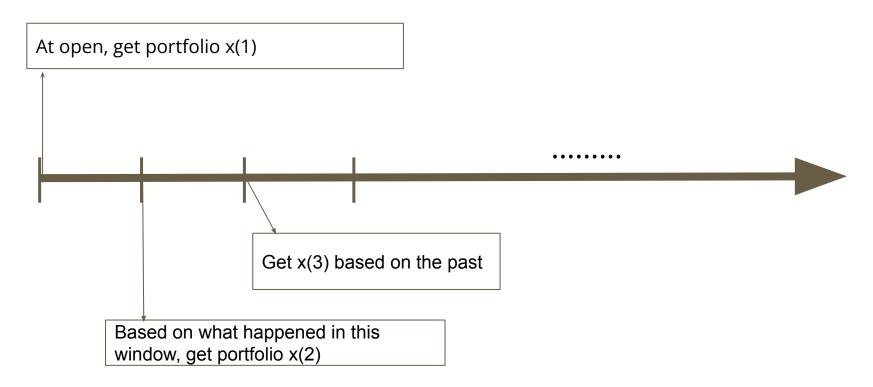
Online Portfolio Selection

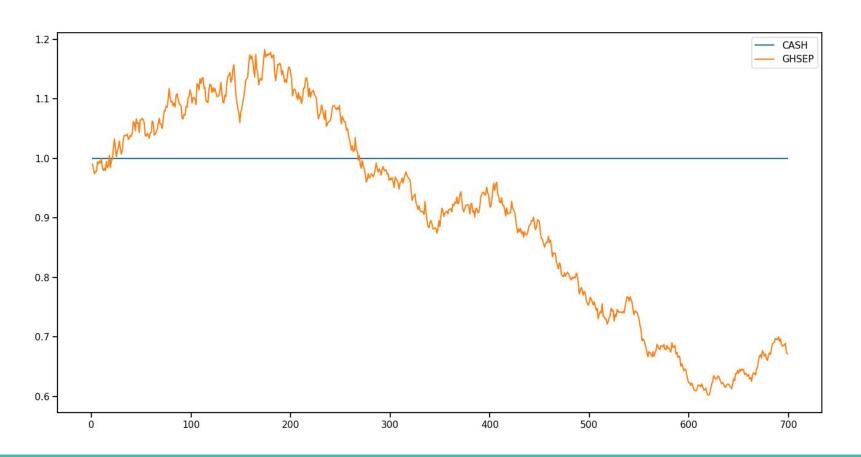
DDMIF

Sudeep

Interaction Protocol



1 asset and cash GBM



Online Portfolio Selection Techniques

- 1. Follow-The-Leader
- 2. Exponentiated Gradient
- 3. Online Newton Step
- 4. Passive Aggressive Mean Revision
- 5. Online Moving Average Revision

Trading Strategy Development

- 1. We run algorithms weekly data.
- 2. Rebalancing portfolios every week.
- 3. We allow shorting.

If we have n assets and cash, the portfolio space with shorting is:

$$\Delta_n = \{x \in \mathbb{R}^{n+1} : x_0 \geq 0, \sum_{i=0}^n (x_i)^+ = 1, \sum_{i=1}^n |x_i| \leq 1 \}$$

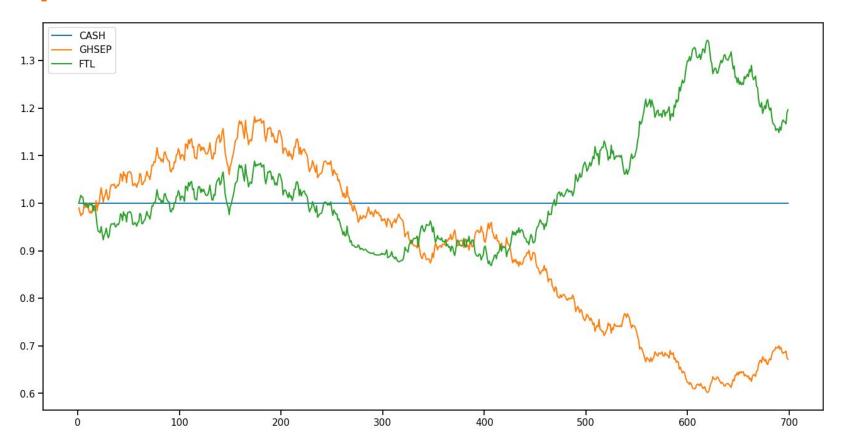
Follow the Leader

At time t, we know the returns from 1 to t-1. So, we optimize:

