The Taylor Rule and Optimal Monetary Policy

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1 Introduction

John Taylor (1993) has proposed that U.S. monetary policy in recent years can be described by an interest-rate feedback rule of the form

$$i_t = .04 + 1.5(\pi_t - .02) + .5(y_t - \bar{y}_t), \tag{1.1}$$

where i_t denotes the Fed's operating target for the federal funds rate, π_t is the inflation rate (measured by the GDP deflator), y_t is the log of real GDP, and $\bar{y}_t$ is the log of "potential output" (identified empirically with a linear trend).). The rule has since been subject to considerable attention, both as an account of actual policy in the U.S. and elsewhere, and as a prescription for desirable policy. Taylor argues for the rule's normative significance both on the basis of simulations and on the ground that it describes U.S. policy in a period in which monetary policy is widely judged to have been unusually successful (Taylor, 1999), suggesting that the rule is worth adopting as a principle of behavior.

Here I wish to consider to what extent this prescription resembles the sort of policy that economic theory would recommend. I consider the question in the context of a simple, but widely used, optimizing model of the monetary transmission mechanism, which allows one to reach clear conclusions about economic welfare. Of course, it will surprise no one that such a simple rule is unlikely to correspond to fully optimal policy in the context of a particular economic model. However, policy optimization exercises are often greeted with skepticism about how robust the advantages may be of the particular complex rule that is shown to be optimal in the particular model. For this reason, the analysis here addresses only broad, qualitative features of the Taylor rule, and attempts to identify features of a desirable policy rule that are likely to be robust to a variety of precise model specifications.

2 The Taylor Principle and Determinacy

A first question about the Taylor rule is whether commitment to an interest-rate rule of this kind, incorporating no target path for any monetary aggregate, can serve to determine

an equilibrium price level at all. According to the well-known critique of Sargent and Wallace (1975), interest-rate rules as such are undesirable, as they lead to indeterminacy of the rational-expectations equilibrium price level. As McCallum (1981) notes, however, their analysis assumes a rule that specifies an exogenous path for the short-term nominal interest rate; determinacy is instead possible in the case of feedback from an endogenous state variable such as the price level. In fact, many simple optimizing models imply that the Taylor rule incorporates feedback of a sort that suffices to ensure determinacy, owing to the dependence of the funds rate operating target upon recent inflation and output-gap measures.

Woodford (2000b) considers this question in the context of a "neo-Wicksellian" model that reduces to a pair of log-linear relations, an intertemporal "IS" equation of the form

$$y_t = E_t y_{t+1} - \sigma(i_t - E_t \pi_{t+1}) + g_t, \tag{2.1}$$

and an expectations-augmented "AS" equation of the form

$$\pi_t = \kappa(y_t - y_t^n) + \beta E_t \pi_{t+1}. \tag{2.2}$$

Here y_t denotes output (relative to trend), i_t the short-term nominal interest rate, and π_t the inflation rate, while g_t and y_t^n represent exogenous disturbances (autonomous spending variation and fluctuation in the "natural rate" of output, respectively). The coefficients σ and κ are both positive, while $0 < \beta < 1$ is the discount factor indicating the rate of time preference of the representative household.

Let monetary policy be specified by an interest-rate feedback rule of the form

$$i_t = i_t^* + \phi_\pi(\pi_t - \pi^*) + \phi_y(y_t - y_t^n - x^*), \tag{2.3}$$

where i_t^* is any exogenous stochastic process for the intercept, ϕ_π and ϕ_y are constant coefficients, and π^* and x^* are constant "target" values for the inflation rate and the output gap respectively. Then using (2.3) to eliminate i_t in (2.1), the system (2.1) – (2.2) can be written in the form

$$E_t z_{t+1} = A z_t + e_t, \tag{2.4}$$

where z_t is the vector with elements π_t and y_t, A is a matrix of constant coefficients, and e_t is a vector of exogenous terms. The system (2.4) has a unique stationary solution

(assuming stationary disturbance processes g_t and y_t^n) if and only if both eigenvalues of the matrix A lie outside the unit circle. If we restrict attention to policy rules with $\phi_\pi, \phi_y \geq 0$, this condition can be shown to hold if and only if

$$\phi_\pi + \frac{1-\beta}{\kappa}\phi_y > 1. \tag{2.5}$$

The determinacy condition (2.5) has a simple interpretation. A feedback rule satisfies the *Taylor principle* if it implies that in the event of a sustained increase in the inflation rate by k percent, the nominal interest rate will eventually be raised by *more than k* percent.[1] In the context of the model sketched above, each percentage point of permanent increase in the inflation rate implies an increase in the long-run average output gap of $(1-\beta)/\kappa$ percent; thus a rule of the form (2.3) conforms to the Taylor principle if and only if the coefficients ϕ_π and ϕ_y satisfy (2.5). In particular, the coefficient values associated with the classic Taylor rule ($\phi_\pi = 1.5$, $\phi_y = 0.5$) necessarily satisfy the criterion, regardless of the size of β and κ. Thus the kind of feedback prescribed in the Taylor rule suffices to determine an equilibrium price level.

