Lecture On The Optimal Tariff

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In this note, I analyze the optimal tariff in a two-country world. I derive a number of classic results, including Lerner symmetry and the optimal tariff being equal to the elasticity of the offer curve.[1] I also generalize the optimal tariff formula for economies with different demographic sizes.

1 The Optimal Tariff in a Two-Country Endowment Economy

Consider a two-country endowment economy. The endowments are Y in the home economy and Y^* in the foreign economy. Preferences for the home and foreign goods in the home and foreign country, respectively, are given by $u(C_H, C_F)$ and $u^*(C_H^*, C_F^*)$, where C_H and C_F denote consumption of home and foreign goods in the home country and C_H^*, and C_F^* denote consumption of the home and foreign good in the foreign country. The problem of the home household is

$$\max_{C_H, C_F} u(C_H, C_F)$$

subject to the budget constraint

$$P_H C_H + P_F C_F = P_H Y + T,$$

where P_H and P_F denote the prices of the home and foreign good in the home country expressed in units of the home country's currency, and T is a lump-sum transfer also expressed in units of the home country's currency. The optimality condition is

$$\frac{u_F}{u_H} = \frac{P_F}{P_H}. \tag{1}$$

A similar optimality condition holds for the foreign household

$$\frac{u_F^*}{u_H^*} = \frac{P_F^*}{P_H^*}, \tag{2}$$

where P_H^* and P_F^* denote the prices of the home good and the foreign good in the foreign economy expressed in units of the home country's currency.

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[1] There are many papers that derive these results. Here, I follow closely Itskhoki and Mukhin (2025).

The government of the home country imposes a gross import tax τ^I and a gross export tax τ^E. We assume that the foreign country imposes no trade taxes. We assume that the law of one price holds for both goods:

$$P_F = \tau^I P_F^*$$

and

$$P_H^* = \tau^E P_H.$$

The above two relations imply that

$$\frac{P_F/P_H}{P_F^*/P_H^*} = \tau^I \tau^E. \tag{3}$$

Note that the distortion of the relative-relative price $\frac{P_F/P_H}{P_F^*/P_H^*}$ depends on the product of the two trade taxes, $\tau^I \tau^E$. So any combination of import and export taxes that have the same product generate the same degree of distortion. Let $\tau \equiv \tau^I \tau^E$.

The resource constraints for home and foreign goods are

$$Y = C_H + C_H^* \tag{4}$$

and

$$Y^* = C_F + C_F^*. \tag{5}$$

The home government transfers to home households all of the revenue generated by trade taxes. Thus, the home government budget constraint is given by

$$T = (\tau^I - 1)P_F^* C_F + (\tau^E - 1)P_H C_H^*.$$

Combine the household and the government budget constraints to get

$$P_H C_H + P_F C_F = P_H Y + (\tau^I - 1)P_F^* C_F + (\tau^E - 1)P_H C_H^*$$

Using the law of one price to eliminate τ^I and τ^E, gives

$$P_H C_H + P_F C_F = P_H Y + P_F C_F - P_F^* C_F + P_H^* C_H^* - P_H C_H^*.$$

Cancel $P_F C_F$ as it is on both sides. Rearrange to get

$$P_H C_H + P_H C_H^* = P_H Y - P_F^* C_F + P_H^* C_H^*$$

By the resource constraint (4), the left-hand side equals $P_H Y$, so it can be cancelled with the first term on the right-hand side. This yields the balanced trade condition

$$P_H^* C_H^* = P_F^* C_F,$$

which says that the value of exports, $P_H^* C_H^*$, must equal the value of imports, $P_F^* C_F$. The balanced trade condition makes sense because this is a one-period economy in which households do not have initial assets or liabilities carried over from the past. Divide by P_H^* to write

$$C_H^* = \frac{P_F^*}{P_H^*} C_F. \tag{6}$$

The international relative price of exports in terms of imports, P_H^*/P_F^*, is known as the external terms of trade.

We are ready to define a competitive equilibrium.

Definition 1 (Competitive Equilibrium) *Given the trade tax τ, a competitive equilibrium is an allocation in the home economy, $C_H > 0$ and $C_F > 0$, and in the foreign economy, $C_H^* > 0$ and $C_F^* > 0$, and domestic and foreign relative prices, $P_F/P_H > 0$ and $P_F^*/P_H^* > 0$, satisfying equilibrium conditions* (1)–(6).

Note that in the complete set of equilibrium conditions, equations (1)–(6), trade taxes appear only as the product $\tau \equiv \tau^I \tau^E$. It follows that the equilibrium allocation and equilibrium relative prices depend only on the overall trade distortion τ, but not on the particular levels of import and export taxes τ^I and τ^E. Put differently, any combination of import and export taxes (τ^I, τ^E) gives rise to the same equilibrium allocation and relative prices as long as their product is the same. This result is known as Lerner symmetry (Lerner, 1936).

Proposition 1 (Lerner Symmetry) *The equilibrium allocation C_H, C_F, C_H^*, C_F^*) and equilibrium relative prices $(P_F/P_H, P_F^*/P_H^*)$ depend on the overall trade distortion $\tau \equiv \tau^I \tau^E$, but not on the individual levels of import and export taxes, τ^I and τ^E.*

Use the foreign household's first-order condition (2) to get rid of P_F^*/P_H^* in (6) to get

$$C_H^* = \frac{u_F^*(C_H^*, C_F^*)}{u_H^*(C_H^*, C_F^*)} C_F.$$

Use the foreign country resource constraint (5) to get rid of C_F^* to get

$$C_H^* = \frac{u_F^*(C_H^*, Y^* - C_F)}{u_H^*(C_H^*, Y^* - C_F)} C_F. \tag{7}$$

This expression is called the implementability constraint.[2]

As we show next, the implemantability condition (7) defines the set of allocations that can be supported as a competitive equilibrium using trade taxes:

Proposition 2 (Implementability Condition) *A pair of quantities $C_F \in (0, Y^*)$ and $C_H^* \in (0, Y)$ can be supported as a competitive equilibrium if and only if it satisfies constraint* (7).

The necessary condition is satisfied by construction, because we derived (7) using only conditions listed in Definition 1. To show sufficiency, take any pair of quantities $C_F \in (0, Y^*)$ and $C_H^* \in (0, Y)$ satisfying (7). We must show that equations (1)–(6) are satisfied. Given these values of C_F and C_H^*, pick C_H and C_F^* to satisfy the resource constraints (4) and (5), respectively. Pick P_F/P_H and P_F^*/P_H^* to satisfy (1) and (2). Pick the trade tax $\tau (\equiv \tau^I \tau^E)$ to satisfy (3). Finally, combine (2) and (7) to obtain (6). This completes the proof of Proposition 2.