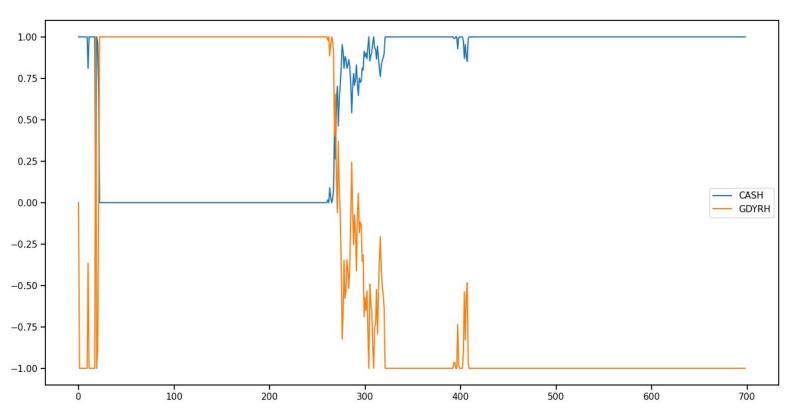
$$x_t = rg \max_{x \in \Delta_n} \sum_{s=1}^{t-1} \log(1 + r_s^ op x)$$

Here r_s is the return vector

FTL performance



FTL portfolio



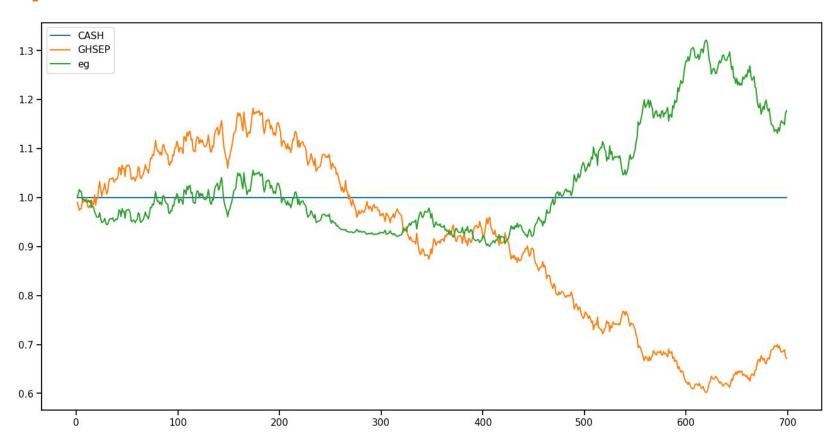
Exponentiated Gradient

Follow the Linearized Leader with Regularization:

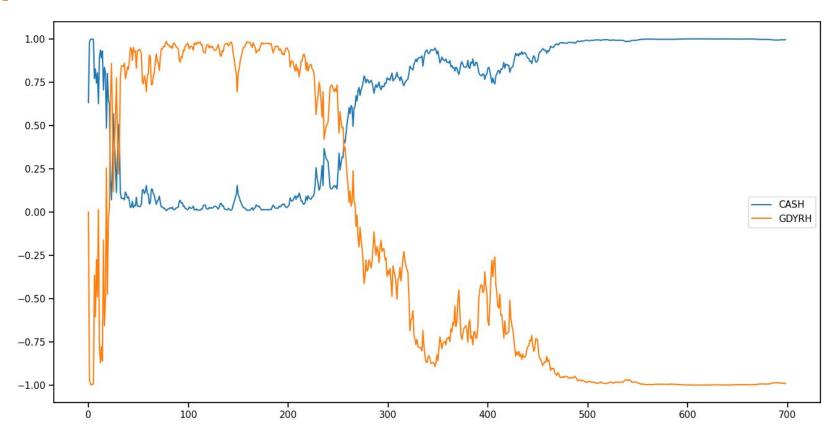
$$egin{aligned} x_t &= rg \min_{x \in \Delta_n} \sum_{s=1}^{t-1} g_s^ op x + \lambda_{t-1} R(x) \ & \ g_s &= rac{-r_s}{1 + r_s^ op x_s} \quad ext{and} \quad \lambda_{t-1} &= \sqrt{\sum_{s=1}^{t-1} \|g_s\|_2^2} \end{aligned}$$

There are different choices for R(x) like entropy and 2-norm. The version with entropy is called Exponentiated gradient.