A similar result is obtained in the case of a rule that incorporates interest-rate inertia of the kind characteristic of estimated Fed reaction functions (*e.g.*, Judd and Rudebusch, 1997). If (2.3) is generalized to the form

$$i_t = i_t^* + \rho(i_{t-1} - i_{t-1}^*) + \phi_\pi(\pi_t - \pi^*) + \phi_y(y_t - y_t^n - x^*), \tag{2.6}$$

where $\rho \geq 0$, then Woodford (2000) shows that equilibrium is determinate if and only if

$$\phi_\pi + \frac{1-\beta}{\kappa}\phi_y > 1 - \rho, \tag{2.7}$$

and ϕ_π and ϕ_y are not both equal to zero. Note that (2.7) once again corresponds precisely to the Taylor principle.

Another argument against interest-rate rules with a venerable history asserts that targeting a nominal interest rate allows for unstable inflation dynamics when inflation expectations extrapolate recent inflation experience. The basic idea, which originates in Wicksell's (1898) description of the "cumulative process", is that an increase in expected

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[1]The importance of this criterion for sound monetary policy is stressed, for example, in Taylor (1999).

inflation, for whatever reason, leads to a lower perceived real interest rate, which stimulates demand. This generates higher inflation, increasing expected inflation still further, and driving inflation higher in a self-fulfilling spiral.[2] But once again, the classic analysis implicitly assumes an exogenous target path for the nominal interest rate. The sort of feedback from inflation and the output gap called for by the Taylor rule is exactly what is needed to damp such an inflationary spiral.

Bullard and Mitra (2000a) consider the "expectational stability" (a sort of stability analysis under adaptive learning dynamics; see Evans and Honkapohja, 1999) of rational-expectations equilibrium in the model sketched above, in the case of a policy rule belonging to the family (2.3). They find that condition (2.5) is also necessary and sufficient for stability in this sense, *i.e.,* for convergence of the learning dynamics to rational expectations. Thus they confirm the Wicksellian instability result in the case of feedback from inflation and/or the output gap that is too weak; but this is not a problem in the case of a rule that conforms to the Taylor principle. Bullard and Mitra (2000b) find that the same is true for rules in the more general class (2.6): the conditions for determinacy of rational-expectations equilibrium coincide with the conditions for "learnability" of that equilibrium. Once again, there is no intrinsic unsuitability of an interest-rate rule as an approach to inflation control; but Taylor's emphasis upon raising interest rates sufficiently vigorously in response to increases in inflation is again justified.

3   Inflation and Output-Gap Stabilization Goals

Even granting that the Taylor rule involves feedback of a kind that should tend to exclude instability due purely to self-fulfilling expectations, one must consider whether the equilibrium determined by such a policy is a desirable one. The dependence of the funds rate target upon the recent behavior of inflation and of the output gap is prescribed, not simply because this is one way to exclude self-fulfilling expectations, but because it is assumed that the Fed wishes to damp fluctuations in both of those variables. This raises two questions. Are inflation and output-gap stabilization in fact sensible proximate goals

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[2]More recent expositions of the idea include Friedman (1968) and Howitt (1992).

for monetary policy? And even if they are, is the kind of feedback prescribed by the Taylor rule an effective way of achieving such goals?

Woodford (1999a) argues that both inflation and output-gap stabilization are sensible goals of monetary policy, as long as the "output gap" is correctly understood. In fact, the paper shows that in the context of the simple optimizing model from which structural equations (2.1) and (2.2) are derived, it is possible to motivate a quadratic loss function as a second-order Taylor series approximation to the expected utility of the economy's representative household. This welfare-theoretic loss function takes the form

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t L_t \right\}, \tag{3.1}$$

where β is the same discount factor as in (2.2), and the period loss function is of the form

$$L_t = \pi_t^2 + \lambda (y_t - y_t^n - x^*)^2, \tag{3.2}$$

for certain positive coefficients λ and x^*. Here y_t^n is the same exogenously varying "natural rate" of output as in (2.2). This is defined as the equilibrium level of output that would obtain in the event of perfectly flexible prices; this will in general not grow with a smooth trend, as a result of real disturbances of many kinds, as has been emphasized by the "real business cycle" literature. (As can be seen from (2.2), in the present model it is also the level of output associated with an equilibrium with stable prices.) It is the gap between actual output and this "natural rate" that one wishes to stabilize.

