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A REGRESSION ANALYSIS OF PROBLEM SOLVING IN A BINARY CHOICE TASK[1]

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This study was designed to investigate three problems concerning human behavior in a two-choice learning situation. Rather than attempt a summary of research on two-choice learning we refer the reader to Bush and Mosteller (1955) and Feldman (1959). The first concern of this study was to develop and test certain trial-to-trial measures of the decision process. The basic trial-to-trial measure is *S*'s choice, left or right, on each experimental trial. Previous studies have utilized various transformations of this basic choice measure, such as the learning parameters of the stochastic learning models (Bush & Mosteller, 1955; Estes, 1950; Luce, 1959), run statistics (Bush & Mosteller, 1955; Goodnow, *et al.*, 1959;[3] Luce, 1959), conditional probabilities (Hake & Hyman, 1953), and stay-shift strategies (Goodnow & Pettigrew, 1955). In an attempt to tap more of the problem solving behavior in the situation we defined some new transformations of the choice data, including a modification of the Goodnow and Pettigrew (1955) strategy measure. Additional raw data were collected in the form of subjective probability estimates. Our aim was to define a criterion of concept attainment in the binary choice situation and to evaluate the power of our measures in predicting this criterion. The statistical technique for evaluating these measures was a regression model. We split the data at the mid-point of the trial series and ran regressions of the attainment criterion for the last half on the values of the selected measures for the first half as a means of testing the predictive power of the measures.

The second concern of the study was to investigate the effect of subjective probability estimation on choice behavior. Originally there was some concern that this additional measure would bias *S*'s conception of the choice situation and modify his pattern of choices. If this were the case the additional measure would change the situation rather than provide supplementary information about the original choice task. A control group, which made no probability estimates, was included to evaluate the extent of the bias.

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[3]Goodnow, J. J., Rubinstein, I., & Lubin, A. Response to changing patterns of events. Unpublished memorandum. Problem Solving Projects, Walter Reed Army Institute of Research, 1959; Goodnow, J. J., Shanks, B., Rubinstein, I., & Lubin, A. What is the human subject responding to in a two-choice task? Unpublished memorandum. Problem Solving Projects, Walter Reed Army Institute of Research, 1959.

The third problem involved the effect of recording materials upon performance in the choice task. Information storage, or memory, is a critical mediating factor for problem solving in a binary choice task. An intuitive supposition would be that performance in a choice situation is a function of storage requirements. Bruner, Goodnow, and Austin (1956) choose to label this factor "cognitive strain." Further, they find that an individual uses information in accordance with a strategy that reduces this "cognitive strain" to a level at which the information may be handled with some degree of efficiency. Thus, one might expect that permitting an individual to record choice-outcome pairs with pencil and paper, and any other information deemed valuable, would remove this constraint, thereby enhancing performance. Surprisingly enough, in an unpublished paper, Bruner, Goodnow, Matter, and Potter (1955)[4] report a categorization experiment in which the use of pencil and paper did little to enhance performance. A median test of their data (performed by us) yielded a chi square of 3.6 which just missed significance at the .05 level. Thus there is some indication that pencil and paper have a facilitating effect but nowhere near as great as one would expect. Bruner, *et al.* explain this finding in terms of lack of knowledge concerning effective use of recording materials on the part of most *S*s. Pencil and paper were used for little more than to keep a record of choices and outcomes. Huttenlocher and Mosteller described, in an unpublished report,[5] a two-armed bandit experiment in which *S*s were provided with full information about the nature of the reward schedule, the payoff probabilities, and previous choice-outcome pairs. But such information did not lead the majority of *S*s to follow the rational strategy suggested to them by the experimenter, although it did have some facilitating effects. This study attempted to investigate in greater detail the effect of recording materials on choice behavior with respect to a wide range of statistics of the trial-to-trial data.

METHOD

Apparatus

The choice apparatus is a two-armed bandit with two alternatives and two possible outcomes for each alternative. The start of each experimental trial is indicated by the activation of a white light located at the center of the apparatus. On each of 100 trials *S* chooses one of the two alternatives, left- or right-hand button, and is rewarded or not rewarded, reward consisting of a poker chip worth one cent. The chip dispensing mechanism, which appears to be automatic, is actually controlled by *E* from an adjacent room.

Experimental Design

Group I.—75:25 random reward schedule with subjective probability estimates. *S* performs the specified choice operations on each of the 100 trials. Prior to making his

[4]Bruner, J. S., Goodnow, J. J., Matter, J., & Potter, M. Determining types of concept. Unpublished paper, 1955.

