Physics UN2801 | Professor Christ
due 10/20/2022

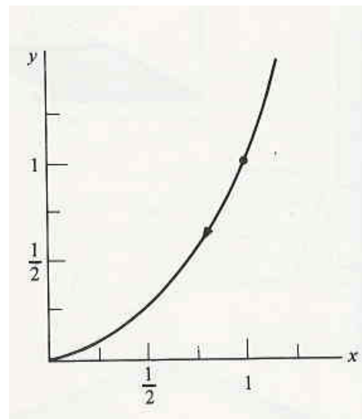
Assignment #5

Reading:

Oct 11	Kleppner and Kolenkow Ch: 6.5, 3.7, 6.1, 6.3, 11.1, 11.2, Note 11.1
Oct 18	Kleppner and Kolenkow Ch:11.3, Note 11.2
Oct 20	Kleppner and Kolenkow Ch:11.4, 11.5, 11.6, Note 11.3

Problems:

34. (Kleppner and Kolenkow 1st edition) A particle of mass m moves in a horizontal plane along the parabola $y = x^2$. At $t = 0$ it is at the point $(1, 1)$ moving downward and to the left with speed v_0 Aside from the force of constraint holding it to the path, it is acted upon by the following external forces:



$$\vec{F}_a = -Ar^3\hat{r}$$
$$\vec{F}_b = B(y^2\,\hat{\imath} - x^2\,\hat{\jmath})$$

where A and B are constants.

(a) Are the forces conservative?

(b) What is the speed v_f of the particle when it arrives at the origin?

35. Kleppner and Kolenkow 4.2

36. Kleppner and Kolenkow 5.7

37. Kleppner and Kolenkow 5.11

38. Kleppner and Kolenkow 5.16

39. (From Kleppner and Kolenkow 1st edition) The potential energy function for a particular two dimensional force field is given by $U(x, y) = Cxe^{-y}$, where C is a constant.

(a) Sketch the constant energy lines.

(b) Show that if a point is displaced by a short distance along a constant energy line, moving a distance dx in the x-direction, then its total displacement must be $\vec{dr} = dx(\hat{\imath} + \hat{\jmath}/x)$.

(c) Using the result of b), show explicitly that $\vec{\nabla}U$ is perpendicular to the constant energy line.

[Note: Don't miss final problem on the second sheet.]

40. Consider two distinct ortho-normal sets of basis vectors, $\{\hat{e}_1, \hat{e}_2, \hat{e}_3\}$ and $\{\hat{e}'_1, \hat{e}'_2, \hat{e}'_3\}$ related by a change of basis matrix, $\{M_{ij}\}_{1\leq i,j\leq 3}$:

$$\hat{e}'_i = \sum_{j=1}^{3} M_{ij}\hat{e}_j.$$

Using these two coordinate systems a vector $\vec{A}$ can be written in terms of two sets of coordinate: $\{a_i\}_{1\leq i\leq 3}$ and $\{a'_i\}_{1\leq i\leq 3}$:

$$\vec{A} = \sum_{i=1}^{3} a_i\,\hat{e}_i \quad \text{and} \quad \vec{A} = \sum_{i=1}^{3} a'_i\,\hat{e}'_i$$

where

$$a'_i = \sum_{j=1}^{3} M_{ij} a_j. \tag{1}$$

Using the chain rule relate the two sets of partial derivatives: $\{\partial V(\vec{a})/\partial a_j\}_{1\leq j\leq 3}$ and $\{\partial V(\vec{a}(\vec{a}\,'))/\partial a'_i\}_{1\leq i\leq 3}$ showing that these three quantities transform between systems like the coordinates of a vector, as in Eq. (1). (Recall that $M^{-1} = M^t$.)