There is a simple intuition for the two stabilization objectives in (3.2). To the degree of approximation discussed in Woodford (1999a), the efficient level of output (which is the same for all goods, in the presence of purely aggregate shocks) varies in response to real disturbances in exactly the same proportion as does the flexible-price equilibrium level of output, y_t^n; the two series differ at all times by a constant factor, indicated by $x^* > 0$. A quadratic approximation to total deadweight loss is then given by the sum over all goods of the squared deviation of the output of each good from the efficient level $y_t^n + x^*$. This sum can then be decomposed into a term equal to the deviation of the average level of output across goods, y_t, from the efficient level, and a term equal to the average deviation of the output of each individual good from the average level of output.

This second term, the dispersion of output levels across goods, is in turn proportional to the dispersion of prices across goods due to imperfect synchronization of price changes. In the case of the particular model (due to Guillermo Calvo, 1983) of price-setting assumed in deriving (2.2), price dispersion is in turn proportional to the square of the inflation rate (whether positive or negative). However, the connection between price dispersion and instability of the general level of prices holds more generally in the presence of delays in price adjustment (as is further illustrated in Woodford, 1999a); thus a goal of inflation stabilization may be justified on more general grounds, even if the welfare-theoretic loss function is not generally of the exact form (3.2).

We thus find that the stabilization goals implicit in the Taylor rule have a sound theoretical basis, subject to two important qualifications. The first is that Taylor's classic formulation of the rule seeks to stabilize inflation around a target rate of two percent per annum. Instead, the welfare-theoretic loss function just referred to implies that the target rate of inflation should be zero, as this is the rate that minimizes relative-price distortions associated with imperfect synchronization of price changes.[3] The simple analysis above ignores various relevant factors that may modify this conclusion. On the one hand, it abstracts from the monetary frictions emphasized by Friedman (1969), that are minimized by anticipated deflation; when these are taken into account as well, the optimal inflation target may be negative, though it will not be as negative as indicated by Friedman's analysis, as deflation causes relative-price distortions in the same way as inflation.[4] On the other hand, the zero lower bound on nominal interest rates limits the ability of the central bank to use monetary policy for stabilization purposes, unless the average nominal interest rate is far enough above zero; Summers (1991) argues that this concern justifies a

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[3]The mere form of the loss function (3.2) may not make this obvious, as it also implies that it is desirable to keep the output gap as close as possible to the positive level x^*, which requires a positive level of average inflation according to (2.2). However, Woodford (1999a) shows nonetheless that the optimal commitment involves a long-run average inflation rate of zero. This is because commitment to a positive inflation rate in any period t raises output in period t (holding fixed expected inflation in period $t + 1$), but also lowers output in period $t - 1$, through the effect of the anticipation of this inflation the period before, and the latter effect lowers welfare as much as the former effect raises it. There is then a net welfare loss, owing to the increased price dispersion.

[4]Khan *et al.* (2000) reach a similar conclusion.

positive inflation target. The numerical analysis presented in Woodford (1999a), drawing upon the estimates of Rotemberg and Woodford (1997), implies that when both of these corrections are made, the optimal inflation target is slightly positive, but still much less than one percent per annum.

The second qualification is that the "output gap" that one should seek to stabilize is the gap between actual output and the "natural rate" of output defined above. This contrasts with the assumption made in Taylor's (1993) comparison between the proposed rule and actual U.S. policy, where the output gap is assumed to be measured by output relative to an exponential trend. In theory, a wide variety of real shocks should affect the growth rate of potential output in the relevant sense; as shown in Woodford (2000b), these include technology shocks, changes in attitudes toward labor supply, variations in government purchases, variation in households' impatience to consume, and variation in the productivity of currently available investment opportunities, and there is no reason to assume that all of these factors follow smooth trends. As a result, the output gap measure that is relevant for welfare may be quite different from simple detrended output. Indeed, the two series might even be negatively correlated, if policies that respond to deviations of output from trend cause output to fall relative to the natural rate, while still rising relative to trend, in periods when the growth in the natural rate is unusually high.[5]

In seeking to construct a theoretically more accurate measure of the extent of demand pressure relative to productive capacity, it is important to recognize that what one really wishes to stabilize, in order to minimize deadweight losses, is the level of real marginal supply cost.[6] In the model of Woodford (2000b), real marginal cost is predicted to covary perfectly (and positively) with the output gap defined above, because the desired markup would be constant under flexible prices; we can then write the loss function (3.2) in terms of inflation and the output gap, just as we can use the output gap as a measure of demand pressure in (2.2). But there are more obvious proxies for real marginal cost than the observed level of real activity. For example, Sbordone (2000) shows that equation

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[5]The residuals of the estimated equations of Rotemberg and Woodford (1997) imply that this has been the case.

