# Optimal Intermediate Good Taxation: A Primal Approach

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## 1 Introduction

In this note, we derive the optimal tax structure in a two-sector closed economy. An upstream sector produces an intermediate good using labor as the sole input. A downstream sector produces a final good using as inputs labor and intermediate materials. Households derive utility from consumption and labor and own the firms. The government consumes an exogenous amount of final goods and does not have access to lump-sum taxation. Thus, it finances its consumption levying taxes on intermediate and final good transactions and on labor income. A central result of the note is the optimality of not taxing intermediate good transactions (Diamond and Mirrlees, 1971). The key difference between the present analysis and that of Diamond and Mirrlees, is that these authors follow a dual approach whereas here we adopt a primal approach, which is more in keeping with modern public finance.

## 2 The Model

This section presents the three economic units in the model economy, households, firms, and the government.

### 2.1 Households

The economy is populated by a large number of identical households with preferences described by the utility function

$$U(c, h),$$

where $c$ denotes consumption, $h$ denotes labor effort, and $U$ is the utility function, assumed to be increasing in its first argument, decreasing in its second argument, and strictly concave. The budget constraint of the household is

$$\tau^c P c = W h + \Pi,$$

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where $P$ is the price of the consumption good, $W$ is the wage rate, $\tau^c$ is the gross consumption tax rate, and $\Pi$ are profits received from firms, which the household takes as given.

The household's problem consists in choosing consumption and labor to maximize its utility. The first-order condition associated with this problem is

$$-\frac{U_h(c,h)}{U_c(c,h)} = \frac{1}{\tau^c}\frac{W}{P}. \tag{1}$$

Here, an increase in the consumption tax rate is akin to a fall in the real wage, discouraging consumption and labor in favor of leisure.

## 2.2 Firms

The upstream firm produces materials, denoted $m$, using labor, denoted $h^u$,

$$m = G(h^u), \tag{2}$$

where $G$ is a production function, assumed to be increasing and concave. Profits are given by

$$\Pi = P_m G(h^u) - \tau^l W h^u,$$

where $P_m$ is the price of the intermediate good, and $\tau^l$ is the gross tax rate on labor. The firm chooses $h^u$ to maximize $\Pi$. The first-order condition of this problem is

$$G'(h^u) = \tau^l \frac{W}{P}\frac{P}{P_m}. \tag{3}$$

An increase in the real wage or in the labor tax rate discourages employment and an increase in the relative price of materials in terms of the final good fosters it.

The downstream firm produces the final good using labor and intermediate goods,

$$y = F(h^d, m),$$

where $y$ denotes output, $h^d$ denotes employment in the downstream sector, and $F$ is assumed to be increasing in both arguments and homogeneous of degree one. Profits are given by

$$PF(h^d, m) - \tau^l W h^d - \tau^m P_m m,$$

where $\tau^m$ is the gross tax rate on purchases of intermediate materials. The downstream firm chooses $h^d$ and $m$ to maximize profits. The first-order conditions associated with this problem are

$$F_h(h^d, m) = \tau^l \frac{W}{P} \tag{4}$$

and

$$F_m(h^d, m) = \tau^m \frac{P_m}{P}. \tag{5}$$

The tax rate $\tau^m$ distorts the price of materials perceived by the downstream firm. The central question this note aims to answer is what is its optimal value in the present environment in which the government needs distortionary taxation to finance its consumption.

Because the production function displays constant returns to scale, profits must be zero in equilibrium

$$PF(h^d, m) - \tau^l W h^d - \tau^m P_m m = 0.$$

## 2.3 The Government

The government consumes a constant amount $g$ of final goods. It finances this expenditure exclusively through distortionary taxes. Its budget constraint is

$$Pg = (\tau^c - 1)Pc + (\tau^l - 1)W(h^u + h^d) + (\tau^m - 1)P_m m.$$

## 2.4 Equilibrium

In equilibrium, the labor market must clear

$$h^u + h^d = h. \tag{6}$$

Combining the budget constraint of the household and the government, the profit equation in the upstream sector, and the zero-profit condition in the downstream sector yields the resource constraint

$$c + g = F(h^d, m). \tag{7}$$

A private-sector competitive equilibrium is defined as follows:

**Definition 1 (Private-Sector Competitive Equilibrium)** *A private-sector competitive equilibrium is a collection of positive scalars* $c$, $h$, $h^u$, $h^d$, $m$, $W/P$, *and* $P_m/P$ *satisfying conditions* (1)-(7)*, given the tax rates* $\tau^c$, $\tau^l$, *and* $\tau^m$ *and the level of government consumption* $g$.

