

EXPONENTIAL DECAY AND INDEPENDENCE FROM IRRELEVANT ASSOCIATIONS IN SHORT-TERM RECOGNITION MEMORY FOR SERIAL ORDER¹

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A test of association between 2 adjacent items in a digit series was provided by giving S a test pair of items and having him decide whether the response member of the pair followed the stimulus member in the preceding digit series. Probabilities of correct and incorrect recognition from various conditions were used to estimate the strength of the associations in short-term memory using the operating characteristic of signal-detection theory. A mathematical model for serial-order recognition memory was proposed which assumed that the strength of interitem associations decays exponentially and that S's response in a recognition test depends on the strength of the test association in relation to a criterion, not upon the strength of that association relative to the strength of other associations to the same test stimulus.

The distinction between memory for items and memory for serial order can be defined more clearly with a recognition test than with a recall test. In recognition memory for items, S decides whether or not the test item was in the preceding list. In recognition memory for serial order, S is given a

pair of items known to be list members, and he decides whether the second item immediately followed the first item in the preceding list.

In a study of recognition memory for items (three-digit numbers), Wickelgren and Norman (1966) found that the strength of an *item* in short-term memory (STM) decayed exponentially with the number of subsequent items. One purpose of the present study was to determine whether the strength of an *association* between two items decays exponentially, where the items are single-digit numbers. If recognition memory for *items* depends

¹This work was supported primarily by Grant No. MH 08890-02, from the National Institute of Mental Health, United States Public Health Service. Further aid was received from a National Aeronautics and Space Administration grant, NsG 496, to Hans-Lukas Teuber. I would like to thank the reviewer of this article for his considerable contribution to the writing.

on the strength of *intraitem* associations, and recognition memory for *serial order* depends on the strength of *interitem* associations, then it is plausible that both might have the same law of decay.

Strength in memory was discussed above as though it were a directly measured variable, which of course it is not. One only measures the probability of one response or another. Such response probabilities are, in general, influenced by both memory and response bias. Thus, estimates of strength in memory require a theory of how strength in memory combines with response bias to determine the probability of a response. Signal-detection theory is one such theory that can be applied to any two-choice situation.

The part of signal-detection theory to be applied to recognition memory is the *criterion decision rule*: respond yes if and only if the strength of the test item (or test association) exceeds a certain cutoff. This assumes that the probability of a "yes" response depends only upon the strength of the test item (association) and is independent of the strength of any other item (association) in memory. If this assumption were true, it would make recognition superior to recall for testing theories of memory, because the probability of *recalling* a response to a stimulus must necessarily depend not only upon its strength in memory but also upon the strength of competing associations.

This assumption is easily tested with recognition memory for serial order, where the assumption might be referred to as the "independence from irrelevant associations." The idea is to establish two associations to the same stimulus item (A-B, A-C) and see whether the estimated strength of the A-B association is the same as when only the A-B association was established, while controlling for the

number of prior and subsequent items in the list. If so, then the hypothesis is supported. However, if *Ss* compare the strength of the test association to the strength of other possible associations to the test stimulus, the estimated strength of the A-B association should decline when an irrelevant A-C association is increased in strength.

Since presenting an item twice in a list may increase *S's* tendency to say "yes" to any test association, one should plot the probability of correct recognition of A-B against the probability of false recognition of A-D to determine whether the strength of the A-B association is the same with or without a competing associate. Such a plot is called an operating characteristic (OC). The distance (d_2') of the OC from the chance diagonal (on normal-normal probability paper) is a measure of the *difference* in strength of the correct and false associations. According to the "independence of irrelevant associations," this difference in strength should be the same with or without a competing associate.

The assumption of independence from irrelevant associations applies only to recognition memory and is independent of the issue of whether strength in memory decays over subsequent items. It is asserted that recognition memory of an A-B pair will be impaired as much by X-Y pairs as by A-C pairs among the prior and subsequent items.

A strength theory of recognition memory for serial order can be formulated as follows:

D1. Let $s(k \rightarrow k+1, L)$ represent the strength of the association between the item in Position k and the item in Position $k+1$, in a list of length L . If the response item was not presented (a new item), let its strength be $s(k \rightarrow *, L)$.