[6]Alternatively, one wishes to stabilize the markup of price over marginal cost, as emphasized by Goodfriend and King (2000).
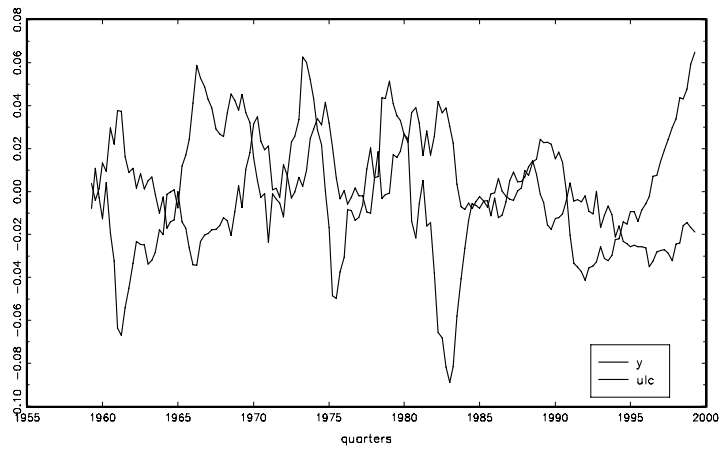
Figure 3.1: Real unit labor costs (relative to mean) compared to detrended real GDP (quadratic trend removed); quarterly US data.

(2.2) gives a very poor account of variations in U.S. inflation over the past several decades when y_t^n is interpreted as the trend in real GDP, but that this model of price-setting can instead explain U.S. inflation history quite well when real marginal supply cost is either directly measured from real unit labor cost, or is instead inferred from the observed paths of real GDP, consumption, hours, and their trends, in accordance with a simple optimizing model of labor supply and demand that allows for stochastic disturbances to both.[7] This suggests the superiority of these alternative measures of real marginal cost. But Sbordone shows that these measures have not even been positively correlated with detrended real GDP over her sample period; note the correlation of -.35 between the two series plotted in figure 1 above. Thus the use of such measures in a Taylor rule would make a significant difference in practice.

A necessary caveat to this conclusion is that the baseline analysis in Woodford (1999a) assumes only real disturbances that shift the efficient level of output and the flexible-price equilibrium level (*i.e.,* the natural rate) to the same extent (to a log-linear approximation, and in the case of small enough distortions). There are a wide variety of real disturbances, of the kinds typically emphasized in models of business fluctuations, that have this property. However, one may imagine various sources of inefficient variation in the natural rate of output (Giannoni, 2000). Examples include variation in marginal tax rates, variation in desired (as opposed to actual) markups, or variation in the distortions resulting from labor-market frictions such as union power, efficiency wages, or wage stickiness. Such disturbances, insofar as they are important, create a time-varying wedge between the real *social* marginal cost of supplying goods and the real *private* supply cost. It is then the social marginal cost that policy should aim to stabilize, while it is the private supply cost

