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Multitrace Strength Theory[1]

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There are many levels at which one can attempt to formulate a theory of memory, ranging from theories of the biochemical and biophysical bases of memory, through anatomical and physiological bases of memory, and finally, through psychological theories of memory. There are also many different sublevels within each of these three major categories of levels. By a theoretical level, I mean the degree of detail with which the memory process is described. One is attempting to describe the memory process in more detail at a molecular level than at a neuronal level than at a psychological (functional, behavioral) level.

Ultimately, we want adequate theories of memory at molecular, neuronal, and psychological levels. It may turn out to be possible to derive the psychological theory from the neuronal theory and the neuronal theory from the molecular theory. Alternatively, one or both of these derivations may be too complicated to be worth the effort. This is not our concern at present.

The concern of the present paper is to develop a possibly adequate theory of memory at a psychological level. The theory, called multitrace strength theory, is rather detailed in that it analyzes the memory trace into components and phases, but both the componential and phase analyses are less detailed (mechanistic) than some might desire. However, multitrace strength theory will attempt to achieve complete generality with respect to the basic functional properties of memory.

The basic properties of multitrace strength theory are as follows: each event and each association between two events is characterized by a vector of unidimensional strength measures for each of four possible time traces (very-short-term memory, VSTM: short-term memory, STM; intermediate-term memory, ITM; and long-term memory, LTM) in each of an unknown number of modalities (visual, auditory, speech-motor, abstract-verbal, etc.).

Each trace in each modality passes through four phases (acquisition, consolidation, decay, and retrieval). The acquisition phase refers to the period of presentation or active rehearsal of events during which the memory traces are initiated. However, acquisition is considered to refer to the establishment of potential traces, not usable (retrievable) traces. The conversion from potential traces to usable traces is accomplished by the consolidation process, which may be a matter of hours or days for LTM, but is on the order of tens of seconds or seconds for ITM and tenths of seconds for STM. After a usable trace has consolidated, it decays exponentially to zero at a rate that may depend on the experimental conditions.

In retrieval, the strengths of all traces in all modalities for an event or association are combined into a single total strength. It is this (unidimensional) total strength which is judged in the retrieval-decision process. In recognition, only the total strength of the test event or association is judged in relation to a criterion to determine the "yes-no" response. In multiple-choice or recall, the total strengths of all alternatives are compared, and the alternative with the maximum strength is selected.

An exponential approach to a limit is chosen as the general form of the acquisition and retrieval functions, and a delayed unit step or ramp function is chosen to represent consolidation. However, the choice of these functional forms is rather arbitrary on the basis of present evidence.

Some consideration is given to the nature of the coding for events and associations in different modalities by making provision for similarity functions between pairs of events and pairs of associations. As an example of event similarity, the letter names "B" and "D" are more similar in phonetic STM than "B" and "S." As an example of the positional similarity of two associations, the similarity between a direct forward association and a direct backward association is greater than the similarity between a direct forward association and a remote backward association.

Comparison to Other Theories

Multitrace strength theory is an extension of the strength theory proposed by Wickelgren and Norman (1966) for item recognition memory. The principal similarities are: (a) the characterization of memory traces by real-valued strengths, with noise added separately, similar to the learning theories of Hull (1943, 1952) and Spence (1956, 1960), (b) the criterion decision rule for recognition memory, first used for this purpose by Egan (1958), (c) the provision for more than one memory trace, (d) the distinction between acquisition, decay, and retrieval phases of memory traces, (e) the additive combination of traces, (f) the provision for strength generalization due to event similarity, and (g) exponential decay.

The principal extensions are: (a) the subdivision of the acquisition phase into acquisition of potential strength and consolidation of retrievable strength, (b) the assumption of an ITM, distinct from STM and LTM, with an approximate specification of its consolidation time and decay rate, (c) the specification of many modalities of memory, (d) the formulation of order memory and its generalization properties, which is a modification of an earlier strength theory of order memory (Wickelgren, 1967a), (e) the specification of the maximum decision rule for recall, which follows Green and Moses (1966), Norman (1966), Kintsch (1968), Wickelgren (1968a), and Norman and Wickelgren (1969), and (f) the particular functions chosen for acquisition, consolidation, and retrieval.

Markov (finite state) models with STM and LTM states (Atkinson and Crothers, 1964; Bernbach, 1965; Calfee and Atkinson, 1965; Waugh and Norman, 1965; Greeno, 1967; Chapter 8) use a very different underlying (state) representation of the memory trace than multitrace strength theory. However, they share the basic idea that there is more than one memory trace, with the different traces having different forgetting (decay) properties. Of course, since no Markov model specifies states corresponding to the VSTM, STM, ITM, and LTM traces in multitrace strength theory, there is far from complete agreement on the number of traces. Furthermore, Markov models necessarily restrict an event or association to be in one state at a time, that is, an event or association could not be in both STM and LTM unless a new compound state is defined. This is clumsy. Also, if one wants to get many gradations of trace strength, this either requires a large increase in the number of states or defining distributions associated with each state (Bernbach, 1967; Kintsch, 1967; Murdock, Chapter 9). Neither alternative seems attractive to me.

Markov models of memory make essentially the same distinction between acquisition, decay, and retrieval phases of memory as multitrace strength theory. However, when consolidation is discussed in the context of a Markov model of memory (Bower, 1967a; Greeno, 1967), consolidation means transfer from STM to LTM. The assumption that an STM trace is "converted" into an LTM trace is not necessarily true, and in fact, some physiological evidence suggests (though it does not prove) that LTM is consolidated independently from STM (or ITM) in rat

(Albert, 1966b). Multitrace strength theory is formulated so as to be able to accommodate either possibility, and, in any case, it is quite easy to have a decaying ITM component at the same time as a consolidating LTM component. This can be represented with a Markov model, but it is awkward.

Recently, a new class of models (Bower, 1967a; Atkinson and Shiffrin, 1965, 1968; Atkinson, Brelsford, and Shiffrin, 1967) has emerged out of the Markov model tradition. These models, called "multiprocess models" by Atkinson *et al.* (1967), postulate VSTM and add considerable structural detail to STM, LTM, the transfer from STM to LTM, and the maintenance of an STM trace by rehearsal. Perhaps the three most basic features of the multiprocess models are: (a) the rehearsal buffer representation of STM, (b) the search representation of LTM, and (c) the distinction between memory structure and control processes such as rehearsal and recoding that can operate on the memory structure. In special cases, multiprocess models can be reduced to Markov models, but multiprocess models have far more flexibility. In particular, traces for a single event can be in both STM and LTM and can have different numbers of copies or degrees of strength. Multiprocess models most often maintain a basically discrete characterization of memory traces, while strength models use a continuous characterization. More important, multiprocess models have been much more concerned with the control processes of rehearsal in STM, and search processes in LTM, than has strength theory. Multitrace strength theory places much greater emphasis on memory structure: the number of traces, acquisition, consolidation, decay, and elementary retrieval-decision processes.

The multicomponent model of Bower (1967b) and the model proposed in this volume by Norman and Rumelhart (Chapter 2) differ from multitrace strength theory by analyzing an item into discrete attributes (components, features) and assuming that memory traces are formed for each attribute. Multitrace strength theory is currently designed to take a continuous similarity-space approach to item analysis.

Information processing models of memory such as EPAM (Feigenbaum, 1963; Chapter 13; Simon and Feigenbaum, 1964) and the model of Judith Reitman (Chapter 5) differ from multitrace strength theory primarily in : (a) their greater emphasis on control processes such as rehearsal and search and (b) in their choice of programming languages as the language for precise expression of the theory, rather than more conventional axiomatic mathematics.