[5]Huttenlocher, J., & Mosteller, F. Two-armed bandit experiment with full information. Unpublished memorandum, Harvard University, 1956.

choice on each trial, *S* records his percentage estimates (subjective probabilities) of the likelihood that each button will pay off on that trial.

Group II.—75:25 random reward schedule, recording materials permitted. *Ss* in this group are given the same instructions as those in Group I with the added provision that they may use pencil and paper to record information concerning the choice-outcome pairs. *E* also records this information, providing *S* with access to it whenever he desires. This condition is designed to investigate choice behavior when there is no limitation on information storage.

Group III.—75:25 random reward schedule without subjective probability estimates. *Ss* are instructed to perform the specified choice operations but do not record subjective probabilities. This is a control group designed to permit investigation of any confounding effects the subjective probability procedure may have upon the choice behavior.

Subjects

Ss were obtained from Harvard and Boston University summer schools. The sample from Boston University Summer School (BU) consisted of 24 students, 12 males and 12 females, providing each experimental group with four males and four females. The sample from Harvard University Summer School consisted of 48 students, 24 males and 24 females, providing each experimental group with eight males and eight females. Assignment to the groups was randomized in blocks of three with separate blocks for BU males, BU females, Harvard males, and Harvard females. In addition, the side of reward, right or left, and the assignment of the observer to the experimental room were randomly assigned in blocks of 12 with the restriction that all units of each block be different with respect to the factors randomized. There were in all 72 *Ss*, 24 in each experimental group.

Results and Discussion

Definition of the Analytic Statistics

Choices of the unfavorable side.—The most sensible strategy in a binary choice situation with random reward schedule is to locate the favorable alternative and choose it consistently. In the light of this strategy, always choosing the more frequently rewarded side can be interpreted as the concept *S* is to attain. Hence, a choice of the less frequently rewarded alternative may be considered an 'error,' and the number of choices of this alternative may be considered a measure of performance in our experimental situation. Let s denote the number of choices of the unfavorable side during the first 50 trials and t the number of such choices during the last 50 trials.

Mean subjective probability estimates.—Interpreting the subjective probabilities as confidence estimates, the question arises whether or not our assessment of an individual's confidence is consistent with his future choice behavior. If such consistency obtains, one would expect central tendency measures of the subjective probability data for the first 50 trials to be reliable predictors of t.

Behavioral strategies.—From a psychological point of view, behavioral strategy refers to an individual's response preferences following a given choice-outcome pair. The stimulus to which *S* is assumed to be responding is the preceding choice and its corresponding outcome. In the light of this interpretation, the binary decision on each trial is whether to stay on or shift from the alternative chosen on the preceding trial. In a manner analogous to that of Goodnow and Pettigrew (1955) the choice data were recoded into the following eight behavioral strategies: win stay on the favorable side, win stay on the unfavorable side, win shift from the favorable side, win shift from the unfavorable side, lose stay on the favorable side, lose stay on the unfavorable side, lose shift from the favorable side, and lose shift from the unfavorable side.

Frequency of behavioral strategies.—On every trial, following the first, S's choice of the right or left button may be interpreted as a selection of a behavioral strategy. Correspondingly, the outcome of this choice may be interpreted as reward or non-reward of the behavioral strategy. In order to investigate the effect of reinforcement of behavioral strategy upon future choice behavior we computed the percentage of time that each behavioral strategy was rewarded.

Prediction of Concept Attainment in the Last 50 Trials (t)

By degree of concept attainment in the first 50 trials (s).—To determine the extent to which s is a reliable predictor of t, we ran a regression of t on s. The regression coefficient is $b = .569$ with a standard error of .098. Using an F test, the reduction due to regression is significant at the .01 level ($F = 33.447$ with $1/70$ df). This implies that individuals who choose the less frequently rewarded side more often during the first 50 trials tend to choose it more often during the last 50 trials. This is not a striking substantive result, given that this criterion is a reasonable measure of concept attainment. However, the significance of this result lies in its validation of the criterion as an attainment measure.

By mean subjective probability estimates.—To evaluate the validity of this estimation data, we ran a regression of t on the subjective probability means for Trials 1 through 50. Only experimental Groups I and II were considered, since Group III was not required to make subjective probability estimates. The regression on the two means yielded an F value of 7.021 with $2/45$ df, significant at the .01 level. This result affirms the consistency of the estimation data with the choice behavior and demonstrates the value of these subjective probability means as predictors of t.

The concept to be learned in this two-choice situation admits reformulation in terms of learning the objective reward probabilities. If subjective probability is a valid measure of S's confidence in his choice, then a measure of central tendency for these confidence estimates should be positively related to future choice of that alternative. It is clearly established that mean subjective probability estimates are related to choices in the predicted direction. Higher estimates for the favorable alternative during the first 50 trials are positively correlated with the number of choices of that alternative during the last 50 trials.

