Data Driven Methods in Finance:

Rebalancing, Transactions Costs, & Tax Management

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Introduction

- In April 2020, Invesco's failure to rebalance its equally-weighted S&P 500 fund resulted in a $105 million loss over 5 days.

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Oops! Invesco Forgot to Rebalance an Index Fund. It's Repaying Investors $105 Million.

By Daren Fonda Follow

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Introduction

- In April 2020, Invesco's failure to rebalance its equally-weighted S&P 500 fund resulted in a $105 million loss over 5 days.
- Frequent trading can result in significant transaction costs, as seen in 2019 when small-cap growth managers with high trading frequencies had annual trading costs over 2.5%.
- Portfolio adjustments, or rebalancing decisions, can be triggered by events like cash inflows/outflows and changes in stock-return parameters. Not all parameter changes necessitate rebalancing due to potential transaction costs.
- Transactions costs can notably influence the returns on stocks within a portfolio, affecting the composition of an optimal portfolio.

Rebalancing decision

- The rebalancing decision is twofold:
  - determining the timing based on the model's periodicity and
  - deciding the method for adjusting model parameters.
- The optimal portfolio’s lifespan corresponds to the periodicity of the model used, requiring rebalancing when it expires.

Rebalancing decision: periodicity

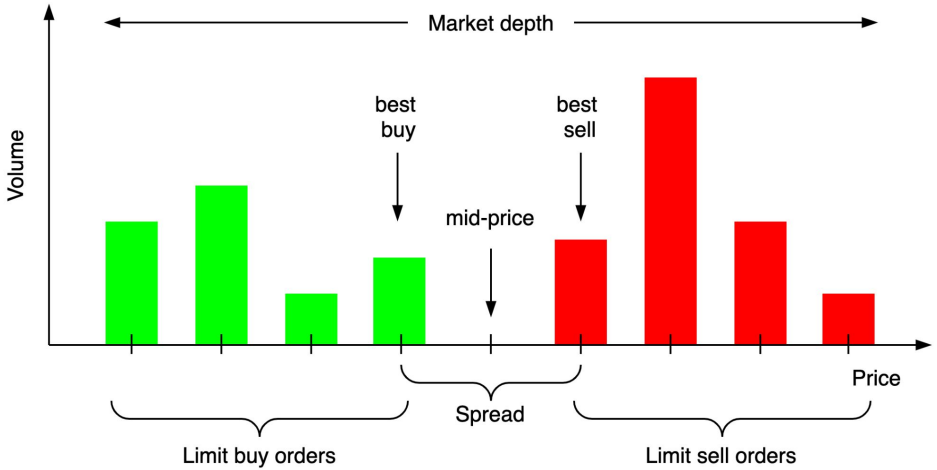
- Stock return models are less effective for daily returns as they're heavily influenced by specific news and events. Predictive powers decrease further for hourly returns.
- **Most financial research focuses on monthly returns due to a balance between predictive power and the risk of changing parameters**, though it's crucial to note that parameters might not remain constant over longer periods.
- Rebalancing frequency depends on market conditions and external factors, like significant cash inflows/outflows.
- It's advisable to update model estimations at least once a month for those using monthly returns, but this doesn't automatically mean rebalancing should occur.

Rebalancing decision: parameters

- Parameters of a stock-return model, like α and factor exposures (β) or premiums (f), can change due to various reasons, necessitating rebalancing.
- Corporate actions, significant changes in the business environment, government restrictions, or even events like a CEO's retirement can impact a stock's parameters.
- Portfolio managers should consistently evaluate stock parameters in the context of their model, especially when faced with new public or private information.
- When parameter changes are suspected, the model should be re-estimated, and based on the updated expected return and risk, the manager decides if rebalancing is beneficial compared to transaction costs.

Rebalancing decision: transaction costs

- Transactions costs refer to **commissions** brokerages charge for executing trades, which can vary based on the broker and specifics of the trade.
- Hidden transaction costs to note are the **bid–ask spread** (difference between buy and sell prices, reflecting liquidity and potentially market maker costs) and **price impact** (how large orders affect stock prices).



Rebalancing decision: transaction costs

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- Hidden transaction costs to note are the **bid–ask spread** (difference between buy and sell prices, reflecting liquidity and potentially market maker costs) and **price impact** (how large orders affect stock prices).
- The bid–ask spread **isn't constant** and discerning it can be challenging due to varying bid and ask prices at different moments.
- Large orders can result in significant price impacts, especially for stocks with limited daily trading volume. The entire order might not be executed at the initially quoted price.
- Estimating the exact transaction cost in advance is difficult, especially concerning price impact and bid–ask spread, but models can help provide educated estimates.

