Multiple Equilibria in Open Economies with Collateral Constraints

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Starting Point

• Open-economy models with collateral constraints have been used to explain:

— sudden stops in response to fundamental shocks
— amplification of business cycles
— overborrowing
— the desirability of capital control taxes

• Open-economy models with collateral constraints are prone to multiple equilibria. This is known. For example, Mendoza (2005) and Jeanne and Korinek (2010) present heuristic analysis of the multiplicity problem.

• However, the related literature (a) does not offer a formal treatment of multiplicity, (b) has not asked whether the overborrowing result is robust to the presence of multiplicity; and (c) does not explore whether self-fulfilling crisis resembe observed sudden stops.

This paper amis to fill this gap.

Specifically, the present paper

- characterizes analytically and numerically equilibrium multiplicity in open economy models with flow collateral constraints.

- shows that self-fulfilling financial crises (sudden stops) can occur in equilibrium.

- shows that such an economy can display underborrowing.

- shows that self-fulfilling crises and underborrowing obtain under plausible calibrations.

An Open Economy Model with a Flow Collateral Constraint (ex: Bianchi 2011)

Households maximize

$$\sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma} - 1}{1-\sigma}$$

subject to

$$c_t = \left[a {c_t^T}^{1-\frac{1}{\xi}} + (1-a) {c_t^N}^{1-\frac{1}{\xi}} \right]^{\frac{1}{1-\frac{1}{\xi}}}$$

$$c_t^T + p_t c_t^N + d_t = y^T + p_t y^N + \frac{d_{t+1}}{1+r}$$

$$d_{t+1} \le \kappa \left(y^T + p_t y^N \right)$$

where c_t = consumption; c_t^T, c_t^N = consumption of tradables, nontradables; d_t = debt due in t; d_{t+1} = debt assumed in t and due in $t+1$; y^T, y^N = endowments of tradables, nontradables; p_t = relative price of nontradables; r = interest rate; $\sigma, \xi, \kappa > 0$, $\beta, a \in (0,1)$ parameters.

Three Equilibrium Conditions of Interest

$$d_{t+1} \leq \kappa(y^T + p_t y^N)$$

$$p_t = \frac{1-a}{a}\left(\frac{c_t^T}{y^N}\right)^{1/\xi}$$

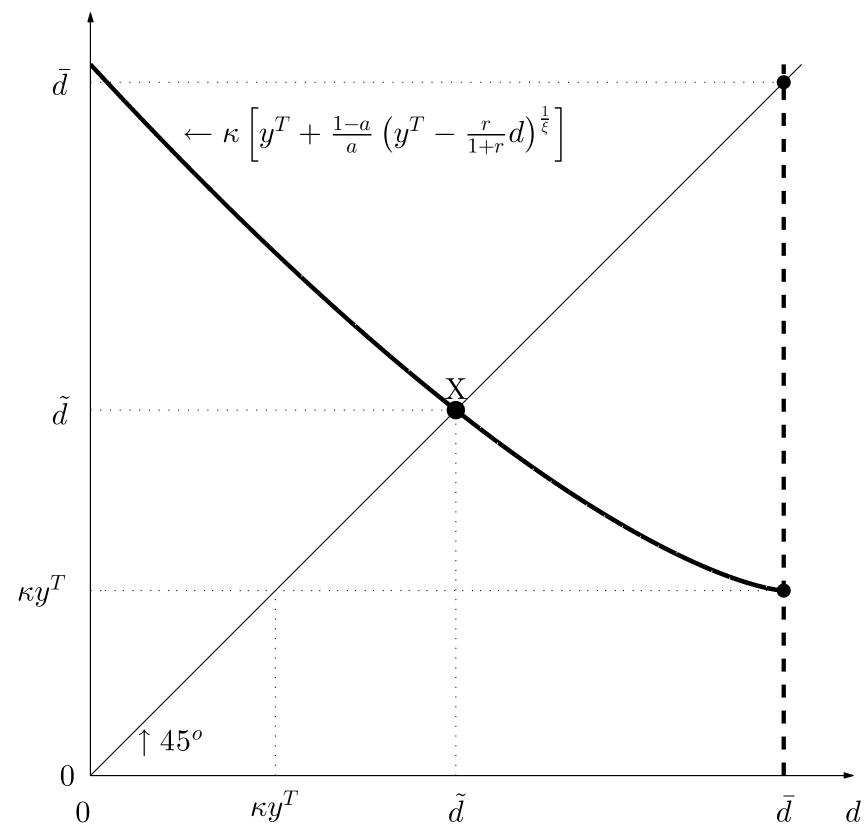
$$c_t^T + d_t = y^T + \frac{d_{t+1}}{1+r}$$

Normalize y^N to 1. Then, these three conditions give rise to the following **equilibrium collateral constraint**

$$d_{t+1} \leq \kappa\left[y^T + \left(\frac{1-a}{a}\right)\left(y^T + \frac{d_{t+1}}{1+r} - d_t\right)^{\frac{1}{\xi}}\right]$$

The Steady-State Collateral Constraint

$$d \leq \kappa \left[y^T + \left(\frac{1-a}{a} \right) \left(y^T - \frac{r}{1+r} d \right)^{\frac{1}{\xi}} \right]$$



where $\bar{d} \equiv y^T(1+r)/r$ is the natural debt limit. We then have that for any $d_0 < \tilde{d}$, the sequence $d_{t+1} = d_0 \ \forall t$ is a candidate for a steady-state equilibrium. Only need to check that the Euler equation is satisfied.

The Euler Equation in a Steady-State Equilibrium

In general, the Euler equation is

$$\frac{\Lambda(c_t^T)}{\Lambda(c_{t+1}^T)} = \frac{\beta(1+r)}{1-\mu_t(1+r)},$$

where $\Lambda(c_t^T) \equiv U'(A(c_t^T, 1))A_1(c_t^T, 1)$, and $\mu_t =$ Lagrange multiplier on the CC. The previous figure shows that in a steady-state equilibrium with $d_0 < \tilde{d}$, the collateral constraint is always slack, so $\mu_t = 0$. Then, the Euler equation becomes

$$\frac{\Lambda(c_t^T)}{\Lambda(c_{t+1}^T)} = \beta(1+r)$$

In a steady-state equilibrium $c_t = c_{t+1}$.

Assume that $\beta(1+r) = 1$. Then it follows that the Euler equation is satisfied $\forall t$ in a steady-state equilibrium. We have therefore established the existence of a steady-state equilibrium for any initial condition $d_0 < \tilde{d}$.