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[7]Figure 2 illustrates this when the data series for real unit labor cost plotted in figure 1 is used. In each panel of the figure, a small, unrestricted VAR is used to forecast the future evolution of the "gap" proxy, and then (2.2) is "solved forward" to obtain the predicted quarterly inflation series. The assumed value of β is .99; in panel (b), the elasticity $\kappa > 0$ is chosen to minimize the mean-squared error of the prediction, while in panel (a) an arbitrary positive value is assumed (since the predicted inflation fluctuations are actually negatively correlated with the actual ones). On the success of this inflation equation when real unit labor costs are used, see also Sbordone (1998), Gali and Gertler (1999), Gali *et al.* (2000), and Batini and Nickell (2000).
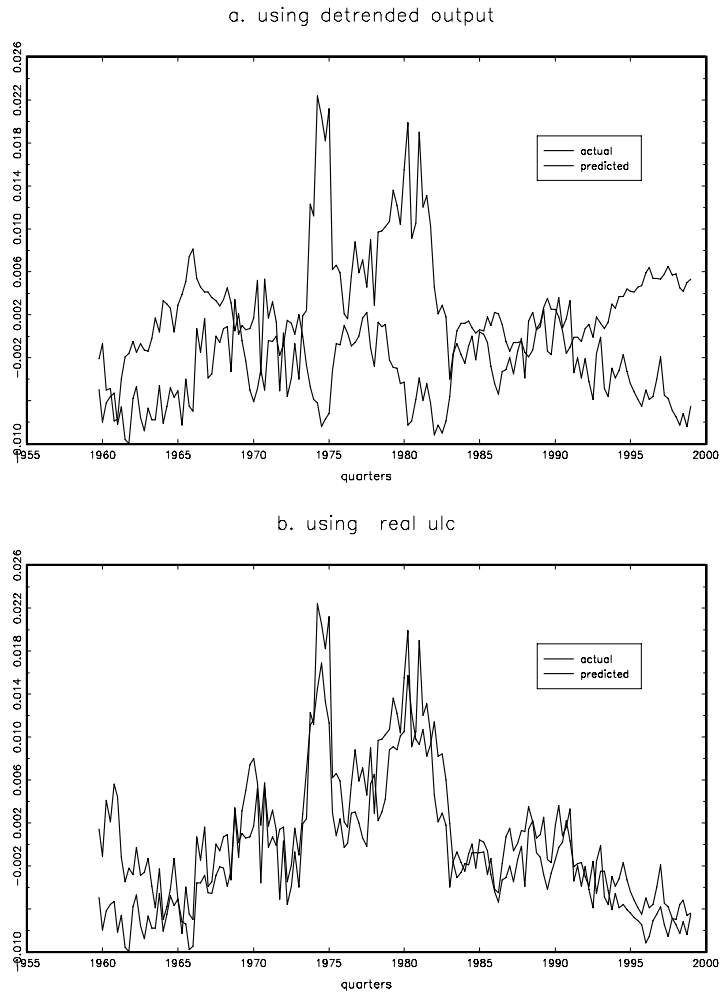
a. using detrended output

b. using real ulc

Figure 3.2: The inflation dynamics predicted by equation (2.2), using two alternative measures of the output gap, compared with actual U.S. quarterly inflation data. (a) The detrended output series plotted in figure 1. (b) The real unit labor cost series plotted in figure 1.

that determines equilibrium inflation. The results of Sbordone and company suggest that real private marginal cost has not covaried closely with detrended output; yet one might conceivably still argue that detrended output is a better proxy for real *social* marginal cost, if moderate-frequency variation in the natural rate is thought to be due largely to inefficient supply disturbances of the kind just mentioned. Analysis of the ultimate sources of random variation in supply costs – not just their time-series properties, but their significance for economic welfare – is thus an important topic for further study.

4 Responding to Variation in the Natural Rate of Interest

We turn now to the question of whether simple feedback from current measures of inflation and the output gap, of the kind prescribed by the Taylor rule, represents a desirable approach to the goal of stabilizing those variables. In at least one simple case, the answer is yes. Suppose that the monetary transmission mechanism is described by (2.1) – (2.2), the objective of policy is described by (3.1) – (3.2), both inflation and the output gap are observed with perfect precision, and a relation of the form (2.3) can be made to hold with perfect precision at all times. Then there is no conflict between the goals of inflation stabilization and output gap stabilization, since by (2.2), complete stabilization of inflation implies complete stabilization of the output gap as well. The optimal equilibrium thus involves a constant inflation rate (zero) and a constant output gap (also zero) at all times. One also observes that it is possible in principle to bring about a determinate equilibrium arbitrarily close to this one through commitment to an interest-rate rule with a constant intercept i^*, as assumed in Taylor's classic formulation. One simply needs to make the coefficients ϕ_π and/or ϕ_y extremely large. Then in the determinate equilibrium, fluctuations in the variable to which the response is extremely strong must be negligible; and since stability of either target variable implies stability of the other, both inflation and the output gap must be stabilized.

But this is not an appealing policy proposal, as even small deviations from the idealized assumptions listed could be quite problematic. In practice, small errors in the inflation and output-gap measures available to the Fed in real time[8] would surely lead to violent interest-

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rate volatility under such a rule. Even a small specification error preventing simultaneous complete stabilization of inflation and the output gap could have the same effect; and even if interest-rate stabilization is not as important a goal as inflation or output-gap stabilization, sufficient volatility of interest rates would surely create problems. It is thus of interest to stabilize inflation and the output gap without extremely large feedback coefficients, if possible. This requires a time-varying intercept i_t^* in the feedback rule (2.3).