EG performance



EG portfolio



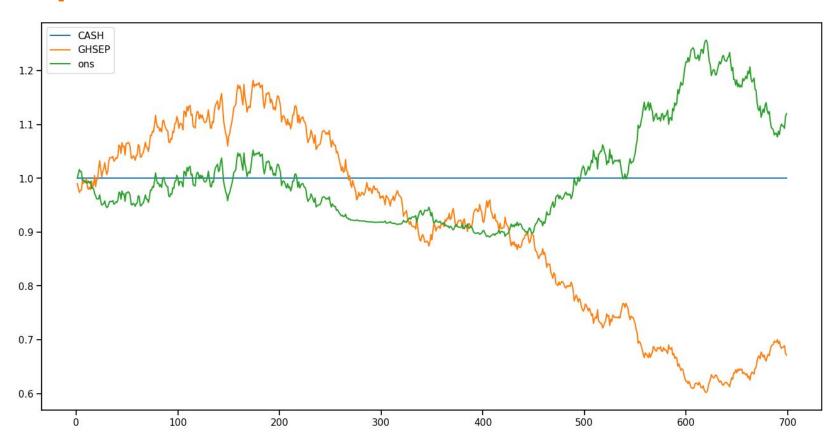
Online Newton Step

Follow the Quadratic Leader with Regularization

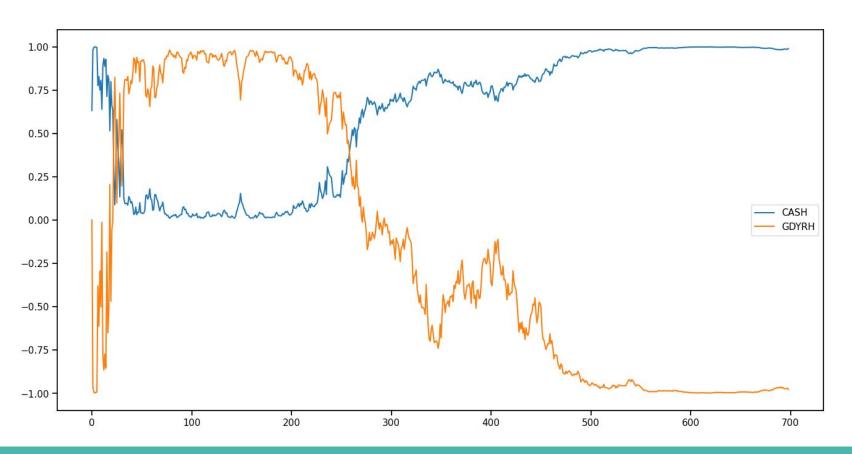
$$egin{aligned} x_t &= rg \min_{x \in \Delta_n} \sum_{s=1}^{t-1} \left(rac{1}{2} x^ op A_s x + b_s^ op x
ight) + \lambda_{t-1} R(x) \ A_s &= rac{r_s r_s^ op}{1 + r_s^ op x_s} \quad , \quad b_s = -r_s \quad ext{and} \quad \lambda_{t-1} = \sqrt{\sum_{s=1}^{t-1} \|g_s\|_2^2} \end{aligned}$$

There are different choices for R(x) like entropy and 2-norm. The version with 2-norm is called Online Newton Step

ONS performance



ONS portfolio



Mean Revision

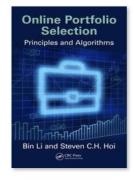
Ada-PAMR

$$\|x_t = rg\min_{x \in \Delta_n} \|x - x_{t-1}\|_2^2 + \eta_t r_{t-1}^ op x$$

Ada-OLMAR

$$egin{aligned} x_t = rg\min_{x \in \Delta_n} \|x - x_{t-1}\|_2^2 - \eta_t ilde{r}_t^ op x \end{aligned}$$

References





Book

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