Y2K

Stephanie Schmitt-Grohé
and
Martín Uribe*

Federal Reserve Board

August 27, 1998

Abstract

As the millennium draws to an end, the threat posed by the Year 2000 (Y2K) problem is inducing vast private and public spending on its remediation. In this paper, we embed the Y2K problem into a dynamic general equilibrium framework. We model the Y2K problem as an anticipated, permanent loss in output whose magnitude can be lessened by investing resources in advance. Our model replicates three observed characteristics of the dynamics triggered by the Y2K bug: (1) Precautionary investment: investment in solving the Y2K problem begins before the year 2000; (2) Investment delay: although economic agents have been aware of the Y2K problem since the 1960s, investment did not begin until recently; (3) Investment acceleration: as the new millennium approaches, the amount of resources allocated to solving the Y2K problem increases. Furthermore, the model predicts that output net of resources devoted to solving the Y2K problem need not decline in 2000.

JEL classification: E22, E32.
Keywords: Y2K problem; investment dynamics.

---

*We would like to thank seminar participants at the Federal Reserve Board for comments. This paper represents the views of the authors and should not be interpreted as reflecting the views of the Board of Governors of the Federal Reserve System or other members of its staff. Address for correspondence: Martín Uribe, International Finance Division, Stop 24, Board of Governors of the Federal Reserve System, Washington, DC 20551. Phone: (202) 452 3780. Fax: (202) 736 5638. E-mail: uribem@frb.gov.

A number of professional economists have compared the expected recession associated with the Year 2000 (Y2K) computer date problem with that caused by the oil price shock in 1973-74.[1] In anticipation of this problem, as the millennium comes to an end, both the government and the private sector have been allocating significant amounts of resources on its remediation. Federal Reserve Governor Edward W. Kelley Jr. estimates that resources allocated to solving the Y2K problem will cost the U.S. economy one tenth of one percent of GDP in 1998.[2] Although at a slower pace, similar efforts are under way in the rest of the world. The Gartner Group has estimated that at a worldwide level the cost associated with solving the Y2K problem will total 300 to 600 billion U.S. dollars.

The macroeconomic dynamics triggered by the Y2K problem are characterized by the following three facts: First, the millennium bug has induced *precautionary investment* in the sense that the allocation of resources aimed at solving the Y2K problem began before the year 2000. Second, there has been *investment delay*. Although economic agents have been well aware of the Y2K problem since the 1960s, the allocation of real resources devoted to its solution did not begin until the 1990s. Third, investment in the Y2K problem has been *accelerating*, particularly since 1997.

Despite the considerable attention that the Y2K problem has received in the policy debate, there has been virtually no theoretical research directed to understanding its macroeconomic consequences. In this paper, we make a first step towards filling this gap by embedding the Y2K problem into a simple dynamic general equilibrium framework. We model the Y2K problem as a situation in which before the year 2000 agents learn that in the year 2000 output will experience a permanent decline. Agents can lessen the output decline by investing resources in advance. This feature, which implies that resources allocated to solving the Y2K problem become productive only in the year 2000 is the key element determining the dynamics of the model.

[1] See, for example, Edward Yardeni, "Year 2000 Recession?," July 1998, Center for Cybereconomics and Y2K.

[2] See his testimony before the Committee on Commerce, Science, and Transportation, U.S. Senate, April 28, 1998.

To highlight the implications of the Y2K problem, we keep other elements of the model fairly simple. In particular, on the demand side, we assume that the economy is populated by identical, utility-maximizing, risk-averse agents that discount future consumption. On the supply side, we study two alternative environments. In the first one, we assume that, in the absence of the Y2K problem, output is an exogenous endowment. In this setup, investment in solving the Y2K glitch is driven not only by supply-side effects, but also by agents' desire to smooth consumption. In the second environment, we assume that agents have access to an accumulation technology that allows them to shift resources across time at a constant rate of return. This technology introduces a separation between the agents' decisions to consume and invest.