Assumptions of Multitrace Strength Theory

Four Phases

The time course of a memory trace M under conditions K has 4 phases acquisition of potential strength, $A(t_A)$, consolidation of actual strength $C(t)$, decay of strength, $D(t)$, and retrieval of strength, $R(t_R)$. Events or the rehearsal of prior events initiate acquisition, and each phase follow after the other in the order: acquisition, consolidation, decay, and re trieval, with overlap being possible between two adjacent phase Judged strength of a memory trace $M = A(t_A)\, C(t)\, D(t)\, R(t_R) + X$, wher X is a normally distributed random variable: $X \sim N[0,\sigma]$. The functio A, C, D, and R and the parameter σ are functions of M and K; t_A is th acquisition (presentation or rehearsal) time, t is the delay since the onse or offset of the acquisition period, and t_R is the time allowed for retrieva

Four Traces per Modality

In each modality of memory, there are as many as four traces with di ferent time courses: very-short-term memory (V), short-term memo (S), intermediate-term memory (I), and long-term memory (L).

Many Modalities

Every sensory, motor, and cognitive modality of performance is modality of memory.

Event Memory

An occurring event i initiates all four memory traces in each relevan modality for that event and for any other event j in proportion to η_{ij} similarity to j in that modality.

Order Memory

A sequence of events, $i = 1, \ldots, n$, initiates all four memory trac in relevant modalities for each direct forward association $i \rightarrow i + 1$ a for each other association $i \rightarrow j (j \neq i + 1)$ in proportion to a weight linear combination of $\eta_{i+1,j}$ (event associative response generalizatio

$\eta_{j-1,i}$ (event associative stimulus generalization), and π_{ij} (positional similarity to a direct forward association). For ungrouped coding, π_{ij} is a monotone decreasing function of $|i-j|$, and $\pi_{i,i+z} > \pi_{i,i-z}$.

Additive Combination of Traces in Retrieval

The sum of all trace strengths in all modalities (total strength) is judged in retrieval.

Criterion Decision Rule for Event and Order Recognition

A subject responds "yes" if the total strength of a test event or test association exceeds a criterion. Confidence ratings are obtained by partitioning the total strength dimension by further criteria. Under conditions K, criteria c_{iK} are normally distributed random variables, $c_{iK} \sim N[0,\sigma_K]$.

Maximum Decision Rule for Recall and Multiple-Choice

A subject chooses the event with the greatest total strength of association to the cue event.

Bounded Exponential Acquisition

$A(t_A) = \alpha(1 - e^{-\theta t_A})$, where α and θ depend on M and K.

Delayed Consolidation

$$C(t) = \begin{cases} 0 & \text{for } t < \tau \\ [(t-\tau)/(\epsilon-\tau)]^{\varphi} & \text{for } \tau \leqslant t \leqslant \epsilon \\ 1 & \text{for } t > \epsilon, \end{cases}$$

where τ, ϵ, and φ depend on M and K. For present purposes, we can assume $\varphi = 1$ so that $C(t)$ is a ramp function from $t = \tau$ to $t = \epsilon$.

Exponential Decay

$$D(t) = \begin{cases} 1 & \text{for } t < \tau \text{ or } \epsilon \\ e^{-\beta(t-\tau)} & \text{for } t \geqslant \tau \text{ or } \epsilon, \end{cases}$$

where β depends on M and K.

Bounded Exponential Retrieval

$$R(t_R) = \rho(1 - e^{-\psi t_R}),$$

where ρ and ψ depend on M and K.

Restrictions in Testing Strength Theory

The foregoing theory is intended to be formulated with sufficient flexibility to be able to handle known behavioral and neurobehavioral (ablation, stimulation, and pharmacological effects on behavior) phenomena of memory, with appropriate choices of the acquisition, consolidation, decay, retrieval, and decision parameters for each trace in any set of conditions. As with any general scientific theory, the predictions of the theory will be very hard to derive for the vast majority of conditions and so only carefully selected conditions are suitable for testing the theory. For ease in testing multitrace strength theory, the important restrictions on conditions are the following:

(*a*) The test event should be sufficiently simple that subjects handle it as a unit, making a single absolute or comparative judgment, not a sequence of elementary decisions combined into an overall decision by means of complicated logical reasoning. In principle, when we have some understanding of the elementary syntactic and semantic units of a phrase, sentence, or sequence of thoughts, stimuli, or responses, and when we know more about the cognitive processes in logical reasoning, then strength theory should be applicable to memory for phrases, sentences, and complex thought, sensory, or motor sequences. However, at present, no application of strength theory to such complex events is possible.

(*b*) For the same purpose of encouraging single-stage decisions, the time for the "yes–no," rating, multiple-choice, or recall response should be very limited, and rapid responding should be encouraged.

(*c*) Only a single response should be required in the retrieval-decision period in order to avoid delay and/or interference effects in the retrieval-decision period, though these effects need not be too difficult to analyze within strength theory. However, if the correctness of previous responses influences later responses, one could get stochastic processes that would needlessly add to the complexity of strength theory. Single-test methods, especially probe methods (e.g., Murdock, 1961a, 196 ; Waugh and Norman, 1965; Norman, 1966; Wickelgren and Norman,

1966) are certainly to be preferred, and complete recall methods, with the order of recall uncontrolled, are too messy to be quantitatively analyzable with strength theory.

(*d*) Recognition tests are theoretically simpler to analyze with strength theory than recall or multiple-choice tests, and so recognition tests are preferable. However, much work must be done with recall to determine the relationship between recall and recognition memory. Omissions greatly complicate the strength-theory analysis of recall experiments. So omissions should not be allowed in recall (or recognition), at least until we understand recall without omissions much better than we do at present.

(*e*) Conscious rehearsal of events to be remembered must be strictly controlled by telling the subject what to rehearse (think of) at every moment. Any controlled method of rehearsal is analyzable by strength theory, but it is easiest to analyze conditions in which the subjects are thinking only of the current event or pair of events and never thinking of previous events. The reason for controlling rehearsal is that strength theory requires that we know at all times what phase of the memory process each event-trace is in. Naturally, control of conscious rehearsal will be less than perfect, but conscientious subjects appear to be able to control rehearsal quite adequately. A small amount of rehearsal will not affect strength decay curves very much and part of the effect is handled by the random noise factor X in the four-phase assumption. The remaining effect of a small probability of uncontrolled rehearsal can be reduced further by increasing the number of events that have to be remembered, since the only time that rehearsal affects the strength decay curve is when a subject rehearses the event to be tested later. Thus, probe methods are superior to presenting and testing a single item. In principle, one could model uncontrolled rehearsal within the context of strength theory. However, this greatly increases the computational complexity of strength theory, and I do not see what one would learn from this that one would not learn much more easily from controlled rehearsal.

(*f*) As we learn more about the properties of different traces in different modalities (particularly the decay rates and what they depend upon), we should try to set up conditions so as to study one trace in one modality at a time. In some cases, it may be possible to study one trace under conditions where the other traces are lower in their contribution to the total strength by a factor of 100 or more yielding simple exponential decay functions (e.g., Wickelgren and Norman, 1966). In other cases, we may have to settle for factors of around 10 (e.g., Wickelgren, 1969).

However, every effort should be made to secure strength decay curv that are very close to simple exponential decay functions, because th precise analysis requires far fewer different delay conditions than dec curves that must be fit by the sum of two exponentials.

Note that, with the exception of (*f*), all the other restrictions on te of multitrace strength theory are concerned with controlling the "str egy" or "control process" of the subject in acquisition, rehearsal duri the storage interval, and retrieval. Experiments on human or anim learning or memory, which have not carefully controlled these acqui tion, storage, and retrieval strategies may provide qualitative tests multitrace strength theory, but quantitative evaluation is generally di cult in these cases. Since verbal instructional control of human strateg is probably much easier to achieve than control of animal strategies pretraining, all quantitative behavioral tests of strength theory to mentioned in this chapter will be on human beings, though some qua tative neurobehavioral findings with animals will also be mention However, there is no reason why strength theory could not be appli to animal learning and memory, at some future time.

Completeness of Multitrace Strength Theory

Although multitrace strength theory applies to all memory situati it requires the estimation of many, possibly different, parameters every situation to which it is applied. Obviously, there must be so parameter invariance over different situations and, failing this, so simple functions for predicting parameters in one situation from par eters in other situations. Efforts to determine parameter invariance other simple parameter functions, within the context of multitr strength theory, have not proceeded far enough to justify including such assumptions in the foregoing statement of the general assumpti of multitrace strength theory. However, the available findings on par eter functions will be discussed later, along with some indication how multitrace strength theory might be completed to include th functions.

