# How Important Is The Commodity Supercycle?


Andrés Fernández
*International Monetary Fund*

Stephanie Schmitt-Grohé
*Columbia University*
*Centre for Economic and Policy Research*
*National Bureau of Economic Research*

Martín Uribe
*Columbia University*
*National Bureau of Economic Research*


World commodity prices are known to display long cycles. These cycles have a periodicity of 20 to 30 years and are called commodity-price supercycles. Figure 1 displays the time paths of eleven commodity prices deflated by the U.S. consumer price index over the period 1960 to 2018. All commodity prices appear to have long cycles in accordance with the supercycle hypothesis. In particular, commodity prices display two peaks post 1960—one in the early 1980s and one in the early 2010s. In the academic and financial-industry literature, the upswing


We are gratefull for the comments of Roberto Chang and seminar participants at Johns Hopkins, Wisconsin, U. Tubingen, Tsinghua, the Macro online seminar series (Hong Kong Baptist, National Taiwan, Yonsei, and Academia Sinica), U. Hamburg, Bank of Canada, the 2020 CEBRA Workshop for Commodities and Macroeconomics (Central Bank of Chile), the 2021 Central Bank of Chile Annual Conference, the 2021 AEA Meeting, LACEA 2021, CEPII-Paris, and the 2022 JPMCC Intl. Commodities Symposium. Daniel Guzman and Ken Teoh provided outstanding research assistance. An earlier version of this paper circulated under the title "Does the Commodity Super Cycle Matter?" Research for this work was conducted while Andrés Fernández was affiliated with the Central Bank of Chile. The information and opinions presented are entirely those of the authors, and no endorsement by the Central Bank of Chile, the IMF or their Board Members or IMF Management is expressed or implied. The replication codes and the time series of the commodity supercycle estimated in our work can be downloaded from http://www.columbia.edu/~mu2166/fsu2/.

*Credibility of Emerging Markets, Foreign Investors' Risk Perceptions, and Capital Flows* edited by Álvaro Aguirre, Andrés Fernández, and Şebnem Kalemli-Özcan, Santiago, Chile.

© 2023 Central Bank of Chile.

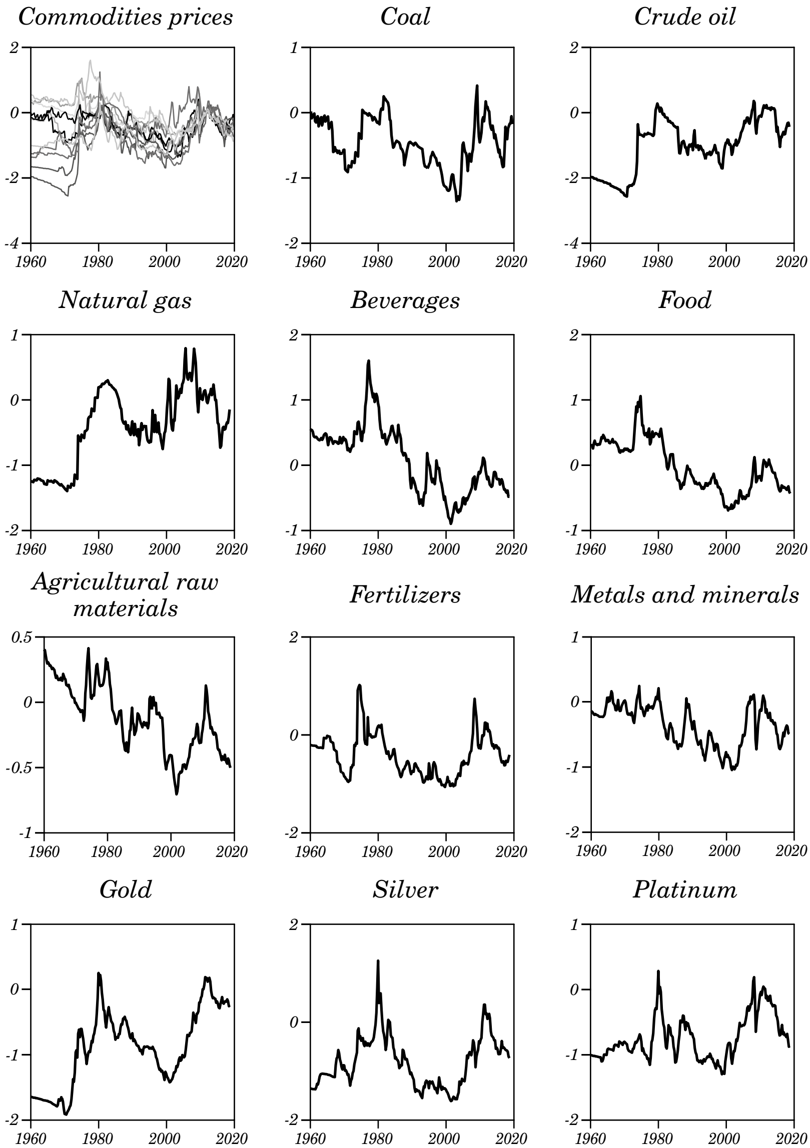
in commodity prices leading to the 1980s peak is typically attributed to the post-World War II reconstruction of Western Europe and Japan and to the cartelization of the crude oil market. The peak in the early 2010s is frequently attributed to the accession of China and other southeast Asian countries to world markets.

The existing literature on commodity-price supercycles has mainly focused on documenting their frequency, amplitude, and turning points. Less work has been devoted to estimating the importance of commodity-price supercycles for economic activity. The contribution of this paper is to identify global disturbances that cause regular cycles and supercycles in world commodity prices and to estimate the contribution of these global shocks to aggregate fluctuations in emerging and developed countries.

The econometric-oriented related literature typically uses spectral analysis to identify commodity-price supercycles. Cuddington and Jerrett (2008) pioneered the use of the asymmetric band-pass filter of Christiano and Fitzgerald (2003) to identify supercycles in commodity prices. They apply this technique to the prices of six metals traded on the London Metal Exchange. Subsequently, Erten and Ocampo (2013) apply this methodology to the identification of supercycles in real non-oil commodity prices.

The present paper proposes a different methodology to identify long cycles in world commodity prices. It identifies the commodity supercycle as a common permanent component in all commodity prices. The proposed common permanent component approach to identifying the commodity-price supercycle has two advantages relative to the spectral analysis approach. First, the spectral approach applies the band-pass filter to individual commodity-price time series separately. As a result, it delivers one supercycle per commodity price. However, the data strongly suggests that long cycles in commodity prices are correlated. The top left panel of figure 1, which plots all eleven commodity prices together, shows that long swings in commodity prices are synchronized. This correlation has been interpreted as reflecting the existence of a common driver. The common permanent component approach we propose delivers this common driver by identifying the nonstationary world shock responsible for the supercycle in all commodity prices.

**Figure 1. Eleven Real Commodity Prices, 1960:I–2018:IV**




Source: Authors' calculations.
Note: The frequency of the data is quarterly. The eleven commodity prices are deflated by the U.S. consumer price index and expressed in logs. All series are normalized to zero in 2010.IV.

A second desirable property of the common permanent component approach proposed in this paper for the identification of the commodity supercycle is that it allows for the joint estimation of transitory and permanent, domestic and world disturbances affecting aggregate activity in individual countries. As a result, the common permanent component approach provides a natural environment for estimating the contribution of the shocks responsible for the supercycle to explaining variations in output at the country level.

The paper formulates an empirical model that includes eleven commodity prices, the world interest rate, and the output of 24 (quarterly sample) or 41 (annual sample) small open developed and emerging economies. All commodity prices are assumed to be cointegrated with a common nonstationary world shock. In addition, commodity prices and the world interest rate are assumed to be buffeted by stationary world shocks. Output at the individual country level is driven by the nonstationary and stationary world shocks, a nonstationary country-specific shock, and a stationary country-specific shock. Thus, a constellation of stationary and nonstationary world and domestic shocks compete to explain movements in country-specific output. The nonstationary world shock is the one responsible for the commodity-price supercycle. Thus, ascertaining the role of the supercycle in accounting for output movements in a given country amounts to estimating the share of the nonstationary world shock in the variance decomposition of the country's output.

The model is cast in terms of deviations of endogenous variables from their respective stochastic trends and exogenous shocks. Since the exogenous shocks and the stochastic trends are unobservable, the variables in the model are latent variables. The estimation exploits the fact that the model delivers precise predictions for variables that are observed. In particular, the observable variables used in the estimation of the model are the growth rates of the eleven commodity prices in figure 1, the level of the world interest rate, and the growth rates of output of the countries included in the sample. The likelihood of the data is computed by using the Kalman filter, and the econometric estimation employs Bayesian techniques. The model is estimated on quarterly data covering the period 1960 to 2018. For countries for which quarterly output data since 1960 is not available, the model is estimated on annual data.

The paper delivers three main results. First, the common permanent component plays an important role in explaining movements in commodity prices at frequencies typically associated with the supercycle.

Specifically, the common permanent component explains on average across commodities between 67 and 91 percent of the forecast error variance of commodity prices at horizons between five and thirty years.

Second, world shocks that drive commodity prices and the world interest rate are major drivers of aggregate fluctuations in developed and emerging small open economies. Jointly the stationary and nonstationary world shocks explain more than half of the variance of output growth on average across countries. Third, and more importantly, the bulk (more than two thirds) of the explanatory power of world shocks stems from stationary shocks. These results obtain not only unconditionally but also conditionally on time horizons. Even at forecasting horizons typically associated with the supercycle (20 years or longer), stationary world shocks play a larger role than the nonstationary world shock in explaining the forecast error variance of the level of output in individual countries. Taken together, these results suggest that the commodity-price supercycle matters for explaining aggregate activity at the country level, but that its contribution is smaller than that of stationary world shocks.

This paper is related to several strands of literature. The empirical model follows recent studies that identify permanent disturbances and their contribution to business cycles (Uribe, 2018). The main results speak to a body of work on the role of world prices as mediators of world shocks for economic outcomes in small open economies. Mendoza (1995) and Kose (2002), by using calibrated real business-cycle models fed with estimated stochastic processes for the terms of trade, find that disturbances to this international price account for more than thirty percent of fluctuations in aggregate activity. More recently, Miyamoto and Nguyen (2017), and Drechsel and Tenreyro (2018) find similar results by using a Bayesian estimation approach. Schmitt-Grohé and Uribe (2018) apply a more agnostic approach based on structural vector autoregressions and find that the contribution of terms-of-trade shocks to explaining aggregate fluctuations in poor and emerging economies is only ten percent. These authors argue for the need to consider more disaggregated measures of world prices to better capture the transmission of world shocks to individual economies. Fernández and others (2017) employ a similar empirical strategy as Schmitt-Grohé and Uribe (2018) but expand the set of world prices from one to four, three commodity prices, and the world interest rate, and find that world shocks mediated by this set of prices explain one third of output fluctuations on average in a set of 138 countries over the period 1960 to 2015. This figure more than doubles when the estimation is conducted on a more recent sample beginning in the late 1990s, as shown by Shousha (2016), Fernández and others (2018), and Fernández and others (2017). In the papers just cited, shocks to commodity prices, if explicitly modeled, are assumed to be stationary, and as a result, this body of work does not speak directly to the importance of commodity-price supercycles.

As mentioned earlier, key references on the use of spectral analysis for the estimation of commodity-price supercycles are Cuddington and Jerrett (2008) and Erten and Ocampo (2013). These papers and the early work by Pindyck and Rotemberg (1990) speculate on the existence of common drivers of commodity prices, which serves as motivation for the common permanent component approach proposed in the present paper. Alquist and others (2020) apply a factor-based identification strategy to estimate the role of commodity prices in explaining the global economic activity. Benguria and others (2018) identify the commodity-price supercycle by HP filtering and analyze its transmission by using firm-level administrative data from Brazil.

The paper also contributes to a literature assessing the role of transitory and permanent shocks in driving business cycles in developed and emerging economies.[1]It finds that, for both developed and emerging countries, transitory shocks play a larger role than permanent shocks, even if one conditions on world shocks or on country-specific shocks.

Finally, the structuralist literature pioneered by Prebisch (1950) and Singer (1950) argues that secular deterioration in the terms of trade plays a central role in the development of emerging countries. The empirical relevance of this hypothesis has been the subject of debate. One reason is that, as figure 1 suggests, it is not clear that overall commodity prices display secular deterioration. In the context of that literature, deterioration of the terms of trade is understood as primary commodity prices growing on average at a slower pace than prices of nonprimary goods. Our commodity-price data is deflated by the U.S. CPI index, and as such secular deterioration should manifest as a downward trend in raw data. Over the sample period considered in the figure, some commodity prices display a downward trend (e.g., agricultural raw materials) but others display an upward trend (e.g., energy and precious metals). The present analysis does not aim to explain the effect of trending real commodity prices on economic

1. See, among others, Aguiar and Gopinath (2007), García-Cicco and others (2010), Chang and Fernández (2013), and Miyamoto and Nguyen (2017).

development. However, our analysis does have a point of contact with the Prebisch-Singer hypothesis in that it allows for innovations in the permanent component of real commodity prices to have an effect on output growth at the country level.