The implementability constraint (7) implicitly defines C_H^* as a function of C_F, which we write as

$$C_H^* = g(C_F). \tag{8}$$

[2]Clearly, we could alternatively define an equilibrium as an allocation $C_H > 0$, $C_F > 0$, $C_H^* > 0$, and $C_F^* > 0$ and relative prices $P_F/P_H > 0$ and $P_F^*/P_H^* > 0$ satisfying equilibrium conditions (1)–(5) and (7).

This expression is known as the offer curve. It indicates how many units of domestic goods must be exported, C_H^*, to obtain a given quantity of imported goods, C_F.

The Ramsey optimal tariff from the point of view of the domestic government is then obtained by solving the problem

$$\max_{C_F} \quad u(Y - g(C_F), C_F).$$

The first-order condition associated with this problem is

$$-u_H g' + u_F = 0.$$

Rearranging

$$\frac{u_F}{u_H} = g'.$$

Use (1) to eliminate u_F/u_H. This gives

$$\frac{P_F}{P_H} = g'.$$

Use (3) to get rid of P_F/P_H, to get

$$\tau \frac{P_F^*}{P_H^*} = g'.$$

Use (6) to get rid of P_F^*/P_H^*. We then have

$$\tau \frac{C_H^*}{C_F} = g'.$$

By (8), $C_H^* = g$. Using this and rearranging, we get

$$\tau = \frac{g'(C_F)C_F}{g(C_F)}.$$

The right-hand side is the elasticity of the offer curve, which we denote ϵ. It measures the percentage change in exports required for the home country to be able to increase imports by one percent in equilibrium. If the external terms of trade, P_H^*/P_F^* where constant, then ϵ would be 1. But in general, as the country increases imports, the external terms of trade can change, giving rise to different values of ϵ. Finally, we write

$$\tau = \epsilon,$$

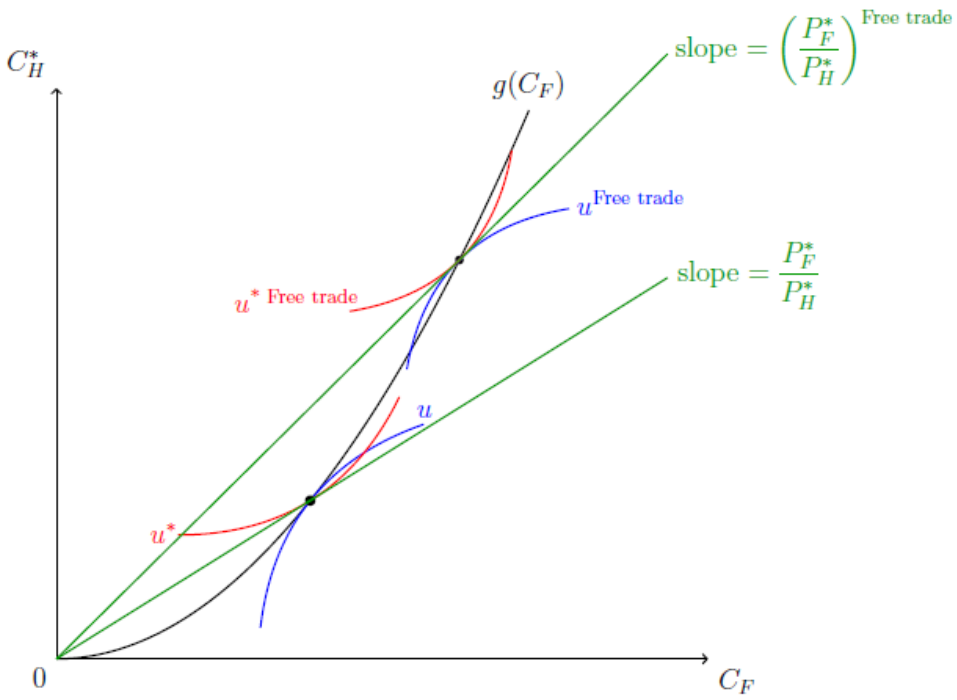
which says that the optimal tariff equals the elasticity of the offer curve. This is a classic result (Johnson, 1950).

1.1 The Optimal Tariff with CES Preferences

Consider the following CES form for the utility function of the foreign country

$$u^*(C_H^*, C_F^*) = \left[{\gamma^*}^{\frac{1}{\eta}} {C_H^*}^{\frac{\eta-1}{\eta}} + (1-\gamma^*)^{\frac{1}{\eta}} {C_F^*}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \tag{9}$$

Figure 1: The Optimal Tariff Graphical Analysis



Notes. Equilibrium under the optimal tariff occurs where an indifference curve of the home country (in the graph marked u in blue) is tangent to the offer curve $g(C_F)$.

It can be shown that with CES preferences, the implementability constraint (7) implies that

$$\epsilon \equiv \frac{d \ln C_H^*}{d \ln C_F} = 1 + \frac{1}{\eta - 1} \frac{Y^*}{C_F^*} \tag{10}$$

See exercise 2. Itskhoki and Mukhin (2025) calibrate η to 4 and the openness ratio of the rest of the world, measured as the export to output ratio, $(Y^* - C_F^*)/Y^*$, to 2 percent, which implies that Y^*/C_F^* is about 1.02. Applying these numbers to equation (10) gives an optimal tariff of 34 percent. We will see shortly that historically the import tariff in the United States has been much lower.

1.2 A Graphical Analysis

Figure 1 displays the offer curve, $g(C_F)$, in the space (C_F, C_H^*). The standard case is that this function is increasing and convex. As we will see below, this is the case, for example, if the utility function of the foreign country is CES with an elasticity of substitution higher than 1. Proposition 2 states that the Home country can pick any point on the offer curve

and supported as a general equilibrium by an appropriate choice of trade taxes, τ. So the question is what point on the offer curve the Home country should pick.

To answer this question, we must introduce the preferences of the Home country. For the home country, C_F, measured on the horizontal axis in the figure, is a good. The other good for the Home country is C_H. But this good does not appear in the graph. Instead, the vertical axis measures C_H^*, which, in equilibrium, is akin to a bad for the home country. This is because, by the resource constraint, $C_F = Y - C_H^*$, so an increase in C_H^* decreases C_H one by one. Thus, in the space (C_F, C_H^*), the indifference curves of the Home country are increasing. If the home utility function has standard properties, they will also be strictly concave.

The utility of the Home country increases in the southeast direction. Thus, the highest level of utility for the Home country occurs when an indifference curve is tangent to the offer curve. In Figure 1 the indifference curve with this property is drawn in blue and marked u. So we have determined the optimal levels of consumption under the optimal tariff.