Rebalancing decision: modeling the transaction costs

- Transactions costs are conventionally modeled as a **fixed proportion** of the total transaction value, often represented by a **constant** like 5 or 10 basis points (1 **bps**=1/100th of 1.0%).
- Let us call this **constant c**. If the transaction value is $10, then the transaction cost is $10c.
- The **transaction value (TV)** is derived from the difference in dollar terms of stocks between the current portfolio (w^b) and the prospective portfolio (w^a): $TV = \Sigma|V_t w^a_i - V_t w^b_i|$ where V_t denotes the dollar value of the current portfolio.
- TV is always positive

Rebalancing decision: modeling the transaction costs

- The **transaction cost (TC)** is a constant fraction of the transaction value ($TV = \Sigma|V_t w^a_i - V_t w^b_i|$) and can be represented as a linear function involving weight vectors of the current and prospective portfolios: $TC = cTV = cV_t\Sigma|w^a_i - w^b_i|$
- The exact value of the constant c for transaction cost varies based on the values of the current and prospective weight vectors, making it not strictly constant.
- Transaction costs can vary per stock, often linked to liquidity; stocks with lower liquidity might incur higher transaction costs due to factors like the bid-ask spread and price impact: $TC = V_t\Sigma c_i|w^a_i - w^b_i|$. Here, c is a vector and elements of c can have both different signs and different absolute values.

Portfolio construction with transactions costs

- Portfolio selection principles remain the same even with transaction costs, aiming for the best mix of expected return and risk.
- The optimization problem in the presence of transaction costs can be approached by maximizing risk-adjusted return, accounting for these costs in the expected return calculation.
- The risk-adjusted return calculation involves considering gross expected return, transaction costs as a fraction of portfolio value, and the time value of these costs, though the latter is often small and can be ignored.
- The optimal portfolio is determined by maximizing a specific equation that includes transaction costs, expected stock returns, and a risk-aversion parameter, but this problem is complex and nonlinear, requiring more than conventional quadratic optimization methods.

Portfolio construction with transactions costs

- If the expected return of the portfolio is μ_P and V_t dollars are invested in it, the value of the portfolio at the end of the holding period equals $V_t(1 + \mu_P)$
- However, the portfolio manager needs to spend $(V_t w_a - V_t w_b)'c$ in transactions costs, and the ending value of the portfolio must take this into account.
- That is, we need to subtract $V_t(w_a - w_b)'c\ (1 + \mu_P)$ from the ending portfolio value.
- Similarly, we need to subtract the transactions costs from the expected return. That is,

  the effective expect return is: $\mu_P - (w_a - w_b)'c - (w_a - w_b)'c\mu_P$
- Since the transactions cost is paid up front rather than at the end of the period, the portfolio manager loses twice, once by paying the cost and once again by not being able to invest and create profit. The second and the third components of the expected return reflect these losses.

Portfolio construction with transactions costs

- Using the preceding notation and considering the transactions costs, **the risk-adjusted return becomes:** $\mu_P - (w_a - w_b)'c - Aw'\Sigma w = w_a'(\mu - c) + w_b'c - Aw_a'\Sigma w_a$
- Note that the transactions cost vector $\mathbf{c}$ may depend on the weight vector $\mathbf{w}_a$
- For the purpose of solving this optimization problem, we can ignore the terms that do not include $\mathbf{w}_a$ or $\mathbf{c}$

Tracking portfolio with transactions costs

- For portfolio managers prioritizing tracking error over overall risk, the focus should be on maximizing the effective tracking-error-adjusted return, not the risk-adjusted return.
- The tracking error is defined as the standard deviation of the difference between portfolio and benchmark returns, but for optimization, only the first two terms of its calculation involving weight vectors are used.
- The effective tracking-error-adjusted return is calculated using a tracking-error-aversion parameter, reflecting the manager's aversion to squared tracking error.
- Finding the optimal tracking-error portfolio involves maximizing this adjusted return under specific constraints. This problem, due to its complexity and the dependency of transaction costs on portfolio weights.