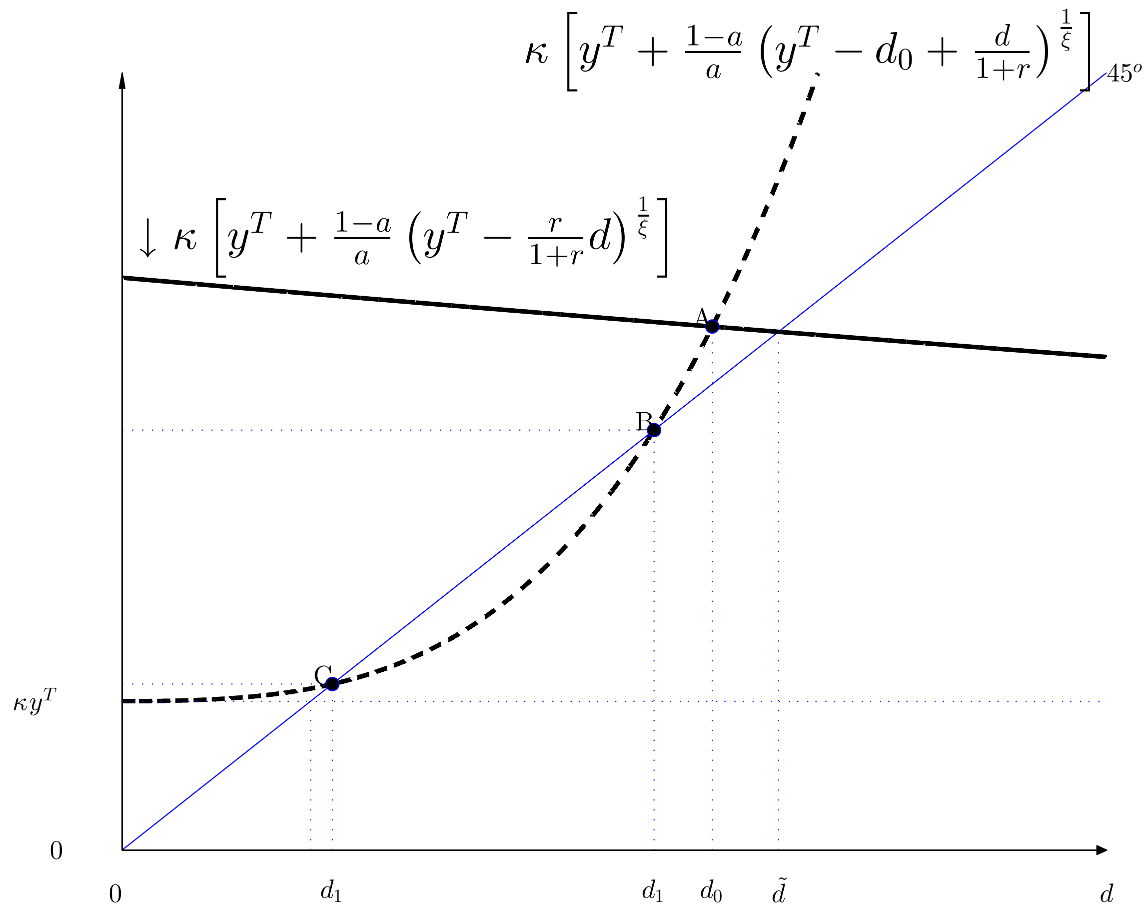
Are there other equilibria?

Yes. We will look for *self-fulfilling financial crisis equilibria* in which a crisis occurs in period 0 and the economy reaches a steady state in period 1.

The period-0 collateral constraint

$$d_1 \leq \kappa y^T + \kappa \frac{1-a}{a}\left(y^T + \frac{d_1}{1+r} - d_0\right)^{\frac{1}{\xi}}$$

Self-Fulfilling Crises



This graph shows that the steady-state equilibrium (point A) may coexist with self-fulfilling financial crises (points B or C). It remains to show that the Euler equation and $\mu_t \geq 0$ are satisfied (next slide).

The Euler Equation at Points B or C

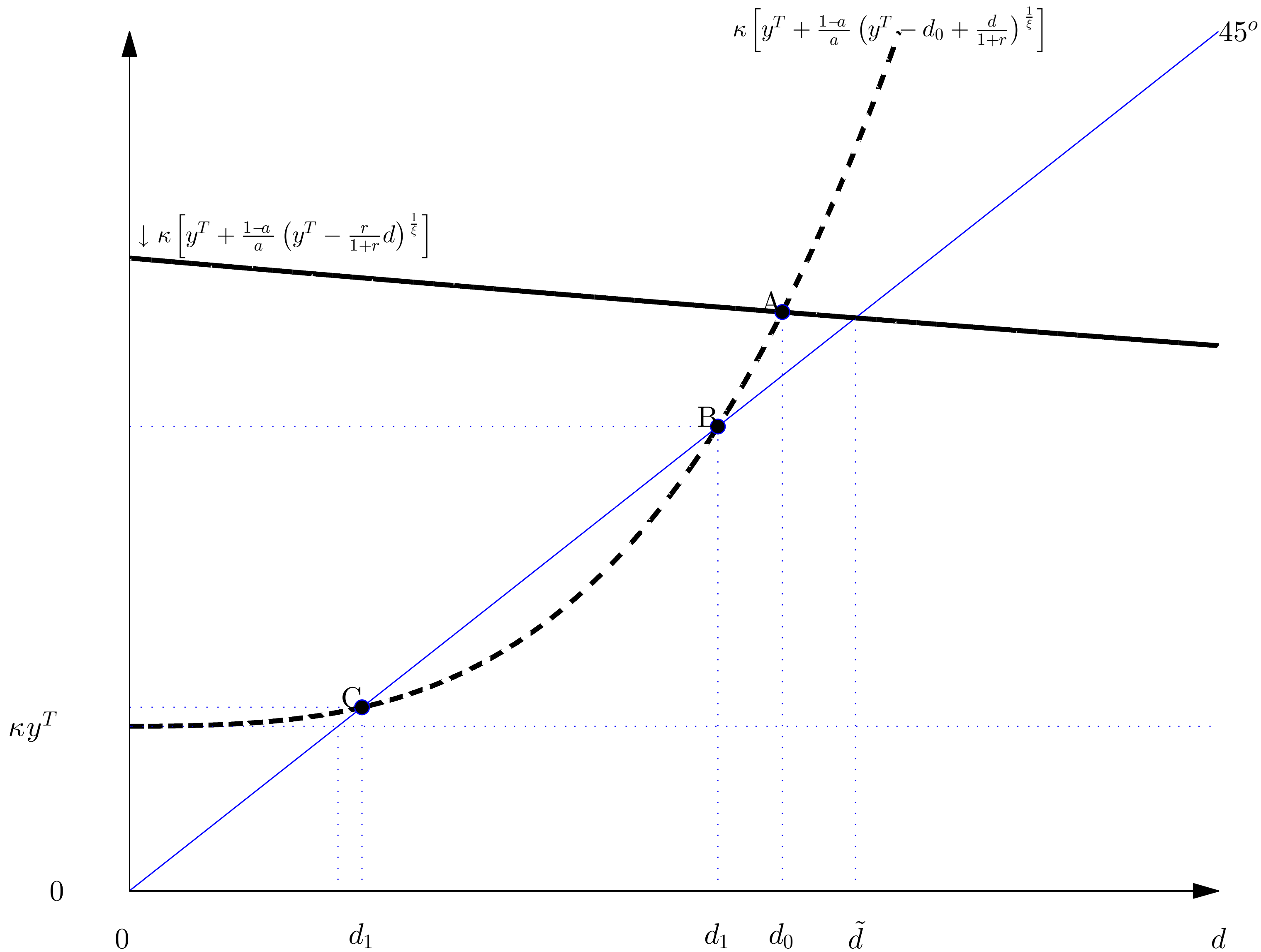
The Euler equation in period 0 is

$$\frac{\Lambda(c_0^T)}{\Lambda(c_1^T)} = \frac{\beta(1+r)}{1-\mu_0(1+r)}$$

In period 0, the economy is at point B or C, where the collateral constraint binds, so $\mu_0 > 0$. Also, $c_0 < c_1$, because the economy deleverages in period 0 and is in a steady state with less debt starting in period 1. It follows that LHS > 1. Pick $\mu_0 > 0$ to make LHS=RHS. Thus the Euler Equation is also satisfied at points B and C.

This establishes the existence of a self-fulfilling crisis.

Sunspots


$\kappa\left[y^T + \frac{1-a}{a}\left(y^T - d_0 + \frac{d}{1+r}\right)^{\frac{1}{\xi}}\right]$
45^o
$\downarrow \kappa\left[y^T + \frac{1-a}{a}\left(y^T - \frac{r}{1+r}d\right)^{\frac{1}{\xi}}\right]$
A
B
C
κy^T
0
0
d_1
d_1
d_0
$\tilde{d}$
d

Sunspots and Persistent Financial Crisis

• under perfect foresight, a self-fulfilling financial crisis can last at most one period.
• with extrinsic uncertainty (sunspots) financial crises can be persistent.
• Let the sunspot variable s_t be either 0 (slack CC) or 1 (binding CC).
• Construct a 2-period financial crisis. Assume: $s_0 = 1$; $s_1 = 1$ with probability π, and $s_t = 0\ \forall t \geq 2$. Let $c^T_{1,0}$ and $c^T_{1,1}$ denote consumption in period 1 if $s_1 = 0, 1$, respectively.
• we then have: $c^T_{1,0} > c^T_0 > c^T_{1,1}$.