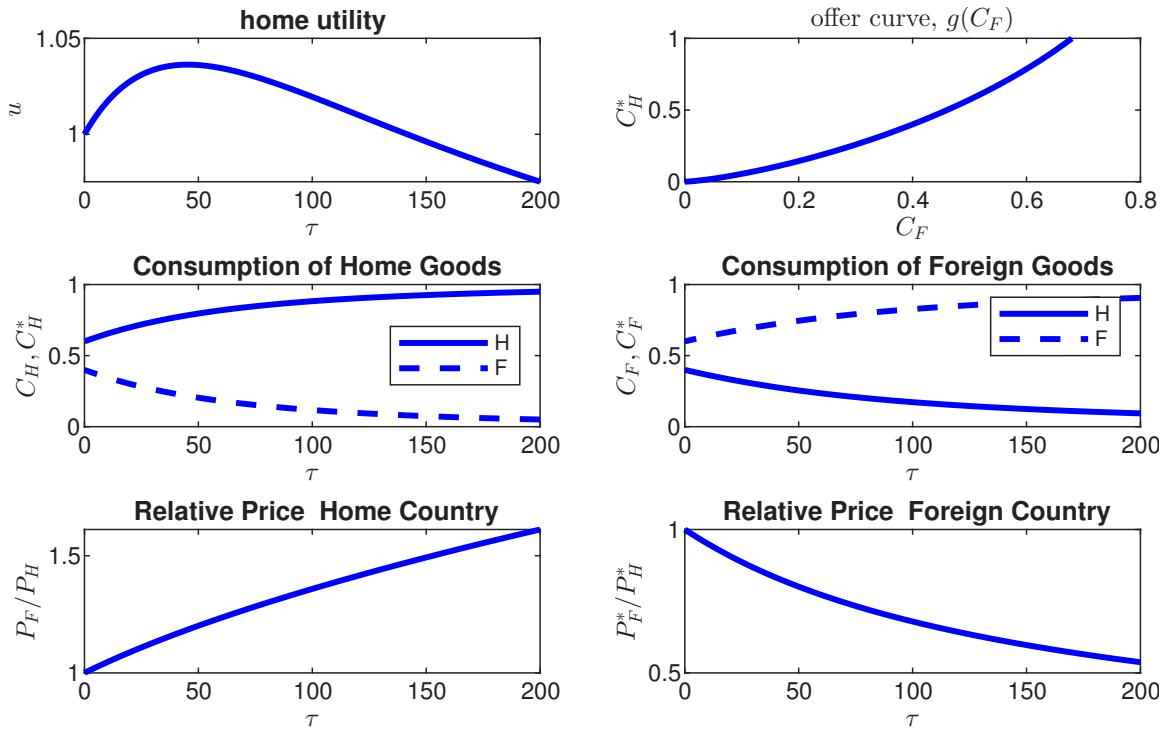
To determine relative prices and the optimal tariff, begin by noting that the domestic relative price, P_F/P_H equals u_F/u_H (equation (1)). Thus, in the figure, P_F/P_H is given by the slope of the indifference curve at the optimal level of consumption. Equivalently, P_F/P_H is given by the slope of the offer curve at the optimal level of consumption.

Now consider the ray that crosses the offer curve at the optimal allocation. Clearly, its slope equals the optimal consumption ratio C_H^*/C_F. But by the balanced trade condition (6) this consumption ratio equals the relative price in the foreign country, P_F^*/P_H^*. Clearly, because $g(C_F)$ at the optimal allocation is steeper than the ray, we have that $P_F/P_H > P_F^*/P_H^*$. The wedge between these two prices is the optimal tariff. Graphically, the optimal tariff is the difference between the slope of the offer curve at the optimal allocation and the slope of the ray that crosses the optimal allocation is the optimal tariff.

The indifference curves of the foreign country are also upward sloping, since for this country C_H^* is a good and C_F is a "bad". In the space (C_F, C_H^*) the indifference curves of the foeign country are convex. Because $u_F^*/u_H^* = P_F^*/P_H^*$, we have that an indifference curve of the foreign country must be tangent to the ray that crosses the optimal allocation. This indifference curve is marked as u^* in redd in the figure.

Where in Figure 1 is laissez faire equilibrium ($\tau = 1$)? Because under free trade $u_F/u_H = P_F/P_H = P_F^*/P_H^* = u_F^*/u_H^*$, we have that the free trade equilibrium happens at a point on the offer curve at which the indifference curves of the Home and Foreign countries are tangent. Clearly, they will also be tangent to the ray that crosses the free-trade equilibrium allocation. The ray an the indifference curves associated with the free-trade equilibrium are labeled "Free trade" in Figure 1. In the figure, free trade occurs at a point on the offer curve located farther from the origin than the optimal-tariff equilibrium. To see why this has to be the case, note that if both goods are normal in both countries, when C_F and C_H^* increase the Home indifference curves become flatter and the Foreign ones become steeper. Since at the optimal-tariff equilibrium the Home indifference curve is steeper than the Foreign one, moving to a point on $g(C_F)$ in which both have the same slope requires moving away from the origin. Because the ray that crosses the free-trade allocation is steeper than the ray that crosses the optimal-tariff equilibrium, it follows that the relative price of the Foreign good is higher in the free-trade equilibrium than in the optimal-tariff equilibrium. This shows that by imposing the optimal tariff, the Home country manipulates the terms of trade in its favor.

Figure 2: Equilibrium as a Function of the Tariff Rate, τ



Notes. The calibration is $Y = Y^* = 1$, $\eta = \theta = 4$, $\gamma = \gamma^* = 0.4$, $n = 1/2$. The tariff rate, τ, is expressed in percent. Its optimal value is 45.4 percent.

Also, in the free-trade equilibrium both C_F and C_H^* are higher, implying that the optimal tariff makes the world more closed to international trade.

1.3 A Numerical Exercise

Suppose that the utility function of the foreign household is of the CES form shown in (9). Assume that the utility function of the home country is also CES and given by

$$u^*(C_H, C_F) = \left[(1-\gamma)^{\frac{1}{\theta}} C_H^{\frac{\theta-1}{\theta}} + \gamma^{\frac{1}{\theta}} C_F^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} \tag{11}$$

Consider the calibration $\eta = \theta = 4$, $\gamma = \gamma^* = 0.4$, and $Y = Y^* = 1$.