Tracking portfolio with transactions costs

- Recall that the tracking error (TE) of the portfolio is defined as the standard deviation of the difference between the portfolio return r_P and the benchmark return r_B:

$$TE = V(\mathbf{r_P}) - 2C(\mathbf{r_P}, r_B) + V(r_B) = \mathbf{w'\Sigma w} - 2\mathbf{w'\gamma} + V(r_B)$$

- The last term does not depend on the weight vector, so only the first two terms will be used in the optimization problem.
- The tracking-error-aversion parameter A measures the manager's aversion to squared tracking error in the portfolio. The effective tracking-error-adjusted return becomes:

$$\mathbf{w_a'(\mu - c) + w_b'c} - A[\mathbf{w_a'\Sigma w_a} - 2\mathbf{w'\gamma}]$$

- For the purpose of solving this optimization problem, we can ignore the terms that do not include $\mathbf{w_a}$ or $\mathbf{c}$

Dealing with cash flows

- To handle cash inflows and outflows, portfolio managers might temporarily invest in index futures or exchange-traded funds (ETFs), waiting for a more opportune time like the next rebalancing period to reintegrate these funds into the portfolio.
- Another method is to purchase specific, possibly more liquid, stocks to reach target portfolio weights efficiently, reducing the number of transactions needed.

Dealing with cash flows: reducing transactions costs

- Portfolio managers dealing with cash flows may opt to invest temporarily in index futures or ETFs, deferring the integration of these funds into the main portfolio until a scheduled rebalancing period.
- This strategy helps in managing the timing of cash flows without disrupting the existing balance of the portfolio.
- Alternatively, they can invest in specific, more liquid stocks to efficiently achieve desired portfolio weights, minimizing the number of transactions required.
- These methods aim to reduce transaction costs and maintain portfolio efficiency in the face of cash inflows and outflows.

Dealing with cash flows: rebalancing towards optimal weights

- During regular rebalancing or periods of cash flow, portfolio managers aim to maintain or achieve optimal stock weights, deciding whether to stick with original target weights or update to new ones based on model reestimation.
- **Standard rebalancing** involves adjusting the portfolio by selling stocks that have exceeded their target weights ("winners") and buying those that have fallen below their target weights ("losers"), focusing on weight balance rather than individual stock performance.
- The rebalancing process uses specific formulas to calculate the difference between current and target portfolio weights, determining the number of shares to buy or sell to align with these target weights.
- While these formulas are effective for rebalancing, they do not account for factors like bid-ask spreads or price impact, which can slightly distort the process and add costs to the portfolio.

Dealing with cash flows: rebalancing towards optimal weights

- The weights of the target portfolio and the current portfolio at any time t + 1 are given by:

  $w^b_{i,t+1}=p_{i,t+1}s^b_{i,t+1}/V_{t+1}$ and $w^a_{i,t+1}=p_{i,t+1}s^a_{i,t+1}/V_{t+1}$

  where: t-time, V-portfolio value, w-weight, s-number of shares, p-price per share, C- any monetary contribution to (or withdrawal from), b-before rebalancing, and a-after rebalancing

- The number of shares of each stock that must be bought or sold at time t + 1 is given by:

  $x_{i,t+1} = s^a_{i,t+1} - s^b_{i,t+1} = w^a_{i,t+1}/p_{i,t+1}(V_{t+1}+C_{t+1}) - s^b_{i,t+1}$

Rebalancing: standard rebalancing

In this illustration of standard rebalancing, we consider a portfolio comprising seven different stocks, labeled A to G. The scenario is analyzed at two points in time: time t and time t + 1.

Initially, at time t, the portfolio's value stands at $100,000. By time t + 1, its value has appreciated to $107,986.68. Initially, at time t, each stock in the portfolio is perfectly weighted as per the optimal target. However, due to fluctuations in stock prices from time t to time t + 1, there is a deviation in the portfolio from these target weights. The table displays the variance in weights between time t (which reflects the optimal target weights) and time t + 1 (where the weights have diverged from the target). The calculation results, indicating the number of shares of each stock to be either bought or sold to realign the portfolio with the optimal target scenario, are shown in the 'Shares to buy/sell' section of the table.

Rebalancing: standard rebalancing

Standard Rebalancing

Date		Stock Holdings						
		A	*B*	*C*	*D*	*E*	*F*	*G*
t	Price per share	$137.45	$30.46	$115	$75	$85	$15	$42.09
	Shares owned	200	200	200	200	200	200	200
	Weight (w_i^a)	0.2749	0.0609	0.2300	0.1500	0.1700	0.0300	0.0842
$t+1$	Price-per-share							
	Shares owned							
	Weight before rebalance (w_i^b)							
	Rebalance shares							
	Shares to buy/sell							
	Rebalance weight (w_i^{RB})							
	Dollar value							