• Euler in period 0

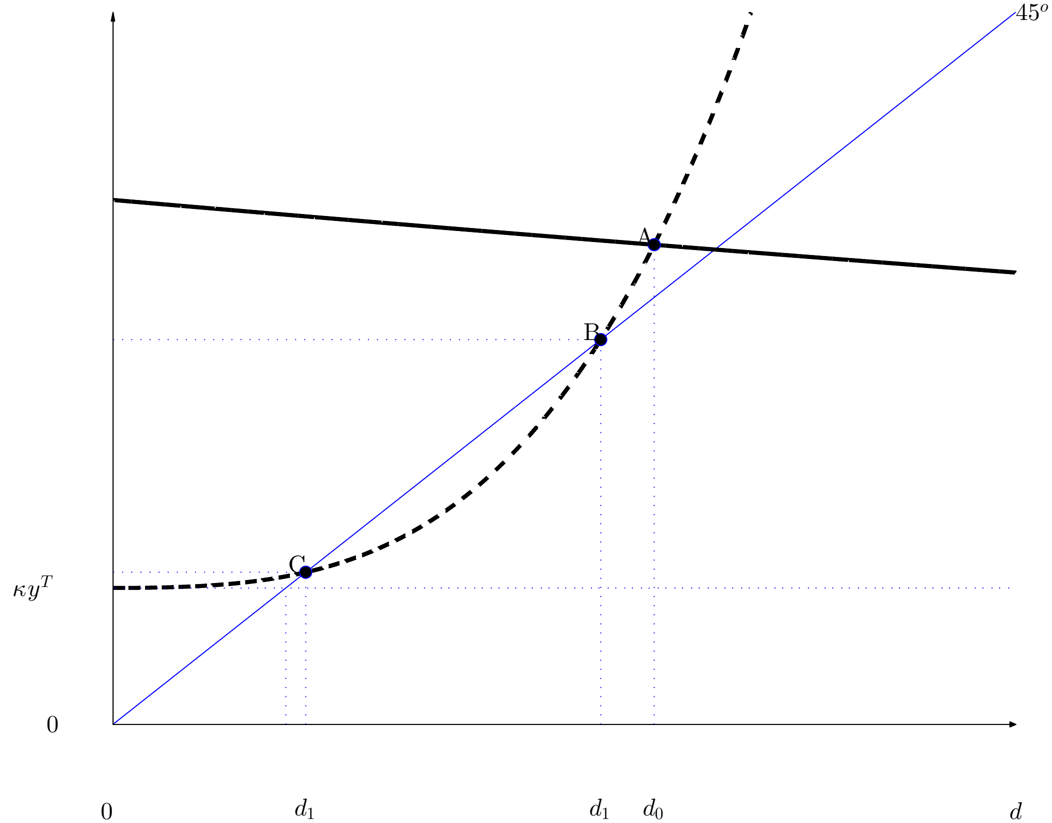
$$[1-(1+r)\mu_0]\Lambda(c^T_0) = \pi\Lambda(c^T_{1,1}) + (1-\pi)\Lambda(c^T_{1,0}).$$

• Persistent financial crisis exist if

$$\pi \in (0, \pi^*], \qquad \text{where } \pi^* \equiv \frac{\Lambda(c^T_0) - \Lambda(c^T_{1,0})}{\Lambda(c^T_{1,1}) - \Lambda(c^T_{1,0})} \in (0,1).$$

Underborrowing

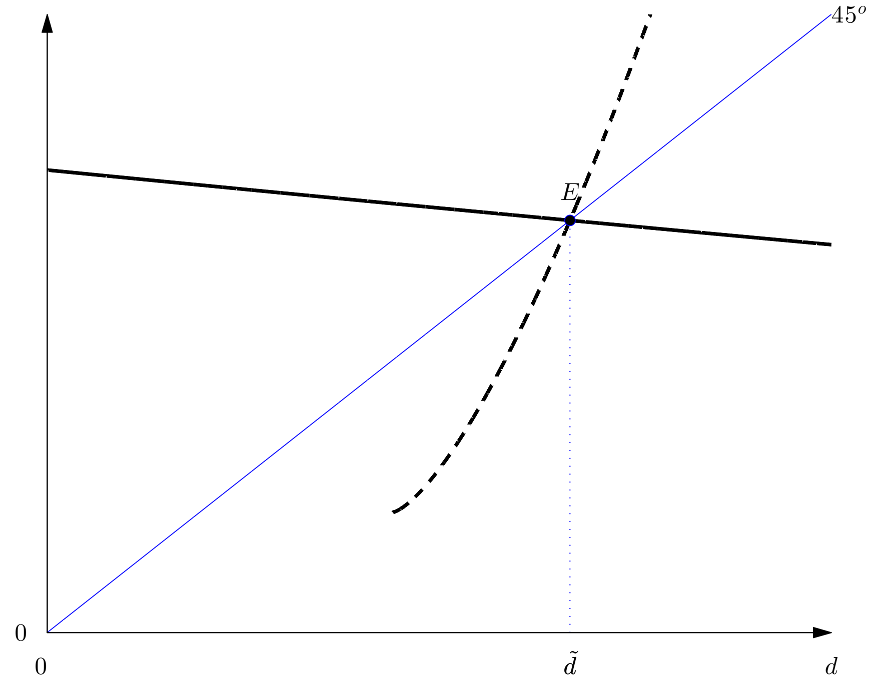
Underborrowing



• Welfare ranking: $C < B < A$.
• The competitive equilibrium at point A is the first-best allocation.
• Therefore, if agents coordinate on equilibrium A, there is neither overborrowing nor underborrowing.
• But if agents coordinate on equilibrium B or C, then the economy suffers from underborrowing.

Plausibility of Self-Fulfilling Crises

Ncessary and sufficient condition for the existence of self-fulfilling crises: the slope of the RHS of the period-0 collateral constraint must be larger than 1 at $d_1 = d_0 = \tilde{d}$ (point E in the figure below).



Notes. The downward-sloping solid line is the right-hand side of the steady-state collateral constraint. The upward-sloping dashed line is the right-hand side of the period-0 collateral constraint for $d_0 = \tilde{d}$. The figure is drawn under the assumption that $0 < \xi < 1$.

Empirical Plausibility of Self-Fulfilling Crises

The Bianchi (2011) Calibration: $\kappa = 0.33$, $\xi = 0.83$, $a = 0.31$, $r = 0.04$, $y^T = y^N = 1$.