Under this calibration, the optimal tariff is 45.4 percent. It is higher than in the Itskhoki and Mukhin (2025) calibration, because we are assuming less home bias. In the present economy, the ratio Y^*/C_F^* is 1.36, compared to 1.02 in the Itskhoki and Mukhin (2025) calibration. Home bias is governed by the parameters γ and γ^*. The value of 0.4 comes from Galí (2008).

Figure 2 displays the equilibrium values of the variables of the model as functions of the import tariff τ. It also displays the offer curve, $g(C_F)$. This function is increasing and convex, implying that the relative price of the foreign consumption good abroad, P_F^*/P_H^*, increases as the home economy increases imports of the foreign good, C_F.

In choosing the optimal tariff, the home government trades off two opposing effects. On the one hand, an increase in the import tariff reduces domestic demand for the foreign good, which lowers the world relative price of imports, P_F^*/P_H^*. This change in relative prices is welfare improving for the home country and is known as terms-of-trade manipulation: the tariff shifts world relative prices in favor of the home economy.

On the other hand, the tariff distorts relative prices within the home economy. In particular, the relative price of foreign goods faced by domestic consumers rises, $P_F/P_H > P_F^*/P_H^*$. As a result, home households consume fewer imported goods than is efficient, generating a welfare loss.

This trade-off is illustrated in the top left panel of the figure, which shows that welfare in the home country is a nonmonotonic function of the tariff. For low tariff levels, home welfare increases with the tariff, as the gains from terms-of-trade manipulation dominate the losses from domestic price distortions. Beyond a certain point, however, welfare declines as the tariff rises further. In this range, the costs associated with distorted domestic relative prices exceed the benefits from improved terms of trade. At the optimal tariff, these two effects exactly offset each other at the margin.

The domestic distortions created by the tariff are illustrated in the bottom left panel of Figure 2. As the tariff increases, the relative price of the foreign good in the home economy, P_F/P_H, rises artificially. The home countrys manipulation of the terms of trade is shown in the bottom right panel, where a higher tariff lowers the relative price at which the foreign economy sells the foreign good to the home economy, P_F^*/P_H^*.

The distortions on aggregate demand created by the tariff are shown in the middle row of the figure. As P_F/P_H increases with the tariff, home households reduce their consumption of foreign goods (solid line in the middle right panel). At the same time, the higher domestic relative price of foreign goods induces home households to increase their consumption of the home good (solid line in the middle left panel). The response of foreign households is symmetric: as P_F^*/P_H^* falls, they increase consumption of the foreign good and reduce consumption of the home good.

Overall, an increase in tariffs makes the world economy more closed, as it induces both countries to consume relatively more of the goods they produce and less of the goods they import.

2 Country Size and the Optimal Tariff

In this section, we consider two notions of country size. One is demographic: how does the optimal tariff change is one country is assumed to be more populated than the other. The second notion of size we consider has to do with market power. A country could be demographically small, but it could still manipulate the world terms of trade if the good it exports is difficult to substitute for other goods.

2.1 Population Size and the Optimal Tariff

Thus far, we have implicitly considered an economy with two countries with equal populations. Suppose now that the population is N in the home economy and N^* in the foreign

economy. Let $n \equiv N/(N+N^*)$ be the fraction of the world population living in the home country. We wish to ascertain how the optimal tariff for the home country is affected by its relative demographic size, n.

Preferences and endowments of individual households are the same as before. The optimization problems of the home and foreign households are unchanged. But the resource constraints in each country now are

$$nY = nC_H + (1-n)C_H^* \tag{12}$$

and

$$(1-n)Y^* = nC_F + (1-n)C_F^*, \tag{13}$$

where Y and Y^* denote endowments per household in the home and foreign country. Note that nY and $(1-n)Y^*$ represent the endowments of home and foreign goods per capita in the world.

As before, we assume that the domestic government transfers to households all of the revenue generated by trade taxes. Thus, the domestic government budget constraint is given by

$$nT = (\tau^I - 1)P_F^* nC_F + (\tau^E - 1)P_H(1-n)C_H^*$$

Combining the household and the government budget constraints, one gets

$$P_H nC_H + P_F nC_F = P_H nY + (\tau^I - 1)P_F^* nC_F + (\tau^E - 1)P_H(1-n)C_H^*,$$

which can be reduced to the balanced-trade condition

$$(1-n)C_H^* = \frac{P_F^*}{P_H^*}\, nC_F. \tag{14}$$

This condition collapses to (6) when $n = 1/2$.

We are ready to define a competitive equilibrium.

Definition 2 (Competitive Equilibrium in Economies of Unequal Sizes) *Given the trade distortion τ, a competitive equilibrium is an allocation in the domestic economy, $C_H > 0$ and $C_F > 0$, and in the foreign economy, $C_H^* > 0$ and $C_F^* > 0$, and domestic and foreign relative prices, $P_F/P_H > 0$ and $P_F^*/P_H^* > 0$, satisfying equilibrium conditions* (1)–(3) *and* (12)–(14).

Note that the equilibrium does not depend on the size of the world population, $N + N^*$, but only on its distribution across countries, n.

Using the foreign household first-order condition (2) to get rid of P_F^*/P_H^* in (14), we get

$$C_H^* = \frac{u_F^*(C_H^*, C_F^*)}{u_H^*(C_H^*, C_F^*)} \frac{n}{1-n} C^F.$$

Use the foreign country resource constraint (13) to get rid of C_F^* to get

$$C_H^* = \frac{u_F^*\left(C_H^*, Y^* - \frac{n}{1-n}C_F\right)}{u_H^*\left(C_H^*, Y^* - \frac{n}{1-n}C_F\right)} \frac{n}{1-n} C_F. \tag{15}$$

This implementability constraint is identical to (7), except that C_F is replaced by $\frac{n}{1-n}C_F$.

We can then write

$$C_H^* = g\left(\frac{n}{1-n}C_F\right). \tag{16}$$

This new offer curve features the same function $g(\cdot)$ as (8), but evaluated at $\frac{n}{1-n}C_F$.