The qualification "private-sector" in Definition 1 is necessary because, in the presence of only distortionary taxes (i.e., in the absence of lump-sum taxation), there is no guarantee that the government's budget constraint will be satisfied for arbitrary tax rates. We therefore introduce the following definition of a competitive equilibrium.

**Definition 2 (Competitive Equilibrium)** *A competitive equilibrium is a private-sector competitive equilibrium for which the tax policy* $(\tau^c, \tau^l, \tau^m)$ *satisfies*

$$g = (\tau^c - 1)c + (\tau^l - 1)\frac{W}{P}(h^u + h^d) + (\tau^m - 1)\frac{P_m}{P}m, \tag{8}$$

*given government consumption* $g$.

It follows from this definition that, even though the government has three taxes at its disposal, it has only two degrees of freedom in choosing them.

# 3 Optimal Taxation

The following proposition shows that, by appropriately setting taxes, the government can achieve any feasible real allocation.

**Proposition 1 (Implementability)** *Any sectoral allocation of labor* $h^u, h^d > 0$ *can be supported as a competitive equilibrium through an appropriate use of tax instruments.*

**Proof** Given $h^u$ and $h^d$, $h$ can be obtained from (6) and $m$ from (2). Then, given $h^d$ and $m$, $c$ can be obtained from (7). Equilibrium conditions (1) and (4) then jointly determine $\tau^c\tau^l$ and $\tau^l W/P$. With this information, one can back out the relative price $P_m/P$ from equation (3) and $\tau^m$ from equation (5). Then, (8) can be used to obtain $\tau^c$. Finally, knowing $\tau^c$, $\tau^l\tau^c$, and $\tau^l W/P$, one obtains $\tau^l$ and $W/P$. ■

Then the Ramsey problem can be written as

$$\max_{\{h^d,h^u\}} U(F(h^d, G(h^u)) - g, h^u + h^d),$$

where we use (2), (6), and (7) to eliminate $c$, $h$, and $m$ from the household's utility function. Note that the Ramsey problem coincides delivers the first-best allocation because it consists in maximizing utility subject only to the production technologies and the resource constraints.

The first-order conditions associated with this problem are (I omit the arguments in the interest of space)

$$U_c F_h = -U_h$$

and

$$U_c F_m G' = -U_h.$$

These are two equations in two unknowns, and the solution yields the Ramsey optimal levels of sectoral employment, $h^u$ and $h^d$. Proposition 1 then indicates how to obtain the Ramsey optimal values of all remaining variables of the model, including the optimal tax rates.

We are ready to derive the main result of this note, namely, the optimality of not taxing transactions in intermediate goods. Dividing the above two Ramsey optimality conditions side by side, one obtains

$$\frac{F_h}{F_m} = G'. \tag{9}$$

Now using (3) to eliminate $\tau^l W/P$ from (4) gives

$$F_h = G' \frac{P_m}{P}.$$

Using this expression to eliminate $P_m/P$ from (5) yields

$$\frac{F_h}{F_m} = \tau^m G'. \tag{10}$$

Comparing (9) and (10), it follows that

$$\tau^m = 1,$$

that is, the Ramsey optimal allocation implies no taxes on purchases of intermediate goods of the downstream firm from the upstream firm.

