Journal of Monetary Economics 45 (2000) 211–246

Journal of MONETARY ECONOMICS

www.elsevier.nl/locate/econbase

# Price level determinacy and monetary policy under a balanced-budget requirement☆

Stephanie Schmitt-Grohé[a], Martín Uribe[b,*]

[a]Rutgers University, New Brunswick, NJ 08901, USA

[b]University of Pennsylvania, Philadelphia, PA 19104, USA

Received 17 April 1998; received in revised form 15 September 1998; accepted 28 September 1998

**Abstract**

This paper analyzes the implications of a balanced-budget fiscal policy rule for price-level determination in a cash-in-advance economy under three alternative monetary policy regimes. It shows that the price level is indeterminate under a nominal interest rate peg and determinate under a money growth rate peg. Under a feedback rule that sets the nominal interest rate as a non-negative and non-decreasing function of the inflation rate, the price level is indeterminate for both low and high values of the inflation elasticity of the feedback rule and determinate for intermediate values. We also study balanced-budget rules that allow for bounded secondary surpluses or deficits. Comparing our results to those emphasized in the fiscal theory of the price level, it becomes clear that a key consideration for price-level determination is whether fiscal policy is specified as an exogenous sequence of primary surpluses/deficits or, alternatively, as an exogenous sequence of secondary surpluses/deficits. © 2000 Elsevier Science B.V. All rights reserved.

*JEL classification:* E63; E52; E31

*Keywords:* Balanced-budget rules; Monetary policy; Fiscal theory of the price level

☆We would like to thank Ken-ichi Akao, Dale Henderson, Eric Leeper, Mike Woodford, an anonymous referee, and seminar participants at the Board of Governors, Indiana University, Michigan State University, Wharton, the University of Minnesota, the NBER's 1997 Summer Institute, the conference on "Indeterminacy and Sunspots in Macroeconomics" held at New York University in November 1997, the 1998 AEA Meetings, and the Federal Reserve Bank of Chicago for comments.

* Corresponding author. Tel.: 215-898-6260; fax: 215-573-2057.

*E-mail address:* uribe@econ.upenn.edu (M. Uribe)



PII: S0304-3932(99)00046-X

## 1. Introduction

In the past decade, the idea of imposing fiscal discipline through a balanced-budget requirement has gained considerable importance in the economic policy debate. This is reflected perhaps most clearly in the proposed balanced-budget amendment that was passed by the United States House of Representatives on 26 January 1995. Yet, little light has been shed on the consequences of balanced-budget rules for business-cycle fluctuations beyond the basic Keynesian insight that balanced-budget rules amplify business cycles by requiring tax increases and expenditure cuts during recessions and the reverse during booms. Even less theoretical work has been devoted to understanding the implications of balanced-budget rules for nominal stability, and, in particular, to understanding the restrictions that such a fiscal policy rule may impose on monetary policy if nominal stability is to be preserved.

This paper is part of a research project that aims to bridge this gap. In Schmitt-Grohé and Uribe (1997), we show in the context of a real economy that a balanced-budget rule can create real instability by making expectations of future income tax increases self-fulfilling. This kind of instability arises for plausible parameter configurations and for income tax structures similar to those observed in the United States and other G7 countries. The present paper embeds a balanced-budget fiscal policy rule into a monetary economy and analyzes its implications for nominal stability, and in particular, for the determinacy of the price level. We model a balanced-budget requirement as a fiscal policy that sets an exogenous path for the secondary surplus, defined as tax revenues net of government expenditures and interest payments on the outstanding public debt. We combine the balanced-budget requirement with three simple monetary policy specifications: a nominal interest rate peg, a money growth rate peg, and a feedback rule whereby the nominal interest rate is set as a non-negative and non-decreasing function of the inflation rate. We conduct the analysis within the cash-in-advance model with cash and credit goods developed by Lucas and Stokey (1987).

We first focus on a specification of the balanced-budget rule in which each period the secondary surplus is required to be zero, that is, the primary surplus (the difference between taxes and government expenditures) is required to be equal to interest payments on the outstanding public debt. We find that under this type of balanced-budget rule, the price level is indeterminate when the monetary authority follows an interest rate peg and is determinate when the monetary authority follows a money growth rate peg. These results are not necessary consequences of the monetary policy specifications alone. For example, Auernheimer and Contreras (1990), Sims (1994), and Woodford (1994) find that if the primary surplus is set exogenously, then an interest rate peg delivers a unique price level. This comparison highlights that given the monetary policy regime the adoption of a balanced-budget rule may have important consequences for nominal stability.