Predictions and Empirical Adequacy of Multitrace Strength Theory

One of the features of a mathematical theory that gives it great ge ality with a small number of axioms is the combining power of the axi with themselves and with the more general axioms of mathematics. In general, the predictions of the theory are theorems derived from several axioms. While this is a desirable feature of a theory, it does make testing the theory somewhat more complicated. Ideally, it would be desirable to test each assumption independently of the others, but this is rarely possible. However, one can attempt to approach this ideal as closely as possible. The present section discusses some of the predictions of multitrace strength theory that have been tested to date, pointing out which assumptions are being tested by each prediction.

Four Traces

Present evidence does not require one to postulate four separate traces with different memory properties. But present evidence does require at least two memory traces, and there is suggestive evidence for four traces. The evidence that compels the assumption of at least two memory traces is that human beings with bilateral mesial temporal ablations can have completely normal short-term memory (STM) with very little ability to form new long-term memories (LTM) (Scoville and Milner, 1957; Milner, 1966; Wickelgren, 1968b). In terms of the present four-trace system, the cut is probably between STM (delays of 1–20 sec) and intermediate-term memory, ITM (delays of 20 sec to minutes or hours), but this is not completely clear. A huge mass of neurobehavioral data on the effects of various drugs, spreading depression, and dc potentials applied to the brain also strongly supports the hypothesis that there are at least two memory traces, though some of these data are more complex to interpret than the neurological data. (See Agranoff and Davis, 1968; Albert, 1966a, 1966b, 1966c; Barondes and Cohen, 1968; Deutsch, Hamburg, and Dahl, 1966; Flexner and Flexner, 1968; for recent representative articles.) Finally, trace strength decay curves for normal subjects often have two components, a rapidly decaying component and a more slowly decaying component (Waugh and Norman, 1965; Wickelgren, 1969).

The argument for distinguishing ITM from STM is that human beings with mesial temporal lesions show pronounced deficits in the level of ITM compared to normal subjects. These deficits appear for delays greater than about 4 sec in those tasks where normal subjects show a very slowly decaying (ITM) component of the trace lasting for minutes, tens of minutes, or more (Milner, 1966).

The argument for distinguishing this ITM from LTM is that many of the neurobehavioral studies indicate that LTM requires a consolidatio period on the order of hours or days to be established. (Agranoff an Davis, 1968; Albert, 1966a, 1966b, 1966c; Deutsch *et al.*, 1966; provid some recent examples.) The ITM which is severely impaired in th subjects with mesial temporal lesions must be established in second so this ITM could not be the same trace as LTM.

The argument for distinguishing between STM and ITM is strong than the argument for distinguishing between ITM and LTM becaus the latter argument requires generalizing from various species o animals to humans. However, there is a factor of about 10^7 betwee the strength decay rates of the fastest decaying STM and the slowe decaying LTM, according to a rough calculation, and only part of th seems to be explainable on the basis of variation in the STM and LT decay rates. Thus, there appears to be a hole which ITM could fill i normal retention curves. The evidence for three traces, STM, ITM, an LTM is not as definitive as that for distinguishing at least two trace STM and ITM-LTM, but the three trace theory does seem more pla sible than the two trace theory.

The evidence for the very-short-term memory (VSTM) trace is sca indeed, but it seems safest to consider the memory for visual or audito material that has not been attended-to (e.g., Sperling, 1960, 1963; Av bach and Sperling, 1961; Broadbent, 1958) to be a different kind memory until and unless it is proven otherwise.

Exponential Decay

One of the most important successes of strength theory is that, so fa strength decay functions have turned out to be either simple expone tials (Wickelgren and Norman, 1966; Wickelgren, 1967a, 1968b, 197 and much unpublished data) or the sum of two exponentials (Wick gren, 1969). For example, Wickelgren and Norman (1966) found simp exponential decay of the strength of the STM trace in a probe stud items (three-digit numbers) in all serial positions of lists from two seven items long. The probe was a single item from the previous presented immediately after the end of the list. Thus, the tempo delay between presentation and test is the number of subseque items times the presentation time for each item (1 second in this stud Semilogarithmic plots of these strength decay curves are shown Fig. 1. Consistent with the assumption of exponential decay, the strength decay curves are well fit by straight lines on semilog plo

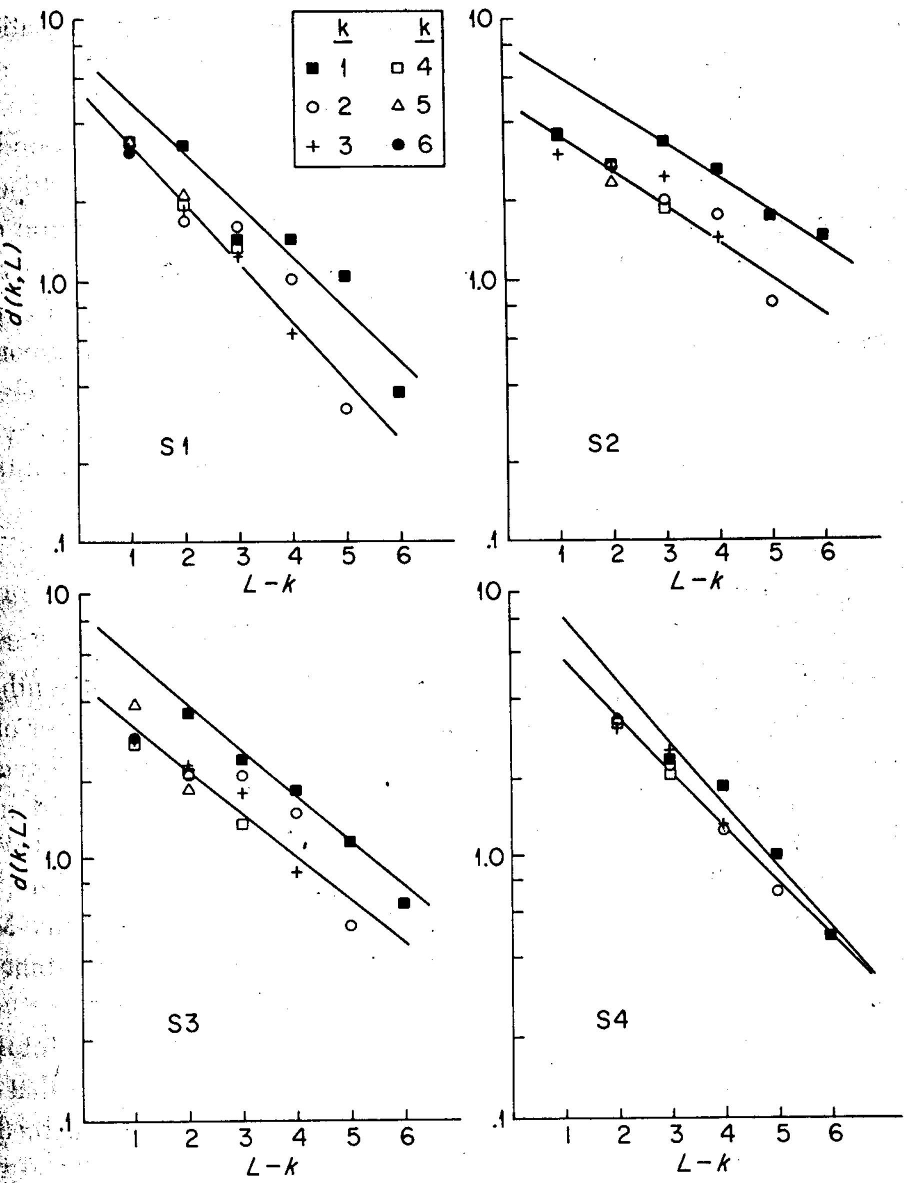

FIG. 1. The $d(k, L)$ values for each serial position (k), in lists of length $L = 2$–7, as a function of the number of subsequent items (L-k). L-k is also equal to the delay in seconds. The two straight lines in each plot are the least-squares fits to the data. The upper line is the fit to the first items ($k = 1$) and the lower line is the fit for all the other items ($k > 1$).