The remainder of the paper is organized into seven sections. Section 1 presents the empirical model. Section 2 introduces the observables, the priors, and the estimation strategy. Section 3 presents the definitions and sources of the quarterly data on commodity prices, world interest rates, and output in 24 predominantly developed economies spanning the period 1960.I to 2018.I. Section 4 analyzes the estimated commodity-price supercycle. Section 5 presents variance decompositions, forecast error variance decompositions, and impulse response analysis to ascertain the importance of the commodity supercycle for aggregate activity in the small open economies considered. Section 6 estimates the model on annual data from 1960 to 2018 for 24 emerging and 17 developed countries[2]. Finally, section 7 concludes.

## 1. An Empirical Model of the Commodity Supercycle

The empirical model consists of a world block and a country-specific block. The world block describes the evolution of the vector $p_t$ containing eleven real commodity prices and the gross real interest rate in quarter $t$, all expressed in logarithms. The commodity supercycle is modeled as a nonstationary exogenous variable $X_t^p$ with the property of being cointegrated with the eleven commodity prices. We can then define a vector of transformed world prices, denoted $\hat{p}_t$, that is stationary as follows:[3]

$$\hat{p}_t = \begin{bmatrix} \hat{p}_t^1 \\ \hat{p}_t^2 \\ \vdots \\ \hat{p}_t^{11} \\ \hat{r}_t \end{bmatrix} \equiv \begin{bmatrix} p_t^1 - X_t^p \\ p_t^2 - X_t^p \\ \vdots \\ p_t^{11} - X_t^p \\ r_t \end{bmatrix}$$

2. For further technicals details see Appendix.

3. For expositional purposes, constant terms are omitted. The model with constant terms is presented in the Appendix.

The identification assumption that all commodity prices have the same cointegrating vector with $X_t^p$ is based on the observation that, in the raw data, commodity prices do not seem to diverge from one another over time (see figure 1, upper left panel).

The vector $\hat{p}_t$ is assumed to be buffeted by a nonstationary shock, given by variations in the growth rate of the permanent component of world prices, $\Delta X_t^p \equiv X_t^p - X_{t-1}^p$, and 12 stationary world shocks denoted $z_t^p$. The vector of world prices evolves according to the following autoregressive process:

$$\hat{p}_t = \sum_{i=1}^{4} B_{pp}^i \hat{p}_{t-i} + C_{pX^p} \Delta X_t^p + C_{pz^p} z_t^p, \tag{1}$$

where $B_{pp}^i$ for i = 1, 2, 3, 4, $C_{pX^p}$, and $C_{pz^p}$ are matrices of coefficients of order 12-by-12, 12-by-1, and 12-by-12, respectively. Without loss of generality, assume that $C_{pz^p}$ is lower triangular with ones on the diagonal. This is not an identification restriction. The elements of $z_t^p$ should be interpreted as combinations of stationary world shocks affecting commodity prices and the interest rate. The present study does not aim to identify these shocks individually, but rather to ascertain their joint contribution to explaining movements in world prices and aggregate activity and to compare it to that of the nonstationary world shock $X_t^p$ driving the commodity supercycle.

The domestic block consists of the vector $y_t$ containing real output for 24 small open economies expressed in logarithms and denoted $y_t^i$, $i$ = 1,...,24. In each country, output is assumed to be cointegrated with a linear combination of a country-specific nonstationary shock, denoted $X_t^i$ for $i$ = 1,...,24 and the nonstationary component of real-world commodity prices, $X_t^p$. The rationale behind the assumption that $X_t^p$ enters in the cointegrating relationship of output is that, in models of small open economies with commodity prices, output inherits their stochastic properties. We note that this long-run relationship between output and the nonstationary component of world shocks is not subject to the observation made by Kehoe and Ruhl (2008) that, depending on how real GDP is measured in the data, terms-of-trade shocks may not act like technology shocks, for their observation has to do with the direct effect of terms-of-trade shocks on measured GDP and not with their indirect effect on quantities. The cointegration relationship between $y_t^i$, $X_t^i$, and $X_t^p$ is estimated, thus allowing the data to choose the strength of the long-run link between output and commodity prices in each country.

Let $\hat{y}_t$ be a 24-by-1 vector of deviations of output from trend. Then,

$$\hat{y}_t = \begin{bmatrix} \hat{y}_t^1 \\ \hat{y}_t^2 \\ \vdots \\ \hat{y}_t^{24} \end{bmatrix} \equiv \begin{bmatrix} y_t^1 - X_t^1 - \alpha^1 X_t^p \\ y_t^2 - X_t^2 - \alpha^2 X_t^p \\ \vdots \\ y_t^{24} - X_t^{24} - \alpha^{24} X_t^p \end{bmatrix}.$$

For each country $i$ the country-specific shocks consist of the growth rate of the permanent component of output, $\Delta X_t^i$, and a stationary shock, $z_t^i$. Detrended output is assumed to evolve according to the following autoregressive process:

$$\hat{y}_t = \sum_{i=1}^{4} B_{yp}^i \hat{p}_{t-i} + \sum_{i=1}^{4} B_{yy}^i \hat{y}_{t-i} + C_{yX^p} \Delta X_t^p + C_{yz^p} z_t^p + C_{yX} \Delta X_t + z_t, \qquad (2)$$

where $\Delta X_t = [\Delta X_t^1 \ldots \Delta X_t^{24}]'$ and $z_t = [z_t^1 \ldots z_t^{24}]'$. $B_{yp}^i$ and $B_{yy}^i$, for $i = 1, \ldots, 4$, are 24-by-12 and 24-by-24 matrices of coefficients, respectively, and $C_{yX^p}$, $C_{yz^p}$, and $C_{yX}$ are matrices of order 24-by-1, 24-by-12, and 24-by-24, respectively. Matrices $B_{yy}^i$ and $C_{yX}$ are assumed to be diagonal.

Note that the world shocks $\Delta X_t^p$ and $z_t^p$ enter directly in the domestic block, as opposed to mediated by $\hat{p}_t$. This flexibility allows for the possibility that $\Delta X_t^p$ and $z_t^p$ capture global and regional shocks affecting individual countries both directly and via world prices, such as exogenous global and regional productivity shocks.

The exogenous shocks, $\Delta X_t^p$, $\Delta X_t$, $z_t^p$, and $z_t$ follow univariate autoregressive processes.

Specifically, let $u_t$ denote the vector of exogenous shocks

$$u_t \equiv \begin{bmatrix} \Delta X_t^p \\ \Delta X_t \\ z_t^p \\ z_t \end{bmatrix}.$$

We assume that $u_t$ obeys the law of motion

$$u_t = \rho u_{t-1} + \psi \nu_t, \tag{3}$$

where $\nu_t \sim i.i.d.N\ (0, I_{61})$. The matrices $\rho$ and $\psi$ are assumed to be diagonal. This implies that the permanent component of world prices, $X_t^p$, is uncorrelated with the stationary world shocks, $z_t^p$; that the permanent and transitory country-specific shocks are uncorrelated with each other and with other country-specific shocks; and that country-specific shocks, $X_t^i$ and $z_t^i$, are uncorrelated with the world shocks, $X_t^p$ and $z_t^p$. The latter assumption is motivated by the fact that the countries in the sample are small open economies and, as such, their idiosyncratic shocks do not affect world prices. We assume that the correlation of output across countries stems from world shocks. Accordingly, the matrices $B_{yy}^i$ for i = 1,..., 4, as well as the matrix $C_{yX}$, are restricted to be diagonal. The assumption that the world shocks $X_t^p$ and $z_t^p$ (as opposed to the contemporaneous world prices $\hat{p}_t$) enter directly in the domestic block, equation (2), allows for the possibility that world shocks affect country-level output, both directly and indirectly, mediated by world prices. A direct effect of world shocks on country-level output could occur, for example, via productivity shocks that are correlated across countries.

## 2. Observables, Priors, and Estimation Strategy

All variables in the system (1), (2), and (3), except for the interest rate, $r_t$, are latent variables and therefore unobservable. As a result, the system cannot be directly estimated on data. However, we will exploit the fact that the model has precise predictions for variables that are observable. Specifically, the data used in the estimation includes the growth rates of the commodity prices, $\Delta p_t^i$ for $i$ = 1,...,11, the level of the world interest rate, $r_t$, and the growth rates of output, $\Delta y_t^i$ for $i$ = 1,...,24. The observable variables are related to the latent variables through the following identities:

$$\Delta p_t^i = \Delta \hat{p}_t^i + \Delta X_t^p; i = 1, \ldots, 11, \tag{4}$$

$$\Delta y_t^i = \Delta \hat{y}_t^i + \Delta X_t^i + \alpha^i \Delta X_t^p; i = 1, \ldots, 24, \tag{5}$$

and

$$r_t = \hat{r}_t. \tag{6}$$

The observable variables are assumed to be measured with error. Letting $o_t$ denote the vector of observed variables, we have that

$$o_t = \begin{bmatrix} \Delta p_t^1 \\ \vdots \\ \Delta p_t^{11} \\ r_t \\ \Delta y_t^1 \\ \vdots \\ \Delta y_t^{24} \end{bmatrix} + \mu_t, \tag{7}$$

where $\mu_t$ is a 36-by-1 vector of measurement errors distributed i.i.d. $N(0, R)$ and $R$ is a diagonal matrix. We restrict the measurement errors to explain no more than ten percent of the variance of the data.

The relationship between the observables and the latent variables, the fact that the model is linear, and that all innovations are Gaussian, make it possible to compute the likelihood of the data, which in turn allows for the estimation of the parameters of the model. To calculate the likelihood, it is convenient to express the model in state-space form. To this end, let

$$\hat{x}_t = \left[ \hat{p}_t \ \hat{y}_t \right]' \text{ and } \xi_t = \left[ \hat{x}_t \ \hat{x}_{t-1} \ \hat{x}_{t-2} \ \hat{x}_{t-3} \ u_t \right]'.$$

Then the state-space representation of the model, equations (1)–(7), is given by

$$\xi_{t+1} = F\xi_t + P\nu_{t+1} \tag{8}$$

and

$$o_t = H'\xi_t + \mu_t, \tag{9}$$

where the matrices $F, P$, and $H$ are known functions of the matrices $B_{pp}^i$, $B_{yp}^i$, $B_{yy}^i$ for $i$ = 1,...,4, $C_{pX^p}$, $C_{pz^p}$, $C_{yX^p}$, $C_{yz^p}$, $C_{yX}$, $\rho$ and $\psi$. The model is estimated with Bayesian techniques. Draws from the posterior distribution are obtained by applying the Metropolis-Hastings algorithm. We construct an MCMC chain of 2.5 million draws and discard the first 1.5 million.

The prior distributions of the estimated parameters are summarized in table 1. We impose normal prior distributions to all elements of $B_{pp}^i$, $B_{yy}^i$, and $B_{yp}^i$, for $i$ = 1,...,4. In accordance with the

Minnesota prior, we assume that, at the mean of the prior parameter distribution, the elements of $\hat{x}_t$ follow univariate autoregressive processes. So, when evaluated at their prior mean, only the main diagonals of $B^1_{pp}$ and $B^1_{yy}$ take nonzero values, and all other elements of $B^i_{pp}$ and $B^i_{yy}$, and all elements of $B^i_{yp}$ for $i = 1,\ldots,4$ are nil. We impose an autoregressive coefficient of 0.95 in all equations so that all elements along the main diagonal of $B^1_{pp}$ and $B^1_{yy}$ take a prior mean of 0.95. We assign a prior standard deviation of 0.5 to these elements, which implies a coefficient of variation close to one half (0.5/0.95). Also, along the lines of the Minnesota prior, we impose lower prior standard deviations on all other estimated elements of the matrices $B^i_{pp}$, $B^i_{yy}$, and $B^i_{yp}$ for $i = 1,\ldots,4$, and set them to 0.25.