The Ramsey optimal tariff from the point of view of the domestic government is then obtained from solving the problem

$$\max_{C_F} \quad u\left(Y - \frac{1-n}{n}g\left(\frac{n}{1-n}C_F\right), C_F\right)$$

The first-order condition associated with this problem is

$$-u_H g' + u_F = 0.$$

This expression is identical to the one that obtains in the symmetric case. (Note that g' is the derivative of g with respect to $\frac{n}{1-n}C_F$, not with respect to C_F.) Rearranging

$$\frac{u_F}{u_H} = g'.$$

Use (1) to eliminate u_F/u_H. This gives

$$\frac{P_F}{P_H} = g'.$$

Use (3) to get rid of P_F/P_H, to get

$$\tau\frac{P_F^*}{P_H^*} = g'.$$

Use (14) to get rid of P_F^*/P_H^*. We then have

$$\tau\frac{C_H^*}{\frac{n}{1-n}C_F} = g'.$$

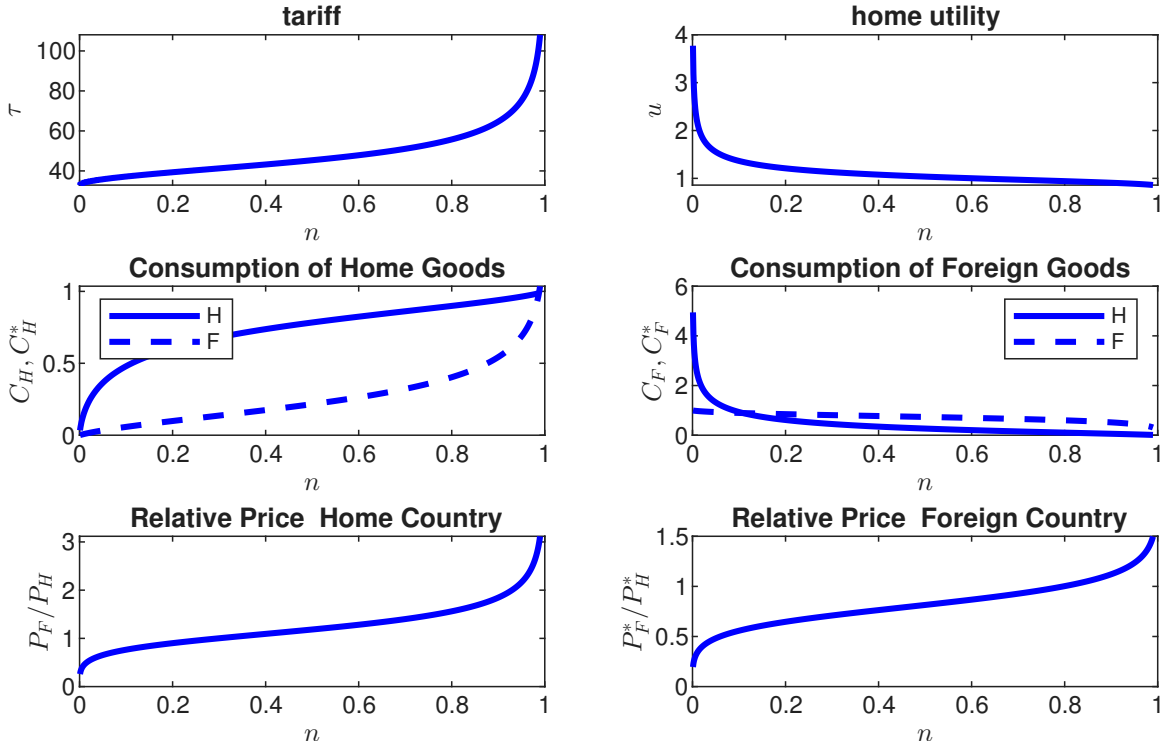
By (16), $C_H^* = g$. Using this and rearranging, we get

$$\tau = \frac{g'\left(\frac{n}{1-n}C_F\right)\frac{n}{1-n}C_F}{g\left(\frac{n}{1-n}C_F\right)}.$$

So we obtain the same result as in the symmetric economy ($n = 1/2$), namely, that the optimal tariff equals the elasticity of the offer curve.

Figure 3 displays the equilibrium of the variables of the model under the optimal import tariff for different values of n. Preferences are CES, and the calibration is the same as before ($\eta = \theta = 4$, $Y = Y^* = 1$, and $\gamma = \gamma^* = 0.4$). The optimal tariff increases with demographic size. But it does not seem to converge to zero as the home country's relative size becomes very small. The top left panel of the figure shows that the optimal tariff is slightly below 40 percent for values of n close to zero. It is of interest that as the country becomes larger

Figure 3: Equilibrium Under the Optimal Tariff as a Function of the Home Population Share



Notes. The calibration is as in Figure 2, except for n.

(n increases) the welfare of the home household goes down, even though the government is setting the import tariff to maximize welfare. The reason is that as the country becomes larger, the global supply of the home good, nY, increases, thus depressing its world relative price, which is detrimental to the home country. This effect is evident from the middle right panel of the figure, which shows that the relative price of the foreign good in the foreign country, P_F^*/P_H^*, increases as the home country becomes larger. This result resembles the immiserizing growth result in classic trade theory (Bhagwati, 1958).

Analytically, we can derive the welfare effect of an increase in country size by applying the envelope theorem. Start with the objective function of the Ramsey Home government.

$$u\left(Y - X^{-1}g\left(XC_F\right), C_F\right),$$

where $X \equiv \frac{n}{1-n}$. Since X is an exogenous parameter, we can determine the equilibrium effect of an increase in demographic size by taking the partial derivative of u with respect to X,

$$\begin{aligned}
\frac{\partial u}{\partial X} &= -u_H\left(-\frac{g}{X^2} + \frac{g'C_F}{X}\right) \\
&= \frac{u_H g}{X^2}\left(1 - \frac{g'XC_F}{g}\right) \\
&= \frac{u_H g}{X^2}\left(1-\tau\right) \\
&< 0,
\end{aligned}$$

when $\tau > 1$ at the Ramsey optimum. Since X is strictly increasing in n, this result says that if the optimal tariff is positive, then an increase in the tariff-setting countrys relative demographic size is welfare-reducing for that country.

This immiserizing effect is not due to the fact that the home good becomes more abundant (its per capita supply in the world, nY, goes up with n), but to the fact that simultaneously the foreign good becomes more scarce (its per capital supply in the world, $(1-n)Y^*$, goes down with n). To see this, consider the equilibrium welfare effect of an increase in the home endowment, Y. This case is similar to an increase in n in that the amount of the home good per capita in the world, nY, increases. But it is different in that the supply of the foreign good per capita in the world, $(1-n)Y^*$, does not change. To determine the welfare effect of an increase in Y, we can apply the envelope theorem once again. Noticing that $g(\frac{n}{1-n}C_F)$ does not depend on Y, we have that

$$\frac{\partial u}{\partial Y} = u_H > 0.$$

So an increase in the home endowment unambiguously increases equilibrium welfare in the home country.

2.2 Market Power as Economic Size

One way in which a country's economic size is measured is by the degree to which movements in domestic activity can induce changes in world prices. The economies studied thus far, are monopolists of the good they are endowed with, so they can exploit this market power to

alter international relative prices through tariffs in a way that is welfare increasing for their households. What if the country has not market power? We can think of such an economy as small in economic terms, even if it is demographically large.