A1. Independence from irrelevant associations:

$$d_2' (k \rightarrow k+1, L) = s(k \rightarrow k+1, L) - s(k \rightarrow *, L)$$

A2. Initial strength: $s(k \rightarrow *, 0) = 0$

A3. Acquisition: $s(k \rightarrow k+1, k+1) = \alpha$

A4. Exponential decay:

$$s(k \rightarrow k+1, L) = \varphi s(k \rightarrow k+1, L-1), \\ 0 < \varphi < 1, \quad k+1 < L$$

This leads to a simple exponential equation for predicting the d' values observed in the present experiment:

$$d_2' (k \rightarrow k+1, L) = \alpha \varphi^{L-k-1} = \alpha \varphi^{1-k}$$

Axiom A1 follows by assuming the criterion decision rule with noise added to an algebraic memory trace. The basic assumption is:

$$Pr(\text{yes} | k, j) = Pr[(s_{kj} - c + X) > 0] \\ = \int_c^\infty N(s_{kj}, 1)$$

That is, the probability of a "yes" response to test pair (k, j) is the probability that the sum of the strength in memory of the association from k to j , s_{kj} , plus a random variable X exceeds a criterion c . X is assumed to be normally distributed with zero mean and unit standard deviation, and s_{kj} and c are measured in units of this standard deviation. From this statement of the criterion decision rule, A1 may be derived (Wickelgren & Norman, 1966).

Assuming the initial strength of associations in STM to be zero is a mathematical convenience, but with the rapid rate of decay observed in STM experiments, it is not unreasonable. The acquisition assumption does not allow differences in learning due to serial position, previous presentation of the cue item, or other factors. The assumption is surely wrong in general,

but it is a reasonable approximation for experiments where efforts are made to equalize degree of initial learning of each item.

METHOD

Procedure.—The trial number was announced, followed by a 1-sec. pause, then a list of 12 digits presented at the rate of 2 digits/sec, then a tone lasting about .5 sec., followed by a test pair of digits presented in 1 sec., followed by a 3-sec. period in which S made a "yes-no" decision regarding whether the response item of the test pair was an immediate successor of the stimulus item in the list just presented, followed immediately by the next trial. The S s also indicated confidence in their decision on a 4-point scale, "1" indicating least confidence and "4" indicating most confidence. They were instructed to give their immediate reaction and to always respond "yes" or "no." All 10 digits were presented at least once in the 12-digit list (2 were repeated in non-adjacent positions), so the stimulus digit of the test pair was always in the list and S s were informed of this.

Independence from irrelevant associations refers to the decision rule used in retrieval from memory during a recognition test. Besides controlling for differences in number of prior and subsequent items, the degree of original learning should be equated over the different conditions.

Several procedures were adopted in an attempt to equate degree of original learning. First, all lists had two repeated digits in them so the only difference in the lists was whether the item to be given as the test stimulus was one of the repeated items. Second, the digits were presented at a relatively fast rate (2 digits/sec). Third, S s were required to copy each digit as it was presented, thus assuring some minimum level of attention to each item. If the pair to be tested was not correctly copied on both occasions, the trial was not scored. The S s covered each item with a card after copying it.

Design.—The stimulus item of the test pair was the third, fifth, seventh, or ninth digit in the list. There were five conditions of repetition of the test stimulus in the list: no repetition (this condition occurred twice as often as each of the other four conditions) or repetition at one of the following positions in the list—first, third, fifth, seventh, or ninth. Depending on which position was used to present the tested occurrence

of the test stimulus, one of these five cannot be used to present another occurrence of the test stimulus. Finally, the test response can be a correct successor to the test stimulus or not. Incorrect test responses were never identical to the test stimulus. The design was factorial, and there were $4 \times 6 \times 2 = 48$ conditions/block (of which $4 \times 5 \times 2 = 40$ were different), 4 blocks in the experiment, or 192 trials altogether. Each trial lasted about 13 sec. and the experiment lasted about 50 min. for each *S*.

Subjects.—The *Ss* were 33 Massachusetts Institute of Technology undergraduates taking psychology courses who participated in the experiment as a part of their course requirements. The *Ss* were run in three approximately equal groups.

RESULTS AND DISCUSSION

Operating characteristics.—Empirical recognition probabilities were determined for the 20 basic conditions in which the response member of the test pair was correct and for the 20 conditions in which the response member of the test pair was false, averaging across *Ss* (total $N = 132$ or 264). For each correct-recognition condition there was a corresponding false-recognition condition, so 20 OCs were plotted, using the method of confidence judgments. Descriptions of this method can be found in Egan, Schulman, and Greenberg (1959), or Norman and Wickelgren (1965).