In spite of their simplicity, the economies described above can explain the three main facts associated with the Y2K problem: precautionary investment, investment delay, and acceleration. In addition, the models predict that investment in the Y2K problem will peak in 1999. Given the specification of the models, a recession in the year 2000 is unavoidable. Interestingly, however, output net of resources allocated to solving the millennium bug need not decline. In fact, in the endowment economy, under certain parameter configurations, consumption booms at the arrival of the new millennium. This possibility arises because in the year 2000 resources that were once allocated to solving the Y2K problem are freed up becoming available for consumption. This expansionary effect may more than offset the contraction in output caused by Y2K itself.

1 The Model

Consider an economy populated by a large number of identical, infinitely-lived consumers with preferences described by the utility function

$$\sum_{t=0}^{\infty} \beta^t u(c_t) \tag{1}$$

where c_t denotes consumption in period t, $\beta \in (0,1)$ denotes the subjective discount factor, and the single-period utility function $u(\cdot)$ is twice continuously differentiable, strictly increasing, and strictly concave, with $u'(0) = \infty$ and $u'(\infty) = 0$.

Each period, the representative consumer is endowed with a constant quantity of goods $\bar{y} > 0$. Beginning in the year 2000, denoted here by $T > 0$, consumers experience a loss in income—the Y2K problem—given by A_t. Agents can invest resources to reduce the magnitude of the Y2K problem. Specifically, we assume that A_t is a decreasing function of the sum of resources spent on solving the Y2K problem

$$A_t = A\Big(\textstyle\sum_{j=0}^{t-1} i_j \Big)$$

where $i_t \geq 0$ denotes the amount of resources invested in solving the Y2K problem in period t. The function $A(\cdot)$ is assumed to satisfy the following assumption:

Assumption 1

$$\begin{array}{ll} \textit{(A1.1)} & A > 0, A' < 0, A'' > 0, \lim_{x\to\infty} A'(x) = 0 \\ \textit{(A1.2)} & \beta[1 - A'(0)] > 1 \\ \textit{(A1.3)} & \bar{y} - A(0) > 0 \end{array}$$

Assumption (A1.1) is self-explanatory. The intuition behind assumption (A1.2) is the following. Suppose that $i_t = 0$ for all t, and let $t' \geq T$. Consider the experiment of increasing investment by one unit in period t' and disinvesting in period $t' + 1$ in such a way that total cumulative investment is zero from period $t' + 1$ on. The marginal cost of forgone consumption in t' is given by $\beta^{t'} u'(\bar{y} - A(0))$, whereas the marginal benefit of increased consumption in period $t' + 1$ is given by $\beta^{t'+1} u'(\bar{y} - A(0))(1 - A'(0))$. Assumption (A1.2) guarantees that the benefit exceeds the cost. Assumption (A1.3) implies that consumers are not forced to invest before the end of the millennium.

The resource constraint of the household is given by

$$c_t = \begin{cases} \bar{y} - i_t & t < T \\ \bar{y} - i_t - A\Big(\sum_{j=0}^{t-1} i_j \Big) & t \geq T \end{cases} \tag{2}$$

The household chooses sequences for consumption and investment so as to maximize (1) subject (2). The first-order conditions of the consumer's problem are:

$$-\beta^t u'(c_t) - \sum_{s=T}^{\infty} \beta^s u'(c_s) A'_s \leq 0 \quad (= 0 \text{ if } i_t > 0); \quad t < T \tag{3}$$

$$-\beta^t u'(c_t) - \sum_{s=t+1}^{\infty} \beta^s u'(c_s) A'_s \leq 0 \quad (= 0 \text{ if } i_t > 0); \quad t \geq T \tag{4}$$

where $A'_t \equiv A'(\sum_{j=0}^{t-1} i_j)$.

Characterization of Equilibrium

Our first result is that there is no investment stoppage before 2000 in the sense that once investment in solving the Y2K problem becomes positive, it must continue to be positive until the year 2000. To see this, assume that $i_t = 0$ and that $i_{t-1} > 0$ for some $t < T$. Then, equation (3) implies that $u'(c_{t-1}) \leq \beta u'(\bar{y})$. But this is impossible because $c_{t-1} = \bar{y} - i_{t-1} < \bar{y}$. We summarize this result in the following proposition.