If the balanced-budget rule is combined with the feedback rule, the price level is determinate when the nominal interest rate is moderately sensitive to the inflation rate, and is indeterminate when the nominal interest rate is either very responsive or little responsive to the inflation rate. Again, this result is driven by the balanced-budget requirement. For the same monetary policy specification we consider, Leeper (1991) shows that when the primary surplus is exogenous – a fiscal policy to which he refers as active – the price level is not indeterminate regardless of how sensitive the interest-rate feedback rule is with respect to inflation.[1] Leeper also shows that if the primary surplus is increasing in and sensitive enough to the stock of public debt – a fiscal policy to which he refers as passive – the price level is indeterminate for relatively insensitive feedback rules and is determinate otherwise. Leeper's passive fiscal policy is similar to our balanced-budget rule because under both policies taxes are an increasing function of the stock of public debt. The reason why in our model, unlike in Leeper's, highly sensitive monetary feedback rules render the equilibrium price level indeterminate is that in our model the nominal interest rate affects the consumption/leisure, or cash/credit, margin. In Leeper's model this effect is not present because in his endowment money-in-the-utility-function model with a separable single-period utility function, the marginal utility of consumption is independent of the nominal interest rate in equilibrium.

In practice, balanced-budget proposals typically allow the fiscal authority to run secondary surpluses, as in the proposed US balanced-budget amendment of 1995, or bounded secondary deficits, as in the Maastricht criteria for membership in the European economic and monetary union. Thus, our benchmark definition of a balanced-budget rule, although analytically convenient, is clearly unrealistic since it forces the government to run a zero secondary surplus on a period-by-period basis. However, it turns out that our main results are not driven by this particular specification of the balanced-budget rule. Specifically, we show that in the case in which fiscal policy takes the form of an exogenous, non-zero, bounded path for either real or nominal secondary surpluses/deficits, the price level remains indeterminate under an interest rate peg. Combining this result with that emphasized by the fiscal theory of the price level – i.e., that an interest rate peg combined with an exogenous path for the primary surplus/deficit delivers nominal determinacy – it becomes clear that a key consideration for price-level determination under an interest rate peg is whether fiscal

[1] Leeper studies local equilibria by characterizing solutions to a linear approximation of the equilibrium conditions near a steady state. By contrast, we perform a global analysis characterizing solutions to the exact equilibrium conditions.

policy is specified as an exogenous sequence of primary surpluses/deficits or alternatively as an exogenous sequence of secondary surpluses/deficits.[2]

We also study the implications of a balanced-budget requirement for optimal monetary policy. We find that under a balanced-budget requirement that eliminates budget surpluses as well as deficits in combination with any of the three monetary regimes described above, there exists no rational expectations equilibrium consistent with the optimum quantity of money advocated by Milton Friedman – a monetary policy consistent with a zero nominal interest rate. If the balanced-budget requirement allows for positive secondary surpluses, an equilibrium consistent with the optimum quantity of money may or may not exist depending on the sign of cumulative fiscal surpluses as well as on whether the fiscal authority specifies an exogenous path for the real or nominal surplus.

In the next section, we describe the formal model and the fiscal policy regime. In Sections 3–5, we analyze the implications of balanced-budget rules for the determination of the price-level when the monetary authority follows, respectively, an interest rate peg, a money growth rate peg, and a feedback rule linking the nominal interest rate to inflation. Section 6 concludes.

## 2. A cash-in-advance economy

### 2.1. Households

In this section, we present a model of a cash-in-advance economy in which public and private consumption are cash goods and leisure is a credit good.[3] The economy is assumed to be populated by an infinite number of identical households with log-linear single-period utility functions defined over consumption, $c_t$, and leisure, $1-h_t$, who seek to maximize their lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta^t [\ln(c_t) + \theta \ln(1-h_t)], \quad \theta > 0, \tag{1}$$

where $\beta \in (0,1)$ denotes the subjective discount factor and $E_t$ denotes the expectation operator conditional on information available in period $t$. Each period $t \geq 0$ is divided into two non-overlapping markets. In the first market,

---

[2] Woodford (1995) defines Ricardian regimes as ones in which the present discounted value of outstanding government debt converges to zero regardless of the path of other endogenous variables, and argues that under such regimes fiscal variables fail to play any role for the determination of the price level. We show below that a balanced-budget rule in combination with an interest rate peg is not Ricardian in the sense of Woodford.