Each OC plots the probability of recognition of the correct associate against the probability of recognition of an incorrect associate for a test stimulus from a given serial position or from a given pair of serial positions in the list. Conditions can be represented by an ordered triple (x, y, z) , where x denotes the serial position of the test stimulus ($x = 3, 5, 7, 9$), y denotes the serial position of another occurrence of the test stimulus ($y = 1, 3, 5, 7, 9, *$), and z is either correct or false depending on whether the test response was the correct successor to

the stimulus from Position x or some other response in the list, excluding x or the successor to y . The symbol $*$ denotes conditions in which the test stimulus occurred only once in the list.

The parameters of the correct and incorrect strength distributions may be estimated by plotting the OC points on normal-normal probability paper. Such plots will be straight lines if the strength distributions are normal. Let s_c and s_i denote the two means, and σ_c and σ_i the two standard deviations. The intersection of the OC with the negative diagonal is entered into Elliott's (1964) tables to estimate

$$d_1' = 2(s_c - s_i)(\sigma_c + \sigma_i)^{-1}.$$

The slope of the OC line estimates $b = \sigma_i/\sigma_c$.

The 20 OCs obtained approximated straight lines on normal-normal paper, supporting the normality assumption. However, the slopes were usually less than unity and were negatively correlated with d_1' . Assuming s_i and σ_i are constant for all conditions, the slope variations indicate a positive correlation between s_c and σ_c . Therefore, to compare the s_c values for different conditions, all are scaled to the common unit σ_i by the transform

$$d_2' = (s_c - s_i)\sigma_i^{-1} = .5(1 + 1/b)d_1'.$$

The d_1' , b and d_2' values for the 20 OCs are given in Table 1. The ordered pair (x, y) in Table 1 denotes a test stimulus from Position x that also occurred in Position y in the previous list. Although the strength distribution of incorrect associations was assumed to be invariant over the different conditions, false recognition tests were run for each of the correct recognition conditions, because the bias to say "yes" might be affected by the number of occurrences and the position

TABLE 1
 D_1' , SLOPE, AND d_2' STATISTICS OF THE
 OPERATING CHARACTERISTIC FOR EACH
 CONDITION

Cond.	d_1'	b	d_2'
(3,1)	.80	.82	.89
(3,5)	.56	.90	.59
(3,7)	.33	.84	.36
(3,9)	.33	.84	.36
(3,*)	.30	.87	.32
(5,1)	.48	.97	.48
(5,3)	.61	.90	.65
(5,7)	.82	.85	.89
(5,9)	.38	1.19	.35
(5,*)	.58	.74	.68
(7,1)	1.38	.70	1.67
(7,3)	.72	.90	.76
(7,5)	1.28	1.00	1.28
(7,9)	1.68	.72	2.00
(7,*)	.91	.84	1.00
(9,1)	1.66	.65	2.11
(9,3)	2.29	.37	4.24
(9,5)	1.90	.59	2.57
(9,7)	2.25	.40	3.94
(9,*)	2.27	.65	2.88

of the occurrences of the test stimulus in the previous list.

Exponential decay of interitem associations.—A semilogarithmic plot of d_2' for each condition against the number of subsequent interfering associations is shown in Fig. 1. There is considerable variation in the d_2' value for the different conditions at the same abscissa value. The method of confidence judgments yields operating characteristics composed of points that are not statistically independent. For this reason no statistical test has yet been devised to determine whether such operating characteristics or their parameters (e.g., d_2') are significantly different. To get some idea of the significance of these differences, one can assume the points to be independent and use a method such as that of Weintraub and Hake (1962). By any such test the operating characteristics for the conditions at any particu-

lar abscissa value would be significantly different. Possible reasons for this variation will be considered in the next section.

However large the variation in d_2' values for the various conditions at a given delay in the retention test, it does not obscure the decline in d_2' with the number of subsequent items. If one compares the five d_2' values for each adjacent pair of delay conditions in Fig. 1, using a Mann-Whitney U test ($n_1 = n_2 = 5$), all three comparisons are in the expected direction of lower d_2' with increasing delay. Two of these comparisons are significantly different at the .01 level, one is insignificant (6 vs. 8 subsequent items), and the overall significance level is .001 (using Fisher's [1938] method for combining significance tests). The results support the assumption that there is decay in STM for serial order.