Under the Ramsey optimal allocation, the labor market is undistorted in the sense that the marginal rate of substitution of consumption for labor is equal to the marginal product of labor in the final good market. This can be seen from the first-order condition of the Ramsey problem with respect to $h^d$, which rearranged gives

$$-\frac{U_h}{U_c} = F_h. \tag{11}$$

This implies that the optimal tax wedge in the labor market, $\tau^c\tau^l$, is equal to unity. To see this, combine equilibrium conditions (1) and (4) to obtain

$$-\frac{U_h}{U_c} = \frac{1}{\tau^c\tau^l} F_h. \tag{12}$$

Comparing (11) and (12) yields that under the Ramsey optimal allocation

$$\tau^c\tau^l = 1.$$

We conclude that the Ramsey planner can achieve the first-best allocation by using only labor and consumption taxes. Note further that the Ramsey optimal consumption and labor tax rates are non-zero, $\tau^c = 1/\tau^l \neq 1$. The Ramsey planner uses these two instruments in a non-distorting way with the sole purpose of raising revenue in the amount $g$. If in the first-best allocation the labor income tax base, $W/Ph$, is larger than the consumption tax base, $c$, the planner will tax labor income and subsidize consumption and vice versa. As it turns out, the government will find it optimal to tax consumption and to subsidize labor. To see this, write equation (8) as

$$\begin{aligned} g &= (\tau^c - 1)c + (\tau^l - 1)\frac{W}{P}(h^u + h^d) \\ &= \left(1 - \frac{1}{\tau^c}\right)\tau^c c + \left(\frac{1}{\tau^c} - 1\right)\frac{W}{P}h \\ &= \left(1 - \frac{1}{\tau^c}\right)\left(\frac{W}{P}h + \frac{\Pi}{P}\right) + \left(\frac{1}{\tau^c} - 1\right)\frac{W}{P}h \\ &= \left(1 - \frac{1}{\tau^c}\right)\frac{\Pi}{P}. \end{aligned} \tag{13}$$

The first equality uses the fact that in the Ramsey allocation $\tau^m = 1$. In the second equality we replace $\tau^l$ by $1/\tau^c$ and $h^u + h^d$ by $h$. And in the third equality, we replace $\tau^c c$ by $W/Ph + \Pi/P$ from the household's budget constraint. Since both $g$ and $\Pi$ are positive, it follows that $\tau^c$ must be bigger than one. The above expression also says that the Ramsey optimal consumption tax rate is approximately equal to the government-spending-to-profit ratio.

## 4 Constant Returns to Scale

Thus far we have considered the case that $G(\cdot)$ is concave. This assumption guarantees that profits $\Pi/P$ are positive. It follows from equation (13) that in this case the consumption tax rate $\tau^c$ and therefore its inverse $\tau^l$ are well defined. If the production technology $G(\cdot)$

is constant returns to scale (linear), then profits are nil and the policy $\tau^c\tau^l = 1$ fails to raise any tax revenues and therefore is infeasible and cannot be the solution to the Ramsey problem.

The procedure of maximizing utility subject to the constraints of the private sector equilibrium and then pick tax rates to make sure that the government constraint is satisfied, which works when the technology in the upstream sector is strictly concave, does not work when this sector has linearly homogeneous technology. The Ramsey problem must take explicitly into account the government's budget constraint to ensure feasibility. This restriction will be an implementability constraint. We wish to express the implementability constraint only in terms of $h^u$ and $h^d$. To this end, begin with equation (8).

$$\begin{aligned} g &= (\tau^c - 1)c + (\tau^l - 1)\frac{W}{P}(h^u + h^d) + (\tau^m - 1)\frac{P_m}{P}m \\ &= (\tau^c - 1)c + (\tau^l - 1)\tau^c c + (\tau^m - 1)\frac{P_m}{P}m \\ &= \tau^l\tau^c c - c + (\tau^m - 1)\frac{P_m}{P}m \\ &= \tau^l\tau^c c - c + F_m m - \frac{F_h}{G'}m \\ &= \tau^l\tau^c c - c + F_m m - F_h h^u \\ &= \tau^l\tau^c c - c + F - F_h(h^d + h^u) \\ &= \tau^l\tau^c c + g - F_h(h^d + h^u). \end{aligned} \tag{14}$$