[3] The presentation of the model follows Woodford (1994).

households use their nominal wealth at the beginning of the period, $W_t$, to pay lump-sum taxes, $T_t$, to acquire money, $M_t^c$, and to purchase state-contingent claims, $D_{t+1}$, which cost $E_t r_{t+1} D_{t+1}$ dollars and pay $D_{t+1}$ dollars in period $t+1$ (i.e., $r_{t+1}$ is the price of a one-period contingent claim divided by the probability of occurrence of that state). The household's budget constraint in the first market is then given by

$$W_t \geq T_t + M_t^c + E_t\{r_{t+1} D_{t+1}\}. \tag{2a}$$

In the second market, goods and labor services are traded. The household purchases consumption goods at a price of $P_t$ dollars per unit using the money balances it held at the beginning of the goods market. Further, the household has access to a linear technology that enables it to produce one unit of the consumption good per unit of labor input. The household sells these consumption goods at a price of $P_t$ dollars per unit. Its nominal asset holdings at the beginning of period $t+1$ are

$$W_{t+1} = D_{t+1} + M_t^c - P_t c_t + P_t h_t. \tag{2b}$$

Purchases of goods are subject to a cash-in-advance constraint of the form

$$M_t^c \geq P_t c_t. \tag{2c}$$

The household chooses sequences for $W_{t+1}$, $D_{t+1}$, $M_t^c$, $h_t$ and $c_t$, given $W_0 > 0$, so as to maximize (1) subject to $c_t$, $M_t^c \geq 0$, $0 \leq h_t \leq 1$, Eqs. (2a), (2b), (2c) and the following borrowing constraint that prevents it from engaging in Ponzi schemes

$$W_{t+1} \geq -q_{t+1}^{-1} \sum_{j=1}^{\infty} E_{t+1}\{q_{t+j+1} P_{t+j} - q_{t+j} T_{t+j}\}, \tag{2d}$$

where $q_t$ denotes the price in period 0 of one dollar in period $t$ in a particular state of the world divided by the probability of occurrence of that state and is defined as

$$q_t \equiv r_1 \; r_2 \ldots r_t \quad \text{with } q_0 \equiv 1.$$

The borrowing limit (2d) ensures that in every state of the world private debt is not greater than the amount an agent would be able to repay, which is equal to the present discounted value of the time endowment net of taxes.

It can be shown (Woodford, 1994), that the set of sequences $\{c_t, h_t, M_t^c\}$ satisfying the budget constraints (2a)–(2d) are equivalent to the set of sequences $\{c_t, h_t, M_t^c\}$ satisfying the cash-in-advance constraint (2c) and the following present-value budget constraint:

$$W_0 + E_0 \sum_{t=0}^{\infty} [q_{t+1} h_t P_t - q_t T_t]$$

$$\geq E_0 \sum_{t=0}^{\infty} [q_t P_t c_t + (q_t - q_{t+1})(M_t^c - P_t c_t)], \tag{3}$$

which states that the household's initial nominal wealth plus the present discounted value of its labor endowment net of taxes must to be greater than or equal to the present discounted value of consumption expenditures plus the opportunity cost of money holdings in excess of consumption.

From the first-order conditions of the household's optimization problem, consumption and hours must satisfy

$$\frac{1}{P_t c_t} r_{t+1} = \beta \frac{1}{P_{t+1} c_{t+1}}, \tag{4}$$

$$\frac{\theta}{1 - h_t} = \beta E_t \frac{P_t}{P_{t+1} c_{t+1}}. \tag{5}$$

The first equation is a standard pricing equation for a one-step-ahead contingent claim and equates the loss in utility from buying a contingent claim in period $t$ with the expected gain in utility realized from consuming its payoff in period $t + 1$. The second equation is a labor supply schedule and says that the disutility of working an extra hour in period $t$ has to equal the utility derived from spending the wage on consumption goods in period $t + 1$. A further requirement for optimality of the household's contingent plan is that the present value budget constraint (3) be satisfied with equality.