Proposition 1 *(No Investment Stoppage before 2000.) If $i_t > 0$, then $i_{t+1} > 0$ for all $t < T - 1$.*

Furthermore, if in any period before the year 2000 investment is positive, then investment increases over time until the end of the millennium. To see this, assume that $i_t > 0$ for some $t < T - 1$. Then Proposition 1 and equation (3) imply that

$$\frac{u'(c_t)}{u'(c_{t-1})} = \beta^{-1}.$$

By the concavity of the single-period utility function, the above expression implies that $c_t < c_{t-1}$, or, equivalently, that $i_t > i_{t-1}$. We formalize this result in the following proposition.

Proposition 2 *(Accelerating Y2K Investment.) If $i_t > 0$, then $i_{t+1} > i_t$ for all $t < T - 1$.*

Next, we establish that, given Assumptions (A1.1) and (A1.2), the model exhibits precautionary investment, in the sense that agents find it optimal to begin to allocate resources to solving the Y2K problem before the arrival of the year 2000.

Proposition 3 *(Precautionary Y2K Investment.) If Assumptions (A1.1) and (A1.2) are satisfied, then there exists a $t < T$ such that $i_t > 0$.*

Proof: To establish the result, it is sufficient to show that $i_{T-1} > 0$. First, we show that if $i_{T-1} = 0$, then $i_t = 0$ for all t. Second, we show that $i_t = 0$ for all t cannot be an equilibrium. By proposition 1, if $i_{T-1} = 0$, then $i_t = 0$ for all $t < T - 1$. Assume now that $i_{T-1} = 0$ and that $i_T > 0$. Evaluating equation (3) at $t = T-1$ and equation (4) at $t = T$ and combining them yields $u'(\bar{y}) \geq \beta[1 - A'(0)]u'(\bar{y} - A(0) - i_T)$, which is a contradiction given Assumptions (A1.1) and (A1.2). A similar argument shows that if $i_{T-1} = 0$, then $i_{T+j} = 0$ for all $j > 0$. Finally, assume that $i_t = 0$ for all t. Then evaluating equation (3) at $t = T - 1$ yields $-\beta^{T-1}u'(\bar{y}) - u'(\bar{y} - A(0))A'(0)\sum_{t=T}^{\infty} \beta^t \leq 0$. This expression reduces to $u'(\bar{y}) \geq -u'(\bar{y} - A(0))A'(0)\beta/(1-\beta)$, which is a contradiction given assumptions (A1.1) and (A1.2). ■

The following two propositions characterize the behavior of investment in the new millennium. The first proposition shows that if investment is positive (zero) in the year 2000 or later, then it must also be positive (zero) in every subsequent period. In addition, it shows that in the year 2000 investment can be positive or zero depending on parameter values. The second proposition establishes that investment falls in the year 2000.

Proposition 4 *(Investment Dynamics in the New Millennium.) Suppose assumptions (A1.1) and (A1.2) are satisfied. Then, (i) if $i_T = 0$, then*

$i_{T+j} = 0$ for all $j > 0$; (ii) if $i_T > 0$, then $i_{T+j} > 0$ for all $j > 0$; and (iii) depending on parameter values, i_T can be positive or nil.

Proof: (i) Note that if $i_t = 0$ for some $t \geq T$, then past cumulative investment in period $t+1$ is the same as in period t. Because after period $T-1$ past cumulative investment is the only predetermined state variable of the economy, it must be the case that if the household finds it optimal not to invest in period t, it must also find it optimal not to invest in period $t+1$. (ii) Assume that $i_T > 0$ and $i_{T+1} = 0$. By (i), $i_{T+j} = 0$ for all $j > 1$. Thus $c_{T+j} = c_{T+1}$ for all $j > 1$. Then, evaluating equation (4) at $t = T+1$ yields $\beta(1 - A'_{T+1}) \leq 1$. At the same time, using equation (4) evaluated at $t = T$ and $t = T+1$, we have that $u'(\bar{y} - A_T - i_T) \leq \beta(1 - A'_{T+1})u'(\bar{y} - A_{T+1})$. Because $A_T + i_T > A_{T+1}$, the above inequality implies that $\beta(1 - A'_{T+1}) > 1$, which is a contradiction. (iii) To show the existence of economies in which $i_T > 0$, let $T = 1$. Then it follows from equation (4) that in order for i_T to be zero it must be the case that $\beta(1 - A'(i_0)) \leq 1$. Because i_0 is bounded above by $\bar{y}$ and A'' is positive, it follows that a sufficient condition for i_T to be positive is that $\beta(1 - A'(\bar{y})) > 1$. We now construct an economy in which $i_T = 0$. Again, let $T = 1$. Assume that $u(c) = \log c$ and $A(x) = (1+x)^{-1}$. Then evaluating equation (3) at $t = 0$ and assuming that $i_t = 0$ for $t > 0$ yields the following Euler equation that uniquely determines i_0:

$$\frac{1}{\bar{y} - i_0} = (1 + i_0)^{-2} \frac{1}{\bar{y} - 1/(1 + i_0)} \frac{\beta}{1 - \beta}$$

Setting $\bar{y} = 1.2$ and $\beta = .7$, the above equation yields $i_0 = .586$. Furthermore, in this case $\beta(1 - A'(i_0)) < 1$ which ensures that $i_1 = 0$. ■

In proving the following proposition, we restrict attention to equilibria in which consumption converges to a positive constant.

Proposition 5 *(Y2K Investment Declines in 2000.) Suppose Assumptions (A1.1) and (A1.2) hold. Then $i_T < i_{T-1}$.*

Proof: If $i_T = 0$, then the result follows trivially because, by Proposition 3, i_{T-1} is positive. On the other hand, if $i_T > 0$, then equations (3) and (4) imply that

$$\frac{u'(c_T)}{u'(c_{T-1})} = \frac{\beta^{-1}}{1 - A'_T} \tag{5}$$

We first show that the right-hand side cannot be larger than one. To see this, suppose that the contrary is true. Note that equations (3) and (4) and the fact that if $i_T > 0$ then $i_{T+j} > 0$ for all $j > 0$ imply that

$$\frac{u'(c_{T+1})}{u'(c_T)} = \frac{\beta^{-1}}{1 - A'_{T+1}}$$

Because $i_T > 0$, $1 - A'_{T+1} < 1 - A'_T$; thus, the right-hand side of the above expression is larger than one. This implies that $c_{T+1} < c_T$ and thus that $i_{T+1} > i_T$. Continuing with this argument, it follows that $i_{T+j} > 0$ and that $1 - A'_{T+j} < 1 - A'_{T+j+1}$ for all $j > 1$. This in turn implies that $u'(c_{T+j}) \rightarrow \infty$ as $j \rightarrow \infty$ and thus $c_{T+j} \rightarrow 0$ as $j \rightarrow \infty$. Therefore, in any equilibrium in which consumption is bounded away from zero, $\beta^{-1}/(1 - A'_T)$ must be less than one. But in this case, equation (5) clearly implies $c_T > c_{T-1}$. Because $c_T = \bar{y} - i_T - A_T$ and $c_{T-1} = \bar{y} - i_{T-1}$ and $A_T > 0$, it follows that $i_T < i_{T-1}$. ■

A direct corollary of the above two propositions is that if the economy continues to allocate resources to solving the Y2K problem in the year 2000, then there is a consumption boom in 2000, and consumption continues to grow as it approaches its long-run level. On the other hand, if investment in the Y2K problem ceases in the year 2000, then consumption may fall with the arrival of the new millennium and will remain constant thereafter.

Proposition 6 *(Consumption Dynamics in the New Millennium.) Suppose Assumptions (A1.1) and (A1.2) are satisfied. Then, if* $i_T > 0$*,* $c_t > c_{t-1}$ *for all* $t \geq T$*, whereas if* $i_T = 0$*,* $c_t = c_{t-1} \leq c_{T-1}$ *for all* $t > T$*.*

Proof: The results follows directly from the proof of propositions 4 and 5. ■

Next, we establish that investment in the Y2K problem peaks immediately before the year 2000.