All estimated elements of the matrices $C_{pX^p}$, $C_{pz^p}$, $C_{yX^p}$, $C_{yz^p}$, and $C_{yX}$ are assumed to have normal prior distributions with mean zero and unit standard deviation, with one exception: the diagonal elements of the diagonal matrix $C_{yX}$, which govern the responses of $\hat{y}^i_t \equiv y^i_t - X^i_t - \alpha^i X^p_t$ to an innovation in $\Delta X^i_t$ for $i = 1,\ldots,24$, are assumed to have a prior mean of –1. This means that a shock that increases output in country $i$ in the long run by one percentage point, under the prior, has a zero-impact effect. This prior is motivated by a strand of the business-cycle literature suggesting that the impact effect on output of a permanent productivity shock could have either sign depending on the strength of the wealth effect on labor supply.[4]

The diagonal elements of the diagonal matrix $\psi$ representing the standard deviations of the innovations in the exogenous shocks are all assigned Gamma prior distributions with mean and standard deviations equal to one. We impose non-negative serial correlations on the exogenous shocks (the diagonal elements of $\rho$) and adopt Beta prior distributions for these parameters. We assume relatively small means of 0.3 for the prior mean of the serial correlations of the nonstationary shocks ($\Delta X^p_t$ and $\Delta X^i_t$ for $i = 1,\ldots,24$) and a relatively high mean of 0.7 for the prior mean of the serial correlations of the stationary shocks ($z^p_t$ and $z_t$). The prior distributions of all serial correlations are assumed to have a standard deviation of 0.2. The variances of all measurement errors (the diagonal elements of the matrix $R$) are assumed to have a uniform prior distribution with lower bound zero and upper bound of ten percent of the sample variance of the corresponding observable indicator. Although not explicitly discussed thus far, the estimated

4. See, for example, Galí (1999).

model includes constants, which appear in the observation equation (9).[5] These constants represent the unconditional means of the 36 observables. They are assumed to have normal prior distributions with means equal to the sample means of the observables and standard deviations equal to their sample standard deviations divided by the square root of the sample length (231 quarters).

**Table 1. Prior Distributions**

| *Parameter* | *Distribution* | *Mean* | *Std. Dev.* |
|---|---|---|---|
| Main diagonal elements of $B_{pp}^1$ and $B_{yy}^1$ | Normal | 0.95 | 0.5 |
| All other estimated elements of $B_{pp}^1$, $B_{yy}^1$, $B_{yp}^1$ | Normal | 0 | 0.25 |
| Estimated elements of $B_{pp}^i$, $B_{yy}^i$, $B_{yp}^i$ $i = 2,3,4$ | Normal | 0 | 0.25 |
| Estimated elements of $C_{pX^p}$ and $C_{pz^p}$ | Normal | 0 | 1 |
| Diagonal of $C_{yX}$ | Normal | –1 | 1 |
| Elements of $C_{yX^p}$ and $C_{yz^p}$ | Normal | 0 | 1 |
| Diagonal of $\rho(1{:}25,1{:}25)$ | Beta | 0.3 | 0.2 |
| All other diagonal elements of $\rho$ | Beta | 0.7 | 0.2 |
| Diagonal of $\psi$ | Gamma | 1 | 1 |
| $\alpha_i$, $i = 1,\ldots,24$ | Normal | 0 | 1 |
| Diagonal elements of $R$ | Uniform $[0, \frac{var(o_t)}{10}]$ | $\frac{var(o_t)}{(10*2)}$ | $\frac{var(o_t)}{(10*\sqrt{12})}$ |
| Elements of $A$ | Normal | mean$(o_t)$ | $\sqrt{\frac{var(o_t)}{T}}$ |

Source: Authors' calculations.
Notes: $T$ denotes the sample length, which equals 231 quarters. The vector A denotes the mean of the vector $o_t$ and is defined in the Appendix.

5. See the Appendix for details.

Posterior means and error bands around the impulse responses shown in later sections are constructed from a random subsample of the MCMC chain of length 100 thousand with replacement.

## 3. The Data

The model is estimated on quarterly data on eleven world commodity prices, the world interest rate, and the gross domestic product of 24 small open economies. The sample period is 1961.I to 2018.IV. The eleven commodity prices included in the estimation are beverages, food, agricultural raw materials, fertilizers, metal and minerals, gold, platinum, silver, coal, crude oil, and natural gas. The raw data is monthly and expressed in current U.S. dollars. The source is the World Bank's Commodity-Price database (the Pink Sheet) except for coal prices, which come from Global Financial Data (GFD). The GFD coal price is identical to the one in the Pink Sheet, except that it begins in 1960.1, whereas it begins only in 1970.1 in the Pink Sheet. Quarterly real commodity-price indices are constructed by first deflating the monthly nominal price indices by the monthly CPI index of the United States, then taking a simple average of the deflated values across the corresponding months in each quarter. The data are normalized by dividing by each series' 2010.IV observation. The World Bank publishes data on the prices of 40 individual commodities. The aggregation into eleven prices responds to the need to economize on degrees of freedom in the estimation. The individual commodities whose prices are aggregates are beverages, food, agricultural raw materials, fertilizers, and metal and minerals. The aggregate price indices are taken from the Pink Sheet. Thus the eleven commodity prices included capture information from all 40 commodity prices in the World Bank's Commodity-Price database.

The quarterly time series for the world real interest rate, $r_t$, is constructed as $1+r_t=\left(1+i_t\right)E_t\frac{1}{1+\pi_{t+1}}$, where $i_t$ denotes the nominal interest rate on three-month U.S. Treasury bills, and $1+\pi_{t+1}=\frac{P_{t+1}}{P_t}$, denotes the gross growth rate of the consumer price index, $P_t$, as measured by the U.S. CPI index. The expected value of the inverse of gross inflation, $E_t\frac{1}{1+\pi_{t+1}}$ is approximated by the fitted component of an OLS regression of $\frac{1}{1+\pi_{t+1}}$ onto a constant, $\frac{1}{1+\pi_t}$ and $\frac{1}{1+\pi_{t-1}}$.

Output is measured by seasonally adjusted real gross domestic product from the quarterly national accounts of the OECD.[6]

For a country to be included in the sample, we require at least 50 years of quarterly observations of real output. The rationale behind this restriction is that identifying the real effects of the commodity supercycle requires observing the behavior of output over a relatively long period. In addition, since commodity prices and the world interest rate are assumed to be exogenous to the country, we exclude large economies. These selection criteria result in the following 24 countries: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Greece, Iceland, Ireland, Italy, Korea, Luxembourg, Mexico, Netherlands, New Zealand, Norway, Portugal, South Africa, Spain, Sweden, Switzerland, Turkey, and the United Kingdom. The panel includes only four emerging economies, Korea, Mexico, South Africa, and Turkey. Section 6 estimates the model on annual data, which allows for the inclusion of a larger number of emerging economies.

## 4. The Commodity-Price Supercycle

Figure 2 displays the estimated common permanent component, $X_t^p$, of real commodity prices. It is constructed by Kalman smoothing at the posterior mean of the parameter estimate. The figure also displays the eleven observed commodity prices. We interpret the variable $X_t^p$ as the commodity supercycle. The figure suggests that this interpretation is sensible as $X_t^p$ appears to capture well the low-frequency comovement of the individual commodity prices. Over the period 1960 to 2018, commodity prices display two distinct supercycles—one peaking in 1980 and the other in 2008. The rapid growth in $X_t^p$ between the early 1970s and 1980 coincides with the OPEC oil-price crises. As the market power of the oil cartel weakened in the 1980s and the supply of other countries (e.g., the United States and those located around the North Sea) rose, the downswing of the supercycle began. The expansionary phase of the second commodity-price supercycle begins around the time of China's accession to the WTO in 2001 and the peak is reached with the onset of the Global Financial Crisis of 2008. The prediction of two commodity supercycles

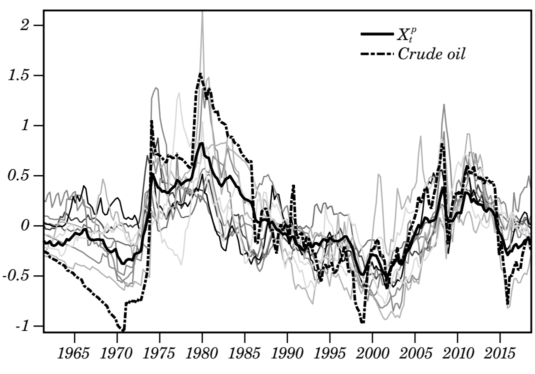
6. The OECD series name is VOBARSA. For Greece and Iceland, the data appears not to have been seasonally adjusted at the source. Therefore, these two series were adjusted by using the X-13 ARIMA-SEATS software produced, distributed, and maintained by the U.S. Census Bureau.

post 1960 and their dating is in line with the estimates reported in Erten and Ocampo (2013) using an asymmetric bandpass filtering approach on real non-oil commodity prices that picks out cycles with periodicity between 20 and 70 years.

The permanent component of commodity prices, $X_t^p$, plays a significant role in explaining movements in these variables. Table 2 displays the fraction of the variance of changes in commodity prices accounted for by changes in their permanent component. On average across prices, $\Delta X_t^p$ explains more than one fourth of the variance of changes in commodity prices. The variance shares are estimated with precision, with standard deviations equal to two percentage points on average. The permanent component plays the largest role in explaining movements in crude oil prices with a variance share of 60 percent.

Estimating the price block of the model separately from the output block (not shown) yields similar results for the time path of $X_t^p$. Also, this estimation approach yields a similar result for the average share of the variance of the growth rates of world prices explained by $\Delta X_t^p$ (21 percent when the price block is estimated separately versus 27 percent when it is estimated jointly with the output block). However, estimating the price block by using only information on prices yields a smaller role for the permanent component, $\Delta X_t^p$, in explaining the variance of the growth rate of crude oil prices (35 versus 60 percent). This finding suggests that, even though the price block is independent of the output block, data on country-level output is informative for the estimation of the parameters governing the dynamics of world prices.

**Figure 2. The Commodity-Price Supercycle**




Source: Authors' calculations.
Notes: The permanent component of the eleven real commodity prices, $X_t^p$, is computed by Kalman smoothing using the posterior mean of the parameter estimates. The thin solid lines are the eleven observed real commodity prices (beverages, food, agricultural raw materials, fertilizers, metal and minerals, gold, platinum, silver, coal, crude oil, and natural gas). All-time series are constructed as cumulative demeaned growth rates.

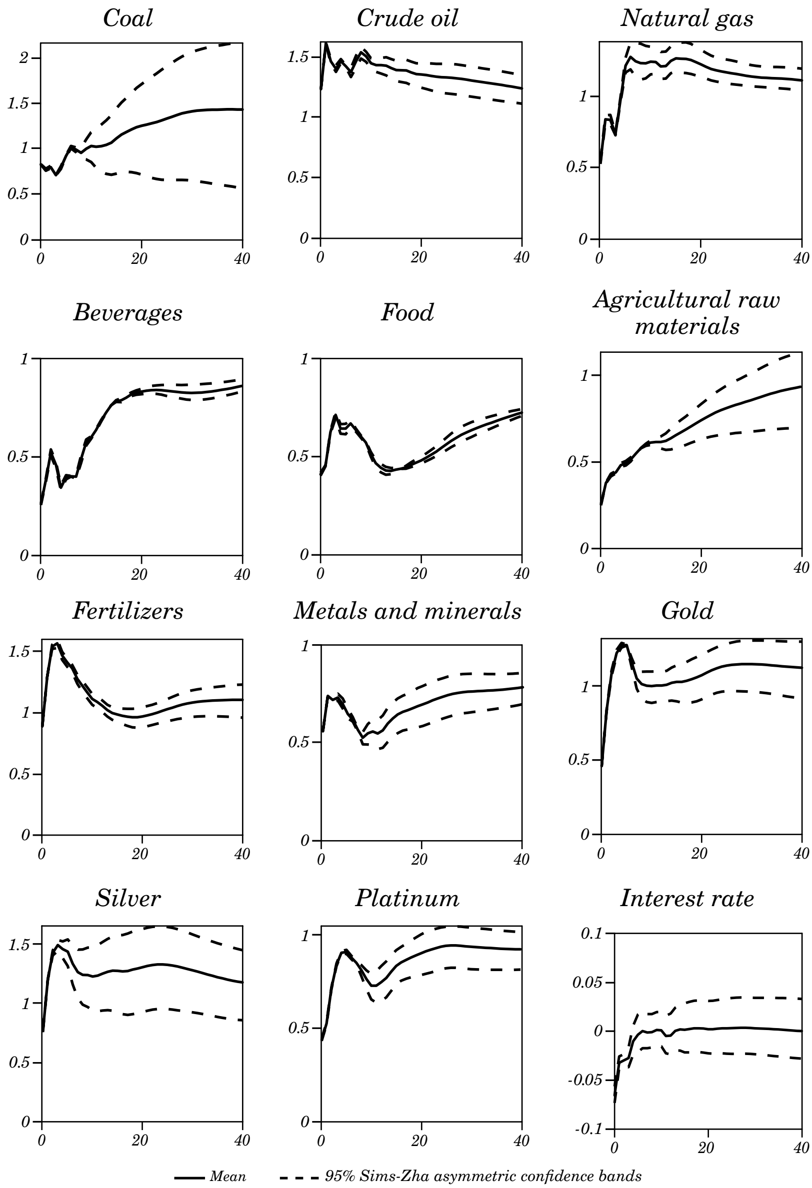
**Table 2. Percent of Variance of the Growth Rate of Real Commodity Prices Explained by $\Delta X_t^p$**

| *Price of* | *Mean* | *Std. Dev.* |
|---|---|---|
| Coal | 26 | 3 |
| Crude Oil | 60 | 2 |
| Natural Gas | 20 | 3 |
| Beverages | 11 | 1 |
| Food | 21 | 2 |
| Agr. Raw Materials | 20 | 3 |
| Fertilizers | 33 | 2 |
| Metal and Minerals | 30 | 2 |
| Gold | 30 | 2 |
| Silver | 28 | 2 |
| Platinum | 19 | 1 |
| Mean across prices | 27 | 2 |
| Median of prices | 26 | 2 |
| Real Rate | 15 | 8 |

Source: Authors' calculations.
Note: The reported figures are based on 100,000 draws from the posterior distribution of the variance decomposition.