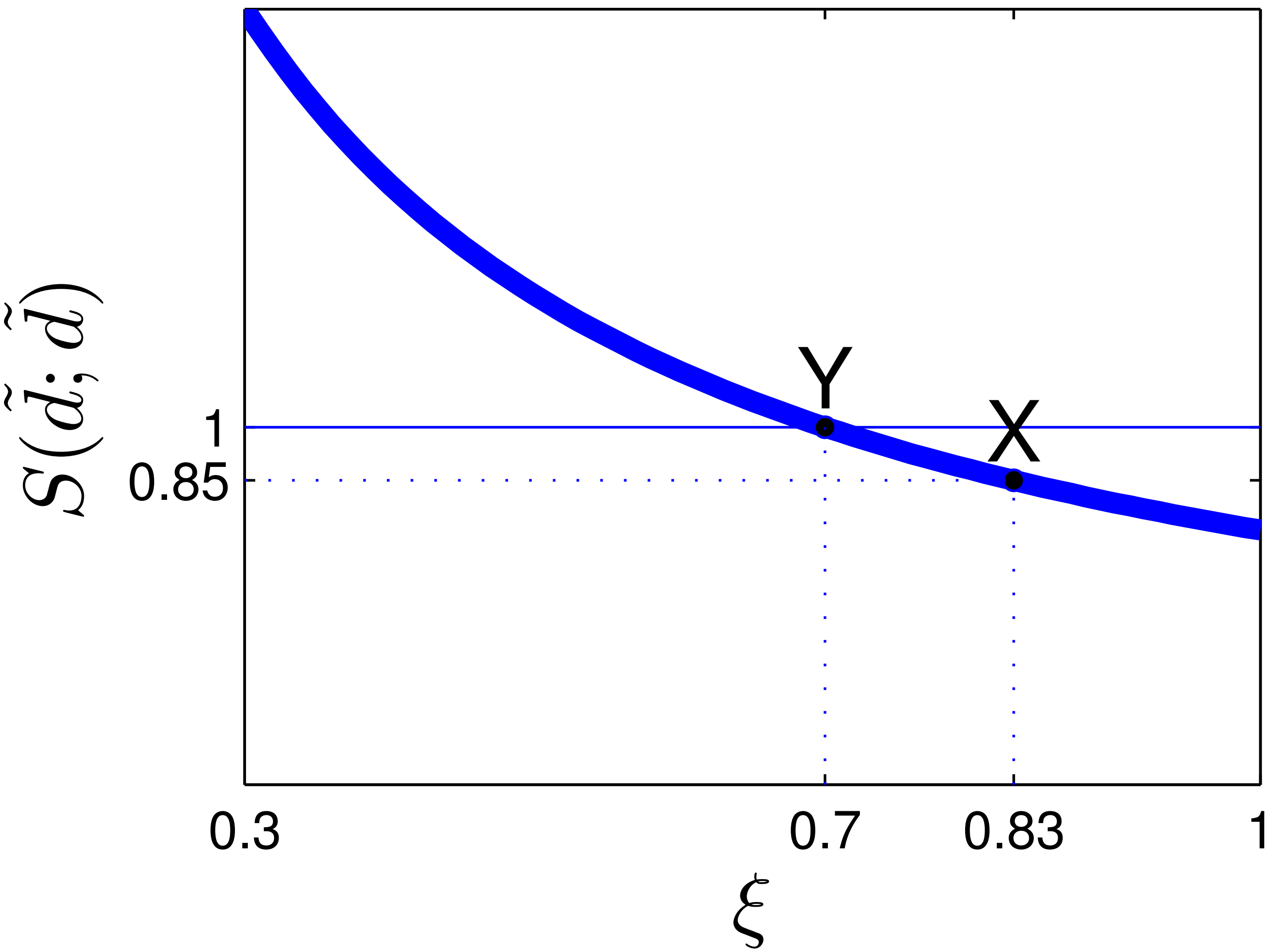
Let $S(\tilde{d}, \tilde{d})$ be the slope of the RHS of the period-0 CC at $d_1 = d_0 = \tilde{d}$. Then,

$$S(\tilde{d}; \tilde{d}) = 0.85$$

Under this calibration, there is a unique equilibrium. However, the empirically plausible range of ξ is [0.4, 0.83].

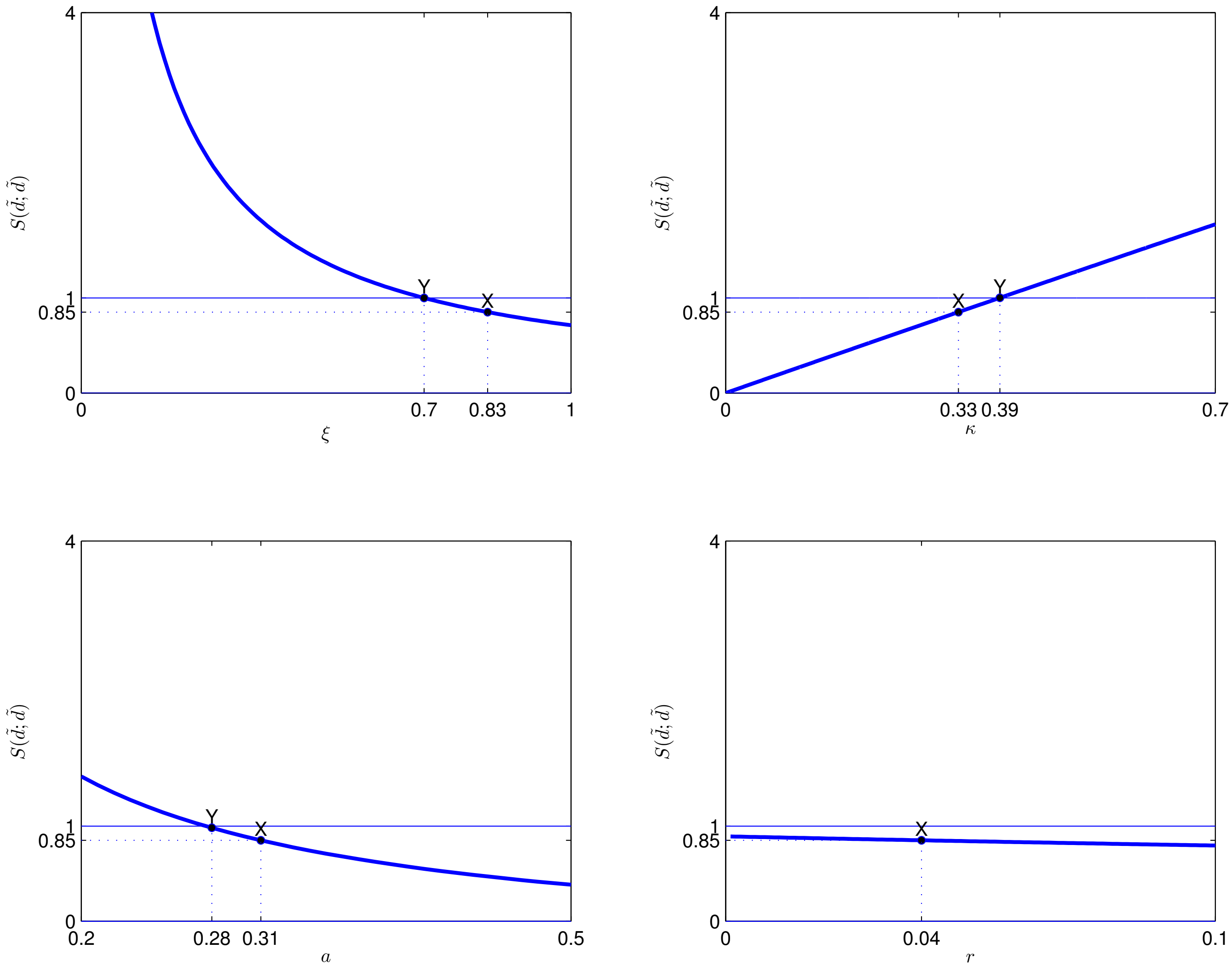
The next slide shows $S(\tilde{d}; \tilde{d})$ for values of ξ in its empirically plausible range.

Empirical Plausibility of Self-Fulfilling Crises(cont.)



