld (1976). The model's predictions are therefore consistent with the empirical studies cited in the introduction that suggest that proxies for financial innovation and financial adaptation help solve the instability problem of standard money demand regressions.

Finally, consider the implications of the model for seignorage revenue. Letting η denote the ratio of seignorage revenue to aggregate domestic spending, the steady-state Laffer curve for the inflation tax takes the following familiar form

$$\eta = \frac{1}{v}\frac{\pi}{1+\pi},$$

which says that the inflation-tax base per unit of domestic spending is the inverse of money velocity and that the inflation-tax rate is $\pi/(1+\pi)$ (Fig. 5). At low and high levels of inflation, v is uniquely determined by π, and therefore so is η. On the other hand, at moderate levels of inflation there are two stable steady-state values of v for each π, and therefore there are also two stable steady-state values of η. Because money velocity is larger and more sensitive to expected inflation in the dollarized steady state than in the non-dollarized steady state, the branch of the Laffer curve corresponding to the dollarized steady state lies below and is flatter than the branch corresponding to the non-dollarized steady state. This result is entirely consistent with the empirical facts associated with the effects of currency substitution on inflationary finance (see Bufman and Leiderman, 1993; Imrohoroglu, 1994; Nichols, 1974).

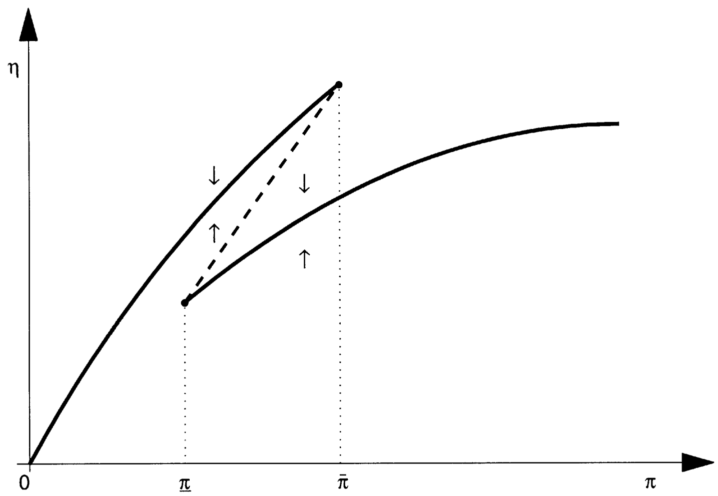

Fig. 5. Inflation-tax Laffer curve.

5. Conclusion

This paper develops a cash-in-advance model of currency substitution in which temporary changes in expected inflation may produce permanent effects on money velocity. The main departure from other models that try to capture this effect is the assumption that the process of financial adaptation involves network effects. Specifically, the network-externality model developed in this paper assumes that the private cost of buying goods with the foreign currency is decreasing in the economy's accumulated experience with the use of the foreign currency as a means of exchange.

The proposed framework explains several empirical facts associated with money demand and inflationary finance. First, money velocity displays hysteresis in the sense that a temporary, large increase in expected inflation can cause a permanent decline in real balances. Second, small disturbances in expected inflation have more persistent effects on money velocity in dollarized economies than in economies in which the only means of exchange is the domestic currency. Third, financial adaptation causes the inflation-tax Laffer curve to shift down and to flatten. Thus, the paper provides a theoretical explanation for the problem of 'missing money' detected in developed and developing countries since the mid-1970s and for the statistical significance of proxies for financial innovation and financial adaptation – such as previous peak interest rates – in money demand regressions.

The model has precise implications for stabilization policies aimed at increasing monetization and seignorage income in financially adapted economies; such policies must include an initial phase of low inflation during which the public, by not using the foreign currency to buy goods, loses their ability to do so. However, the stabilization policy does not need to be as tough as is typically believed, in the sense that complete de-dollarization can be induced even when domestic inflation exceeds foreign inflation.

The analysis presented in this paper could be extended in several directions. First, the network-externality model implies a simple money demand function whereby money velocity depends on the nominal interest rate and a latent variable that reflects the level of experience accumulated by the public in using a foreign currency as a means of payment. The latent variable, in turn, depends on the entire past history of money velocity and interest rates. These relations could be estimated to assess the importance of network effects in the process of financial adaptation. Second, the multiplicity of steady-state values of seignorage revenue suggests that augmenting the model with uncertainty and optimizing behavior on the side of the government could lead to a theory of endogenous inflation cycles.

Appendix

Claim: $\bar{\theta}(\cdot,\cdot)$ is continuous.

Proof: Check for continuity in the following three cases: (i) Points (k,π) such that $\phi(0,k)>\pi$. In this case $\bar{\theta}(k,\pi)=0$. Take any sequence (k_n,π_n) converging to (k,π). Since ϕ is continuous, $\exists N$ such that $\phi(0,k_n)>\pi_n$ $\forall n>N$, so $\bar{\theta}(k_n,\pi_n)=0$ $\forall n>N$ and $\bar{\theta}(k_n,\pi_n)\to 0=\bar{\theta}(k,\pi)$. (ii) Points (k,π) such that $\phi(0,k)<\pi$. Since ϕ is continuous, there exists an open ball B containing (k,π) such that $\phi(0,k')<\pi'$ for all $(k',\pi')\in B$. Since $\phi(\theta,k)-\pi=0$ satisfies all the conditions of the implicit function theorem, $\bar{\theta}$ is continuous (indeed continuously differentiable) over B. (iii) Finally, consider points (k,π) such that

$$\phi(0,k)=\pi. \tag{A.1}$$

Then $\bar{\theta}(k,\pi)=0$. Let $(k_n,\pi_n)\to(k,\pi)$. Consider the subsequence (k'_n,π'_n) containing all points of (k_n,π_n) for which $\phi(0,k'_n)<\pi'_n$. To this subsequence corresponds a subsequence $\theta'_n\equiv\bar{\theta}(k'_n,\pi'_n)$, that is implicitly given by $\phi(\theta'_n,k'_n)=\pi'_n$. To show that θ'_n converges to zero, suppose for the moment that it does not. Then $\exists\varepsilon>0$ such that for any n there exists an $N>n$ for which $\theta'_N\geq\varepsilon$. Consider now the subsequence (k''_n,π''_n) of (k'_n,π'_n) such that

$$\theta''_n\equiv\bar{\theta}(k''_n,\pi''_n)>\varepsilon, \tag{A.2}$$

where $\bar{\theta}(k''_n,\pi''_n)$ is implicitly given by

$$\phi(\theta''_n,k''_n)=\pi''_n. \tag{A.3}$$

Subtract (A.1) from (A.3) to get

$$\pi_n'' - \pi = \phi(\theta_n'', k_n'') - \phi(0,k) > \phi(\varepsilon, k_n'') - \phi(0,k), \tag{A.4}$$

where the last inequality follows from (A.2) and the fact that $\phi_\theta > 0$. Taking limits in (A.4) yields

$$0 = \lim_{n\to\infty}[\phi(\theta_n'', k_n'') - \phi(0,k)] \geq \phi(\varepsilon, k) - \phi(0,k) > 0,$$

which is a contradiction. Then $\bar{\theta}(k_n'', \pi_n'') \to 0 = \bar{\theta}(k, \pi)$. All other elements of (k_n, π_n) are associated with $\bar{\theta}(k_n, \pi_n) = 0$. $\square$

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