Figure 3 presents the impulse responses of the eleven real commodity prices and the world interest rate to a unit long-run increase in the permanent component $X_t^p$ along with 95-percent asymmetric confidence bands, computed by using the methodology proposed by Sims and Zha (1999). For most commodity prices, a positive innovation in $X_t^p$ has a positive but less than unity impact effect and induces a slow convergence to the permanently higher level, which by construction is equal to one. Exceptions are crude oil, which displays overshooting on impact and convergence from above, and natural gas, fertilizers, gold, and silver, which display delayed overshooting.

**Figure 3. Impulse Responses of World Prices to a Long-Run Increase in $X_t^p$ of Unity**




Source: Authors' calculations.

Notably, an increase in the permanent component of commodity prices has a negative effect on the world interest rate. The estimated negative conditional comovement between commodity prices and interest rates is important for commodity exporters with external debt because it suggests that, when commodity prices increase, the country also benefits from favorable conditions in international financial markets. Similarly, during a downturn in commodity prices, the costs of external debt rise. This result is in line with the work of Shousha (2016), who finds that movements in the interest rate are in part driven by variations in commodity prices. The novel aspect of the result documented here is that the negative comovement between commodity prices and interest rates is conditional on a permanent change in commodity prices. The finding that interest rates fall when commodity prices increase can also be interpreted as representing a particular manifestation of a phenomenon that Kaminsky and others (2005) refer to as 'When it Rains, it Pours.'

## 5. Importance of the Commodity Supercycle for Economic Activity

Thus far we have documented that the permanent component of commodity prices explains a sizeable fraction, over one fourth, of movements in commodity prices. In other words, we have documented that there is a significant commodity supercycle. We now wish to ascertain the role of the commodity supercycle in explaining business-cycle fluctuations in individual countries.

Table 3 displays the variance decomposition of output growth for the 24 countries in the sample. On average across countries, the permanent component of commodity prices explains only eight percent of the overall volatility of output growth. By contrast, the transitory components of commodity prices jointly explain 62 percent of the variance of output growth. This result suggests that world shocks are important in explaining output movements in small open economies. However the vast majority of the movements stems from stationary world disturbances. In this sense, the role of the commodity supercycle is modest in accounting for business cycles. The importance of the commodity supercycle in explaining output fluctuations does not vary much across countries. The cross-sectional standard deviation of the variance share of output growth accounted for by $\Delta X_t^p$ is only 2.4 percentage points. This means that the relatively modest role played by the supercycle is not just valid on average but applies to most countries in the sample.

Table 3 also speaks to a large literature assessing the role of permanent versus transitory shocks in accounting for aggregate fluctuations in emerging and developed countries. [7] It shows that, in the present sample of 24 countries, the vast majority of fluctuations in output growth is driven by stationary shocks. Jointly the domestic and world stationary shocks ($z_t^i$ and $z_t^p$) explain 80 percent of the variance of output growth on average across countries. Noticeably this result is obtained not only for the developed countries in the sample but also for the emerging ones (Korea, 80 percent; Mexico, 93 percent; South Africa, 91 percent; and Turkey, 95 percent). The finding that stationary shocks explain the lion's share of output fluctuations in emerging countries is in line with those reported in Garcia-Cicco and others (2010), Chang and Fernández (2013), and Singh (2020).

It is of interest to ascertain the effects of the commodity-price supercycle on commodity prices and aggregate activity at different time horizons. Table 4 presents forecast error variance decompositions of the level of commodity prices and the level of output at horizons of 5, 10, 20, and 30 years computed at the posterior mean of the parameter estimate. The top panel of the table shows that the commodity supercycle plays a sizeable role in explaining the level of commodity prices at all forecasting horizons considered. The median share of $X_t^p$ in the forecast error across the eleven commodity prices ranges from 67 percent at the five-year horizon to 93 percent at the 30-year horizon. This suggests that the commodity supercycle affects commodity prices not just at its own frequency of 20 years or higher but also at shorter frequencies of five to ten years.

By contrast, the commodity supercycle appears to play a secondary role in explaining movements in the level of output at horizons of five and ten years, which are typically associated with business-cycle fluctuations. The bottom panel of the table shows that the contribution of $X_t^p$ in accounting for the forecast error variance is at most 12 percent at horizons of ten years or less. At horizons of 20 and 30 years, which fall into the range of frequencies of the commodity supercycle itself, the contribution of $X_t^p$ to explaining the variance of forecast errors of output increases to 19 percent. By contrast, stationary world shocks, the elements of the vector $z_t^p$, account for the majority of the forecast error variance of output at all horizons considered. Their median

7. See, for example, Aguiar and Gopinath (2007).

contribution ranges from 75 percent at the five-year forecasting horizon to 58 percent at the 30-year horizon. This indicates that the economic impact of the commodity supercycle on output, relative to that of stationary world shocks, is small at business-cycle frequencies (ten years or less) and moderate at its own frequency (20 years or more).

**Table 3. Variance Decomposition of Output Growth**

| | *Shock* | | | |
|---|---|---|---|---|
| *Country* | $\Delta X_t^p$ | $z_t^p$ | $\Delta X_t^i$ | $z_t^i$ |
| Australia | 7 | 61 | 1 | 32 |
| Austria | 10 | 67 | 1 | 22 |
| Belgium | 8 | 84 | 7 | 1 |
| Canada | 10 | 71 | 1 | 19 |
| Denmark | 7 | 65 | 0 | 28 |
| Finland | 6 | 68 | 17 | 8 |
| France | 8 | 60 | 1 | 31 |
| Greece | 7 | 63 | 30 | 0 |
| Iceland | 5 | 47 | 45 | 2 |
| Ireland | 6 | 42 | 51 | 2 |
| Italy | 10 | 74 | 0 | 17 |
| Korea, Rep. | 11 | 60 | 10 | 20 |
| Luxembourg | 10 | 50 | 23 | 18 |
| Mexico | 7 | 71 | 0 | 22 |
| Netherlands | 8 | 58 | 33 | 1 |
| New Zealand | 5 | 51 | 36 | 8 |
| Norway | 4 | 55 | 19 | 22 |
| Portugal | 13 | 63 | 0 | 24 |
| South Africa | 9 | 61 | 0 | 29 |
| Spain | 12 | 69 | 0 | 19 |
| Sweden | 8 | 54 | 0 | 37 |
| Switzerland | 6 | 62 | 0 | 31 |
| Turkey | 4 | 51 | 0 | 44 |
| United Kingdom | 7 | 74 | 1 | 19 |
| Mean | 8 | 62 | 12 | 19 |
| Median | 8 | 62 | 1 | 20 |

Source: Authors' calculations.

Notes: The table presents the share (expressed in percent) of the total variance of output growth explained by shocks to the permanent component of commodity prices, $\Delta X_t^p$, all 12 transitory commodity-price shocks taken together, $z_t^p$, the country-specific nonstationary shock, $\Delta X_t^i$, and the country-specific stationary shock, $z_t^i$. The reported numbers are averages over 100,000 draws from the posterior distribution of the variance decomposition.

## Table 4. Forecast Error Variance Decomposition of the Level of Commodity Prices and Output

| *Shock* | $X_t^p$ | | | | $z_t^p$ | | | | $X_t^i$ | | | | $z_t$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *Horizon (in years)* | 5 | 10 | 20 | 30 | 5 | 10 | 20 | 30 | 5 | 10 | 20 | 30 | 5 | 10 | 20 | 30 |
| Coal | 52 | 66 | 77 | 82 | 48 | 34 | 23 | 18 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Crude Oil | 86 | 89 | 93 | 95 | 14 | 11 | 7 | 5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Natural Gas | 73 | 83 | 90 | 93 | 27 | 17 | 10 | 7 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Beverages | 46 | 70 | 85 | 91 | 54 | 30 | 15 | 9 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Food | 38 | 52 | 76 | 85 | 62 | 48 | 24 | 15 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Agr. Raw Materials | 54 | 74 | 87 | 92 | 46 | 26 | 13 | 8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Fertilizers | 68 | 78 | 87 | 91 | 32 | 22 | 13 | 9 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Metal and Minerals | 47 | 63 | 79 | 87 | 53 | 37 | 21 | 13 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Gold | 71 | 83 | 90 | 93 | 29 | 17 | 10 | 7 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Silver | 67 | 78 | 86 | 89 | 33 | 22 | 14 | 11 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Platinum | 67 | 81 | 90 | 93 | 33 | 19 | 10 | 7 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Median | 67 | 78 | 87 | 91 | 33 | 22 | 13 | 9 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Interest Rate | 8 | 7 | 6 | 7 | 92 | 93 | 94 | 93 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Australia | 5 | 2 | 2 | 7 | 82 | 89 | 87 | 80 | 5 | 6 | 8 | 11 | 8 | 3 | 2 | 2 |
| Austria | 19 | 27 | 33 | 36 | 74 | 70 | 65 | 63 | 1 | 1 | 1 | 1 | 6 | 2 | 1 | 1 |
| Belgium | 9 | 10 | 13 | 13 | 88 | 87 | 82 | 79 | 3 | 3 | 5 | 8 | 0 | 0 | 0 | 0 |
| Canada | 2 | 12 | 33 | 41 | 92 | 85 | 64 | 56 | 3 | 2 | 3 | 3 | 3 | 1 | 0 | 0 |
| Denmark | 6 | 13 | 22 | 24 | 88 | 84 | 76 | 74 | 0 | 0 | 0 | 0 | 6 | 3 | 2 | 2 |
| Finland | 5 | 8 | 11 | 10 | 60 | 59 | 50 | 42 | 34 | 32 | 39 | 48 | 1 | 0 | 0 | 0 |
| France | 20 | 27 | 36 | 41 | 70 | 69 | 61 | 56 | 2 | 2 | 2 | 2 | 8 | 2 | 1 | 1 |
| Greece | 1 | 2 | 5 | 5 | 95 | 96 | 92 | 91 | 3 | 2 | 2 | 3 | 0 | 0 | 0 | 0 |
| Iceland | 0 | 1 | 1 | 1 | 25 | 22 | 16 | 12 | 65 | 70 | 78 | 84 | 9 | 8 | 5 | 4 |
| Ireland | 4 | 2 | 1 | 2 | 26 | 18 | 11 | 7 | 69 | 79 | 88 | 91 | 1 | 1 | 0 | 0 |

## Table 4. Forecast Error Variance Decomposition of the Level of Commodity Prices and Output (continued)

| *Shock* | $X_t^p$ | | | | $z_t^p$ | | | | $X_t^i$ | | | | $z_t$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *Horizon (in years)* | 5 | 10 | 20 | 30 | 5 | 10 | 20 | 30 | 5 | 10 | 20 | 30 | 5 | 10 | 20 | 30 |
| Italy | 12 | 14 | 15 | 14 | 81 | 83 | 83 | 84 | 0 | 0 | 0 | 1 | 7 | 3 | 1 | 1 |
| Korea, Rep. | 1 | 0 | 1 | 2 | 19 | 11 | 6 | 4 | 80 | 88 | 93 | 94 | 1 | 0 | 0 | 0 |
| Luxembourg | 29 | 35 | 36 | 33 | 43 | 36 | 27 | 22 | 25 | 28 | 36 | 44 | 3 | 1 | 1 | 1 |
| Mexico | 1 | 1 | 1 | 3 | 85 | 93 | 94 | 93 | 0 | 0 | 1 | 1 | 13 | 5 | 3 | 3 |
| Netherlands | 19 | 27 | 30 | 29 | 75 | 67 | 60 | 55 | 4 | 5 | 10 | 16 | 1 | 0 | 0 | 0 |
| New Zealand | 1 | 1 | 3 | 5 | 21 | 18 | 11 | 8 | 74 | 80 | 85 | 86 | 3 | 2 | 1 | 1 |
| Norway | 6 | 4 | 5 | 4 | 77 | 77 | 70 | 63 | 14 | 17 | 24 | 32 | 3 | 2 | 1 | 1 |
| Portugal | 24 | 23 | 25 | 25 | 69 | 74 | 74 | 73 | 0 | 0 | 0 | 0 | 7 | 3 | 2 | 1 |
| South Africa | 43 | 42 | 38 | 50 | 49 | 55 | 59 | 47 | 1 | 1 | 1 | 1 | 6 | 3 | 1 | 1 |
| Spain | 7 | 18 | 30 | 33 | 87 | 80 | 69 | 65 | 0 | 0 | 1 | 1 | 6 | 1 | 1 | 1 |
| Sweden | 8 | 36 | 59 | 67 | 74 | 58 | 38 | 30 | 2 | 2 | 1 | 2 | 16 | 5 | 2 | 1 |
| Switzerland | 8 | 15 | 29 | 33 | 75 | 75 | 65 | 61 | 1 | 1 | 2 | 3 | 16 | 9 | 4 | 3 |
| Turkey | 1 | 7 | 7 | 9 | 67 | 74 | 77 | 75 | 0 | 0 | 1 | 1 | 31 | 19 | 15 | 15 |
| United Kingdom | 17 | 28 | 39 | 42 | 77 | 67 | 56 | 52 | 2 | 3 | 4 | 5 | 4 | 2 | 1 | 1 |
| Mean | 10 | 15 | 20 | 22 | 67 | 64 | 58 | 54 | 16 | 18 | 20 | 22 | 7 | 3 | 2 | 2 |
| Median | 6 | 12 | 19 | 19 | 75 | 72 | 65 | 58 | 3 | 2 | 2 | 3 | 6 | 2 | 1 | 1 |

Source: Authors' calculations.
Note: All shares are computed at the posterior mean of the estimated parameters and are expressed in percentage points.