Proposition 7 *(Y2K Investment Peaks in 1999.) If Assumption (A1.1) and (A1.2) are satisfied, then* $i_{T-1} > i_t$ *for all* $t \neq T - 1$*.*

Proof: For $t < T-1$, the result follows directly from proposition 2. For $t = T$, the result follows from proposition 5. Consider now the case $t > T$. If $i_t = 0$, the result follows trivially because, by proposition 3, $i_{T-1} > 0$. If $i_t > 0$, then, by proposition 6, $c_t > c_{T-1}$, or, equivalently, $\bar{y} - i_t - A_t > \bar{y} - i_{T-1}$. The result follows from the fact that $A_t \geq 0$. ■

One characteristic of the Y2K problem is that firms seem to have delayed the decision to allocate resources to solving it. Specifically, investment in solving the Y2K problem did not begin until the early 1990s, even though firms were aware of the Y2K problem well before that time. The following proposition shows that the model can explain the observed investment delay.

Proposition 8 *(Investment Delay.) Assume that Assumption 1 is satisfied and that the single-period utility function is of the form $u(c) = (c^{1-\sigma} - 1)/(1-\sigma)$, with $\sigma > 0$. Let T be the date at which the Y2K problem occurs. If T is sufficiently large, then $i_0 = 0$.*

Proof: Suppose that $i_0 > 0$ for all T. Then equation (3) must hold with equality for all $t < T$ and all T. Evaluating that equation at $t = 0$ and $t = T-1$ yields $c^{\sigma}_{T-1} = \beta^{T-1} c^{\sigma}_0$. Because c_0 is bounded by $\bar{y}$, the above expression implies that c_{T-1} converges to zero as $T \to \infty$. This, in turn, implies that $\lim_{T\to\infty} i_{T-1} = \bar{y}$ and that $\lim_{T\to\infty} \sum_{j=0}^{T-1} i_j = \infty$. Therefore, by Assumption (A1.1), $\lim_{T\to\infty} A'_T = 0$ and thus $\lim_{T\to\infty} \beta(1 - A'_T) = \beta < 1$. Then, by equation (4) it follows that one can choose T large enough so that $i_T = 0$ and, by proposition (4), $i_{T+j} = 0$ for all $j > 0$. In this case, evaluating equation (3) at $t = T-1$ yields $u'(c_{T-1}) = -A'_T\beta/(1-\beta)u'(c_T)$, which implies that $c_{T-1} \geq c_T$, or, equivalently, that $\bar{y} - i_{T-1} \geq \bar{y} - A_T$. Because i_{T-1} converges to $\bar{y}$ as $T \to \infty$, the above inequality implies that $\bar{y} - A_T$ converges to zero as $T \to \infty$, which violates assumptions (A1.1) and (A1.3). ■

2 An economy with capital

In this section, we analyze the dynamics triggered by the Y2K problem in an economy in which in equilibrium agents choose to completely smooth consumption across time. To obtain this separation between consumption and investment decisions, we assume that households have access to an accumulation technology displaying a constant rate of return $r > 0$. Let w_t denote the stock of wealth held by the representative household at the beginning of period t. Then, the household's budget constraint is given by

$$w_{t+1} = \begin{cases} (1+r)w_t + \bar{y} - i_t - c_t & t < T \\ (1+r)w_t + \bar{y} - i_t - c_t - A_t & t \geq T \end{cases} \tag{6}$$

In order to eliminate inessential dynamics in consumption, we assume that $\beta(1+r) = 1$. As before, A_t is meant to represent the Y2K problem and is given by

$$A_t = A(I_t),$$

where I_t is a function of past investment. Because agents can now transfer resources across time at a constant rate of return, it is clear that if, as in the previous section, I_t was assumed to be simply equal to past accumulated investment, then agents would optimally choose to lump investment in period $T-1$. To avoid this unrealistic implication, we consider the following generalization of the specification used in the previous section:

$$I_t = \sum_{j=0}^{t-1} (1 + \gamma i_j)^{\alpha} - t; \quad \gamma > 0,\ \alpha \in (0,1) \tag{7}$$