$w^a_{i,t}=p_{i,t}s^a_{i,t}/V_t$

=137.45 x 200 / 100,000

=0.2749

Rebalancing: standard rebalancing

Standard Rebalancing

Date		Stock Holdings						
		A	*B*	*C*	*D*	*E*	*F*	*G*
t	Price per share	$137.45	$30.46	$115	$75	$85	$15	$42.09
	Shares owned	200	200	200	200	200	200	200
	Weight (w_i^a)	0.2749	0.0609	0.2300	0.1500	0.1700	0.0300	0.0842
$t+1$	Price-per-share	$154.88	$32.53	$125.35	$76.50	$89.47	$15.37	$45.83
	Shares owned	200	200	200	200	200	200	200
	Weight before rebalance (w_i^b)	0.2869	0.0602	0.2322	0.1417	0.1657	0.0285	0.0849
	Rebalance shares							
	Shares to buy/sell							
	Rebalance weight (w_i^{RB})							
	Dollar value	$29,686	$6,579	$24,837	$16,198	$18,358	$3,240	$9,090

$w^b_{i,t+!}=p_{i,t+1}s^b_{i,t+1}/V_{t+1}$

=154.88 x 200 / 107,986.68

=0.2869

Rebalancing: standard rebalancing

Standard Rebalancing

Date		Stock Holdings						
		A	*B*	*C*	*D*	*E*	*F*	*G*
t	Price per share	$137.45	$30.46	$115	$75	$85	$15	$42.09
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	Weight before rebalance (w_i^b)	0.2869	0.0602	0.2322	0.1417	0.1657	0.0285	0.0849
	Rebalance shares	191.66	202.23	198.14	211.73	205.19	210.76	198.34
	Shares to buy/sell							
	Rebalance weight (w_i^{RB})							
	Dollar value	$29,686	$6,579	$24,837	$16,198	$18,358	$3,240	$9,090

$w^b_{i,t+1} = w^a_{i,t} / p_{i,t+1} V_{t+1}$

=0.2749 / 154.88 x 107,986.68

=191.66

Rebalancing: standard rebalancing

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	Weight before rebalance (w_i^b)	0.2869	0.0602	0.2322	0.1417	0.1657	0.0285	0.0849
	Rebalance shares	191.66	202.23	198.14	211.73	205.19	210.76	198.34
	Shares to buy/sell	−8.34	2.23	−1.86	11.73	5.19	10.76	−1.66
	Rebalance weight (w_i^{RB})							
	Dollar value	$29,686	$6,579	$24,837	$16,198	$18,358	$3,240	$9,090

$x_{i,t+1} = s^a_{i,t+1} - s^b_{i,t+1}$

= 191.66 - 200 = -8.34

Rebalancing: standard rebalancing

Standard Rebalancing

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$w_i^{RB} = w_i^a = 0.2749$

Rebalancing: standard rebalancing

Standard Rebalancing

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191.66 x 154.88 = 29,686

Rebalancing without selling

- Portfolio rebalancing without selling can be cost-effective, using cash inflows to adjust stock weights towards optimal targets while avoiding the tax and transaction costs associated with selling securities.
- One method is to distribute cash inflows proportionally across all securities based on target weights, gradually moving the portfolio closer to these targets depending on the relative size of the cash injection.
- Alternatively, cash can be allocated only to stocks below their target weights, accelerating the portfolio's convergence to the optimal weights more effectively than proportional distribution.
- This approach also includes a calculation method to determine the minimum cash inflow needed to rebalance the portfolio perfectly without selling, providing a systematic way to handle net cash inflows and corresponding strategies for net withdrawals.

Tax Management

- James Garland's study on taxable investing highlights that minimizing taxes and fees is crucial for maximizing investment returns, with the typical mutual fund losing about 24% of its gross return to taxes from 1996 to 2020.
- Effective tax management strategies can significantly increase the amount of earnings retained in a portfolio, as investment decisions greatly influence the tax burden and, consequently, the net investment return.
- Tax management techniques are divided into passive and active approaches. **Passive** techniques focus on reducing tax impact during necessary cash outflows, including dividend-income, tax-lot, and capital gain and loss management.
- **Active** tax management techniques proactively create additional tax advantages. The principles and techniques discussed are primarily relevant to U.S. investors but are also applicable globally due to similar tax structures in many countries.

Tax Management: overview

Category	Name of Technique	Description
Passive	Dividend management	How to create the portfolio to have the right dividend yield desired by the investor
	Tax-lot management	Which tax lots to sell given that a withdrawal of cash is necessary and a specific stock is chosen
	Capital gain/loss management	Which stocks to sell when it is necessary for a cash withdrawal
Active	Loss harvesting	How to rebalance the portfolio when the amount to be withdrawn is zero

Tax Management: dividends, capital gains, and capital losses

- Investment income is subject to three main types of taxes:
  - dividends,
  - long-term capital gains/losses, and
  - short-term capital gains/losses, with a
  - distinct approach to treating capital gains and losses.