**Figure 4. Impulse Response of Output to a Unit Long-run Increase in $X_t^p$**

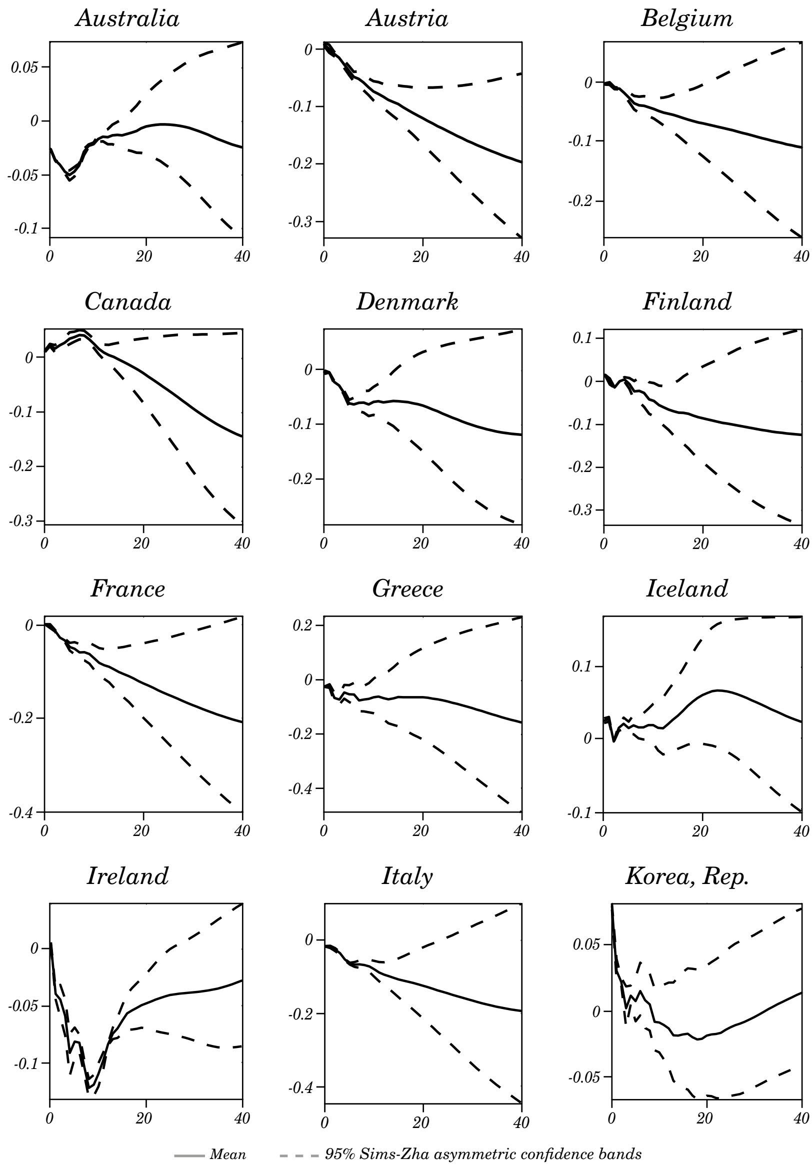
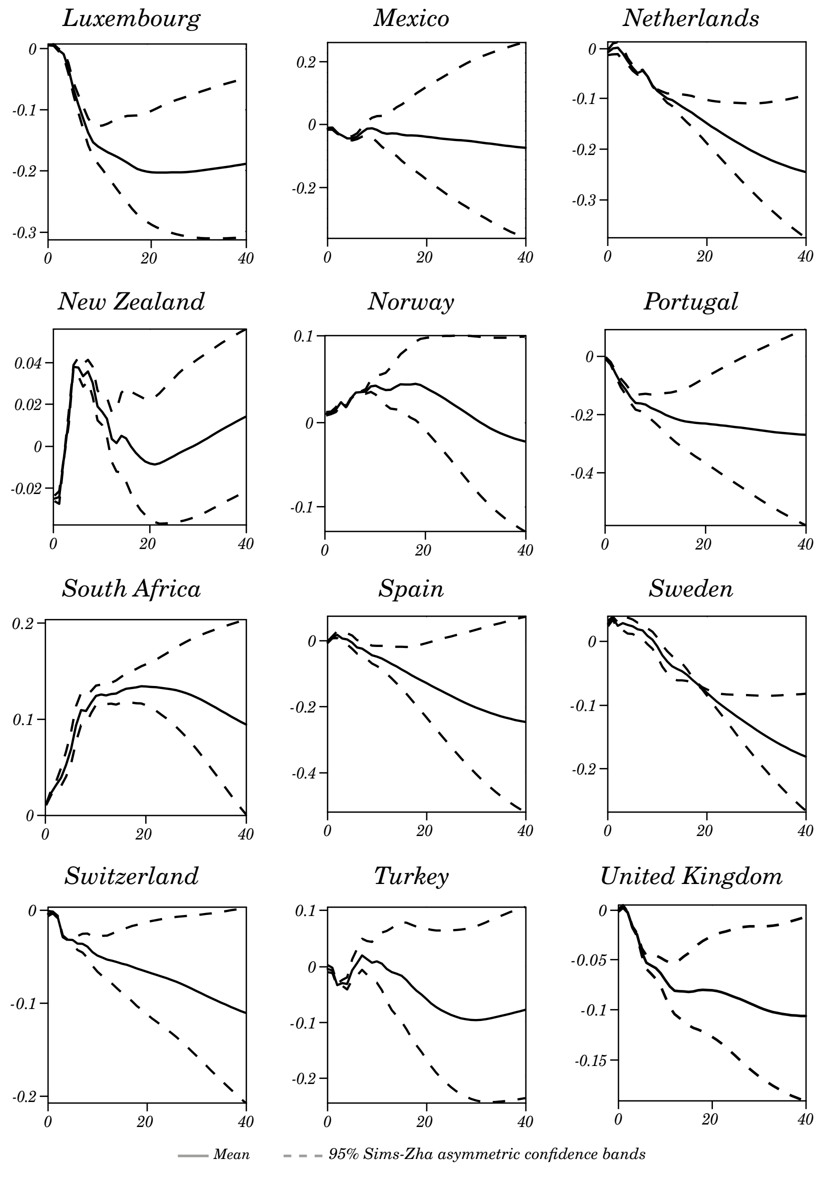


**Figure 4. Impulse Response of Output to a Unit Long-run Increase in $X_t^p$ (continued)**




Source: Authors' calculations.

The fact that the world and domestic stationary shocks, $z_t^p$ and $z_t^i$, jointly explain the majority of the forecast error variance of output even at horizons of 20 and 30 years, 65 and 60 percent on average, respectively, indicates that the world and domestic nonstationary components, $X_t^p$ and $X_t^i$, are not the dominant drivers of movements in output.

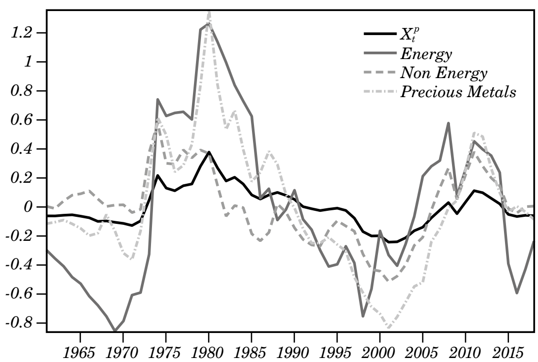
Figure 4 displays the impulse responses of the level of output, $y_t$, in each of the 24 countries, to a permanent world shock, $X_t^p$, that increases commodity prices in the long run by one percent. In most countries, the permanent commodity-price increase is contractionary. One possible explanation for this finding is that the sample includes mostly developed open economies that are not important primary commodity producers. As we will see in section 6, in emerging countries the output response to an increase in the permanent component of world prices is in general positive. But even for primary commodity producers, an increase in $X_t^p$ could have an ambiguous effect on output for at least two reasons. One is that when $X_t^p$ goes up, all commodity prices go up. To the extent that some commodities are imported and used as intermediate inputs in domestic production, an increase in $X_t^p$ would result in an increase in marginal costs, which in turn, may lower domestic employment. The second reason is that because $X_t^p$ represents a permanent increase in real commodity prices, it might entail a large positive wealth effect for the commodity-producing country. In turn, this positive wealth effect could lead to a contraction in labor supply and in this way lower equilibrium employment.

## 6. Emerging Countries

As mentioned earlier, long quarterly time series for output are available mostly for developed countries. As a result, emerging countries are underrepresented in the sample. To shed light on the importance of the commodity supercycle in emerging countries, this section turns to an analysis based on annual data for which the coverage of this group of countries is more comprehensive. The empirical model is the one described in section 1 except for the number of lags and the number of commodity prices included. Because the data is annual, the model includes only one lag of prices and output, $\hat{p}_t$ and $\hat{y}_t$. To economize on the number of parameters estimated, the eleven commodity prices are aggregated into three indices—energy, non-energy, and precious metals—, following the Pink Sheet aggregation scheme. Energy commodities include coal, crude oil, and natural gas. Non-energy commodities include beverages, food, agricultural raw materials, fertilizers, and metals and minerals. And precious metals comprise gold, silver, and platinum. The sample includes 24 emerging countries and 17 developed countries, which are listed in table 6. The output data comes from World Development Indicators. The selection of countries follows a number of criteria which include data availability since 1960, a population of more than three million in 2018, not having transitioned from a planned to a market economy, and having a common secondary data source for output. As in the analysis using quarterly data, the model is estimated using Bayesian techniques. The prior distributions for the model parameters are the same as those presented in table 1.

Figure 5 plots with thin lines the three real commodity-price indices and with a thick line their estimated permanent component, $X_t^p$. As in the case of the estimation on quarterly data, the commodity supercycle is a smooth stochastic trend of the three prices and displays two peaks since 1960—one in 1980 and the other in 2012. The peaks and troughs of the estimated commodity-price supercycle line up with the ones identified using quarterly data on the more disaggregated commodity prices plotted in figure 2.

**Figure 5. The Commodity-Price Supercycle in Annual Data**




Source: Authors' calculations.
Notes: The permanent component of the three aggregate commodity-price indices, $X_t^p$, is computed by Kalman smoothing using the posterior mean of the parameter estimates. All time series are constructed as cumulative demeaned growth rates.

**Table 5. Percent of Variance of the Growth Rate of Annual World Prices Explained by $\Delta X_t^p$**

| *Price of* | *Mean* | *Std. Dev.* |
|---|---|---|
| Energy Commodities | 98 | 1 |
| Non-Energy Commodities | 94 | 2 |
| Precious Metals | 94 | 1 |
| Mean | 95 | 1 |
| Median | 94 | 1 |
| Real Rate | 25 | 18 |

Source: Authors' calculations.
Note: The reported figures are based on 100,000 draws from the posterior distribution of the variance decomposition.