By behavioral strategies.—We are interested in partial prediction of t as a function of these eight behavioral strategies, calculated from the first 50 trials for each S. To test their predictive power, we ran regressions of t on each of the eight behavioral strategies (see Table 1).

Bearing in mind that t represents the last 50 trials, and bearing in mind that the eight statistics are estimated on only the first 50 trials, it is apparent that an individual's behavioral strategy preferences are reliable predictors of at least some gross aspects of future choice behavior, namely, the number of choices of the unfavorable alternative. From a substantive point of view, a

negative relationship with t implies that the behavioral strategy leads to positive learning in this situation, i.e., fewer choices of the less frequently rewarded side. Initially, it may be surprising to note from Table 1 that only two of these strategies are negatively correlated with t, win stays and lose stays on the favorable side. Yet, *Ss* do learn to decrease their choices of the unfavorable side in the last 50 trials as compared with the first 50 trials. For experimental Groups I, II, and III the mean value of s is 17.57, while the mean value of t is 11.12. Explanation for this apparent contradiction derives from the greater frequency of win stays on the favorable side.

It is of interest to note that no behavioral strategy following a choice of the unfavorable side is associated with positive learning. Only the strategies of staying on the more frequently rewarded side, regardless of outcome, lead to positive learning. These two strategies combine to yield the best asymptotic strategy for the choice situation—locate the favorable side and choose it consistently.

This pattern of relationship between behavioral strategy and learning suggests three classes of decision makers in the binary choice task (see Table 1).

TABLE 1

RESULTS OF REGRESSIONS OF t ON EIGHT BEHAVIORAL STRATEGIES

Statistics	Correlation with t	F value*	p	
Win stays on favorable side	−.583	36.894	.01	Class I
Lose stays on favorable side	−.395	5.914	.05	
Win shifts from favorable side	+.472	18.579	.01	Class II
Lose shifts from favorable side	+.027	2.588	ns	
Win shifts from unfavorable side	+.421	13.127	.01	
Lose shifts from unfavorable side	+.316	7.590	.01	
Win stays on unfavorable side	+.206	2.803	ns	Class III
Lose stays on unfavorable side	+.413	14.809	.01	

*The F value refers to the significance in reduction of variance due to the regression.

The first class of individuals stay on the favorable side once it has been located, win stays and lose stays on the favorable side. An individual in this class might have decided that no sequences occur and that the best he can do is choose the side with higher payoff probability. Another member of the first class may never have checked the possibility of a sequence obtaining but is satisfied with the rate of reward and, consequently, sees no reason for shifting. The second class of *Ss* consists of people who shift frequently, irrespective of alternative and outcome. Individuals who are pattern testing might fall into this class of learners, since pattern testing often involves frequent alternation. In addition, a strong negative recency effect would lead one to expect a moderate number of alternations. Lastly, *Ss* who stay on the less frequently rewarded alternative,

regardless of outcome, constitute the third class. A very large negative recency effect might induce this strategy as a result of the feeling that the unfavorable side 'is due to pay off.' Thus, in terms of our measure of learning, t, individuals in the first class are assessed as having more successfully attained the concept, since they choose the less frequently rewarded side less often in the last 50 trials. Individuals in the second and third classes choose the unfavorable side significantly more often and, therefore, are assessed as having learned less than the people in the first class. We conclude that an individual's preference for different classes of behavioral strategies has a marked effect on his later choice behavior.

TABLE 2

RESULTS OF REGRESSIONS OF t ON EIGHT RATIOS

Statistics	Correlation with t	Regression F value	p
$\frac{\text{rewarded win stays on favorable side}}{\text{total number of win stays on favorable side}}$	−.090	0.565	n.s.
$\frac{\text{rewarded win stays on unfavorable side}}{\text{total number of win stays on unfavorable side}}$	+.122	0.707	n.s.
$\frac{\text{rewarded win shifts from favorable side}}{\text{total number of win shifts from favorable side}}$	+.089	0.565	n.s.
$\frac{\text{rewarded win shifts from unfavorable side}}{\text{total number of win shifts from unfavorable side}}$	−.060	0.281	n.s.
$\frac{\text{rewarded loss stays on favorable side}}{\text{total number of lose stays on favorable side}}$	+.155	1.721	n.s.
$\frac{\text{rewarded lose stays on unfavorable side}}{\text{total number of lose stays on unfavorable side}}$	+.035	0.007	n.s.
$\frac{\text{rewarded lose shifts from favorable side}}{\text{total number of lose shifts from favorable side}}$	+.035	0.007	n.s.
$\frac{\text{rewarded lose shifts from unfavorable side}}{\text{total number of lose shifts from unfavorable side}}$	−.066	0.281	n.s.