### *2.2. The government*

We assume that the government issues a riskless one-period pure discount bond, that is, a bond that pays one dollar in the following period regardless of the state realized. The government's period-by-period budget constraint is

$$M_t + \frac{B_{t+1}}{R_t} = B_t + M_{t-1} + P_t g - T_t, \tag{6}$$

where $M_t$ denotes the money supply, $B_t$ bonds maturing in period $t$, $g$ constant real government purchases, and $R_t$ the gross nominal interest rate paid on the riskless bond, which must satisfy the arbitrage condition

$$R_t = \frac{1}{E_t r_{t+1}}. \tag{7}$$

The government, like households, is subject to a cash-in-advance constraint on its purchases of goods

$$M_t^g \geq P_t g.$$

The central element that distinguishes this paper from other studies of price level determination is the specification of fiscal policy. We assume that the government is subject to a balanced-budget requirement whereby the primary surplus must be equal to interest payments on the outstanding public debt, that is,

$$T_t - P_t g = (R_{t-1} - 1)\frac{B_t}{R_{t-1}}. \tag{8}$$

This specification of the balanced-budget rule implies that seignorage income cannot be used to finance current spending or to pay interest on the debt. Combining Eqs. (6) and (8) yields

$$M_t + \frac{B_{t+1}}{R_t} = M_{t-1} + \frac{B_t}{R_{t-1}}, \tag{9}$$

which says that end-of-period total nominal government liabilities are constant over time; that is, under the balanced-budget rule, changes in the stock of money are implemented exclusively through open market operations.

We consider three alternative monetary policy regimes: (i) a pure interest rate peg, (ii) a money growth rate peg, and (iii) a feedback rule whereby the nominal interest rate is set as an increasing function of the inflation rate. Under policy regimes (i) and (iii), the central bank sets the nominal interest rate by fixing the price of the riskless one-period nominal bond and standing ready to exchange money for bonds in any quantities demanded. This means that $M_t$ and $B_{t+1}$ are endogenous. Under policy regime (ii), the government specifies a deterministic path for the money supply, so that $B_{t+1}$ and $R_t$ are endogenous.

### 2.3. Equilibrium

In equilibrium, the product and money markets clear, that is,

$$h_t = c_t + g \tag{10}$$

and

$$M_t = M_t^c + M_t^g = M_t^c + P_t g, \tag{11}$$

where the last equality follows from the assumption that the government's cash-in-advance constraint is always binding. Because all agents are identical and government bonds are the only financial assets in positive aggregate net supply, it must be the case that $D_{t+1} = B_{t+1}$ and $W_0 = M_{-1} + B_0$.

In equilibrium, the nominal interest rate must be non-negative. When the nominal interest rate is positive ($R_t > 1$), the household's cash-in-advance constraint holds with equality ($M_t^c = P_t c_t$) and when the nominal interest rate is zero ($R_t = 1$), consumption is equal to the social optimum,

$$\hat{c} \equiv \underset{c}{\operatorname{argmax}} \, [\ln c_t + \theta \ln(1 - c_t - g)].$$

Therefore, in equilibrium

$$c_t = \min(\hat{c}, m_t - g), \tag{12}$$

where

$$m_t \equiv \frac{M_t}{P_t}. \tag{13}$$

Using Eqs. (12) and (13) and the definitions

$$F(m) \equiv \frac{\theta m}{1 - \min(m, \hat{c} + g)}$$

and

$$G(m) \equiv \frac{m}{\min(m, \hat{c} + g) - g},$$

one can write the first-order conditions (4) and (5) as

$$G(m_t) r_{t+1} = \beta G(m_{t+1}) M_t / M_{t+1} \tag{14}$$

and

$$F(m_t) = \beta E_t [G(m_{t+1}) M_t / M_{t+1}]. \tag{15}$$

Taking expected values of both sides of Eq. (14) and substituting (7) and (15) implies a demand for real balances of the form

$$\frac{G(m_t)}{F(m_t)} = R_t. \tag{16}$$

The function $G(\cdot)/F(\cdot)$ is continuous at all $m > g$. For $m < \hat{c} + g$, $G(m)/F(m)$ is greater than one and strictly decreasing in $m$, and as $m$ approaches $g$ from above,