Table 5 shows that the permanent component, $X_t^p$, explains more than 90 percent of the variation in the growth rate of the three commodity indices. Thus, as in the quarterly estimation, the commodity supercycle is an important driver of commodity prices. A difference is that now the share of the variance of the growth rate of prices explained by the permanent component is much larger than the one estimated in quarterly data. This is to some extent expected since aggregation across time and commodities tends to average away the effects of commodity-specific and transitory disturbances. The table also shows that the commodity supercycle explains 25 percent of movements in the world interest rate, a share somewhat higher than the one obtained in the estimation on quarterly data (15 percent).

Table 6 displays the variance decomposition of output growth in the 24 emerging and 17 developed countries considered. As in the case of the analysis using quarterly data on a sample of mostly developed economies, all world shocks taken together, $X_t^p$ and $z_t^p$, play a major role in explaining the variance of output growth. It also continues to be the case that, of the contribution of world shocks to output fluctuations, the majority is attributed to stationary disturbances, $z_t^p$.

This pattern applies to a large extent when one limits attention to emerging countries. Within this group, on average world shocks explain more than 50 percent of the variance of output and, of this fraction, almost two thirds are attributable to stationary world shocks.[8]

The fact that stationary world shocks, $z_t^p$, explain a much larger share of the variance of output growth than of the variance of the growth rate of prices indicates that these world shocks may be only partially mediated through commodity prices. An example of a world shock that could have an output effect both directly and through world commodity prices are productivity shocks that are correlated across countries.

Table 6 also speaks to the literature on the role of stationary and nonstationary shocks in explaining business cycles in emerging countries. The posterior mean joint contribution of stationary shocks, $z_t^p$ and $z_t^i$, to the variance of output growth is 57 percent with the remaining 43 percent explained by nonstationary shocks, $X_t^p$ and $X_t^i$. This result suggests that the majority of fluctuations in aggregate activity in the emerging countries considered stems from stationary domestic and world disturbances.

The preponderance of stationary world shocks in accounting for movements in output in emerging economies also manifests itself at different forecasting horizons. Table 7 displays the forecast error variance decomposition of the level of output at horizons 5, 10, 20, and 30 years. At forecasting horizons of five and ten years, which are typically associated with business-cycle frequencies, the mean share of variance explained by the stationary world shocks, $z_t^p$, is 40 and 38 percent, respectively, compared to 19 and 24 percent explained by the nonstationary world shock, $X_t^p$. At longer forecasting horizons of 20 and 30 years, the role of nonstationary world shocks increases, as expected, but does not clearly dominate that of stationary world shocks. Specifically, the variance of the forecasting error of output explained by $X_t^p$ has a mean of 30 and 34 percent at horizons 20 and 30 years, compared to 34 and 31 percent for stationary world shocks.

8. The results are robust to estimating the model by maximum likelihood, which indicates that the findings are not due to the choice of priors. The maximum likelihood estimate assigns more importance (about 10 percentage points) to world shocks in explaining the variance of output growth. For emerging countries, of this, about one third is accounted for by the supercycle and two thirds by stationary world shocks. For developed countries, the relative importance of the supercycle is somewhat larger, with innovations to $\Delta X_t^p$ explaining about forty percent of the variance of output growth accounted for by world shocks.

## Table 6. Variance Decomposition of Output Growth — Annual Data

| | Shock | | | |
|---|---|---|---|---|
| *Country* | $\Delta X_t^p$ | $z_t^p$ | $\Delta X_t^i$ | $z_t^i$ |
| Mean Emerging | 18 | 32 | 24 | 25 |
| Mean Developed | 19 | 48 | 13 | 20 |
| Argentina | 16 | 8 | 74 | 1 |
| Bangladesh | 8 | 17 | 73 | 1 |
| Bolivia | 26 | 55 | 0 | 19 |
| Brazil | 21 | 33 | 0 | 45 |
| Chile | 10 | 18 | 0 | 71 |
| Colombia | 26 | 28 | 2 | 44 |
| Costa Rica | 25 | 42 | 28 | 4 |
| Dominican Republic | 8 | 8 | 0 | 84 |
| Ecuador | 34 | 32 | 33 | 1 |
| Guatemala | 20 | 78 | 1 | 1 |
| India | 10 | 24 | 63 | 2 |
| Indonesia | 15 | 50 | 32 | 2 |
| Korea, Rep. | 19 | 55 | 26 | 1 |
| Malaysia | 24 | 54 | 0 | 21 |
| Mexico | 12 | 40 | 47 | 1 |
| Pakistan | 8 | 35 | 57 | 1 |
| Panama | 14 | 16 | 0 | 70 |
| Paraguay | 30 | 19 | 0 | 50 |
| Peru | 18 | 19 | 0 | 63 |
| Philippines | 20 | 16 | 60 | 2 |
| South Africa | 35 | 27 | 1 | 36 |
| Thailand | 13 | 60 | 6 | 20 |
| Turkey | 4 | 16 | 79 | 0 |
| Uruguay | 18 | 21 | 0 | 61 |
| Australia | 6 | 27 | 64 | 4 |
| Austria | 25 | 51 | 1 | 22 |
| Belgium | 21 | 64 | 1 | 13 |
| Canada | 16 | 43 | 1 | 40 |
| Denmark | 21 | 54 | 2 | 21 |
| Finland | 17 | 60 | 19 | 3 |
| France | 18 | 77 | 2 | 4 |
| Greece | 22 | 47 | 0 | 30 |
| Iceland | 8 | 17 | 0 | 75 |
| Italy | 25 | 58 | 0 | 17 |
| Luxembourg | 28 | 22 | 50 | 1 |
| Netherlands | 18 | 50 | 0 | 32 |
| Norway | 10 | 39 | 3 | 49 |
| Portugal | 19 | 58 | 22 | 1 |
| Spain | 23 | 53 | 0 | 24 |
| Sweden | 14 | 63 | 21 | 2 |
| United Kingdom | 30 | 33 | 34 | 2 |

Source: Authors' calculations.

Notes: The table presents the share (expressed in percent) of the total variance of output growth explained by shocks to the permanent component of commodity prices, $\Delta X_t^p$, all stationary world price shocks taken together, $z_t^p$, the country-specific nonstationary shock, $\Delta X_t^i$, and the countryspecific stationary shock, $z_t^i$. The reported numbers are averages over 100,000 draws from the posterior distribution of the variance decomposition.

Figures 6 and 7 display the impulse response of output in the 17 developed and 24 emerging economies, respectively, to a shock in $X_t^p$ that increases energy, non-energy, and precious metal prices in the long run by one percent. The figures also include 95-percent confidence bands. In line with the results obtained in section 5 using quarterly data (figure 4), in developed economies, a permanent increase in world commodity prices is contractionary for most countries. By contrast, for most emerging countries a permanent increase in commodity prices is expansionary. As pointed out in section 5, a possible explanation for this difference could be that in emerging countries the production of primary commodities represents a larger share of total output than it does in developed countries.

## Table 7. Forecast Error Variance Decomposition of the Level of Output — Annual Data

| *Shock* | $X_t^p$ | | | | $z_t^p$ | | | | $X_t^i$ | | | | $z_t$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *Horizon (in years)* | 5 | 10 | 20 | 30 | 5 | 10 | 20 | 30 | 5 | 10 | 20 | 30 | 5 | 10 | 20 | 30 |
| Argentina | 45 | 33 | 20 | 14 | 10 | 6 | 4 | 3 | 45 | 61 | 76 | 83 | 1 | 0 | 0 | 0 |
| Bangladesh | 15 | 12 | 9 | 9 | 26 | 41 | 38 | 33 | 55 | 45 | 52 | 57 | 4 | 2 | 1 | 1 |
| Bolivia | 1 | 6 | 48 | 65 | 63 | 69 | 41 | 30 | 0 | 1 | 1 | 1 | 36 | 25 | 10 | 5 |
| Brazil | 23 | 28 | 31 | 32 | 39 | 51 | 57 | 58 | 0 | 0 | 1 | 1 | 38 | 21 | 12 | 9 |
| Chile | 5 | 5 | 11 | 20 | 15 | 16 | 15 | 14 | 0 | 1 | 3 | 4 | 80 | 78 | 71 | 63 |
| Colombia | 34 | 42 | 58 | 69 | 29 | 30 | 25 | 18 | 1 | 2 | 2 | 2 | 37 | 26 | 15 | 10 |
| Costa Rica | 11 | 10 | 38 | 60 | 68 | 61 | 35 | 18 | 17 | 26 | 26 | 21 | 3 | 2 | 1 | 0 |
| Dominican Republic | 20 | 33 | 43 | 49 | 15 | 19 | 19 | 17 | 0 | 0 | 0 | 1 | 65 | 48 | 38 | 34 |
| Ecuador | 59 | 67 | 66 | 63 | 34 | 26 | 23 | 21 | 7 | 7 | 11 | 16 | 1 | 1 | 0 | 0 |
| Guatemala | 17 | 13 | 10 | 11 | 82 | 86 | 86 | 84 | 1 | 1 | 3 | 5 | 1 | 1 | 0 | 0 |
| India | 9 | 10 | 16 | 17 | 54 | 64 | 64 | 61 | 32 | 24 | 20 | 21 | 5 | 2 | 1 | 1 |
| Indonesia | 21 | 32 | 44 | 46 | 66 | 57 | 39 | 32 | 12 | 10 | 16 | 21 | 2 | 1 | 1 | 0 |
| Korea, Rep. | 9 | 7 | 5 | 9 | 77 | 54 | 49 | 45 | 12 | 38 | 45 | 46 | 3 | 1 | 0 | 0 |
| Malaysia | 16 | 18 | 31 | 42 | 50 | 43 | 36 | 32 | 0 | 1 | 2 | 2 | 33 | 38 | 31 | 24 |
| Mexico | 19 | 22 | 24 | 24 | 70 | 64 | 57 | 53 | 10 | 14 | 19 | 23 | 1 | 0 | 0 | 0 |
| Pakistan | 0 | 2 | 16 | 28 | 71 | 67 | 48 | 37 | 27 | 29 | 35 | 35 | 2 | 2 | 1 | 0 |
| Panama | 13 | 26 | 36 | 36 | 14 | 14 | 13 | 14 | 0 | 1 | 1 | 2 | 73 | 59 | 50 | 47 |
| Paraguay | 37 | 54 | 65 | 68 | 18 | 21 | 22 | 22 | 0 | 0 | 0 | 0 | 45 | 25 | 13 | 10 |
| Peru | 24 | 38 | 38 | 35 | 13 | 12 | 20 | 27 | 0 | 0 | 1 | 1 | 63 | 50 | 42 | 37 |
| Philippines | 19 | 19 | 12 | 10 | 5 | 5 | 10 | 14 | 74 | 75 | 77 | 75 | 2 | 1 | 1 | 1 |
| South Africa | 45 | 54 | 59 | 60 | 20 | 18 | 17 | 16 | 1 | 1 | 2 | 4 | 35 | 27 | 22 | 20 |
| Thailand | 1 | 1 | 6 | 14 | 66 | 63 | 67 | 65 | 1 | 3 | 7 | 8 | 32 | 32 | 20 | 13 |
| Turkey | 4 | 3 | 4 | 6 | 35 | 22 | 12 | 8 | 61 | 75 | 84 | 86 | 1 | 0 | 0 | 0 |