This aggregator function collapses to the one used in the previous section when $\gamma = \alpha = 1$. The function $A(\cdot)$ is assumed to satisfy Assumptions (A1.1) and

$$\begin{array}{ll} \text{(A1.2')} & \beta[1 - \alpha\gamma A'(0)] > 1 \\ \text{(A1.3')} & rw_0 + \bar{y} - A(0) > 0, \end{array}$$

which are the natural counterparts of assumptions (A1.2) and (A1.3) in the economy analyzed here. Households choose sequences for consumption, investment, and wealth so as to maximize (1) subject to (6), (7), and a no-Ponzi-game borrowing constraint of the form

$$\lim_{t\to\infty} \frac{w_t}{(1+r)^t} \geq 0,$$

given w_0. The optimality conditions are (6) and

$$u'(c_t) = \lambda \tag{8}$$

$$-\beta^t(1+\gamma i_t)^{1-\alpha} - \gamma\alpha \sum_{s=T}^{\infty} \beta^s A'_s \leq 0 \quad (= 0 \text{ if } i_t > 0); \quad t \leq T-1 \tag{9}$$

$$-\beta^t(1+\gamma i_t)^{1-\alpha} - \gamma\alpha \sum_{s=t+1}^{\infty} \beta^s A'_s \leq 0 \quad (= 0 \text{ if } i_t > 0); \quad t \geq T-1 \tag{10}$$

$$\lim_{t\to\infty} \frac{w_t}{(1+r)^t} = 0 \tag{11}$$

where λ is the Lagrange multiplier associated with the period-by-period budget constraint (6) and is constant because of the assumed equality between the pecuniary and subjective rates of discount.

Note first that, unlike in the endowment economy studied in the previous section, consumption is constant over time. However, the dynamics of the flow of resources allocated to solving the Y2k problem are similar to those arising in the endowment economy. In particular, the model implies precautionary investment, investment delay, and acceleration. Further, investment peaks in the period preceeding the year 2000. contrary to the endowment economy, in the economy with capital investment is always positive in the new millennium. We summarize these results in the following proposition.

Proposition 9 *(Investment Dynamics in the Economy with Capital.) Suppose Assumptions (A1.1), (A1.2'), and (A1.3') hold. Then*

Precautionary investment: There exists a $t < T$ *such that* $i_t > 0$.

Investment acceleration: If $i_t > 0$, *then* $i_{t+1} > i_t$ *for all* $t < T-1$.

Investment dynamics in the new millennium: $0 < i_t < i_{t-1}$ *for all* $t \geq T$.

Investment peaks in 1999: $i_{T-1} > i_t$ *for all* $t \neq T-1$.

Investment delay: Let T be the date a which the Y2K problem occurs. If T is sufficiently large, then $i_0 = 0$.

Proof:

No investment stoppage before 2000: We first prove that if $i_t > 0$, then $i_{t+1} > 0$ for all $t < T-1$. Suppose $i_t > 0$ and $i_{t+1} = 0$ for $t < T-1$. Then evaluating equation (9) at t and $t+1$, it follows that $(1+\gamma i_t)^{1-\alpha} \leq \beta$, which is impossible.

Precautionary investment: It is sufficient to show that $i_{T-1} > 0$. To do this, we first show that if $i_{T-1} = 0$, then $i_t = 0$ for all t, and then show that $i_t = 0$ for all t is impossible. By the fact that there is no investment stoppage, if $i_{T-1} = 0$, then $i_t = 0$ for all $t < T-1$. Assume that $i_{T-1} = 0$ and $i_T > 0$. Then evaluating equation (9) at $t = T-1$ and (10) at $t = T$, it follows that $(1+\gamma i_{T-1})^{1-\alpha} \geq \beta[(1+\gamma i_T)^{1-\alpha} - \alpha\gamma A'_T]$. Since $i_{T-1} = 0$, $A'_T = A'(0)$, and $(1+\gamma i_T)^{1-\alpha} > 1$, this inequality violates Assumption (A1.2'). By a similar argument, it follows that if $i_{T-1} = 0$, then $i_{T+j} = 0$ for all $j > 0$. Assume now that $i_t = 0$ for all t. Then evaluating equation (9) at $t = T-1$ yields $\beta^{T-1} \geq -\alpha\gamma A'(0)\beta^T/(1-\beta)$, which violates Assumption (A1.2').