A straightforward way to see that if the rest of the world can readily substitute the good supplied by the home economy by other goods the optimal tariff is low, is by going back to the economy with CES preferences, and in particular to equation (10), showing the optimal tariff. Holding constant the openness ratio in the rest of the world, Y^*/C_F^*, that equation shows that as the elasticity of substitution in the rest of the world goes to infinity, $\eta \to \infty$, the net optimal tariff , $\tau - 1$, reduces to zero. In this case, the utility function of the foreign country is linear and equal to $C_H^* + C_F^*$. So if both goods are consumed in the foreign country, the relative price P_F^*/P_H^* is constant and equal to 1, independently of the tariff imposed by the domestic country. Since the domestic country cannot alter its terms of trade via trade policy, the best it can do is laissez faire.

Another way to see this result is without making any reference to preferences in the rest of the world. Consider a small economy in the sense that it faces constant world relative prices P_F^*/P_H^*. The resource constraint of this economy is

$$Y = C_H + X, \tag{17}$$

where X is the number of units of home goods exported. The budget constraint of the government is

$$T = (\tau^I - 1)P_F^* C_F + (\tau^E - 1)P_H X,$$

Using this expression to eliminate T from the household budget constraint $P_H C_H + P_F C_F = P_H Y + T$, yields the balanced trade condition

$$X = \frac{P_F^*}{P_H^*} C_F \tag{18}$$

We can now define a competitive equilibrium as follows:

Definition 3 (Competitive Equilibrium in a Price-Taking Economy) *Given P_F^*/P_H^* and τ, a competitive equilibrium is a quadruplet $(C_H, C_F, X, P_F/P_H)$ satisfying* (1), (3), (17), *and* (18).

Because the international relative price P_F^*/P_H^* is given, the balance-trade condition (18) is also the implementability condition: Any allocation (X, C_F) satisfying (18) can be supported as a competitive equilibrium. To see this, proceed as follows: Given X, C_H can be obtained residually from (17); the home relative price P_F/P_H can be backed out from optimality condition (1); and the tariff that supports the equilibrium can be obtained from (3).

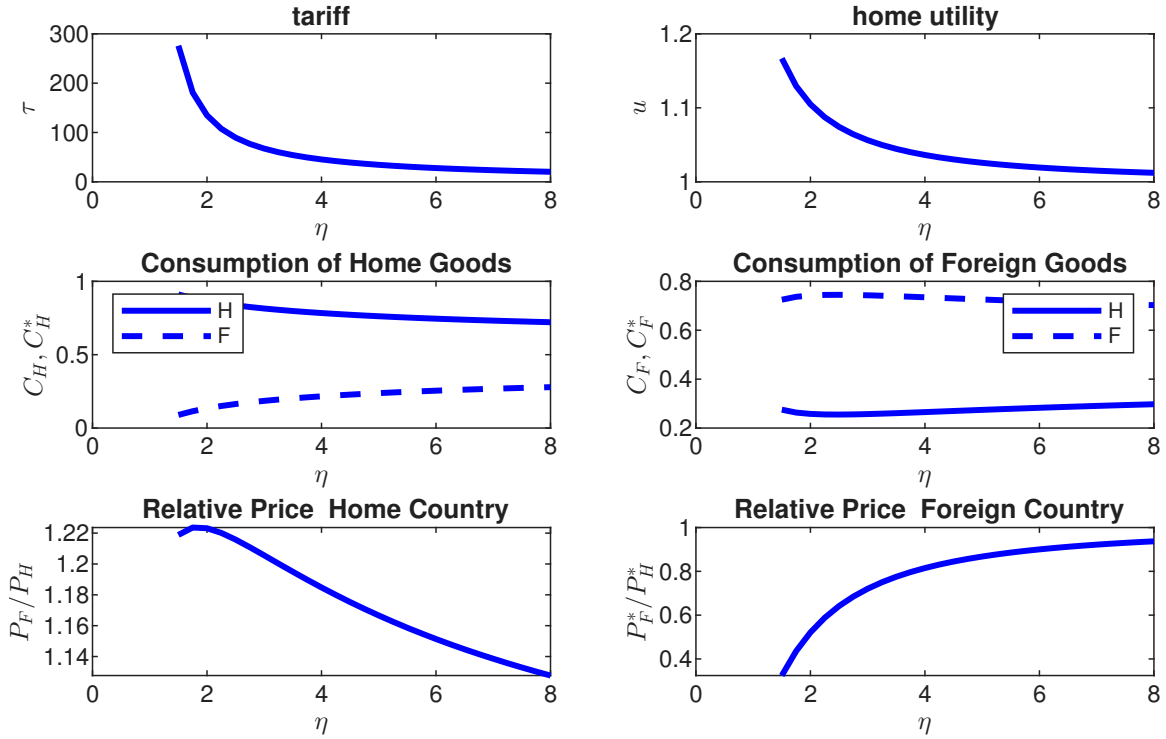
The optimization problem of the Ramsey planner is then

$$\max_{C_F} u\left(Y - \frac{P_F^*}{P_H^*} C_F, C_F\right)$$

The first-order condition associated with this problem is

$$\frac{u_F}{u_H} = \frac{P_F^*}{P_H^*}$$

Figure 4: Equilibrium Under the Optimal Tariff as a Function of the Foreign Elasticity of substitution



Notes. The calibration is as in Figure 2, except for η.

Comparing this expression with optimality conditions (1) and (3), it follows immediately that $\tau = 1$, that is, the optimal import tariff is zero.

Proposition 3 (Optimality of Free Trade in a Price-Taking Economy) *If the country takes the international relative price P_F^*/P_H^* as given, then the optimal import tariff is zero.*

Figure 4 displays the equilibrium values of the variables of the model under the optimal tariff as a function of the elasticity of substitution in the foreign country, η. As before, the economy features CES preferences and the calibration $\theta = 4$, $Y = Y^* = 4$, $\gamma = \gamma^* = 0.4$, and $n = 1/2$. As the elasticity of substitution abroad becomes large, the optimal tariff imposed by the home country converges to 0, which is the limit case studied in this section. The welfare of the home country's household falls with η, because, as the home good becomes more substitutable abroad, the home country losses the capacity to manipulate the terms of trade in its favor. This can be seen from the bottom right panel of the figure, which shows that as η increases, the relative price of the foreign good in the foreign country increases, that is, the terms of trade move against the home economy.