$G(m)/F(m)$ becomes arbitrarily large. For $m \geq \hat{c} + g$, $G(m)/F(m)$ equals one. Thus, for any $R_t > 1$, there exists a unique $g < m_t < \hat{c} + g$ satisfying (16), while for $R_t = 1$, any $m_t \geq \hat{c} + g$ satisfies (16).[4]

From (14) we can express the present value deflator as

$$q_t = \beta^t M_0/M_t \frac{G(m_t)}{G(m_0)}. \tag{17}$$

Substituting the market clearing conditions (10) and (11) into (3), which in equilibrium must hold with equality, yields

$$M_{-1} + B_0 = E_0 \sum_{t=0}^{\infty} q_t[T_t - P_t g + (1 - q_{t+1}/q_t)M_t]. \tag{18}$$

This expression says that in equilibrium total nominal liabilities of the government at the beginning of period zero must equal the present discounted value of primary surpluses plus interest savings from the issuance of money. It is equivalent to (6) and the transversality condition

$$\lim_{t\to\infty} E_0 q_{t+1}(M_t + B_{t+1}) = 0,$$

which can also be written as

$$\lim_{t\to\infty} E_0 q_t[M_t + B_{t+1}/R_t - (1 - q_{t+1}/q_t)M_t] = 0. \tag{19}$$

Note that under the assumed balanced-budget requirement, the sum of the first two terms within the square brackets is constant. Using (9) and (17) to eliminate $M_t + B_{t+1}/R_t$ and $q_t$ from this transversality condition yields

$$\lim_{t\to\infty} E_0 \beta^t [G(m_t)A_{-1}/M_t + F(m_t) - G(m_t)] = 0, \tag{20}$$

where $A_{-1} \equiv M_{-1} + B_0/R_{-1}$ denotes total nominal government liabilities at the end of period $-1$ and is assumed to be positive.

A rational expectations monetary equilibrium is a set of processes $m_t > g$, $M_t > 0$, and $R_t \geq 1$ satisfying (15), (16), (20), and one additional equation

[4] Such a liquidity preference relation arises for more general preferences than the log-log form considered here. Specifically, it obtains for single-period utility functions of the form $U(c,1-h)$ that satisfy the following assumptions: $U$ is strictly concave and twice continuously differentiable, $U_1, U_2 > 0$, $c$ and $1-h$ are normal goods, $\lim_{c\to 0} U_1(c,1-g-c)/U_2(c,1-g-c) = \infty$, and $g < \text{argmax}_c\, U(c,1-g-c) < 1-g$. In this case, $F(m) \equiv mU_2(\min(\hat{c}, m-g), 1-g-\min(\hat{c}, m-g))$ and $G(m) \equiv mU_1(\min(\hat{c}, m-g), 1-g-\min(\hat{c}, m-g))$. It can also be shown that the assumption that government purchases are cash goods is not essential in deriving a money demand relation of this form.

specifying the monetary policy regime, given $A_{-1} > 0$. Given a rational expectations monetary equilibrium, one can uniquely determine $c_t$ from (12); $h_t$ from (10); $P_t$ from (13); $r_{t+1}$ from (14); $q_t$ from (17); $B_{t+1}/R_t$ from (9); and $T_t$ from (6).[5]

## 3. Equilibrium under an interest rate peg

The monetary policy regime considered in this section is an interest rate peg of the form

$$R_t = R,$$

where $R$ is a constant satisfying $R \geq 1$.

Consider first the case $R > 1$. To establish the existence of a rational expectations equilibrium, recall that for any $R_t > 1$, there exists a unique $m_t$ satisfying Eq. (16). Thus in any rational expectations equilibrium real balances are unique and constant. Let $m$ denote the value of $m_t$ that solves (16) when $R_t = R$. Use Eqs. (15) and (16) to replace $E_0[G(m_t)A_{-1}/M_t]$ by $(\beta R)^{-t}G(m)A_{-1}/M_0$ in Eq. (20) to obtain

$$\lim_{t \to \infty} R^{-t}G(m)A_{-1}/M_0 + \beta^t[F(m) - G(m)] = 0. \tag{21}$$

This equation is satisfied for any $M_0 > 0$. Therefore, a rational expectations equilibrium exists, and nominal balances are indeterminate. Since real balances are unique, the indeterminacy of nominal balances implies that the price level, $P_0 = M_0/m$, is also indeterminate.[6] The fact that $R > 1$ implies that $g < m < \hat{c} + g$ and that the household's cash-in-advance constraint is binding. Thus, $c_t = m - g$. From market clearing in the product market, it follows that $h_t = m$.