It is easily seen from equations (2.1) – (2.2) that in the optimal equilibrium, the nominal interest rate will satisfy $i_t = r_t^n$, where

$$r_t^n = \sigma^{-1}[g_t + E_t(y_{t+1}^n - y_t^n)] \tag{4.1}$$

is the Wicksellian *natural rate of interest, i.e.,* the equilibrium real rate of interest in the case of perfectly flexible prices.[9] In our simple model, the natural rate of interest, like the natural rate of output, is an exogenous state variable – a function of the real shocks that is independent of monetary policy. The optimal equilibrium is then consistent with the policy rule (2.3) if and only if $i_t^* = r_t^n$ at all times – that is, the intercept term is adjusted one-for-one with variation over time in the natural rate of interest. Conversely, if the intercept varies in this way, then for *any* feedback coefficients satisfying (2.5), there is a determinate rational expectations equilibrium, and it is one in which inflation and the output gap are completely stabilized; if in addition $\pi^* = 0$ and $x^* = 0$ in (2.3), the equilibrium furthermore involves the optimal inflation rate (zero). Thus the optimal equilibrium is consistent with quite modest feedback coefficients, for example those suggested by Taylor.

However, achieving this result requires that the intercept term in the feedback rule vary over time, in response to real disturbances. Taylor (1993) assumes by contrast a constant intercept term, equal to the sum of the "the central bank's estimate of the equilibrium real rate of interest" and the inflation target π^*. By the "equilibrium" rate Taylor presumably means the flexible-price market-clearing rate, or what I have called the natural rate of

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[8]See Orphanides (2000) for documentation of the substantial revisions that have been made in these measures relative to the data available when policy decisions were made.

[9]See Woodford (2000b) for further discussion of this concept.

interest; however, he assumes that this term is a constant (two percent per annum) in his discussion of U.S. policy under Greenspan, while the natural rate should vary over time in response to many types of real disturbances, as discussed in Woodford (2000b). Failure to adjust the intercept i_t^* in the policy rule to track variation in the natural rate of interest will result in fluctuations in inflation and the output gap, for any finite values of the coefficients ϕ_π and ϕ_y — just as in the classic analysis of Wicksell (1898).

A rule of the kind just proposed — a rule of the form (2.3) with $i_t^* = r_t^n$, $\pi^* = x^* = 0$, and coefficients ϕ_π, ϕ_y satisfying (2.5) — is not the unique policy consistent with the optimal equilibrium, but it is a formulation that is particularly robust to possible mis-specifications in the assumed model of the economy. In particular, this rule is relatively independent of the assumed details of price adjustment, which is surely one of the most controversial features of our model. The coefficients in (4.1), indicating how the intercept term should be adjusted in the case of real disturbances, are independent of the assumed nature or degree of price stickiness; for the natural rate of interest is determined by relations that assume flexible prices. (The same is true of the definition of the natural rate of output y_t^n, which must also be computed in order to implement the rule.) The optimality of the target values $\pi^* = x^* = 0$ similarly follows from considerations that are independent of the assumed price stickiness; for example, Woodford (1999a) shows how the same conclusion is obtained under each of three alternative models of price-setting that correspond to alternative specifications of the aggregate supply relation. The determinacy condition (2.5) does depend upon the degree of price stickiness (as seen, for example, by the presence of the coefficient κ); but this is only an *inequality* that must be satisfied, and so it is possible for a given set of coefficients ϕ_π, ϕ_x to lie within the region of determinacy under any of a range of assumptions regarding price stickiness. The robustness of this proposal suggests that the development of empirically validated quantitative models of the natural rate of interest, and the preparation of real-time estimates of the current natural rate, should be an important task for central-bank staffs.[10]

The discussion thus far has assumed that the optimal equilibrium involves complete stabilization of inflation and output. There are various reasons why this need not be so.

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[10]See Neiss and Nelson (2000) for an early attempt.