Furthermore, while the first item established a stronger trace in STM, it had approximately the same rate of decay as subsequent items in the list.

Probability decay curves are usually S-shaped with rate of decay first increasing and then decreasing. Strength theory may be able to account for all memory decay curves using one or two component traces at any given delay, though there are three different possible combinations of two traces, VSTM and STM, STM and ITM, ITM, and LTM.

The importance of this cannot be overemphasized. When the decay (forgetting) rate is not constant as a function of delay (under conditions which are homogeneous during all delays), it is extremely difficult to make meaningful comparisons among decay rates as a function of different conditions. When one has an analysis of the total memory trace into one or two components with each component having a constant decay rate at every delay, one has a general framework for memory and can hope for some success in determining the more specific laws of acquisition, consolidation, decay, and retrieval for each memory trace in each modality. Without such a framework, one is just stumbling in the dark.

STM and ITM

Since both STM and ITM are consolidated in seconds or tenths of a second, they are both potentially present whether the number of presentations is one or many. The basic idea that many studies have substantial amounts of both STM and ITM was first expressed in the Markov-model framework by Atkinson and Crothers (1964) and Waugh and Norman (1965). Using this theory, Waugh and Norman (1965) analyzed the decay curves for many tasks into two components. Multitrace strength also yields such an analysis, although it is somewhat different from a Markov analysis.

Besides being able to analyze a composite trace decay curve into components, it is also very desirable to design experiments so that only STM or ITM is being studied in an experiment. How can this be done according to multitrace strength theory?

Studying ITM independently of STM is simple: just make use of the fact that ITM decays much more slowly than STM. Do not use immediate retention tests. Do use a variety of longer retention intervals on the order of minutes and hours. At least the first 20 sec of the retention interval should be filled with rehearsal preventing activity to eliminate the STM traces, but it is also highly desirable to do all one can to minimize rehearsal throughout the entire span of any retention interval. Finally, although some kinds of material and tasks give substantial levels of ITM with a single presentation of a few seconds per item, ITM often requires numerous presentations to build up to a substantial level. According to the present theory, all verbal learning tasks with filled (rehearsal-preventing) retention intervals of minutes or hours are studying verbal ITM.

Studying STM independently of ITM also makes use of the fact that ITM decays much more slowly than STM, but it makes use of this fact in a somewhat subtler way. If one uses a small population of items, say digits, letters, or a small set of words, over and over again on closely spaced trials, the ITM traces for all items and perhaps also all associations between items will be approximately equal. Only the STM trace will differentiate the items or associations on the last trial from those on previous trials. Thus, since ITM traces are equal for correct and incorrect events, one is studying only the STM traces on all trials after the first few.

Another potential way to study STM independently of ITM is to present material to be remembered so rapidly that the ITM trace does not have time to be acquired or cannot be consolidated. There is some reason to think this may be possible, but it is too early to tell for sure.

Free Recall. The present theory of STM and ITM explains many phenomena. First, it explains the two-component decay curves for free recall (e.g., Deese and Kaufman, 1957; Murdock, 1962b; Waugh, 1962) in a manner very similar to the analysis done with a different theory by Waugh and Norman (1965).

Continuous Recognition Memory. Second, it explains the large, slowly decaying component in continuous (steady-state) verbal recognition memory studies (Shepard and Teghtsoonian, 1961; Shepard and Chang, 1963; Donaldson and Murdock, 1968). In a simple yes–no continuous recognition memory study that I have done, items (words or a complex pattern composed of three consonants followed by three digits, abbreviated CCC–DDD) were presented at a rate of 3.5 sec per item. Subjects indicated whether or not they had seen an item previously, with delays between presentation and test ranging from immediate to about 12 minutes. The strength decay curves (log strength vs. delay) for one subject in this experiment are shown in Fig. 2. The straight lines are the theoretical, exponentially decaying, STM and ITM traces. The curved theoretical lines represent the total memory strength (sum of STM and ITM strengths). The component traces were derived under the assumptions that (*a*) STM decay rates are identical for words and CCC–DDD complexes, (*b*) ITM decay rates are identical for words and CCC–DDD complexes, (*c*) STM consolidates essentially immediately for both kinds of items, and (*d*) ITM consolidates linearly over

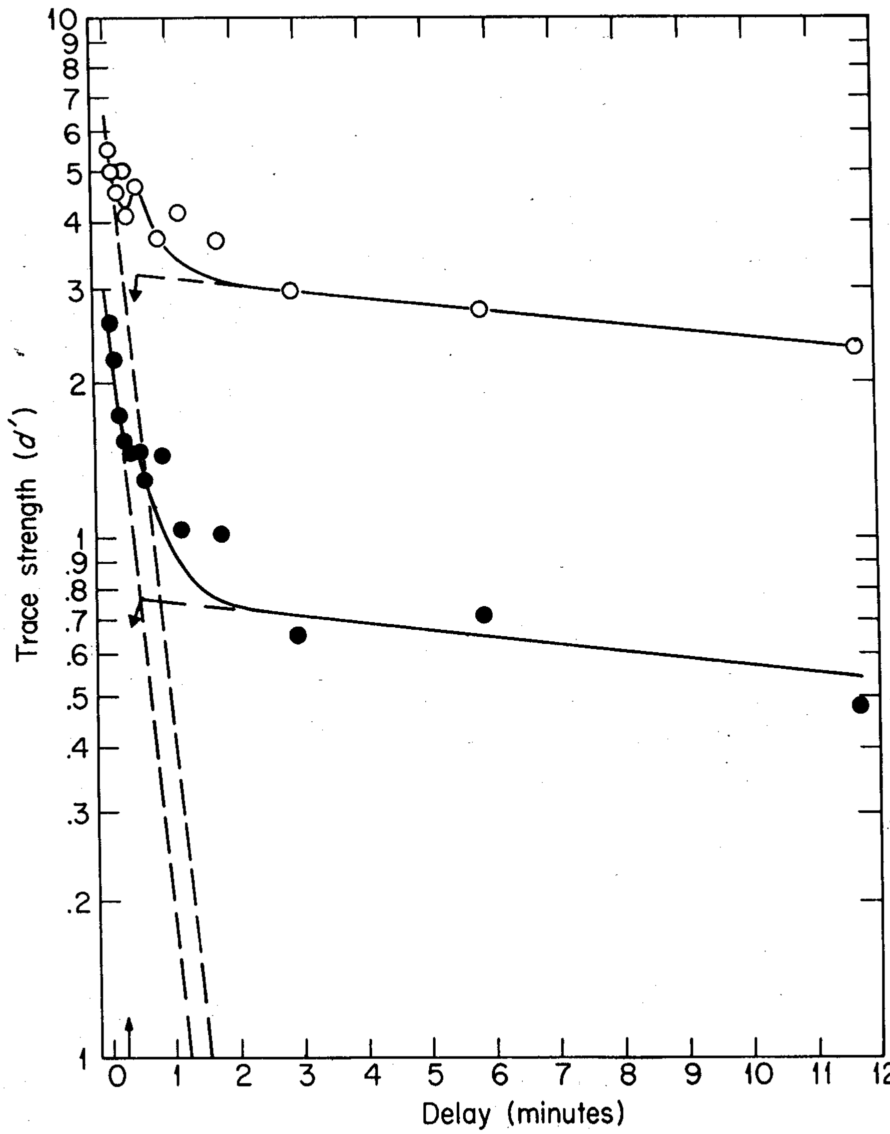

FIG. 2. Strength-decay curves for one subject in a continuous recognition memory for words or CCC–DDD complexes. The dashed lines represent the best-fitting S and ITM components of the trace, with the ITM component assumed to be consolid over the period from 10 to 30 sec after presentation. The solid lines represent the memory strength, the sum of the STM and ITM strengths.

the period from 10 to 30 sec following presentation for both kind items.

Note that depending on the form and rate of the consolidation fu tion for ITM in relation to the rate of decay for STM, one can ob short sections of the decay curve for total strength which are increas in strength (reminiscence), though this need not occur and would generally occur over a very long period of time. The fit of multitrace strength theory to these data is quite good with the exception of a few points at delays between 1 and 2 minutes. These modest deviations between theory and data could be due to averaging over items with very different STM or ITM decay curves, and work is in progress to determine if this is happening.