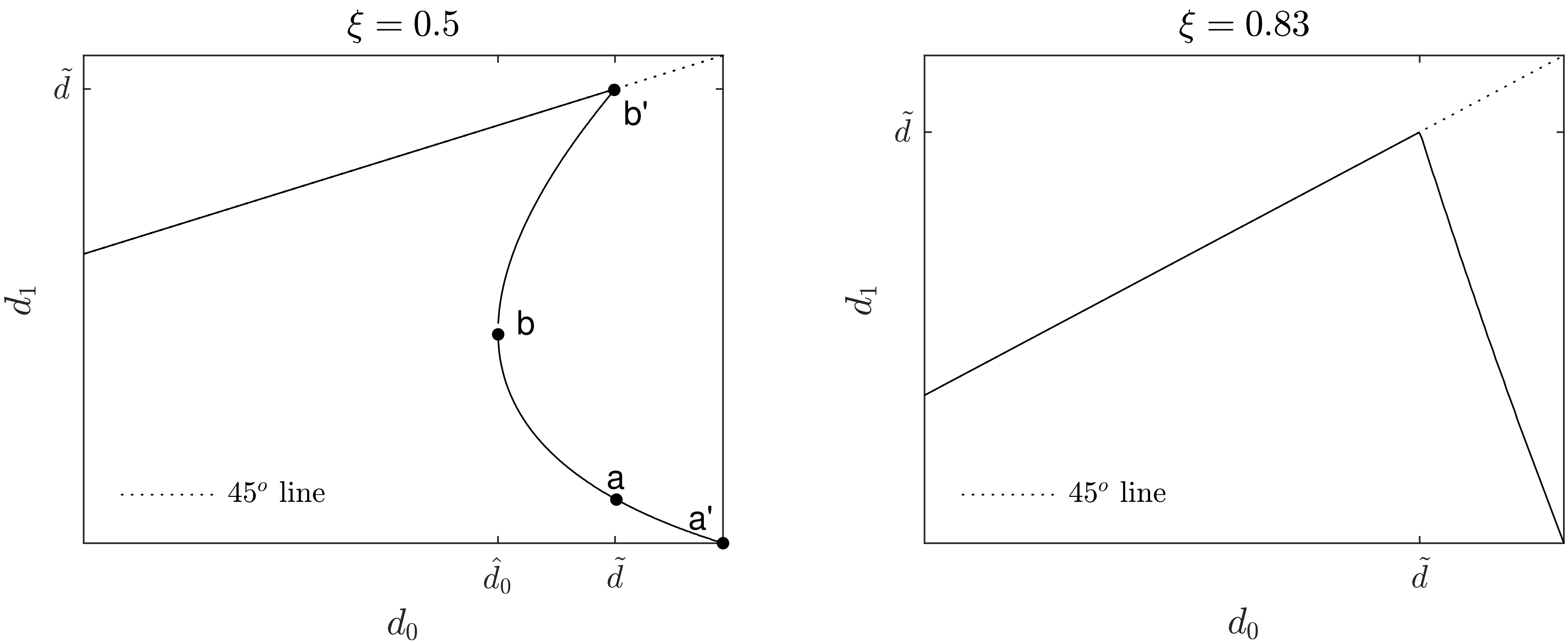
Note. The other parameters are $\kappa = 0.33$, $a = 0.31$, $r = 0.04$, $y^T = y^N = 1$ (Bianchi, 2011).

Plausibility of Self-Fulfilling Crises (cont.)



Notes. X baseline value; Y value at which $S(\tilde{d};\tilde{d}) = 1$ (multiplicity if $S(\tilde{d};\tilde{d}) > 1$). Other parameters: $\xi = 0.83$, $\kappa = 0.33$, $a = 0.31$, $r = 0.04$, and $y^T = y^N = 1$.

The Debt Policy Function for $\xi = 0.5$ and $\xi = 0.83$



Notes. Each panel displays the equilibrium value of d_1 as a function of d_0. The figure is drawn using the parameter values $\kappa = 0.33$, $a = 0.31$, $r = 0.04$, and $y^T = y^N = 1$. When $\xi = 0.5$, $S(\tilde{d}, \tilde{d}) > 1$, and when $\xi = 0.83$, $S(\tilde{d}, \tilde{d}) < 1$.

Implementation

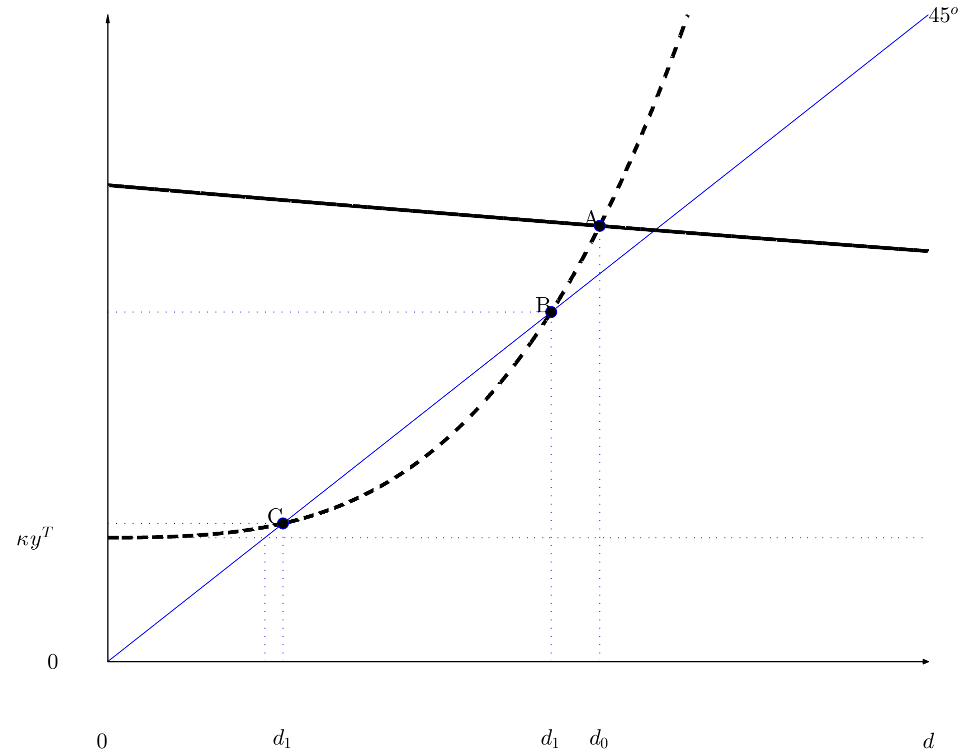
Implementation with Capital Control Taxes

Suppose that the government imposes a proportional tax on debt, τ_t. The budget constraint of the household becomes

$$c_t^T + p_t c_t^N + d_t = y^T + p_t y^N + \frac{1-\tau_t}{1+r} d_{t+1}$$

Capital control taxes can be rebated lump-sum or through income-based transfers.

Ramsey Optimal Taxation Versus Implementation



• The Ramsey optimal tax rate in this economy is $\tau_t = 0$ at all times. This is because the unregulated competitive equilibrium delivers the first-best allocation (point A).

• However, announcing the policy $\tau_t = 0$ for all t does not guarantee that the Ramsey optimal equilibrium will emerge. Indeed, this tax policy also supports the deleveraging equilibria (points B or C).

Implementing the Ramsey Optimal Allocation

What capital control policy can induce the Ramsey-optimal equilibrium? Consider capital-control policy rules of the form

$$\tau_t = \tau(d_{t+1}, d_t)$$

satisfying $\tau(d,d) = 0$ and $\tau_1 < 0$. This policy is consistent with the Ramsey equilibrium, since under the Ramsey equilibrium $d_{t+1} = d_t$ for all $t \geq 0$, and therefore $\tau_t = 0$ for all $t \geq 0$.

Off-Equilibrium Threat: By this tax rule, the government sets $\tau_t = 0$ if the economy stays at point A, but threatens to tax capital outflows if the economy enters in a financial panic and deleverages to point B or C.

Result: If the capital outflow tax is strong enough, then points B and C are ruled out, point A is the unique equilibrium, and capital controls are never imposed. The next slide shows why.

Implementation (continued)

Under this tax-policy rule, the Euler equation in period 0 becomes:

$$\left(\frac{c_1^T}{c_0^T}\right)^\sigma = \frac{1}{1-\tau(d_1,d_0)-(1+r)\mu_0}$$

(1) In the intended (Ramsey) equilibrium, $c_1^T/c_0^T = 1$, $d_1 = d_0$, $\mu_0 = 0$, and $\tau(d_1,d_0) = 0$, so the Euler equation holds.