To get some idea of how well the results are fit by a single exponentially decaying trace, the (visually estimated) best-fitting straight line was drawn in Fig. 1. Estimates of the acquisition and decay parameters obtained from the line in Fig. 1 are $\alpha = 5.8$, $\varphi = .70$. Exponential decay appears to provide a reasonable first approximation to the data. However, there may be a small, longer-term component of the trace, as suggested by the slower-than-exponential decay from 6 to 8 subsequent items. Furthermore, the small number of delay intervals and the considerable variation in d_2' values at each delay interval restricts the power of the test. Thus, the data unequivocally support the assumption that there is decay in STM, but the exact form of this decay requires further investigation. At present, exponential decay of the association between items should be considered a first approximation.

Regardless of whether or not the

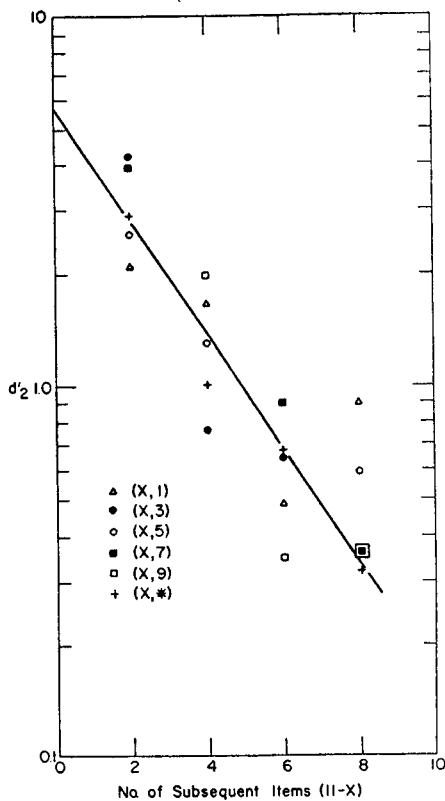


FIG. 1. d_2' values for each condition (x,y) as a function of the number of subsequent items ($11-x$). (The straight line is visually determined.)

decay is exactly exponential, the rate of decay in recognition memory for serial order is quite similar to that obtained by Wickelgren and Norman (1966) in recognition memory for items. The general similarity of the form and rate of decay for both item memory and serial-order memory suggests that both are performed by the same memory system, with item memory based on the strength of intraitem associations and serial-order memory based on the strength of interitem associations.

However, it should be made clear that nothing in the present results allows one to distinguish between direct, item-to-item, associations and indirect item-to-serial-position-to-item associations. The present results imply

nothing about the question of the degree to which serial-order memory is mediated by serial-position cues.

Also, since time and number of subsequent items completely covary in the present experiment, it is not possible to decide whether the decay results from the passage of time without rehearsal or from the presentation of interfering items.

Independence from irrelevant associations.—The main point of the present study was to determine whether the measured difference in strength (d_2') between an A-B and an A-D association was influenced by the strength of an irrelevant A-C association. If it were, this could account for the differences in the d_2' values for the same delay value, as shown in Fig. 1. It does not. If strengthening an irrelevant A-C association decreased the d_2' value for A-B vs. A-D associations, then the d_2' values for each abscissa value would be ordered from greatest to least as follows: ($x,*$), ($x,1$), ($x,3$), ($x,5$), ($x,7$), ($x,9$). Clearly they are not so ordered, nor is there any tendency in that direction. Furthermore, the * condition is not consistently high or low.

There is some suggestion that the degree of learning of the A-B association was greater when the A-C pair occurred immediately adjacent to the A-B pair, particularly near the beginning of the list. However, the variation in d_2' values is not completely explained by this effect either. There appears to be some residual variation in the average difficulty of digit pairs tested in the different conditions at the same delay.

In any event there is no indication that A-C associations have any systematic effect on the measured strength of A-B associations, when a recognition test is used. A recall test with A as the cue and but one response permitted would surely have shown differences between presenting A-B vs. presenting A-B and A-C. Associations to a common stimulus would very likely compete in recall

even if S were told to give only the more recent response. Allowing S to give more than one response might or might not avoid this competition. The present results indicate that a recognition test avoids this competition, whether the competition is viewed as conflict concerning which of two available associates occurred more recently or as blocking of weak associations by stronger ones.

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(Received January 3, 1966)