For example, there may be inefficient variation in the natural rate of output, as discussed above; in this case, the welfare-theoretic loss function will be of the form

$$L_t = \pi_t^2 + \lambda(y_t - y_t^e)^2, \tag{4.2}$$

where y_t^e is the efficient level of output. It is then no longer possible to fully stabilize both inflation and the welfare-relevant output gap, $y_t - y_t^e$. Alternatively, it may not be feasible to fully stabilize inflation at a zero rate, because of the zero bound on nominal interest rates; in this case, incomplete inflation stabilization (but a low average inflation rate) will typically be preferable to complete stabilization around a higher rate. Or the high nominal interest rates required for complete inflation stabilization when the natural rate of interest is temporarily high may be undesirable, on account of the distortions stressed by Friedman (1969). Both of these latter considerations imply that one should prefer incomplete inflation and output-gap stabilization for the sake of less volatility of the short-term nominal interest rate. Optimal policy is then one that minimizes a loss function of the form

$$L_t = \pi_t^2 + \lambda_y(y_t - y_t^n - x^*)^2 + \lambda_i(i_t - i^*)^2, \tag{4.3}$$

for some weights $\lambda_y, \lambda_i > 0$, and a target interest rate i^* that need not be exactly consistent with zero average inflation. Once again the loss function contains a term that is not minimized through complete stabilization of inflation.

The optimal responses to shocks in these more complicated cases are characterized in Woodford (1999b) and Giannoni (2000). Here it suffices to note that it is still possible to achieve the optimal equilibrium through commitment to a rule of the form (2.3), but where i_t^* is now a more complicated function of the history of exogenous disturbances, and not simply equal to the current natural rate of interest. Once again, any values of ϕ_π and ϕ_y consistent with (2.5) are suitable — though the optimal process i_t^* now depends upon the values chosen for the feedback coefficients, since neither inflation nor the output gap is completely stabilized. The optimal process i_t^* is also no longer independent of the assumed degree and character of price stickiness.

However, the evolution of the natural rate of interest will continue to be a critical determinant of the optimal adjustment of the intercept i_t^*. For example, in the case that

all fluctuations in the natural rate of output are efficient, but a tradeoff exists between inflation stabilization and an interest-rate stabilization goal (as assumed in Woodford, 1999b), the optimal paths of inflation, the output gap, and the nominal interest rate are all functions solely of current and lagged values of state variables relevant for forecasting the natural rate of interest. It follows that under an optimal policy rule of the kind just described, i_t^* will be a function solely of that same information.

5 Advantages of Policy Inertia

Another limitation of the classic formulation of the Taylor rule is its suggestion that policy need respond only to the currently observed values of the target variables (inflation and the output gap). It is true that it is desirable to seek to stabilize these variables (when appropriately measured), and it is also true that the kind of contemporaneous response called for by the Taylor rule tends to stabilize them — to a greater extent the larger the positive values assigned to ϕ_π and ϕ_y (Woodford, 2000b). But it is not generally true that an optimal rule should make the nominal interest rate a function solely of the current values of these variables, or even of their current values and the current values of the exogenous states that determine them (such as the natural rate of interest). It is true that in the special case in which complete stabilization of inflation and the output gap is optimal, an optimal rule can take this simple form (though there is nothing uniquely optimal about this form — the rule might equally well respond to lagged inflation or to a forecast of future inflation, as none of these vary in the optimal equilibrium). But in general no optimal rule can take such a simple form. Instead, it is generally optimal for policy to respond *inertially* to fluctuations in the target variables and/or their determinants, so that policy will continue for some time to depend upon past conditions, even when these are irrelevant to the determination of current or future values of the target variables.

It may seem counter-intuitive that it is not optimal to instead "let bygones be bygones." But when the effects of policy depend crucially upon private sector expectations about *future* policy as well, it is generally optimal for policy to be history-dependent, so

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that the anticipation of later policy responses can help to achieve the desired effect upon private-sector behavior (Woodford, 2000a). For example, the policy that is optimal from the point of view of minimizing the objective (3.1) with period loss function (4.3) makes the nominal interest rate an increasing function of a moving average of current and past values of the natural rate of interest,

$$i_t = A(L)r_t^n, \tag{5.1}$$

rather than of the current natural rate alone (Woodford, 1999).[11] This is desirable because (2.1) implies that aggregate demand depends as much on expected future short real rates of interest as on the current short rate.[12] An inertial policy allows monetary policy to counteract the effects upon the output gap of an increase in the natural rate of interest by raising *both* current and expected future short rates, thus requiring less volatility of short-term interest rates to achieve a given degree of stabilization.

This inertial response of interest rates might be achieved by making the intercept i_t^* a function of lagged as well as current real disturbances, as discussed above. Alternatively, it may be achieved by introducing feedback from lagged endogenous variables, in particular lags of the interest-rate instrument itself, as in (2.6) and many estimated Fed reaction functions.[13] In this case, it may be possible to achieve the necessary interest-rate variations solely through feedback from the history of the target variables. For example, in the model sketched above, in the presence both of inefficient variation in the natural rate and a concern to reduce interest-rate volatility, Giannoni (2000) shows that optimal

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[11]In the numerical examples presented in Woodford (1999b), the optimal lag polynomial $B(L)$ can be fairly well approximated by an exponential moving average of the form $b\sum_j(\rho L)^j$, where ρ is approximately .5 for a quarterly model.