3 Retaliation

Up until this point, we have assumed that the government of the foreign country passively tolerates the commercial policy of the home country. Suppose now that the foreign country also imposes import and export tariffs, denoted τ^{I*} and τ^{E*}. The law of one price then implies that

$$P_F = \tau^I \tau^{E*} P_F^*$$

and

$$P_H^* = \tau^{I*} \tau^E P_H$$

These two expressions imply that the distortion in relative-relative prices across countries is

$$\frac{P_F/P_H}{P_F^*/P_H^*} = \tau^I \tau^{E*} \tau^{I*} \tau^E. \tag{19}$$

The optimization problems of the domestic and foreign households are unchanged. In particular, optimality conditions (1) and (2) continue to hold. Let's now derive the balanced trade condition. As before, the government in the home country is assumed to rebate tax revenues to the households through the lump-sum transfer T. Its budget constraint is

$$T = (\tau^I - 1)\tau^{E*} P_F^* C_F + (\tau^E - 1) P_H C_H^*$$

Combining this constraint with the household's budget constraint, which is unchanged, to eliminate T gives

$$P_H C_H + P_F C_F = P_H Y + (\tau^I - 1)\tau^{E*} P_F^* C_F + (\tau^E - 1) P_H C_H^*$$

Use the resource constraint (4) to get rid of Y and cancel $P_H C_H$,

$$P_F C_F = P_H C_H^* + (\tau^I - 1)\tau^{E*} P_F^* C_F + (\tau^E - 1) P_H C_H^*$$

Now cancel $P_H C_H^*$

$$P_F C_F = (\tau^I - 1)\tau^{E*} P_F^* C_F + \tau^E P_H C_H^*$$

Multiply through by τ^{I*}

$$\tau^{I*} P_F C_F = (\tau^I - 1)\tau^{I*}\tau^{E*} P_F^* C_F + \tau^{I*}\tau^E P_H C_H^*$$

Use the law of one price to replace $\tau^{I*}\tau^E P_H C_H^*$ by P_H^*

$$\tau^{I*} P_F C_F = (\tau^I - 1)\tau^{I*}\tau^{E*} P_F^* C_F + P_H^* C_H^*$$

Replace P_F by $\tau^I \tau^{E*} P_F^*$

$$\tau^{I*}\tau^I \tau^{E*} P_F^* C_F = (\tau^I - 1)\tau^{I*}\tau^{E*} P_F^* C_F + P_H^* C_H^*$$

Cancel $\tau^{I*}\tau^I\tau^{E*} P_F^* C_F$, which appears on both sides, to finally get the balanced trade condition

$$\tau^{I*}\tau^{E*} P_F^* C_F = P_H^* C_H^*$$

It is identical to the one without retaliation, except that the trade distortion in the rest of the world appears augmenting the dollar value of imports in the home country. Now divide through by P_H^* to get

$$\tau^{I*}\tau^{E*}\frac{P_F^*}{P_H^*}C_F = C_H^* \tag{20}$$

To obtain the implementability condition, proceed as before: use the optimality condition (**??**) to eliminate the relative price and the resource constraint (5) to eliminate C_F^* from that optimality condition, Also, use the notation $\tau^* \equiv \tau^{I*}\tau^{E*}$ to refer to the external trade distortion. This yields

$$C_H^* = \tau^*\frac{u_F^*(C_H^*, Y^* - C_F)}{u_H^*(C_H^*, Y^* - C_F)}C_F \tag{21}$$

This is the implementability condition.

We will consider a Nash equilibrium, in which the home government maximizes home welfare taking the foreign commercial policy, τ^*, as given. Then, given τ^*, one can show that any allocation (C_F, C_H^*) satisfying (21) can be supported as an equilibrium. The proof is identical to the its counterpart in the economy without retaliation (Proposition 2). As before, we write the implementability constraint in a compact form as follows

$$C_H^* = g(C_F, \tau^*) \tag{22}$$

The home Ramsey planner then solves the problem

$$\max_{C_F} u(Y - g(C_F, \tau^*), C_F)$$

This gives the optimal value of C_F given τ^*. Equation (22) then yields the optimal level of C_H^* from the perspective of the home government, given τ^*. The resource constraints (4) and (5) then give C_H and C_F^*. With the optimal consumption levels at hand, the optimality conditions (1) and (2) give the Ramsey optimal relative prices from the perspective of the home government given the foreign commercial distortion, P_F/P_H and P_F^*/P_H^*. Finally, these relative prices and equation (19) yields the home country's best response function, i.e., the optimal home tariff τ given the foreign tariff τ^*, which we write as

$$\tau = \mathcal{T}(\tau^*) \tag{23}$$

Consider now the optimal tariff problem from the perspective of the foreign country. Use (19) to rewrite the balanced trade condition (20) as

$$C_F = \tau\frac{P_H}{P_F}C_H^*. \tag{24}$$

Use the optimality condition (1) to eliminate the relative price and get

$$C_F = \tau\frac{u_H(C_H, C_F)}{u_F(C_H, C_F)}C_H^*$$

Finally, use the resource constraint (4) to eliminate C_H. This gives the foreign government's implementability constraint

$$C_F = \tau \frac{u_H(Y - C_H^*, C_F)}{u_F(Y - C_H^*, C_F)} C_H^* \tag{25}$$

We write this condition as

$$C_F = g^*(C_H^*, \tau). \tag{26}$$

The Ramsey problem of the foreign government is

$$\max_{C_H^*} u^*(C_H^*, Y^* - g^*(C_H^*, \tau))$$

The solution to this problem gives the optimal value of C_H^*, given the home country's trade distortion τ. The implementability constraint (26) then gives the optimal value of C_F. The resource constraints (4) and (5) then give C_H and C_F^*, and the optimality conditions (1) and (2) give the Ramsey optimal relative prices, P_F/P_H and P_F^*/P_H^*. These relative prices and equation (19) yield the foreign country's best response function (the optimal foreign tariff τ^*, given the home tariff τ), which we write as

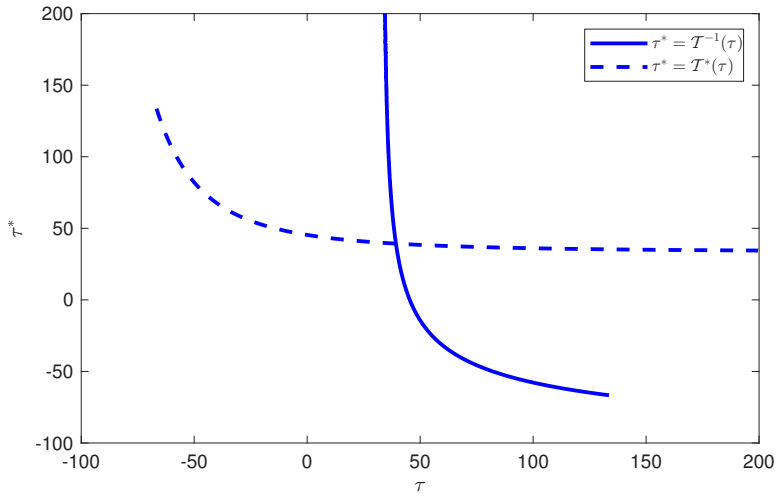
$$\tau^* = \mathcal{T}^*(\tau) \tag{27}$$