Consider now the evolution of real debt and real taxes. Using the facts that real balances are constant, that total nominal government liabilities are constant, and that $E_0(1/M_t)$ is equal to $(\beta R)^{-t}/M_0$, the expected value of real government debt outstanding in period $t$ can be expressed as

$$E_0 \frac{B_{t+1}}{RP_t} = E_0 \frac{A_{-1} - M_t}{P_t} = m[(\beta R)^{-t}A_{-1}/M_0 - 1].$$

---

[5] Note that in order to determine $T_0$ it is necessary to know the composition of initial government liabilities, $M_{-1}$ and $B_0/R_{-1}$, as well as $R_{-1}$.

[6] Clearly, this result also obtains for the more general preference specification described in footnote 4. Further, the assumption that government purchases are subject to a cash-in-advance constraint does not affect this result in any important way.

According to this expression, the expected stock of real debt depends on the initial money supply $M_0$ and is therefore not unique. The expected long-run level of real debt depends on the level of the nominal interest rate. If the monetary authority pegs the nominal interest rate at a level higher than the real interest rate (i.e., $R > \beta^{-1}$), then the stock of real debt is expected to converge to $-m$. That is, the government is expected to become a net lender to the public. Alternatively, if the monetary authority pegs the nominal interest rate at a value below the real interest rate (i.e., $1 < R < \beta^{-1}$), then the stock of real debt is expected to grow without bound.[7] The reason for this explosive behavior is that when the nominal interest rate is below the real interest rate, both prices and nominal balances are falling and therefore seignorage income is negative. These seignorage losses must be financed with new debt because, by the balanced-budget rule, the government is only allowed to raise taxes to cover government spending and interest on the outstanding debt. Private agents are willing to hold the ever increasing government debt because they face a path of real lump-sum taxes which is also expected to increase over time (Eq. (8)).

We have established the main result of this section, namely, that if the government pegs the nominal interest rate at a positive level ($R > 1$), a balanced-budget rule leads to price level indeterminacy. The intuition for this result is the following. In equilibrium the present discounted value of nominal government liabilities net of interest savings from the issuance of money must converge to zero as $t$ approaches infinity. Under a balanced-budget rule, nominal government liabilities are constant, thus their present discounted value approaches zero for any positive nominal interest rate path. In addition, interest savings from the issuance of money are proportional to nominal balances. Since the nominal interest rate is constant and positive, real balances are constant, thus nominal balances grow at the rate of inflation. But the inflation rate is less than the nominal interest rate because the real interest rate, given by the subjective discount factor, is positive. Therefore, regardless of their initial level, nominal balances converge to zero in present discounted value.

This finding recovers the Sargent and Wallace (1975) result of price-level indeterminacy under an interest rate peg. However, unlike in Sargent and Wallace's model, here price-level indeterminacy obtains in an economy with a fully specified fiscal regime. On the other hand, our finding contrasts with the central result of the fiscal theory of the price level, namely, that if the policy regime consists of an interest rate peg and an exogenous path for the primary surplus, the equilibrium price level is determinate (Auernheimer and Contreras, 1994; Sims, 1994; Woodford, 1994). To see that the price level is uniquely determined if the path for the primary surplus is exogenous, use Eqs. (7) and (17)

[7] Note that although the expected stock of real debt is increasing over time, its present discounted value, $E_0 \beta^t B_{t+1}/(RP_t)$, converges to zero.

and the fact that for $R > 1$ real balances are constant to express the transversality condition (18) as

$$\frac{M_{-1} + B_0}{P_0} = \sum_{t=0}^{\infty} \beta^t [E_0(T_t/P_t - g) + (1 - R^{-1})m].$$

When the process for the real primary surplus, $T_t/P_t - g$, is exogenous, the above expression uniquely determines $P_0$, provided its right-hand side exists and is positive and finite. The price level is also uniquely determined in the case in which the sequence of nominal primary surpluses, $T_t - P_t g$, is exogenous and deterministic. To verify this, rewrite the above expression as

$$\frac{M_{-1} + B_0}{P_0} = \sum_{t=0}^{\infty} \beta^t \left[ \frac{T_t - P_t g}{(\beta R)^t P_0} + (1 - R^{-1})m \right].$$

This expression can be solved for a unique positive $P_0$ provided that $M_{-1} + B_0 > \sum_{t=0}^{\infty} R^{-t}(T_t - P_t g)$.