Investment acceleration: If $i_t > 0$ for some $t < T-1$, then by equation (9) if follows that $(1+\gamma i_{t+1})^{1-\alpha} = \beta^{-1}(1+\gamma i_t)^{1-\alpha}$, so $i_{t+1} > i_t$.

Investment dynamics in the new millennium: We first show that if $i_t = 0$, then $i_{t+1} = 0$ for all $t \geq T$. Assume that $i_{t-1} > 0$, $i_t = 0$, and $i_{t+1} > 0$. Evaluate (10) at $t-1$ and t to get $\beta[1 - \alpha\gamma A'_t] \geq (1+\gamma i_{t-1})^{1-\alpha}$. Evaluate (10) at t and $t+1$ to get $\beta[(1+\gamma i_{t+1})^{1-\alpha} - \alpha\gamma A'_t] \leq 1$, which contradicts the previous expression. Because $i_{T-1} > 0$, the result follows by induction. Next, we show that $i_t > 0$ for all $t \geq T$. Let $t \geq T$. Suppose $i_{t-1} > 0$ and $i_t = 0$. Evaluate (10) at $t-1$ to get $(1+\gamma i_{t-1})^{1-\alpha} = -\beta/(1-\beta)\alpha\gamma A'_t$. Now evaluate (10) at t to get $1 \geq -\beta/(1-\beta)\alpha\gamma A'_t$. Clearly, the above two expressions represent a contradiction. The result follows by induction because $i_{T-1} > 0$.

Investment peaks in 1999: We now show that $i_{T-1} > i_t$ for all $t \neq T-1$. As shown above, if $i_t > 0$ for some $t < T-1$, then $i_{t+1} > i_t$. For $t \geq T-1$, equation (10) implies that $(1+\gamma i_{t+1})^{1-\alpha} = \beta^{-1}(1+\gamma i_t)^{1-\alpha} + \alpha\gamma A'_{t+1}$. Let $x_t \equiv (1+\gamma i_t)^{1-\alpha} > 0$ and $a_{t+1} \equiv \alpha\gamma A'_{t+1} < 0$. Then $x_{t+1} = \beta^{-1}x_t + a_{t+1}$. Note that a_{t+1} is strictly increasing in t. Clearly, for any $t \geq T-1$, if $x_{t+1} > x_t$, then x_t converges to infinity at a rate that approaches β^{-1}. Thus, i_t converges to infinity at a rate that approaches $\beta^{-1/(1-\alpha)} > \beta^{-1}$. Such a trajectory for i_t implies a path for wealth that violates the transversality condition (11). Similarly, if $x_{t+1} = x_t$, then $a_{t+2} > a_{t+1}$ and thus $x_{t+2} = \beta^{-1}x_{t+1} + a_{t+2} = \beta^{-1}x_t + a_{t+2} > \beta^{-1}x_t + a_{t+1} = x_{t+1}$, which, as already established, cannot be supported as an equilibrium outcome. Thus, in equilibrium $x_t > x_{t+1}$ for all $t \geq T-1$.

Investment delay: Finally, we prove that if T is sufficiently large, then $i_0 = 0$. Suppose that $i_0 > 0$ for all T. Then $i_t > 0$ for all t and all T. Letting $x_t \equiv (1+\gamma i_t)^{1-\alpha}$, it follows from equation (9) that $x_{t+1} = \beta^{-1}x_t$ for $t < T-1$. Thus, $x_{T-1} \to \infty$ as $T \to \infty$. Using equation (10) and taking into account that $A'_T \to 0$ as $x_{T-1} \to \infty$, it follows that the law of motion of x_t for $t \geq T$ converges to $x_t = \beta^{-1}x_{t-1}$ as $T \to \infty$. Thus, if T is sufficiently large, $i_0 > 0$ implies a path for i_t that violates the transversality condition. ■