However, it is clear that, at least within the context of strength theory, the data in Fig. 1 and the continuous recognition data of Shepard and Teghtsoonian (1961) cannot possibly be fit by a single exponentially decaying trace. Rather, what seems to be required is to assume that performance is mediated by two traces, one with a decay rate similar to (though somewhat slower than) that found in probe studies of STM (Wickelgren and Norman, 1966; Wickelgren, 1970) and one with a decay rate slower by a factor of 10^2.

A study in progress on a subject with a bilateral mesial temporal lesion indicates that he has a normal or slightly reduced STM component in continuous recognition memory tasks, but has a much reduced ITM component in these tasks. This provides further support for the hypothesis that continuous recognition memory studies are studying both STM and ITM, and these studies should not be lumped together, willy-nilly, with pure STM studies that use a small population of items over and over again in rapid proximity.

Most continuous recognition memory studies using a large population of items have employed very slow presentation rates (around 5 seconds per item) though Howe (1967) got a moderate amount of ITM for pictures presented at 1.5 sec per picture. It is possible that acquisition of the ITM trace, while very rapid, is nevertheless on the order of a second or more (possibly varying with the type of material and the conditions of presentation). On the other hand, the STM trace clearly can be acquired in tenths of a second, since good STM is obtained for lists of items presented at four items per second. Thus, presenting one three-digit number per second, as in Wickelgren and Norman (1966) or four words per second in an unpublished study of mine appears to produce little or no ITM. This permits study of STM with larger populations of items.

Furthermore, use of a small population of events does not guarantee an uncomplicated study of STM, if the rate of presentation is slow enough to permit use of complicated coding or rehearsal strategies. This appears to have occurred in a continuous recognition memory study by Katz (1966) which employed a rather small population of associations (20 letter–number pairs), but used a very slow rate of pre- sentation (8 seconds for each test plus new presentation of a pair). Katz (1966) instructed his subjects not to rehearse prior pairs, but this was not sufficient to eliminate the more slowly decaying component of the memory trace in his situation.

Recognition Memory for Pitch. Third, the present theory of STM and ITM explains the two-component trace-strength decay curves, invariably found for normal subjects in short-term recognition memory for pitch using the delayed comparison procedure (Wickelgren, 1966a, 1969). The only subject who has not shown some ITM in this situation is a neurological subject with a bilateral mesial temporal lesion (Wickel- gren, 1968b). The level of the ITM trace for normal subjects in this situation is generally fairly low, but given the frequency of presenting each tone from the rather small population of tones (usually 10 or 2 tones), this is probably reasonable. Here again, we find that some ITM can be formed for each item from a small population of items, when the rate of presenting new items to be learned is slow enough (on the order of one new item ever 10–40 sec in my pitch-memory experi- ments). Since the rate of decay of ITM for different kinds of material under different conditions is not well established, it is not yet possible to make a definitive quantitative check on whether the level of ITM found in these pitch-memory studies is reasonable or not. On the basis of the estimated rate of decay for the ITM trace obtained from one experiment in Wickelgren (1969) and other unpublished studies, the level of ITM found in the pitch-memory studies appears to be of about the right order of magnitude, but further studies are necessary to clinch the point.

Three-Phase Studies. Fourth, multitrace strength theory explains the frequent (but not invariable) presence of a more slowly decaying com- ponent of the verbal memory trace in the "three-phase" or "distractor" (Murdock, 1967) design originated by Brown (1958) and Peterson and Peterson (1959). In this design, a single item or short list of items is presented followed by rehearsal-preventing activity followed by a test of some or all of the items in the short list.

In the three-phase design, acquisition (perhaps including some or all of the consolidation), storage (decay, perhaps preceded by some con- solidation), and retrieval-decision phases are all distinguished to the subject and independently manipulable by the experimenter. This is the advantage of the three-phase method over the probe (two-phase method, which confounds acquisition and storage, and the continuous (one-phase) method, which confounds all three. Probe and continuous methods have compensating advantages for certain purposes, and, as will be apparent from the present discussion, the three-phase design has some disadvantages, so no one should conclude that one of these methods is always to be preferred.

Just as in the delayed comparison of pitch studies (which use the three-phase design), the frequency with which each item is presented to be learned has generally been much lower than in probe studies. Thus, according to the present theory, it is not surprising that strength decay curves for three-phase recall studies (Peterson and Peterson, 1959; Murdock, 1961b; Hellyer, 1962; Melton, 1963) frequently require both an STM and an ITM component to achieve a good fit. In Figs. 3 and 4, strength decay curves have been plotted for the three-phase recall study of Hellyer (1962) and the vocal rehearsal condition of the Peterson and Peterson (1959) study. The parameter is the number of repetitions or amount of rehearsal time prior to beginning the backward counting that filled the delay interval. These strength-decay curves were derived from the probability decay curves by assuming that the consonant trigrams to be remembered came from a population of about 1000 and that all incorrect trigrams had approximately equal strength.

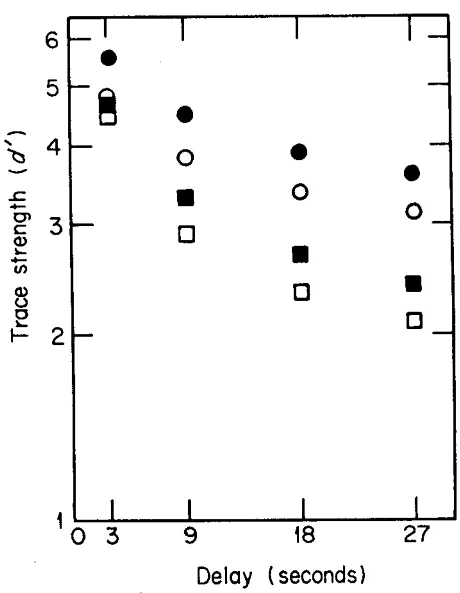

FIG. 3. Strength-decay curves for three-phase recall of consonant trigrams as a function of the number of repetitions (from Hellyer, 1962).

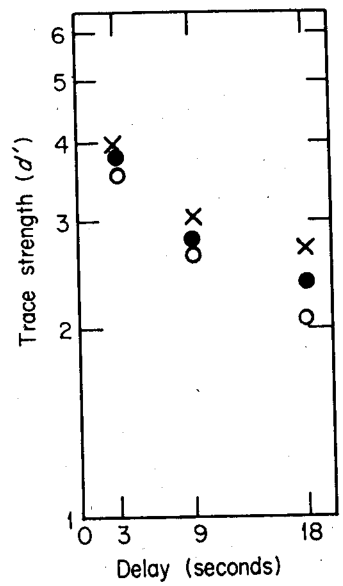

FIG. 4. Strength-decay curves for three-phase recall of consonant trigrams, as a functi
of rehearsal time (from Peterson and Peterson, 1959).

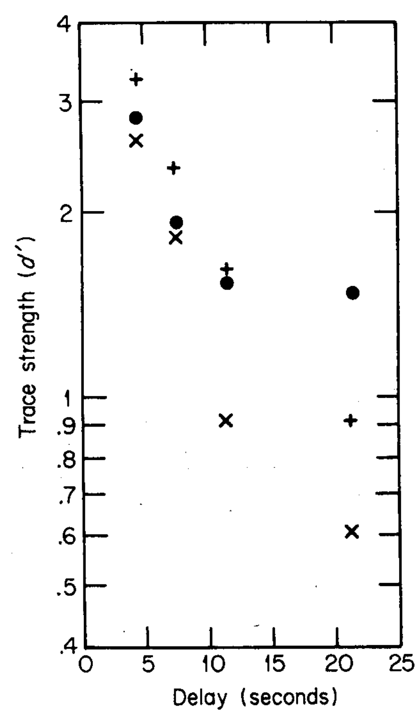

FIG. 5. Strength-decay curves for three subjects in three-phase recognition of a si
consonant letter.