The second equality uses the household's budget constraint to replace $W/P(h^u + h^d)$ by $\tau^c c$ (recall that $\Pi = 0$). The third equality is a rearrangement. The fourth equality uses (5) to replace $\tau^m P_m/P$ by $F_m$ and (3) and  (4) to replace $P_m/P$ with $F_h/G'$. The fifth equality uses the linearity of $G$ to replace $m/G'$ by $h^u$. The sixth equality uses the fact that $F = F_m m + F_h h^d$, as $F$ is linearly homogeneous. And the seventh and last line replaces $-c + F$ by $g$ invoking the resource constraint (7). Cancelling $g$ from both sides and combining (1) and (4), to replace $\tau^c\tau^l$ by $-F_h U_c/U_h$, we can write

$$0 = -F_h U_c/U_h c - F_h(h^d + h^u).$$

Multiplying both sides by $-U_h/F_h$ we obtain

$$0 = U_c(c, h)c + U_h(c, h)h. \tag{15}$$

This implementability condition is ubiquitous in Ramsey problems without lump-sum taxation. Condition (15) is necessary and sufficient to support a competitive equilibrium. Since we derived it using only equilibrium conditions, the necessary condition is satisfied by construction. To show sufficiency assume that $c$ and $h$ satisfy (15). Pick $h^u$ to satisfy $c + g = F(h - h^u, G(h^u))$. Then pick $m$ to satisfy $m = G(h^u)$ and $h^d$ to satisfy $h^d = h - h^u$. It follows that equilibrium conditions (1), (6), and (7) hold. Pick $\tau^c\tau^l = -(U_c(c, h)/U_h(c, h))F_h(h^d, m)$. Set $P_m/P = F_h(h^d, m)/G'(h^u)$. Set $\tau^m = F_m(h^d, m)P/P_m$. Set $\tau^l W/P = F_h(h^d, m)$. The latter four restrictions ensure that equilibrium conditions (1) and (3)-(5) hold. Finally, to show that equilibrium condition (8) holds, retrace your steps from (15) to the top of equation (14).

The Ramsey problem with constant returns to scale in production is then

$$\max_{\{h^d,h^u\}} U(c(h^u,h^d),h(h^u,h^d))$$

subject to

$$U_c(c(h^u,h^d),h(h^u,h^d))c(h^u,h^d)+U_h(c(h^u,h^d),h(h^u,h^d))h(h^u,h^d)=0,$$

where $c(h^u,h^d)=F(h^d,G(h^u))-g$ and $h(h^u,h^d)=h^d+h^u$. Let the Lagrangian of this problem be

$$L(c(h^u,h^d),h(h^u,h^d),\lambda).$$

Then, the first-order conditions with respect to $h^d$ and $h^u$ are

$$L_cF_h+L_h=0$$

and

$$L_cF_mG'+L_h=0.$$

Combining these two expressions, we obtain

$$\frac{F_h}{F_m}=G'.$$

Comparing this expression with (10), we obtain

$$\tau^m=1.$$

Thus, the result that it is optimal not to tax transactions between downstream and upstream firms is robust to assuming that both sectors have constant returns to scale technologies.

One difference with the case of decreasing returns to scale is that with constant returns to scale the planner will not achieve the first-best allocation in general. The government cannot set $\tau^c\tau^l=1$ because with constant returns to scale this combination collects zero taxes, so it must distort the labor market in order to raise resources to finance its consumption. Because the equilibrium conditions (including the government's budget constraint) depend only on $\tau^l\tau^c$, it follows that the solution to the Ramsey problem pins town $\tau^l\tau^c$ but not $\tau^l$ and $\tau^c$ individually. Without loss of generality, one could therefore eliminate one of these two taxes. For example, one could set $\tau^c=1$. Then the Ramsey optimal allocation can be obtained through labor income taxation alone.

Finally, the result that it is Ramsey optimal not to tax intermediate goods does not depend on the assumption that the government is benevolent, that it, that it maximizes the utility of the representative household. This is clear from the fact that one can write the Lagrangian of the Ramsey planner in the generic form $L(c,h,\lambda)$ without reference to the specific form of the government's welfare function.

# References

Diamond, Peter A., and James A. Mirrlees, "Optimal Taxation and Public Production I: Production Efficiency," *The American Economic Review 61,* 1971, 8–27.