**Table 7. Forecast Error Variance Decomposition of the Level of Output — Annual Data (continued)**

| *Shock* | $X_t^p$ | | | | $z_t^p$ | | | | $X_t^i$ | | | | $z_t$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *Horizon (in years)* | 5 | 10 | 20 | 30 | 5 | 10 | 20 | 30 | 5 | 10 | 20 | 30 | 5 | 10 | 20 | 30 |
| Uruguay | 22 | 31 | 33 | 30 | 13 | 12 | 18 | 25 | 0 | 1 | 1 | 2 | 64 | 56 | 48 | 43 |
| Mean-Emerging | 19 | 24 | 30 | 34 | 40 | 38 | 34 | 31 | 15 | 17 | 20 | 21 | 26 | 21 | 16 | 13 |
| Australia | 0 | 0 | 0 | 0 | 69 | 62 | 46 | 36 | 24 | 34 | 52 | 62 | 7 | 4 | 2 | 1 |
| Austria | 4 | 2 | 1 | 2 | 86 | 93 | 95 | 94 | 1 | 1 | 1 | 2 | 9 | 4 | 2 | 2 |
| Belgium | 9 | 7 | 9 | 13 | 86 | 90 | 88 | 83 | 1 | 1 | 2 | 3 | 4 | 2 | 1 | 1 |
| Canada | 15 | 15 | 15 | 14 | 50 | 59 | 64 | 65 | 1 | 1 | 2 | 3 | 35 | 25 | 19 | 17 |
| Denmark | 13 | 12 | 8 | 7 | 68 | 75 | 81 | 82 | 2 | 3 | 5 | 6 | 16 | 9 | 6 | 5 |
| Finland | 7 | 4 | 3 | 3 | 86 | 87 | 85 | 81 | 3 | 6 | 10 | 14 | 4 | 3 | 2 | 2 |
| France | 2 | 1 | 5 | 13 | 94 | 96 | 92 | 84 | 1 | 2 | 2 | 2 | 3 | 2 | 1 | 1 |
| Greece | 21 | 30 | 38 | 41 | 55 | 59 | 57 | 55 | 0 | 0 | 0 | 0 | 24 | 11 | 5 | 4 |
| Iceland | 8 | 14 | 26 | 35 | 27 | 41 | 46 | 43 | 0 | 0 | 1 | 1 | 65 | 44 | 28 | 21 |
| Italy | 5 | 5 | 18 | 29 | 84 | 90 | 80 | 70 | 0 | 0 | 0 | 0 | 11 | 4 | 2 | 1 |
| Luxembourg | 28 | 37 | 33 | 27 | 47 | 32 | 22 | 18 | 23 | 31 | 45 | 55 | 1 | 1 | 0 | 0 |
| Netherlands | 3 | 5 | 6 | 6 | 72 | 81 | 85 | 86 | 0 | 1 | 1 | 1 | 24 | 13 | 8 | 7 |
| Norway | 1 | 6 | 17 | 24 | 48 | 66 | 69 | 65 | 3 | 4 | 4 | 4 | 48 | 23 | 10 | 7 |
| Portugal | 4 | 3 | 2 | 3 | 91 | 92 | 92 | 90 | 4 | 4 | 5 | 7 | 1 | 1 | 0 | 0 |
| Spain | 6 | 15 | 21 | 22 | 69 | 72 | 72 | 72 | 0 | 0 | 1 | 1 | 25 | 12 | 7 | 6 |
| Sweden | 4 | 3 | 6 | 10 | 89 | 88 | 77 | 66 | 5 | 8 | 16 | 23 | 2 | 2 | 1 | 1 |
| United Kingdom | 29 | 36 | 37 | 35 | 52 | 39 | 31 | 26 | 17 | 24 | 32 | 39 | 2 | 1 | 0 | 0 |
| Mean-Developed | 9 | 12 | 14 | 17 | 69 | 72 | 70 | 66 | 5 | 7 | 11 | 13 | 17 | 9 | 5 | 4 |

Source: Authors' calculations.
Note: All shares are computed at the posterior mean of the estimated parameters and are expressed in percentage points.

**Figure 6. Impulse Responses of Output to a Unit Long-Run Increase in $X_t^p$ — Developed Economies**

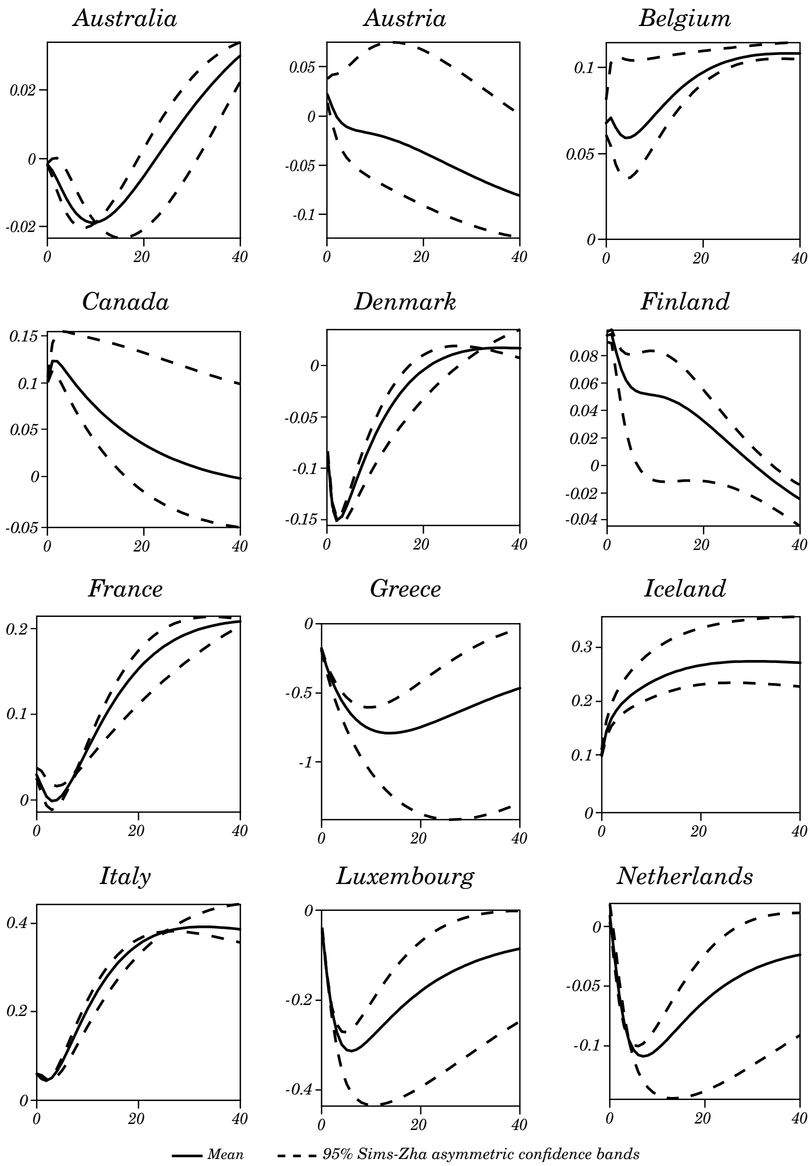
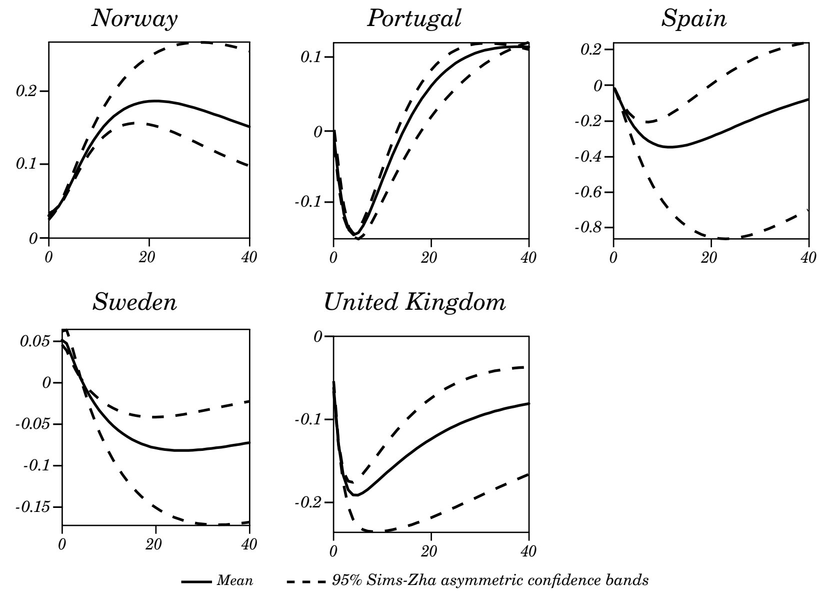


**Figure 6. Impulse Responses of Output to a Unit Long-Run Increase in $X_t^p$ — Developed Economies (continued)**




Source: Authors' calculations.

**Figure 7. Impulse Response of Output to a Unit Long-run Increase in $X_t^p$: Emerging Economies**

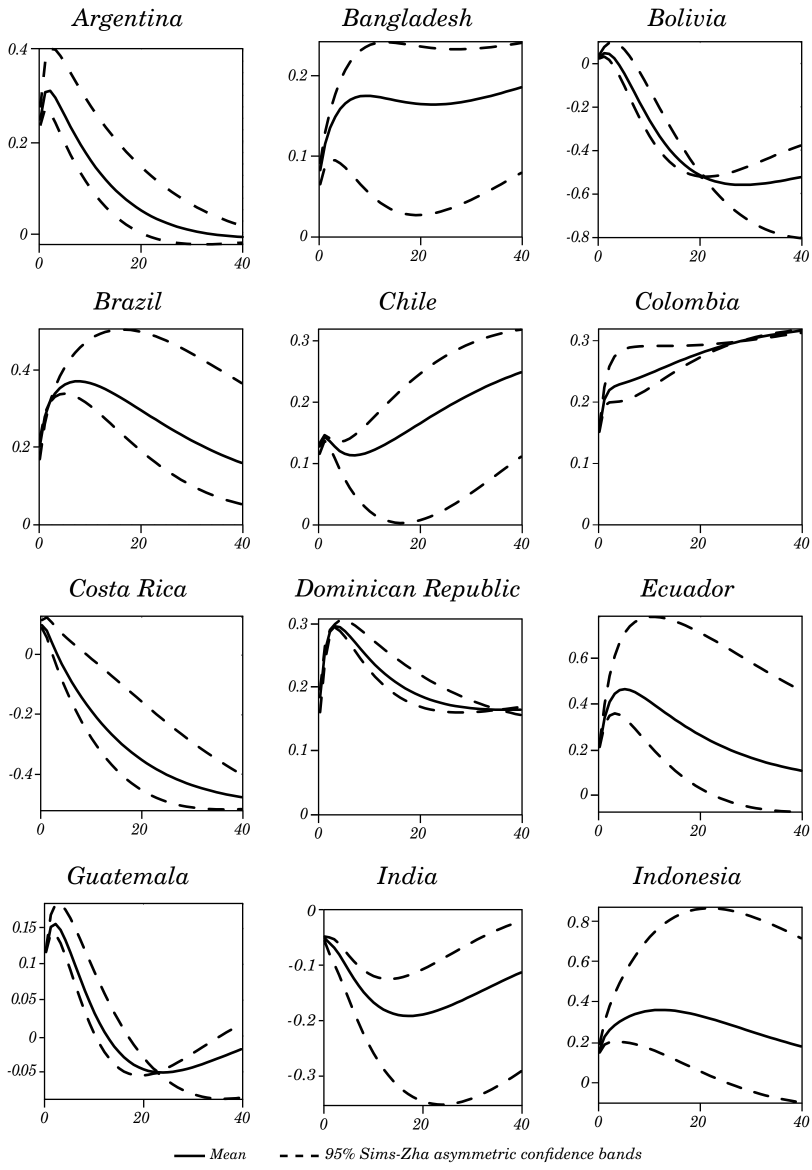
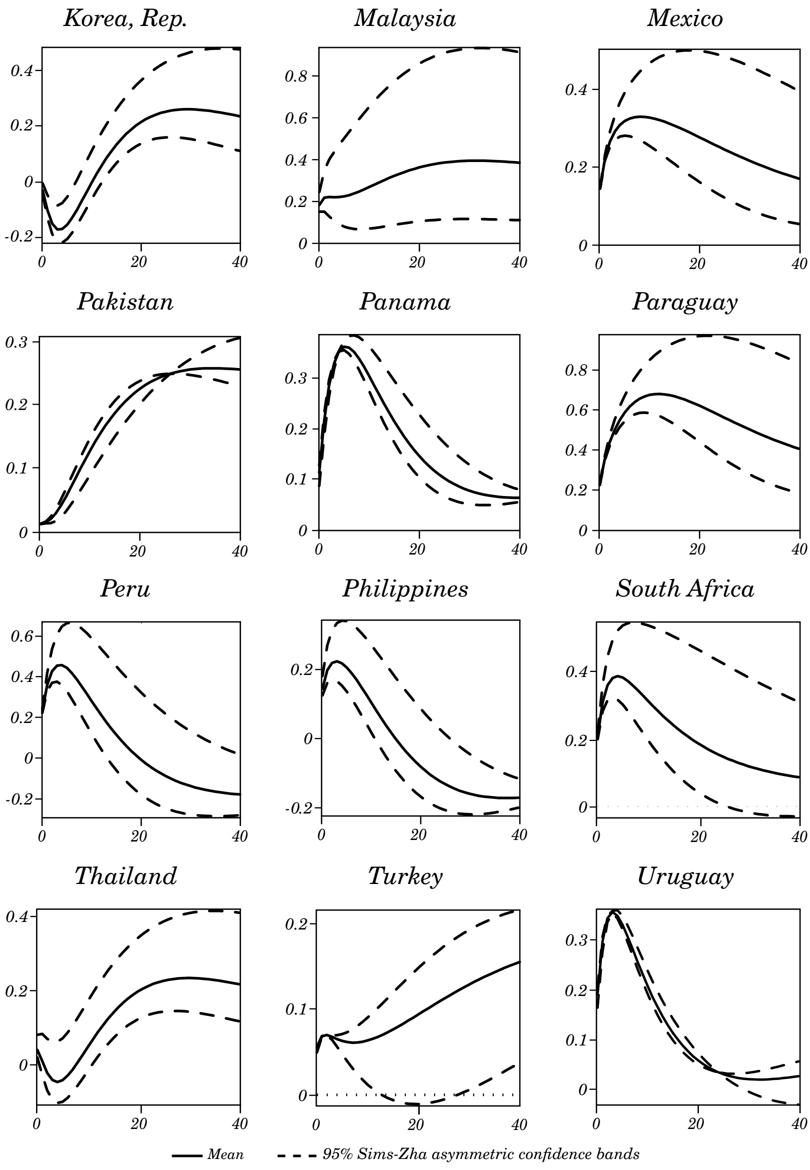


**Figure 7. Impulse Response of Output to a Unit Long-run Increase in $X_t^p$: Emerging Economies (continued)**




Source: Authors' calculations.