[12]An alternative argument would be that monetary policy affects aggregate demand mainly through its effect upon long rates and upon the exchange rate, and both of these are affected as much by expected future short rates as by current short rates.

[13]The advantages of intrinsic interest-rate inertia in a generalized Taylor rule has been shown through numerical analysis in the context of a variety of estimated models that include realistic degrees of forward-looking private-sector behavior. See, *e.g.*, Levin *et al.* (1999), Rotemberg and Woodford (1999), and Williams (1999). Woodford (1999b) discusses the issue analytically in the context of the simpler model treated here.

policy can be described by a feedback rule of the form

$$i_t = (1 - \rho_1)i^* + \rho_1 i_{t-1} + \rho_2(i_{t-1} - i_{t-2}) + \phi_\pi \pi_t + \phi_x(x_t - x_{t-1}), \qquad (5.2)$$

where $x_t \equiv y_t - y_t^e$ is the welfare-relevant output gap, the intercept i^* is now a constant (equal to the *long-run average* level of the natural rate of interest), and the coefficients $\rho_1, \rho_2, \phi_\pi, \phi_x > 0$ are functions only of the parameters $\beta, \kappa, \sigma, \lambda_y$ and λ_i. An advantage of this representation of optimal policy is that the coefficients of the policy rule are independent of the assumed statistical properties of the exogenous disturbances (the relative variances of different types of shocks, their serial correlation properties, and so on). It is also a policy rule that can be implemented without the central bank's having to determine the current values of any of the exogenous shocks, except insofar as it must estimate the change in y_t^e in order to estimate the change in the gap x_t.[14]

In practice, there are likely to be advantages to combining these two approaches to optimal interest-rate adjustment. Any rule of the form

$$i_t = \theta \bar{\imath}_t + (1 - \theta)\tilde{\imath}_t, \qquad (5.3)$$

where $\bar{\imath}_t$ is the right-hand side of (5.1), $\tilde{\imath}_t$ is the right-hand side of (5.2), and θ is an arbitrary parameter, will be consistent with the optimal equilibrium. Cases with θ too close to 1 are likely to be less robust to uncertainty about the current exogenous disturbances,[15] while cases with θ too close to zero are likely to be less robust to uncertainty about the nature of price adjustment. An intermediate value of θ — and therefore a rule that incorporates both an intercept that varies in response to real disturbances, and interest-rate inertia — is thus likely to be the most prudent approach.

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[14]It is worth noting here that in practice there is probably much less uncertainty about the quarter-to-quarter *change* in the efficient level of output than there is about its absolute level.

[15]A value of θ too close to 1 will also imply indeterminacy, while a value close enough to zero implies determinacy. But the problem of indeterminacy can be dealt with without having to introduce interest-rate inertia (though interest-rate inertia helps, as noted above).

6 Conclusions

The Taylor rule incorporates several features of an optimal monetary policy, from the standpoint of at least one simple class of optimizing models. The response that it prescribes to fluctuations in inflation or the output gap tends to stabilize those variables, and stabilization of both variables is an appropriate goal, at least when the output gap is properly defined. Furthermore, the prescribed response to these variables guarantees a determinate rational expectations equilibrium, and so prevents instability due to self-fulfilling expectations. Under at least certain simple conditions, a feedback rule that establishes a time-invariant relation between the path of inflation and of the output gap and the level of nominal interest rates can bring about an optimal pattern of equilibrium responses to real disturbances.

At the same time, the rule as originally formulated suffers from several defects. The measure of the "output gap" suggested in Taylor's analysis of the rule's empirical fit may be quite different from the theoretically correct measure, as the efficient level of output should be affected by a wide variety of real disturbances. The rule assumes a constant intercept, but a desirable rule is likely to require that the intercept be adjusted in response to fluctuations in the Wicksellian natural rate of interest, and this too should vary in response to a variety of real disturbances. Finally, the classic formulation assumes that interest rates should be set on the basis of current measures of the target variables alone, but an optimal rule will generally involve a commitment to history-dependent behavior; in particular, more gradual adjustment of the level of interest rates than would be suggested by the current values of either the target variables or their exogenous determinants has important advantages. These considerations call for a program of further research, to refine the measurement of the appropriate defined output gap and natural rate of interest, and to analyze the consequences of the refinements of the policy rule proposed here in the context of more realistic models.

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