We are ready to define a Nash equilibrium

Definition 4 *A Nash equilibrium is a pair of tariffs in the home and foreign countries,* $(\widetilde{\tau}, \widetilde{\tau}^*)$ *satisfying equations* (23) *and* (27)*.*

Figure 5 displays the best response functions of the home and foreign governments. They are computed for the calibration $Y = Y^* = 1$, $\eta = \theta = 4$, $\gamma = \gamma^* = 0.4$, and $n = 1/2$. Both best responses are decreasing functions. Because the two economies are identical (including the same degree of home bias), the Nash-optimal tariff is the same for both countries, 39.2 percent, lower than the optimal tariff without retaliation of 45.4 percent. However, the world distortion, given by $\tau\tau^*$, is 94 percent, much higher than in the absence of retaliation. It is worth noting that at least in this case, the function $\tau = \mathcal{T}(\mathcal{T}^*(\tau))$ is a contraction, so it can be used to solve for the Nash equilibrium.

Figure 5: Nash-Equilibrium Tariff Rates



Notes. The calibration is $Y = Y^* = 1$, $\theta = \eta = 4$, $\gamma = \gamma^* = 0.4$, $n = 1/2$. The tariff rates, τ and τ^*, are expressed in percent. Their Nash-equilibrium value is 39.2 percent, implying a world distortion of 94 percent.

4 Exercises

Exercise 1 (A log-log Economy) *Suppose that preferences in the Home and Foreign countries are given by*

$$u(C_H, C_F) = \ln C_H + \ln C_F$$

and

$$u^*(C_H^*, C_F^*) = \ln C_H^* + \ln C_F^*.$$

The home country imposes import and export tariffs τ^I and τ^I, and the foreign country does not retaliate.

1. *Derive the equilibrium levels of consumption, C_H, C_F, C_H^*, and C_F^*, and the external terms of trade, denoted $TOT_t^* \equiv P_F^*/P_H^*$, as functions of the endowments, Y and Y^*, and of the overall trade distortion $\tau \equiv \tau^I \tau^E$.*

2. *Provide intuition for the equilibrium effect of an tariff increase on the external terms of trade.*

3. *What can you say about the optimal tariff from the perspective of the home country?*

4. *What is the laissez-faire allocation, i.e., the allocation associated with $\tau = 1$?*

5. *Derive the optimal tariff τ for an altruistic home government that maximizes world welfare, defined as $u(C_H, C_F) + u(C_H^*, C_F^*)$.*

Exercise 2 (Optimal Tariff with CES Preferences) *Consider the following form for the utility function of the foreign country*

$$u^*(C_H^*, C_F^*) = \left[{\gamma^*}^{\frac{1}{\eta}} {C_H^*}^{\frac{\eta-1}{\eta}} + (1-\gamma^*)^{\frac{1}{\eta}} {C_F^*}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$$

Derive step by step the result that the elasticity of the offer curve, $\epsilon \equiv \frac{d \ln C_H^*}{d \ln C_F}$, is given by

$$\epsilon = 1 + \frac{1}{\eta - 1} \frac{Y^*}{C_F^*}$$

Exercise 3 (Demographic Size and Equilibrium Tariffs) *Suppose that the allocation $(\bar{C}_H^*, \bar{C}_F)$ is an equilibrium in an asymmetric economy with $n = \bar{n} \neq 1/2$.*

1. *Is the allocation $(\bar{C}_H^*, \widetilde{C}_F)$ with $\widetilde{C}_F = \frac{\bar{n}}{1-\bar{n}} \bar{C}_F$, an equilibrium in the symmetric economy with $n = 1/2$? Show your work.*

2. *Compare the equilibrium tariff in the asymmetric economy ($n = \bar{n} \neq 1/2$), denoted $\bar{\tau}$, with the corresponding one in the symmetric economy ($n = 1/2$), denoted $\widetilde{\tau}$.*

Exercise 4 (Demographic Size and the Optimal Tariff) *Consider the economy with CES preferences and the calibration $\eta = \theta = 4$, $Y = Y^* = 1$, and $\gamma = \gamma^* = 0.25$. Produce a table, using the computing software of your choice, showing optimal tariff (τ) and the associated levels of per capita consumption (C_H, C_F, C_H^*, C_F^*), and relative prices (P_F/P_H and P_F^*/P_H^*) for $n = 0.001, 0.25, 0.5, 0.75,$ and 0.99.*

Exercise 5 (Parameterization of the Elasticity of Substitution) *We have shown that the elasticity of substitution η is a key determinant of the optimal tariff. How big is it? Write a survey of no more than two pages (not counting references) reviewing the values assigned to η in the related literature. Distinguish between Trade and Macro studies, and indicate whether the parameter is estimated econometrically or calibrated. Include a "References" section at the end.*

Exercise 6 (The Balanced-Trade Condition Under Retaliation) *Derive the balanced-trade condition* (24) *step by step starting with the budget constraints of the foreign household and the foreign government.*

Exercise 7 (Implementing a Social Planner's Allocation) *Consider a two-country economy with CES preferences given by* (9) *and* (11)*, with $\eta = \theta = 4$, $\gamma = \gamma^* = 0.5$. Suppose that the endowments are $Y = 32$ and $Y^* = 2$. The two economies have the same population, $n = 1/2$. Suppose that the world social planner is the WTO, and that its welfare criterion is the sum of the two utility functions.*

1. *What are the values of consumption of each good under the social planner's allocation?*

2. *Can the WTO implement this allocation as a competitive equilibrium through import taxes or subsidies? If so, assuming that the WTO does not use export taxes ($\tau^E = \tau^{E*} = 1$), what are the values of the import taxes/subsidies in the home and foreign countries, τ^I and τ^{I*} that support the desired competitive equilibrium?*

3. *Suppose all prices are measured in the same currency. Is one of the countries more expensive than the other in both goods? If so, which one, and by what percentage? If you find that this situation can never happen in equilibrium, explain why.*

4. *Compute consumption and prices in the laissez-faire competitive equilibrium and compare the result with the competitive equilibrium associated with the social planner's allocation.*

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