The same deviation from simple exponential decay is found in recognition memory, where it is not necessary to make assumptions about population size or equivalence of incorrect strength distributions. See Fig. 5 for the strength decay curves of three subjects in an unpublished study of mine where single letters were presented and tested for recognition memory after delays filled with rapid backward counting.

For a variety of reasons (the main one being the absence of really long delays with the three-phase method), it is not yet possible to draw quantitative conclusions regarding the rates of decay for verbal STM and ITM in three-phase situations. However, there is every reason to hope that the strength decay rates for STM and ITM in three-phase studies will be consistent with the STM decay rates found in probe studies and the STM and ITM decay rates found in continuous studies.

Proactive Interference in STM. Fifth, multitrace strength theory explains why the decay of the memory trace for once presented material is much slower on the first few trials of an STM experiment (Keppel and Underwood, 1962) or on the first few trials after the type of material is changed (Wickens, Born, and Allen, 1963; Loess, 1968). According to the present theory, this "proactive interference" effect is due to the first few trials having substantial levels of ITM, in addition to STM. On later trials, the ITM for items on previous trials has not decayed very much, and competition from these items in a recall test substantially reduces the value of the ITM component of the trace for items on later trials, since the cues for each trial are not very distinct. According to multitrace strength theory, the contribution of the ITM trace to the discriminability of correct and incorrect items on the previous trial decreases rapidly over the first few trials of an experiment to an asymptote that depends on the average time between successive presentations of the same item. Many factors affect the average time between successive presentations of the same item: the number of items presented per trial, the size of the item population, and the intertrial interval.

In accord with the prediction of multitrace strength theory, longer intertrial intervals are known to produce less "proactive interference" from previous trials (better memory) in "STM" experiments where few items are presented to be learned on each trial (Peterson and Gentile, 1965). According to the theory, intrusions from items in the same position on previous trials should decrease with increasing distance from the present trial, measuring distance in either time or trials, and this prediction is also known to be valid (Melton and Von Lackum, 1941;

Conrad, 1959, 1960; Peterson and Gentile, 1965; Peterson and James, 1967).

The one possibly discrepant finding is that Conrad (1960) found no net improvement in memory performance with longer intertrial intervals, though he did find intrusions from items in the same position on the previous trial to decrease with intertrial interval. The reasons for this discrepant finding are not clear. However, it is likely that Conrad's (1960) experiment involved very little ITM compared to the experiment of Peterson and Gentile (1965), since Conrad presented a much longer list to be remembered and presented it at a fairly rapid rate (two items per second).

Finally, it should be noted that all of the STM "proactive interference" studies have used recall to test retention. It should be possible to analyze the "proactive interference" effect much more precisely with recognition, carefully controlling the similarity of incorrect test items to items correct for previous trials.

VSTM and STM

If there is an auditory or visual VSTM that should be distinguished from STM, then probably the last item or two in a list, when tested immediately, should not be considered in fitting an STM trace to the strength decay curve.

ITM and LTM

Besides the neurobehavioral evidence for distinguishing ITM and LTM, strength decay curves for visual memory (Shepard, 1967; Nickerson, 1968) show a much more rapid decay in the first week or two than from two weeks to one year. Until more is known concerning the rate of decay of the ITM trace and the rate of consolidation of the LTM trace for different types of materials and conditions, one must be cautious in interpreting retention data at delays of two hours to two weeks. At some delays in this interval, one either gets substantial overlap of ITM and useful LTM or else it should be possible to show reminiscence between carefully selected delays.

Incidentally, if one does get reminiscence here, it suggests that the LTM trace consolidates independently of the ITM trace, at least to some extent.

Four Phases

The distinction between the acquisition, decay (storage), and retrieval-decision phases of memory seems to be necessarily valid for any device with memory. In the present formulation of multitrace strength theory, acquisition refers to establishing a potential, but not yet usable, memory trace. Consolidation converts this potential trace into an actual (usable) trace. The distinction between acquisition and consolidation is somewhat questionable for VSTM and STM, and the evidence is not yet conclusive for distinguishing acquisition and consolidation in ITM. However, the neurobehavioral evidence just cited seems to indicate a relatively substantial consolidation phase for LTM.

Consolidation of VSTM and STM in all modalities may proceed simultaneously with acquisition or occur so rapidly after acquisition that there is no need to recognize it as a separate phase of these memory traces. However, this is easily handled by the present theory through choice of a consolidation function that approaches asymptote so quickly that it is essentially a step function for our purposes. If this is so, then the only advantage of including a consolidation phase in VSTM and STM is to be able to handle all four traces in the same framework. Alternatively, we may, one day, find phenomena that require STM consolidation times of seconds or tenths of a second. Multitrace strength theory can handle either eventuality.

Independence from Irrelevant Strengths

The principal qualitative component of the criterion decision rule for recognition memory is the assumption of "independence from irrelevant strengths" (similar to the analogously named notions of Arrow, 1951; Luce, 1959). The assumption is that in a recognition test, the subject judges only the strength of the test event (item or association), without considering the strength of other events. This assumption means that there is no retrieval interference (competition) in a recognition test, an assumption made without proof by many workers in verbal learning.

I have not been able to think of a good test of this assumption for item memory, but a powerful test is possible for order memory. The test is to determine if the strength discriminability of a correct A–B association from an incorrect A–D association is affected by the presence or absence of a strong A–C association. According to strength theory, the difference in strength of A–B and A–D associations should be unaffected by the strength of an "irrelevant" A–C association. This strong prediction of strength theory appears to be valid, at least for STM (Wickelgren, 1967a; Bower and Bostrom, 1968). Since the assumption of the independence from irrelevant strengths applies only to the retrieval-decision phase of recognition memory, proving it for STM very strongly indicates that it holds when ITM or LTM traces are being judged also.

However, a successful test, of the type discussed above, of the assumption of independence from irrelevant strengths in retrieval requires that there also be no reduced acquisition or consolidation and and no increased decay of an A–B association when preceded or followed by an A–C association. This equivalence of acquisition, consolidation and decay for A–B, A–C and A–B, C–D is known to be false for verbal ITM (e.g., McGovern, 1964; Postman, 1965) and LTM (Houston, 1967). In these verbal learning studies, the effect is usually called "unlearning," though "storage interference" might be a better term; recognition tests show that the effect holds when the "irrelevant" A–C association is presented before the A–B association. "Unlearning" is a fine term for a retroactive interference effect, but not for an effect due to proactive interference.

The report by Houston (1968) of an unlearning effect in STM did not use a recognition test and also employed a paradigm conducive to the presence of large ITM traces, according to the classification scheme presented here.

Either no storage interference (unlearning) occurs in STM or, more likely, as stated by Wickelgren (1967a), the strength in STM of an A–B association is impaired as much by C–D pairs as by A–C pairs among the prior and subsequent items. There is quite solid evidence that the number of subsequent items plays an important role in decay in STM even when the temporal delay between presentation and test is held constant and rehearsal is presumed to be minimal (Waugh and Norman, 1965; Norman, 1966). The Waugh and Norman (1965) and Norman (1966) studies were recall studies, and so could be affected by retrieval interference. However, I have evidence showing that the number of intervening items also plays an important role, in addition to temporal delay, using a recognition test of STM (Wickelgren, 1970).

Invariance of Decay Rates

Although the decay rate for a single trace for a particular subject appears to be invariant with delay, it does not appear to be invariant under other conditions. For example, in a probe recognition study, I have found decay in STM to be a function of both temporal delay and the number of intervening items, making the rate of decay in STM different for different rates of presenting the items to be learned (Wickelgren, 1970). Furthermore, the rate of temporal decay of the STM trace in three-phase or continuous memory tasks appears to be somewhat slower than the rate of temporal decay in probe memory tasks, though it may be possible to account for this under the rubric of "rate of presentation of new material to be learned."

Analyzing these tasks in terms of the number of intervening items, rather than temporal delay, fails to produce invariance in STM decay rates, and, in my opinion, item decay functions provide a far less satisfactory framework in which to analyze what STM decay rates depend upon than do temporal decay functions (one example of this is found in Wickelgren, 1970).