By frequency of reward of behavioral strategies.—To evaluate the predictive ability of this statistic, we ran regressions of t on each of the eight ratios (see Table 2). The percentage of trials on which each behavioral strategy was rewarded during the first 50 trials seems to have no effect upon the percentage of choices of the unfavorable side during the last 50 trials. Individuals who happened to be frequently rewarded for win staying or lose staying on the favorable side during the first 50 trials made no more choices of the favorable side during the last 50 trials than individuals who were less often rewarded for these strategies. Also, frequent reward of the remaining six behavioral strategies did not lead to fewer choices of the favorable side during the last 50 trials. Whatever leads to learning in a binary choice situation, it is not the learning of the reward frequency for behavioral strategy.

A comparison of Tables 1 and 2 seems to imply that an individual's

strategy preferences are of great predictive value, but the degree of reward for behavioral strategy has little relationship to future choice behavior. By itself, this tends to indicate that what happens to an individual in a choice situation is of much less significance than his strategy preferences. Thus, it is not the case that what is being learned in the choice situation is the relative success of different behavioral strategies, but rather that individuals with certain strategy preferences have differential success in attaining the concept of choosing the more frequently rewarded side. It appears that behavioral strategy is not something that is learned in a choice situation so much as it is a personality characteristic of the individual decision maker.

Differences Between Experimental Groups

Comparison of Groups I and III.—Groups I and III were given the same 75:25 random reward schedule, but differed in that Group I was required to make subjective probability estimates while Group III was not. Comparison of these groups assesses the extent to which the estimation procedure influences the trial-to-trial decision process. The following statistics were used for this comparison: s, t, $s + t$, number of runs on the favorable side, length of longest run on the favorable side, mean run length on the favorable side, variance of run length on the favorable side, alternations, and the eight behavioral strategies. On the basis of all the statistics used, there are no significant differences in choice behavior between Groups I and III. This implies that the estimation technique does not confound choice behavior.

Comparison of Groups I and II.—The only difference between the two groups is that *S*s in Group II were permitted the use of pencil and paper, while those in Group I were not. In a larger sense this comparison allows limited investigation of the role of information storage capacity in binary choice behavior. Individuals in Group II have available to them complete information with respect to previous choice-outcome pairs, but individuals in Group I must operate within the limitations of their memory capacity.

The comparison of Groups I and II was made on the basis of the same statistics used in the preceding comparison with the addition of mean subjective probability estimates and estimation score (which measures the closeness of the estimate to the true probability). With one exception, there are no significant differences between Groups I and II on choice statistics and behavioral strategies. The exception, variance of run length, implies that *S*s using pencil and paper have more long runs and short runs on the more frequently rewarded side ($F = 4.23$ with $23/23$ df, $p \leq .01$). Our substantive interpretation is based on the observation that pencil and paper encourages more pattern testing during early trials, but also encourages asymptoting on the more frequently rewarded side after the possibility of a sequence has been eliminated.

The over-all estimates of payoff percentages, made at the conclusion of the experiment,

and also the mean trial-to-trial estimates of subjective probability yielded no significant differences between the two groups. However, estimation score, which measures the closeness of the estimates to the true probability, is significantly higher for Group II (Kolmogorov-Smirnov two-sample test $K_D = 10$, $p < .05$). Higher estimation score results from a number of individuals in Group II who counted the payoff frequencies. Apparently the use of pencil and paper has little effect on the choices but tends to improve subjective probability estimates. We observed, as did Bruner, Goodnow, Matter, and Potter,[4] that *S*s make little use of the information they record in determining future choices.

SUMMARY

Three experimental groups performed the same binary choice task. The groups differed with respect to subjective probability estimation and the use of recording materials. The estimation procedure did not influence choice behavior. Use of pencil and paper improved subjective probability estimates but did not affect choices. Choice of the less frequently rewarded side was defined as the performance criterion. An alternative criterion defined in terms of subjective probability was highly correlated with this choice criterion. Subjects' preferences for patterns of choices (behavioral strategies), assessed in the first 50 trials, proved to be significant predictors of the performance criterion for the last 50 trials. However, the frequency of reward of these behavioral strategies during the first 50 trials was not significantly related to the performance criterion for the last 50 trials. The results indicate that individual strategy preferences are of greater significance than experience in the choice situation.

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