(2) In the unintended equilibrium (points B or C), $c_1^T/c_0^T > 1$, and $d_1 < d_0$, and $\tau(d_1,d_0) > 0$. Make $\tau(d_1,d_0)$ so large that μ_0 has to be negative for the Euler equation to hold. Since μ_0 must be nonnegative, this capital-control policy rules out the unintended equilibrium.

Result: To rule out undesired self-fulfilling crises, the Ramsey planner commits to impose sufficiently large capital controls in case of speculative capital outflows.

Quantitative Analysis

The Environment

- A stochastic version of the economy, following Bianchi (2011).

- The time unit is one year.

- The economy is driven by tradable and nontradable endowment shocks (y_t^T and y_t^N).

- Shocks are estimated on Argentine data and assumed to follow a bivariate AR(1) process.

- Stationarity is induced by assuming that $\beta(1 + r) < 1$.

Equilibrium Selection

(A) For a given current state (y_t^T, y_t^N, d_t) if there is a value of d_{t+1} for which all equilibrium conditions are satisfied and the collateral constraint is not binding, pick it.

(B) If for a given current state (y_t^T, y_t^N, d_t) there are one or more values of d_{t+1} for which all equilibrium conditions are satisfied pick the largest one for which the collateral constraint is binding.

(C) If for a given current state (y_t^T, y_t^N, d_t) there are one or more values of d_{t+1} for which all equilibrium conditions are satisfied pick the smallest one for which the collateral constraint is binding.

Criteria (A), (B), and (C) favor equilibria like points A, B, and C, respectively, in the figures shown earlier.

Parameter Values

$\kappa = 0.33$
$\xi = 1/\sigma = 0.5$
$a = 0.31$
$r = 0.04$
$\beta = 0.91$

Discretization: 800 equally spaced points for d, 50 for y^T, and 50 for y^N.

How to define underborrowing in the present setup? Underborrowing obtains if borrowing in the unregulated economy is less than in the constrained optimum.

Constrained Optimal Problem

$$v(y^T, y^N, d) = \max_{c^T, d'} \left\{ U(A(c^T, y^N)) + \beta \mathbb{E} \left[v(y^{T\prime}, y^{N\prime}, d') \,\middle|\, y^T, y^N \right] \right\}$$

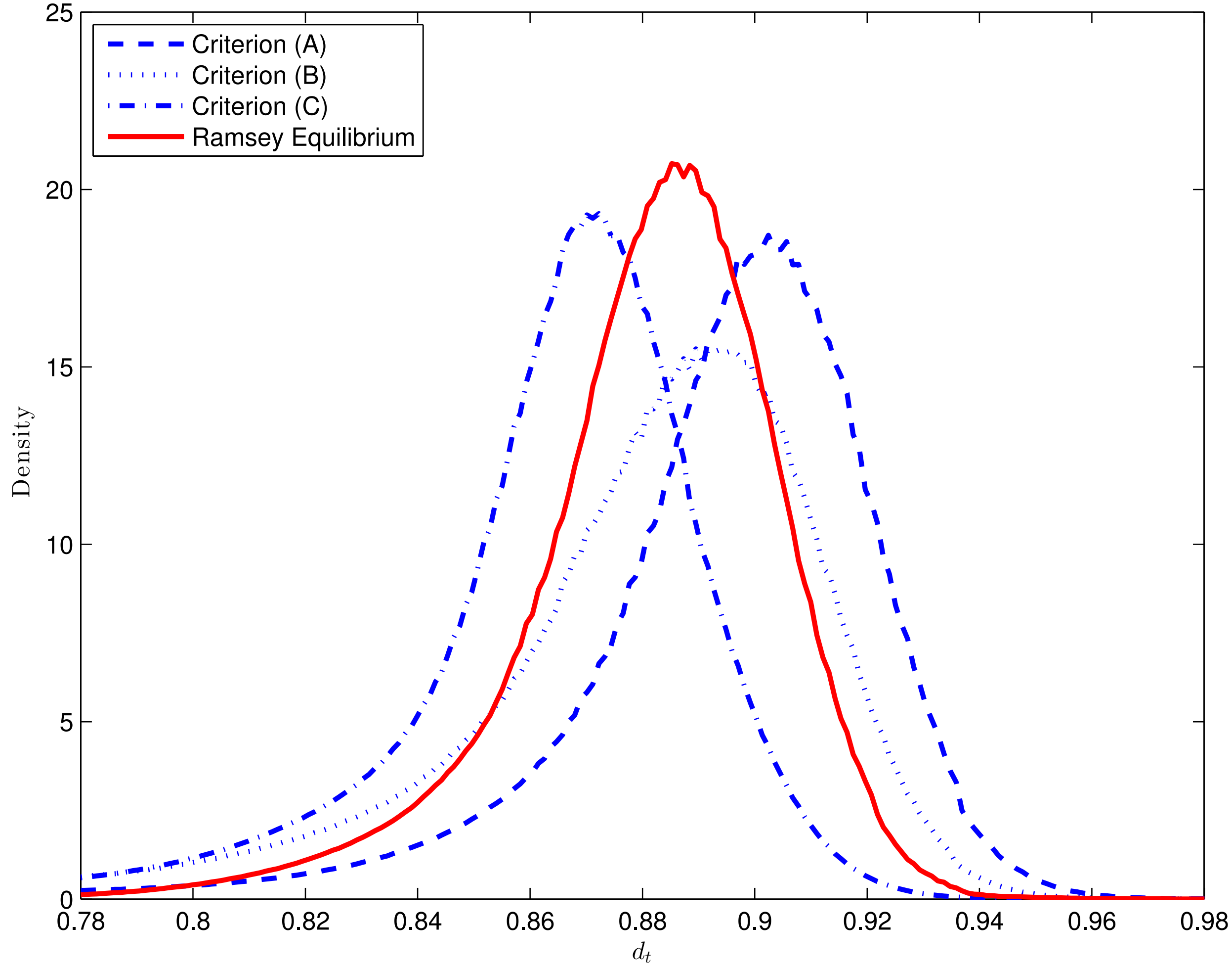
subject to

$$c^T + d = y^T + \frac{d'}{1+r}$$

$$d' \leq \kappa \left[y^T + \frac{1-a}{a} \left(\frac{c^T}{y^N} \right)^{\frac{1}{\xi}} y^N \right]$$

Note. Though, the constraints of this control problem may not represent a convex set in tradable consumption and debt, the equilibrium is generically untique.