Examples of the lack of invariance of strength decay rates for ITM and LTM can undoubtedly also be found in verbal learning studies, where a number of factors appear to affect decay rate, such as the amount and similarity of interpolated learning and the degree of learning of the original list. However, virtually all of the relevant studies were done using recall to measure retention, and permitting omissions. Such studies are very difficult to analyze with strength theory. Furthermore, intrusion frequencies are often not reported in enough detail to make any strength-theory analysis possible. One exception is the set of recognition-matching studies on unlearning that were referred to already.

Finally, there is an intuitive argument against invariance of the STM, ITM, and LTM decay rates, which derives from the factor of 10^7 between the decay rates of the fastest decaying STM and the slowest decaying LTM. Intuitively, it seems as if some memories last for seconds, others for minutes, others for hours, others for days, others for weeks or months, and others for years. To make memories last for each of these different periods of time, even with three traces, would require factors of e^{100} or e^{1000} in degree of acquisition above that required for perfect performance with an immediate retention test. This is undoubtedly a biophysical and biochemical impossibility for the nervous system. If such smooth variation in the duration of different memories is to be achieved by the nervous system with three traces, it must be done by varying the decay rate for one or more traces, not the degree of acquisition. This variability of decay rate is precisely what Melton (1963) claims for a single-trace theory.

Form of the Acquisition, Consolidation, and Retrieval Functions

No deep significance is attached to the form I have chosen for these functions: (*a*) exponential approach to a positive limit, starting from zero for the acquisition and retrieval functions and (*b*) a delayed bounded ramp function for consolidation.

The assumption of zero starting value in acquisition and consolidation means that, in separated multiple-presentation situations, we are always focusing on the increment in trace strength contributed by the last presentation. At present, this seems to me to be the simplest way to handle multiple presentation. However, it should be noted that this requires us to consider some aspects of the history of prior presentations as part of the conditions that determine the parameters in the acquisition and consolidation functions. This could be a mess.

The upper bound on the degree of acquisition, consolidation, and retrieval is a completely reasonable constraint. The provision for some delay before consolidation begins is probably absolutely necessary for ITM and LTM, but slight delays may also be found in the onsets of other processes.

The degree of empirical support for the chosen form of these functions is almost nonexistent. Some weak evidence that the form of the acquisition function is approximately an exponential approach to a limit was found in a study of STM for pitch (Wickelgren, 1969).

Spacing of Multiple Presentation

The effects of the spacing of multiple presentations have been admirably reviewed by Bjork (Chapter 10), and this review will not be repeated here. I will content myself with three empirical generalizations that Bjork has derived from previous experimental studies, citing one representative study to support each generalization: (*a*) Massed presentations lead to superior memory at delays of less than 4 sec (Peterson, Hillner, and Saltzman, 1962); (*b*) Spaced presentations lead to super memory at delays of 8 sec or more (Peterson *et al.*, 1962); (*c*) As the delay between presentations increases from 0 to somewhere between 16 and 32 sec, memory assessed 16 sec after the second presentation increases, but as the delay between presentations increases beyond this point memory (still assessed 16 sec after the second presentation) decreases (Peterson, Wampler, Kirkpatrick, and Saltzman, 1963).

These three effects are nicely explained by multitrace strength theory. First, the beneficial effects of massed presentation are obtained at just those brief delays at which the STM component of the total memory trace is most important. Since the STM trace consolidates and decays rapidly, one expects optimal STM from massed presentation. Second, the advantages of spaced presentation appear when ITM is beginning to play a much larger role in memory performance. It is quite reasonable to suppose that ITM requires a much longer consolidation time than STM and that the optimal level of ITM would be obtained from two presentations, when the second presentation occurred after the first had had time to consolidate. Third, one expects to find an optimal spacing time at around the time consolidation of the first presentation is complete or almost complete, because the ITM trace does decay after it consolidates. Although it may be a complete coincidence, the improvement in ITM performance with spacing appears to be greater than 16 and less than 32 sec, and in the section on continuous recognition memory studies, there seemed to be some advantage in assuming that consolidation of ITM took place over the period from 10 to 30 sec following presentation.

Independence of the Phases

This is sort of a "catch-all" title under which to include a lot of formally similar, but substantively different, properties (of an extended version of strength theory) about which little is known at present. As formulated in the present paper, multitrace strength theory, while making many definite predictions, still has considerable flexibility in many other predictions because of the unstated dependencies of acquisition, consolidation, decay, retrieval, and noise parameters on the conditions *K*. When one has not even specified the aspects of the experimental conditions that influence each parameter, this leaves a lot of flexibility. A completely extended version of multitrace strength theory must specify all of these parameter functions. When this is done, an important factor in evaluating the simplicity of strength theory will be the degree to which the parameters for each phase depend only upon the conditions during that phase, in addition to depending on the type of trace.

For example, one would like the decay rate for a particular trace to be independent of the degree of acquisition and consolidation. This has been found in STM for pitch (Wickelgren, 1969), where acquisition was manipulated by varying either the duration of the standard tone or the frequency difference between the standard tone and the comparison tone. In verbal STM, Wickelgren and Norman (1966) found the same decay rate for the first item in a list as for other items, even though the first item had a higher degree of learning. Unpublished data of mine in verbal ITM (1–12 minutes) and verbal LTM (weeks to years) also show decay rate to be independent of degree of acquisition.

Less is known concerning acquisition functions, but Wickelgren (1969) has found the acquisition function in STM for pitch to have approximately the same form and rate of approach to a limit, irrespective of the delay time.

Relation between Event and Order Memory

Essentially nothing is known about the relation between event and order memory. If the same kinds of traces mediate both event and order memory, then there ought to be a considerable degree of functional form and parameter invariance. Failing complete parameter invariance, there might be other simple relations between comparable parameters for item and order memory.

Relation between Recognition, Multiple-Choice, and Recall

It would also be very desirable if the functional form and parameters of acquisition, consolidation, decay, retrieval, and noise were invariant over recognition, multiple-choice, and recall methods of testing memory. Failing complete invariance, there might still be some fairly simple relations that would enable an extended version of multitrace strength theory to predict performance on one test from performance on another test. Little is known about the relations between parameters for recall, multiple-choice, and recognition. However, for verbal STM, Norman (1966) obtained some support for the invariance of STM decay rate across recall and recognition.

When all the relations between two tests of memory are known for all phases of memory, it is then possible to predict performance on one test from performance on the other test. Even assuming that the basic memory traces are identical for two methods of testing retention, there are still a number of possible complications in making these predictions, which are discussed more extensively in Wickelgren (1968a) and Norman and Wickelgren (1969). First, there is the question of whether the retrieval noise is the same for two different methods of testing retention, especially when the number of traces to be retrieved is different. Second, there is the question of whether there is a noise source in recall and multiple-choice comparable to the criterion noise in recognition. Third, there is the question of whether there is increased time for decay when the number of traces to be retrieved is increased. Fourth, there is the question of whether noise distributions for different traces are uncorrelated.

Considering all these possible complications, it is surprising that two rather straightforward strength theory predictions of multiple-choice from recognition have been completely successful, in what was probably verbal ITM in one case (Green and Moses, 1966) and a mixture of verbal ITM and STM in the other case (Kintsch, 1968). The relation between recognition, multiple-choice, and recall was not quite so simple in a study of verbal STM by Norman and Wickelgren (1969).

Systematic Errors in Event Memory

Errors in recall or recognition of items (events) using verbal STM tend to be phonetically similar to the correct item (Conrad, 1964; Wickelgren, 1965a, 1965b, 1965c, 1966b, 1966c). Errors for more obviously compound items, such as digit pairs (Norman and Wickelgren, 1965; Wickelgren, 1966d), are also more frequent for compound items that have elements in common with the correct items. There appear to be two basic approaches to a mathematical theory of this kind of data: the discrete component approach (such as the multicomponent theory of Bower, 1967b, or Norman and Rumelhart, Chapter 2) and the generalization gradient (similarity space) approach taken by the present version of multitrace strength theory.