## 7. Conclusion

This paper aims to fill a gap in the literature on the transmission of world shocks through commodity prices in open economies. An existing literature has documented the presence of a commodity-price supercycle. An empirical technique employed in many of these studies is based on spectral analysis and identifies one supercycle per commodity price. The resulting supercycles are positively correlated across commodities suggesting a common driver.

The first contribution of the present paper is to propose an alternative definition of the commodity-price supercycle consisting in representing it as the common stochastic trend in all commodity prices. The so-identified supercycle turns out to share a number of key characteristics with the ones obtained using spectral analysis. An advantage of the common permanent component approach is that it lends itself to a joint estimation of the contributions of domestic and foreign transitory and permanent shocks to aggregate fluctuations in open economies.

The results of the paper suggest that world shocks are responsible for more than half of observed variations in aggregate activity in developed and emerging economies. However, more than two thirds of the contribution of world shocks is due to temporary disturbances, leaving less than one third to the permanent world shock that drives the commodity supercycle. This result is obtained both unconditionally and conditional on forecasting horizons. Importantly, even at horizons of 20 and 30 years, which are typically associated with the periodicity of the commodity-price supercycle, the permanent world shock does not clearly dominate temporary world shocks in accounting for variations in aggregate activity.

Taken together, these findings indicate that the permanent world shock that drives the commodity-price supercycle does matter but does not play the central role in shaping short- or medium-run business-cycle fluctuations.

## References

Aguiar, M. and G. Gopinath. 2007. "Emerging Market Business Cycles: The Cycle is the Trend." *Journal of Political Economy* 115: 69–102.

Alquist, R., S. Bhattarai, and O. Coibion. 2020. "Commodity-Price Comovement and Global Economic Activity." *Journal of Monetary Economics* 112: 41–56.

Benguria, F., F. Saffie, and S. Urzúa. 2018. "The Transmission of Commodity Price Super-Cycles." Unpublished manuscript, University of Maryland.

Cuddington, J.T. and D. Jerrett. 2008. "Super Cycles in Real Metals Prices?" IMF Staff Papers 55: 541–65.

Chang, R. and A. Fernández. 2013. "On the Sources of Aggregate Fluctuations in Emerging Economies." I*nternational Economic Review* 54: 1265–93.

Christiano, L.J. and T.J. Fitzgerald. 2003. "The Band Pass Filter." *International Economic Review* 44: 435–65.

Drechsel, T. and S. Tenreyro. 2018. "Commodity Booms and Busts in Emerging Economies." *Journal of International Economics* 112: 200–18.

Erten, B. and J.A. Ocampo. 2013. "Super Cycles of Commodity Prices since the Mid-Nineteenth Century." *World Development* 44: 14–30.

Fernández, A., S. Schmitt-Grohé, and M. Uribe. 2017. "World Shocks, World Prices, and Business Cycles: An Empirical Investigation." *Journal of International Economics* 108: S2–S14.

Fernández, A., A. González, and D. Rodríguez. 2018. "Sharing a Ride on the Commodities Roller Coaster: Common Factors in Business Cycles of Emerging Economies." *Journal of International Economics* 111: 99–121.

Galí, J. 1999. "Technology, Employment, and the Business Cycle: Do Technology Shocks Explain Aggregate Fluctuations?" *American Economic Review* 89: 249–271.

García-Cicco, J., R. Pancrazi, and M. Uribe. 2010. "Real Business Cycles in Emerging Countries?" *American Economic Review* 100: 2510–31.

Kaminsky, G.L., C.M. Reinhart, and C. Vaegh. 2004. "When It Rains, It Pours: Procyclical Capital Flows and Macroeconomic Policies." In NBER Macroeconomics Annual 2004 19: 11–53

Kehoe, T.J. and K.J. Ruhl. 2008. "Are Shocks to the Terms of Trade Shock to Productivity?" *Review of Economic Dynamics* 11: 804–19.

Kose, M.A. 2002. "Explaining Business Cycles in Small Open Economies, How Much Do World Prices Matter?" *Journal of International Economics* 56: 299–327.

Mendoza, E. 1995. "The Terms of Trade, the Real Exchange Rate, and Economic Fluctuations." *International Economic Review* 36: 101–37.

Miyamoto, W. and T.L. Nguyen. 2017. "Business Cycles in Small Open Economies: Evidence from Panel Data between 1900 and 2013." *International Economic Review* 58: 1007–44.

Pindyck, R.S. and J.J. Rotemberg. 1990. "The Excess Co-Movement of Commodity Prices." *Economic Journal* 100: 1173–89.

Prebisch, R. 1950. "The Economic Development of Latin America and Its Principal Problems." *Economic Bulletin for Latin America* 7: 1–12.

Schmitt-Grohé, S. and M. Uribe. 2018. "How Important Are Terms-of-Trade Shocks?" *International Economic Review* 59: 85–111.

Shousha, S. 2016. "Macroeconomic Effects of Commodity Booms and Busts: The Role of Financial Frictions." Working paper, Board of Governors of the Federal Reserve System.

Sims, C. and T. Zha. 1999. "Error Bands for Impulse Responses." *Econometrica* 67: 1113–56.

Singer, H. 1950. "The Distribution of Gains between Investing and Borrowing Countries." *American Economic Review*, *Papers and Proceedings* 40: 473–85.

Singh, A. 2020. "Clustered Default." Working paper, ITAM.

Uribe, M. 2018. "The Neo-Fisher Effect: Econometric Evidence from Empirical and Optimizing Models." NBER Working Paper 25089. *American Economic Journal: Macroeconomics* 14 (3): 133–62.

## APPENDIX

In section 1, the presentation of the model omitted constant terms to facilitate the exposition. This Appendix presents the model including those omitted constant terms. As we will see, this will introduce a vector of constants, denoted $A$, into the observation equation (9). We will also derive expressions for the matrices $F$, $P$, and $H$ of the state-space representation of the model, equations (8) and (9).

Redefine the vectors $\hat{p}_t$, $\hat{y}_t$, and $u_t$ as deviations from their respective means:

$$\hat{p}_t = \begin{bmatrix} p_t^1 - X_t^p - E\left(p_t^1 - X_t^p\right) \\ p_t^2 - X_t^p - E\left(p_t^2 - X_t^p\right) \\ \vdots \\ p_t^{11} - X_t^p - E\left(p_t^{11} - X_t^p\right) \\ r_t - Er_t \end{bmatrix};$$

$$\hat{y}_t = \begin{bmatrix} y_t^1 - X_t^1 - \alpha^1 X_t^p - E\left(y_t^1 - X_t^1 - \alpha^1 X_t^p\right) \\ y_t^2 - X_t^2 - \alpha^2 X_t^p - E\left(y_t^2 - X_t^2 - \alpha^2 X_t^p\right) \\ \vdots \\ y_t^{24} - X_t^{24} - \alpha^{24} X_t^p - E\left(y_t^{24} - X_t^{24} - \alpha^{24} X_t^p\right) \end{bmatrix}$$

and

$$u_t \equiv \begin{bmatrix} \Delta X_t^p - E\left(\Delta X_t^p\right) \\ \Delta X_t - E\left(\Delta X_t\right) \\ z_t^p - E\left(z_t^p\right) \\ z_t - E\left(z_t\right) \end{bmatrix}.$$

The evolution of the vector $\hat{p}_t$ is

$$\hat{p}_t = \sum_{i=1}^{4} B_{pp}^i \hat{p}_{t-1} + C_{pX^p}\left(\Delta X_t^p - E\left(\Delta X_t^p\right)\right) + C_{pz^p}\left(z_t^p - E\left(z_t^p\right)\right). \tag{10}$$

The evolution of the vector $\hat{y}_t$ is

$$\hat{y}_t = \sum_{i=1}^{4} B_{yp}^{i} \hat{p}_{t-i} + \sum_{i=1}^{4} B_{yy}^{i} \hat{y}_{t-i}$$
$$+ C_{yX^p}\left(\Delta X_t^p - E\left(\Delta X_t^p\right)\right) + C_{yz^p}\left(z_t^p - E\left(z_t^p\right)\right) \tag{11}$$
$$+ C_{yX}\left(\Delta X_t - E\left(\Delta X_t\right)\right) + \left(z_t - E\left(z_t\right)\right).$$

The evolution of the exogenous shocks, $u_t$, is

$$u_t = \rho u_{t-1} + \psi \nu_t. \tag{12}$$

Let

$\hat{x}_t = \left[\hat{p}_t \ \hat{y}_t\right]'$; and $\xi_t = \left[\hat{x}_t \ \ \hat{x}_{t-1} \ \ \hat{x}_{t-2} \ \ \hat{x}_{t-3} \ \ u_t\right]'$.

The system of equations (10), (11), and (12) can then be expressed as:

$$\hat{x}_{t+1} = B\begin{bmatrix} \hat{x}_t \\ \hat{x}_{t-1} \\ \hat{x}_{t-2} \\ \hat{x}_{t-3} \end{bmatrix} + C u_{t+1}, \tag{13}$$

where

$$B \equiv \begin{bmatrix} B_{pp}^1 & \varnothing_{12\times24} & B_{pp}^2 & \varnothing_{12\times24} & B_{pp}^3 & \varnothing_{12\times24} & B_{pp}^4 & \varnothing_{12\times24} \\ B_{yp}^1 & B_{yy}^1 & B_{yp}^2 & B_{yy}^2 & B_{yp}^3 & B_{yy}^3 & B_{yp}^4 & B_{yy}^4 \end{bmatrix}$$

and

$$C \equiv \begin{bmatrix} C_{\hat{p}X^p} & \varnothing_{12\times24} & C_{\hat{p}z^p} & \varnothing_{12\times24} \\ C_{\hat{y}X^p} & C_{\hat{y}X} & C_{\hat{y}z^p} & I_{24\times24} \end{bmatrix}.$$

The vector $\xi_t$ evolves over time as

$$\xi_{t+1} = F\xi_t + P\nu_{t+1},$$

where

$$F = \begin{bmatrix} B & C\rho \\ I_{108\times144} & \varnothing_{108\times61} \\ \varnothing_{61\times144} & \rho \end{bmatrix}; \text{ and } P = \begin{bmatrix} C\psi \\ \varnothing_{108\times61} \\ \psi \end{bmatrix}.$$

Given the redefinition of $\hat{p}_t$, the observation equations (4), (5), and (6) become

$$\Delta p_t^i = \Delta \hat{p}_t^i + \left(\Delta X_t^p - E\left(\Delta X_t^p\right)\right) + E\left(\Delta X_t^p\right); i = 1,\ldots,11,$$

$$\Delta y_t^i = \Delta \hat{y}_t^i + \left(\Delta X_t^i + \alpha^i \Delta X_t^p - E\left(\Delta X_t^i + \alpha^i \Delta X_t^p\right)\right)$$
$$+ E\left(\Delta X_t^i + \alpha^i \Delta X_t^p\right); i = 1,\ldots,24,$$

and

$$r_t = \left(r_t - Er_t\right) + Er_t.$$

In vector form, the observation equations can be expressed as

$$o_t = A' + H'\xi_t + \mu_t$$

where

$$A = \left[E\left(\Delta X_t^p\right)\ldots E\left(\Delta X_t^p\right)\ Er_t\ \ E\left(\Delta X_t^1 + \alpha^1 \Delta X_t^p\right)\ \ldots\ E\left(\Delta X_t^{24} + \alpha^{24} \Delta X_t^p\right)\right],$$

$$H' = \left[I_{36\times36} \quad H_2 \quad \varnothing_{36\times72} \quad H_4 \quad H_5 \quad \varnothing_{36\times36}\right]$$

with

$$H_2 = -\begin{bmatrix} I_{11\times12} & \varnothing_{11\times24} \\ \varnothing_{1\times12} & \varnothing_{1\times24} \\ \varnothing_{24\times12} & I_{24\times24} \end{bmatrix}$$

$$H_4 = \left[1_{1\times11}\ \ 0\ \ \alpha^1\ \ \ldots\ \alpha^{24}\right]'$$

$$H_5 = \begin{bmatrix} \varnothing_{12\times24} \\ I_{24\times24} \end{bmatrix}.$$