Multiple Equilibria, $\xi = 0.5$

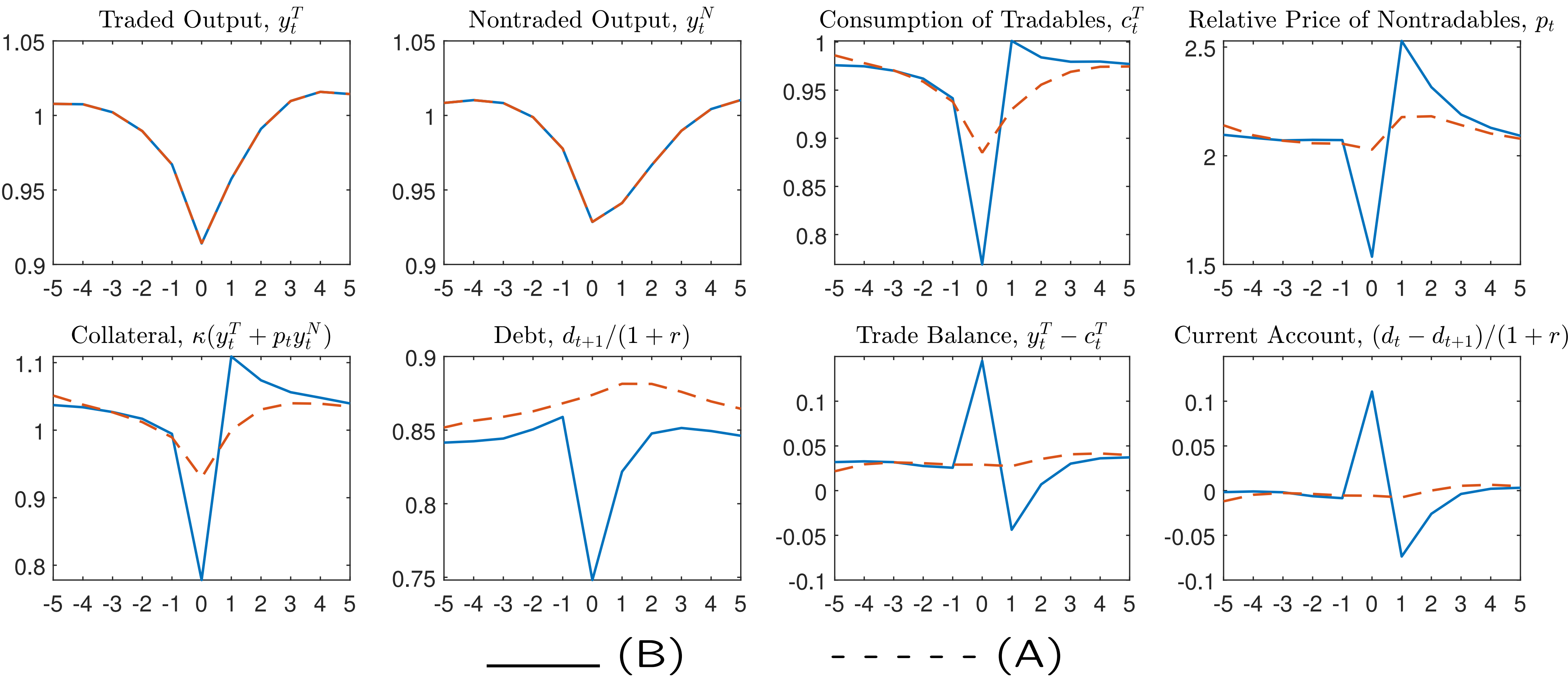


Notes. The figure shows the debt density for the unregulated economy under equilibrium selection criteria (A), (B), and (C) and under the Ramsey planner allocation. Replication program plotd.m.

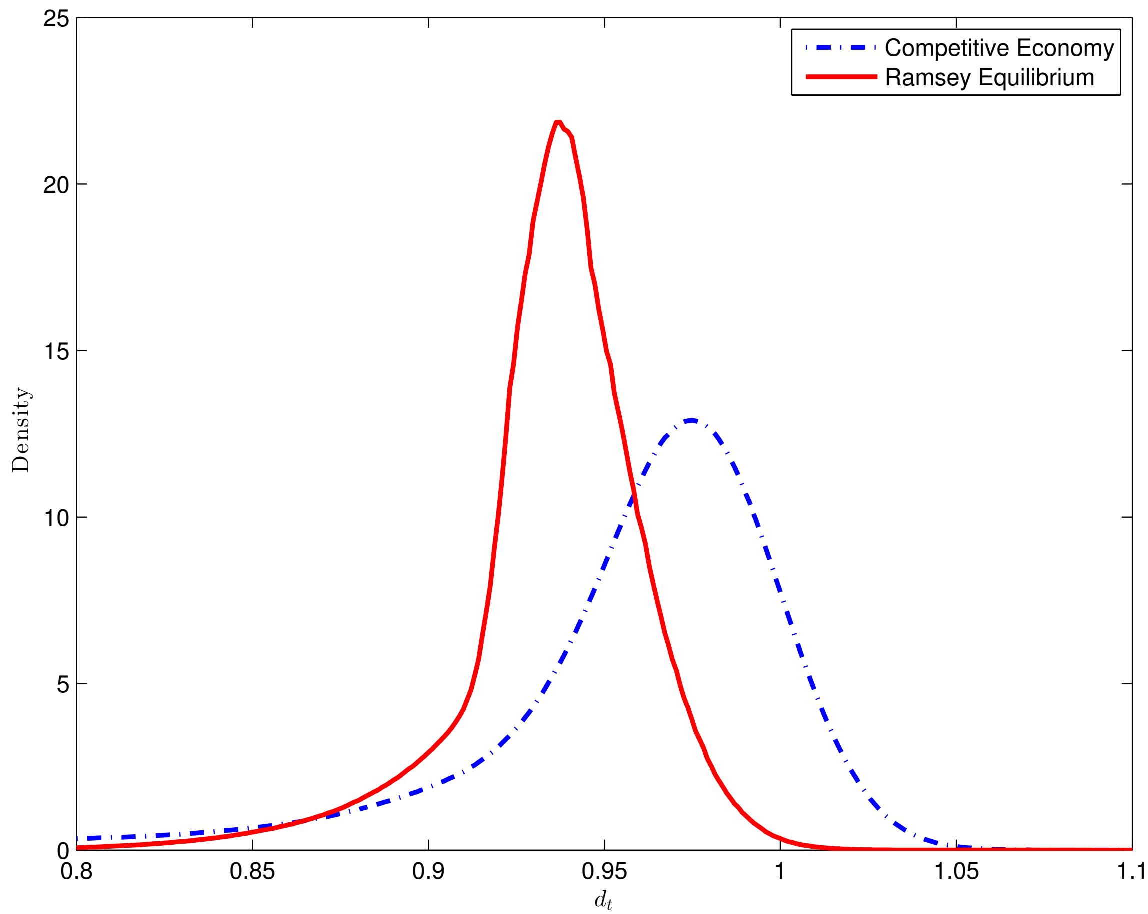
The typical self-fulfilling financial crisis

looks like a sudden stop

Typical Self-Fulfilling Crisis: Equilibrium Selection Criterion (B)



Overborrowing, $\xi = 0.83$



Notes. Debt distributions. Bianchi (2011) calibration ($\xi = 0.83$)

Conclusions

- This paper shows that open economies with flow collateral constraints display multiple equilibria.