Tax Management: dividends, capital gains, and capital losses

Dividends:

- Individual Taxpayers: Initially taxed at ordinary personal income rates, leading to double taxation (corporate level and personal level). Since 2003, "qualified dividends" are taxed at lower rates (0% to 23.8% in 2020) to reduce double taxation.
- Corporations Receiving Dividends: Can deduct a portion (50% to 65% in 2020) from taxable income to avoid triple taxation, effectively reducing the tax rate on dividends to around 13%.
- Taxation of Reinvested Dividends: Dividends are taxed whether reinvested or not, which is seen as unfair by some because reinvesting dividends without gaining additional value still incurs taxes.

Tax Management: dividends, capital gains, and capital losses

Capital Gains and Losses:

- Long-term vs. Short-term: Long-term capital gains (holdings over a year) are taxed at a maximum rate of 20% in 2020, whereas short-term gains (under a year) are taxed at the ordinary income rate. This system encourages long-term investing.
- Taxation Upon Sale: Only realized gains (profit from sold stock) are taxed. Unrealized gains ("paper profit") are not taxed until the stock is sold.
- Losses: Both long-term and short-term losses can be deducted from respective gains. Any remaining losses can offset other types of income for individuals, but corporations cannot use capital losses to offset other types of income. Unused losses can be carried forward to future years.

Tax Management: dividends, capital gains, and capital losses

Asymmetric Treatment of Gains/Losses:

- Capital gains and losses are treated differently, impacting how investors manage their portfolios and tax liabilities.

Principles of Tax Management

- Avoid Dividend Income for Individuals: Individual taxpayers should avoid dividend income due to higher tax burdens, but corporations benefit more from dividends as they are taxed less.
- Prefer Long-Term Capital Gains: It's better to realize long-term capital gains over short-term gains, as long-term gains are taxed at a lower rate.
- Prioritize Realizing Capital Losses: Realizing capital losses is preferable as they reduce taxable income, whether they are short-term or long-term.
- Delay Capital Gains, Expedite Capital Losses: Investors should delay realizing capital gains and expedite realizing capital losses, leveraging the time value of money to maximize investment returns and minimize taxes.

Dividend management

Dividend Management Strategies:

- For Individuals: Create portfolios with minimal dividends. This involves adding a no-dividend constraint in portfolio optimization to avoid unnecessary tax burdens, especially when cash withdrawal is not needed.

Portfolio Optimization:

- Individuals: Adjust the portfolio by penalizing dividend-paying stocks, reducing expected returns by the tax burden of dividends. For example, $\mathbf{w'\mu} - A\mathbf{w'\Sigma w} - \tau_d\mathbf{w'd}$

Tax Implications:

- Selling Part of the Portfolio: If dividends are insufficient for cash needs, selling part of the portfolio might be necessary, considering the tax consequences. For example, $\mathbf{w'\mu} - A\mathbf{w'\Sigma w} - (\tau_d-\tau_l)\mathbf{w'd}$

Tax-lot management

Tax-Lot Management:

- Prioritize realizing all capital losses before gains, and long-term gains before short-term gains.
- Utilize tax-lot accounting to manage tax burdens when selling shares, focusing on selling those with the highest losses first and then those with the lowest tax burden.

Tax-Lot Mathematics:

- Introduce a streamlined notation system to manage tax lots more effectively.
- Keep track of stocks, purchase times, and current times to accurately calculate tax implications.

Capital gain and loss management

Capital Gain and Loss Management:

- Focus on selling stocks with the greatest capital losses or smallest capital gains to minimize tax burden.
- Implement a systematic approach for selecting which stocks and specific tax lots to sell.

Loss Harvesting:

- Actively manage portfolio to generate gains from tax management by harvesting capital losses.
- Implement different strategies like selling losing stocks and holding winning ones, or replacing sold stocks with similar ones to comply with wash-sale rules.

Gains from tax management

Gains from Tax Management:

- Simulations show significant gains from efficient tax management strategies.
- Demonstrated differences in effective tax rates and after-tax returns between investors practicing tax management and those who don't.

Disclaimer

This course is for educational purposes only and does not offer investment advice or pre-packaged trading algorithms. The views expressed herein are not representative of any affiliated organizations or agencies. The main objective is to explore the specific challenges that arise when applying Data Science and Machine Learning techniques to financial data. Such challenges include, but are not limited to, issues like short historical data, non-stationarity, regime changes, and low signal-to-noise ratios, all of which contribute to the difficulty in achieving consistently robust results. The topics covered aim to provide a framework for making more informed investment decisions through a systematic and scientifically-grounded approach.