More recently, there has been an effort to classify policy regimes according to whether equilibrium conditions involving fiscal variables are needed for price-level determination. For example, Benhabib et al. (1998a) refer to regimes in which fiscal policy fails to play a role for price-level determination as Ricardian. Specifically, they define a Ricardian policy regime as one in which the present discounted value of total end-of-period government liabilities converges to zero regardless of the behavior of endogenous variables. Clearly, the policy regime considered in this section – a balanced-budget rule in combination with an interest rate peg – is Ricardian in the sense of Benhabib, Schmitt-Grohé and Uribe since by the balanced-budget rule total government liabilities are constant and by the assumed monetary policy the nominal interest rate is strictly positive.[8]

[8] Woodford (1995, p. 27) defines a Ricardian regime as one in which the exogenous sequences and feedback rules that specify the policy regime imply that the present discounted value of outstanding nominal government debt necessarily converges to zero regardless of the behavior of other endogenous variables and argues that under such regimes the price level is typically not uniquely determined. According to Woodford's definition, a balanced-budget rule of the type considered in this section in combination with an interest rate peg is not Ricardian. To see this, consider for simplicity a perfect-foresight economy and note that under a balanced-budget rule, the present discounted value of outstanding government debt can be written as

$$\lim_{t \to \infty} R^{-t} \frac{B_{t+1}}{R} = \lim_{t \to \infty} R^{-t} [A_{-1} - M_t].$$

The right side of this expression does not necessarily converge to zero for any path of endogenous variables. For example, consider an off-equilibrium path in which $M_t$ grows at a rate equal to or greater than $R$.

We now extend the class of balanced-budget rules by allowing for non-zero secondary surpluses. This type of budget rules is clearly more realistic than our baseline specification of a zero period-by-period secondary surplus. For example, the proposed balanced-budget amendment that was passed in 1995 by the US House of Representatives allows for positive secondary surpluses. Another example of a budget rule of this type is the 'Excessive Deficit Procedure' of the Maastricht Treaty on Economic and Monetary Union which requires that a country's secondary budget deficit for each fiscal year be no larger than 3 percent of its gross domestic product.

For simplicity, we assume that the time path of either the real secondary surplus, $s_t$, or the nominal secondary surplus, $S_t$, is exogenous and bounded. Under either assumption, lump-sum taxes are given by

$$T_t = P_t g + (R_{t-1} - 1)\frac{B_t}{R_{t-1}} + S_t.$$

Combining this expression with the government's sequential budget constraint (6), the evolution of total nominal government liabilities, $M_t + B_{t+1}/R_t$, becomes

$$M_t + \frac{B_{t+1}}{R_t} = M_{t-1} + \frac{B_t}{R_{t-1}} - S_t = \cdots = A_{-1} - \sum_{j=0}^{t} S_j.$$

Using this expression to eliminate $M_t + B_{t+1}/R_t$ in the transversality condition (19) yields

$$\lim_{t\to\infty} E_0 q_t\left[A_{-1} - \sum_{j=0}^{t} S_j - (1 - q_{t+1}/q_t)M_t\right] = 0, \tag{22}$$

which together with (17) replaces equilibrium condition (20).