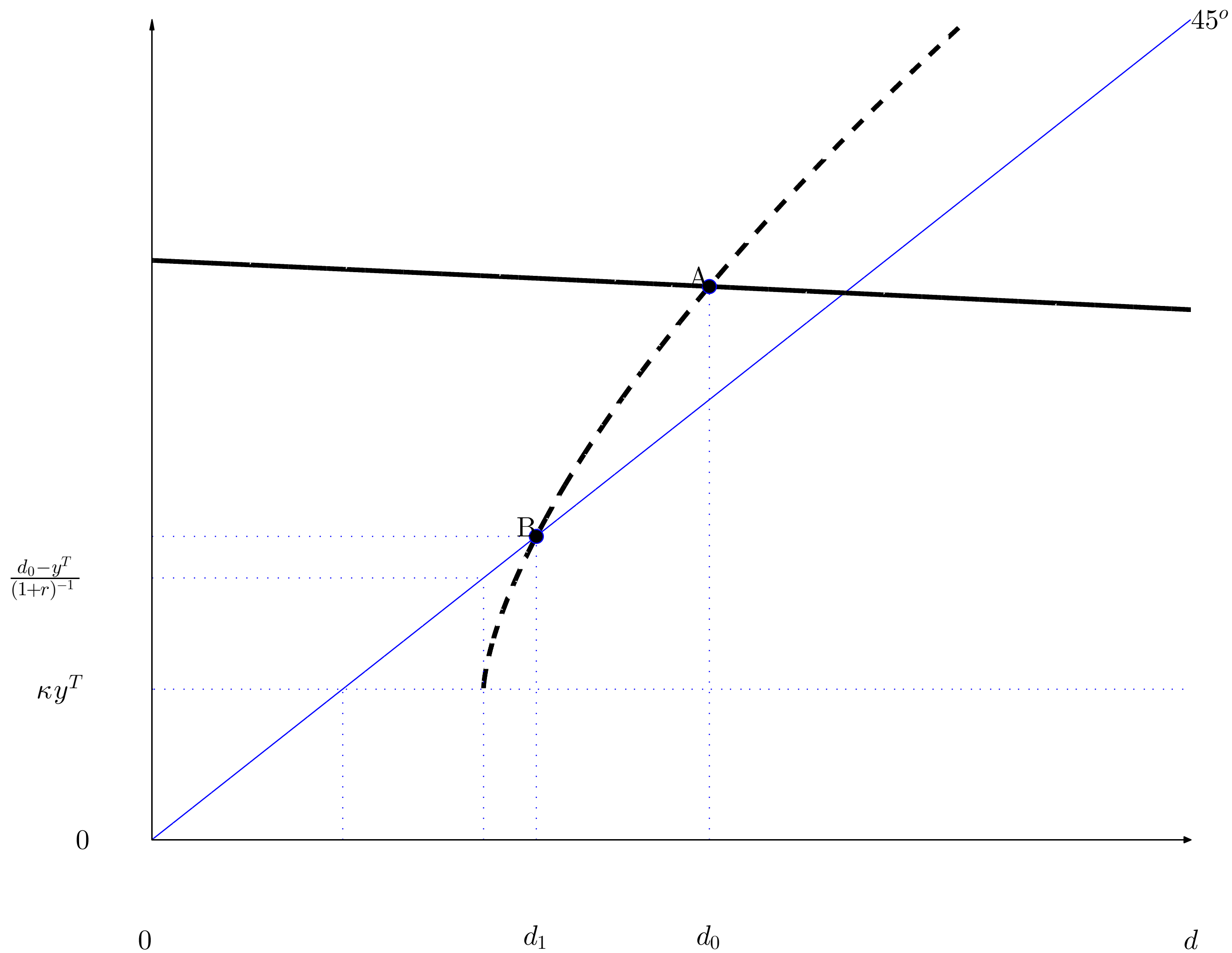
- In particular, they are prone to self-fulfilling crises in which deleveraging and Fisherian deflations take place in the absence of changes in fundamentals.

- The competitive equilibrium can display underborrowing, as agents deleverage in anticipation of a financial crisis.

- Averting self-fulfilling crises requires the threat of taxing non-fundamental bursts of capital outflows.

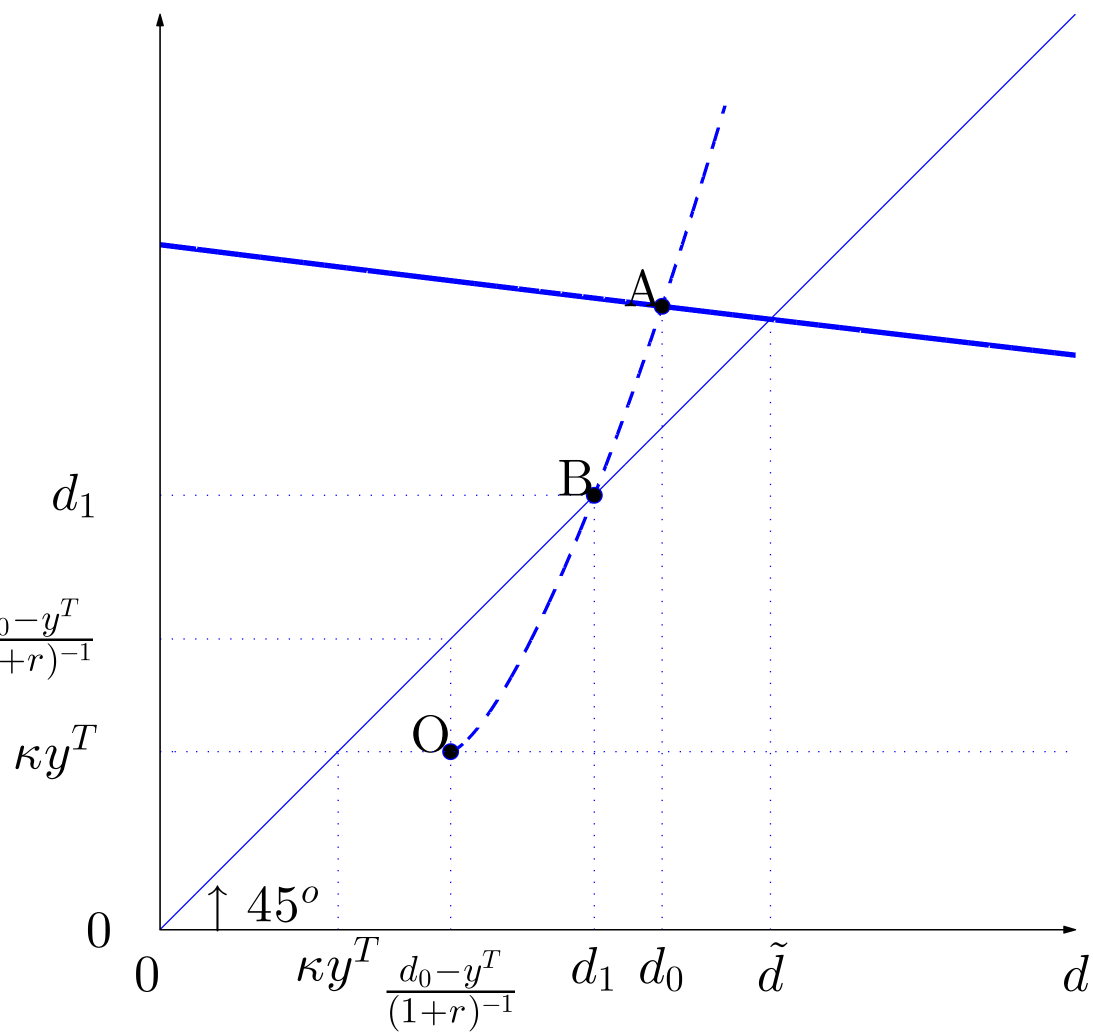
EXTRAS

Self-fulfilling financial crisis equilibria also exist for $\xi > 1$:



Are Points Between A and B Equilibria? No.

Take another look at the figure



At any such point: (a) The collateral constraint is satisfied (indeed it is slack); (b) the resource constraint is satisfied with positive consumption.

However, because at any such point $c_0^T < c_1^T$, the Euler equation is satisfied only if $\mu_t > 0$.

But this violates the slackness condition, which requires $\mu_0 = 0$, because the collateral constraint is slack in period 0.

Constructing the Transition Probability Matrix of the Exogenous States

Construct the transition probability matrix of the state $(\ln y_t^T, \ln(y_t^N))$ using a simulation approach. The Matlab code `tpm.m` implements a procedure consisting in simulating a time series of length 2,000,000 drawn from the AR(1) system above and allocating each draw in the time series with one of the 2500(=50×50) possible discrete states by distance minimization (see Schmitt-Grohé and Uribe, 2009, for details),

The resulting discrete-valued time series is used to compute the probability of transitioning from a particular discrete state in one period to a particular discrete state in the next period.

The resulting transition probability matrix, stored in `tpm.mat`, captures well the covariance matrices of order 0 and 1.

An alternative method for computing the transition probability matrix of the exogenous state is the quadrature based method proposed by Tauchen and Hussey (1991).

Note. Some combinations of (y_i^T, y_i^N) are never visited. We remove those states, resulting in 2,189 possible pairs (y_i^T, y_i^N) instead of 2,500. Thus we have $ny = 2,189$ grid points for the exogenous state.

How to pick the grid for debt, d_t

Use $nd = 800$ equally spaced points for d_t in the interval $[\underline{d}, \overline{d}]$

How to pick the first and last points of the grid? Upper bound taken from Bianchi: $1.02(1+r)$. First point is 0.2;

Overall state space, (y_t^T, y_t^N, d_t)

Overall grid size: $n = ny \times nd = 2,189 \times 800 = 1,751,200$ points.