Accounting for systematic error data with multitrace strength theory can be done at two levels. At the more superficial level, the similarity parameters, η_{ij}, for all pairs of items can be estimated from the data, and a variety of predictions, such as invariances of decay rates and other rate parameters, can be tested. At a deeper level, one could attempt to extend multitrace strength theory to include a theory of the similarity parameters, deriving them from some underlying space characteristics of the modality of the memory trace. Ideally, there should be some relation between the memory similarity space for a modality and the perceptual similarity space for the same modality. But since multitrace strength theory has been (successfully) tested against error data only at the more superficial level and only in the case of STM for pitch (Wickelgren, 1969), this is all far in the future.

Systematic Errors in Order Memory

In testing memory for the item that followed another item in a list, errors tend to be from similar serial positions to that of the correct item and the similarity function π_{ij} can be one dimensional in ungrouped coding of a list (Norman, 1966) or two dimensional in grouped coding of a list (Wickelgren, 1964, 1967b).

In addition, there is evidence for systematic errors based on item-to-item associations: (*a*) stimulus generalization (a similar item, or the same item in a different position, evoking the response appropriate to the cue item, Wickelgren, 1965d, 1966e) and (*b*) response generalization (the cue item evoking response items that are similar to the response item, Conrad, 1964; Wickelgren, 1965a, 1965b, 1965c, 1966. These item-to-item associative effects should be handled by the same event similarity functions, η_{ij}, as before. In the absence of any evidence on how positional similarity, stimulus generalization, and response generalization are to be combined, I have just assumed a weighted average.

Latency in Memory Judgments

The present statement of multitrace strength theory does not make predictions about the latency distributions of responses in recognition, multiple-choice, and recall. A beginning effort to handle latencies with strength theory was made by Norman and Wickelgren (1969). At present little can be said concerning the ultimate success of such an extension of strength theory.

Noise and Operating Characteristics

For reasons of simplicity, multitrace strength theory is a real-variable theory almost everywhere, with zero-mean random variables added at two places: (*a*) in the four-trace assumption, to handle the sum of the noise in acquisition, consolidation, decay, and retrieval and (*b*) in the criterion decision rule, to handle criterion noise. Both random variables are assumed to be normally distributed, but only the unimodality property of the (normal) probability density functions is important at the present level of precision in theories of memory.

Strength theory follows Thurstonian scaling (Thurstone, 1927; Torgerson, 1958) and signal detection theory (Tanner and Swets, 1954; Swets, Tanner, and Birdsall, 1961; Green and Swets, 1966) in using the standard deviation of the total noise in all phases of the process as the unit by which strengths are measured. Assuming that only one trace is substantially above zero under the conditions K, this means that in recognition the unit of strength measurement is $(\sigma_{MK}^2 + \sigma_K^2)^{1/2}$ and in recall or multiple-choice the unit of strength measurement is σ_{MK}. Since K is a subscript standing for all of the conditions of the memory task, there is no assurance that the unit of strength measurement remains constant across different conditions. In particular, one cannot be sure that σ_{MK} for recognition equals σ_{MK} for recall or multiple-choice, even when all other aspects of the conditions are identical. In addition, there is the σ_K term for recognition, which may have no analogue in the maximum decision rule for recall and multiple-choice (though a criterion-noise term can be incorporated into the maximum rule). When σ_{MK} is not invariant over different conditions, one must be careful to measure all strengths with the same unit. Sometimes it is necessary to estimate $\sigma_{MK_i}/\sigma_{MK_j}$ ratios. Problems in using the standard deviation of the noise as the unit of psychological measurement are discussed at length in Wickelgren (1968a).

Strength theory also follows Thurstonian scaling and signal-detection theory in having no true zero strength. Only the difference in trace strength between two conditions is meaningful, and this difference is measured in units of the noise in one of the conditions, i.e., $D(K_i, K_j) = (M_{K_i} - M_{K_j})/\sigma_{MK_j}$. Usually, one looks at the difference in strength between a correct item or association and an incorrect item or association. This difference can be thought of as the discriminability of correct and incorrect events (items or associations), and this discriminability is formally identical to d' in signal detection theory and the analogous concept in Thurstonian scaling.

Thus, strength is measured on an interval scale. The criterion and maximum decision rules found in Thurstonian scaling, signal detection theory, and strength theory both imply measurement on an interval scale, and one which uses the standard deviation of the total noise as the unit of measurement.

Testing the assumption that the noise is normally (unimodally) distributed, determining the ratios of noise standard deviations under different conditions, and determining the strength discriminability values for pairs of conditions are all most easily accomplished with a special plot called an operating characteristic (OC). Descriptions and proofs of the properties of OC's can be found in Green and Swets (1966) in the context of signal detection theory and in Wickelgren and Norman (1966) and Wickelgren (1968a) in the context of strength theory of memory.

OC's are only applicable to recognition, two-alternative multiple-choice, or two-alternative recall experiments, and are most efficiently and accurately derived from experiments using confidence ratings, in addition to the "yes–no" or other two-choice response. Everyone grants that ratings are the most efficient method of generating OC's. Some people think that ratings are less accurate than other methods of generating OC's, but the reverse is more likely to be true (Wickelgren, 1968a). For these reasons, OC's in memory experiments testing strength theory have always been derived from ratings.

In almost all tests of strength theory to date, the assumption of normally distributed noise has been validated by the absence of any systematic deviation of the OC's from straight lines on normal–normal plots. Systematic deviations of OC's from that expected for overlapping unimodal distributions have occurred in only two cases.

Once was in STM for pairs of digits from a serial list, where it appeared that about half of the old (presented) pairs were not distinguishable in trace strength from new pairs, leading to a bimodal distribution of trace strength for old pairs (Norman and Wickelgren, 1965). This is presumably due to the fact that the subjects coded the list into nonoverlapping pairs, leading to an incremented trace for coded pairs and little or no increment for uncoded pairs. This two-state (nonnormal) acquisition noise source can be eliminated by a variety of methods. Two methods which are known to work are to use paired-associate presentation so that pairs are never tested unless they were in fact coded as pairs

(Murdock, 1965) or just to present a single pair to be remembered (Wickelgren, 1966d).

The second case was in "higher-same-lower" judgments of recognition memory for pitch, where the deviation of the OC from a straight line was predicted by strength theory on the grounds that these judgments, under the conditions of that experiment, resulted from a multistage decision procedure, whose forced unidimensional representation led to bimodal distributions (Wickelgren, 1969). In tens of other experiments on recognition memory for pitch, one obtains OC's indicating no departure from unimodality (normality) in the underlying distributions.

The point is that there is no reason to doubt that the uncontrollable internal noise in the memory system is approximately normally distributed. When gross departures from normally distributed noise have been detected by OC's, it has been possible to determine the reasons. Since the reasons have nothing to do with the intrinsic nature of the memory system, but rather depend on the subjects' strategies (acquisition or decision, in the two cases), it is possible to study the same memory traces under conditions where the noise is normally distributed, as required in the Thurstonian scaling used by strength theory. There is no reason to think that this will not always be possible.

Now that a maximum likelihood method of estimating the intercept and slope parameters and testing goodness-of-fit for single rating OC's has been developed for rating data (Dorfman and Alf, 1968), it will probably be possible to definitely reject the assumption of normally distributed noise in many cases with a large enough sample. In my opinion, little will be gained from this, since the normal distribution assumption is merely a computational convenience, not an essential part of strength theory, and it is my guess that the accuracy of strength theory in predicting trace strength differences can not be improved substantially by assuming other noise distributions. Certainly, nothing will be gained from a mere rejection of the normal distribution assumption, without deriving a distribution that works better.

Conclusion

The present paper has demonstrated how multitrace strength theory handles a variety of memory phenomena. Emphasis has been placed on human studies of STM and ITM, because that is where the most appropriate experiments have been done to test strength theory. How-ever, the theory is applicable to all phenomena of learning and memory and to my knowledge, there are no phenomena that contradict multi-trace strength theory. Nevertheless, much testing of the basic frame-work remains to be done, especially for VSTM, ITM, and LTM in variety of modalities, and much theoretical work remains to be done to complete the theory. It remains to be seen how simple and accurate the theory will be, when it is more complete and more extensively tested.

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