We continue to assume that the nominal interest rate is pegged at a strictly positive value $R > 1$. As shown earlier, in this case real balances and consumption are uniquely determined and constant, and $\lim_{t\to\infty} E_0 q_t[A_{-1} - (1 - q_{t+1}/q_t)M_t] = 0$. Thus Eq. (22) collapses to

$$\lim_{t\to\infty} E_0 q_t \sum_{j=0}^{t} S_j = 0. \tag{23}$$

Assume first that the fiscal authority sets an exogenous and deterministic path for the real secondary surplus that is bounded above by $\bar{s}$ and below by $\underline{s}$. Using the facts that $q_t = \beta^t M_0/M_t$ and $E_0 M_j/M_t = (\beta R)^{j-t}$, the left side of Eq. (23) can be written as

$$\begin{aligned}\lim_{t\to\infty} E_0 q_t \sum_{j=0}^{t} S_j &= \lim_{t\to\infty} E_0 \beta^t \frac{M_0}{M_t} \sum_{j=0}^{t} P_j s_j \\ &= \lim_{t\to\infty} \beta^t \frac{M_0}{m} \sum_{j=0}^{t} E_0 \frac{M_j}{M_t} s_j \\ &\leq \lim_{t\to\infty} \beta^t \frac{M_0}{m} \sum_{j=0}^{t} E_0 \frac{M_j}{M_t} \bar{s} \\ &= \lim_{t\to\infty} \beta^t \frac{M_0 \bar{s}}{m} \sum_{j=0}^{t} (\beta R)^{j-t} \\ &= \lim_{t\to\infty} R^{-t} \frac{M_0 \bar{s}}{m} \quad \frac{1-(\beta R)^{t+1}}{1-\beta R} \\ &= 0.\end{aligned}$$

Similarly,

$$\begin{aligned}\lim_{t\to\infty} E_0 q_t \sum_{j=0}^{t} S_j &\geq \lim_{t\to\infty} R^{-t} \frac{M_0 \underline{s}}{m} \frac{1-(\beta R)^{t+1}}{1-\beta R} \\ &= 0,\end{aligned}$$

so that the transversality condition (22) is satisfied for any $M_0 > 0$. Given a value for $M_0$, the price level is given by $M_0/m$. Thus, as in the case in which the balanced-budget rule requires the secondary surplus to be exactly equal to zero at all dates and all states, a rational expectations equilibrium exists and the price level is indeterminate.

Assume now that the fiscal authority sets an exogenous and deterministic path for the nominal secondary surplus that is bounded above by $\bar{S}$ and below by $\underline{S}$. In this case the left side of Eq. (23) satisfies

$$\begin{aligned}\lim_{t\to\infty} E_0 q_t \sum_{j=0}^{t} S_j &\leq \lim_{t\to\infty} E_0 q_t \sum_{j=0}^{t} \bar{S} \\ &= \lim_{t\to\infty} \bar{S} t E_0 q_t \\ &= \lim_{t\to\infty} \bar{S} t R^{-t} \\ &= 0,\end{aligned}$$

and

$$\lim_{t \to \infty} E_0 q_t \sum_{j=0}^{t} S_j \geq \lim_{t \to \infty} \underline{S} t R^{-t}$$
$$= 0.$$

This implies that the transversality condition will be satisfied for any $M_0 > 0$ and hence the initial price level is again indeterminate.

We have demonstrated that if the monetary authority pegs the nominal interest rate at a strictly positive value, a balanced-budget rule induces price-level indeterminacy even if it allows for bounded secondary deficits or surpluses. Comparing this result with those emphasized in the fiscal theory of the price level it becomes clear that a key consideration for price-level determination is whether the fiscal policy is described as an exogenous sequence of primary deficits/surpluses or, alternatively, as an exogenous sequence of secondary deficits/surpluses.

### *3.1. The optimum quantity of money*

Consider now a monetary policy whereby the nominal interest rate is set at zero ($R = 1$). This policy corresponds to the optimum quantity of money advocated by Milton Friedman. Assume first that the government follows a period-by-period state-by-state balanced-budget rule as described by Eq. (8). From (16) it follows that $F(m_t) = G(m_t)$. Thus (20) becomes $\lim_{t \to \infty} E_0 \beta^t G(m_t) A_{-1}/M_t = 0$, which using (15) and (16) can be expressed as $\lim_{t \to \infty} G(m_0) A_{-1}/M_0 = 0$. This equality is a contradiction because any solution to (16) satisfies $m \geq \hat{c} + g$, so that $G(m_0) > 0$. Consequently, the optimum quantity of money cannot be brought about under a balanced-budget rule.

The intuition behind this result is simple. In equilibrium, initial government liabilities must be equal to the present discounted value of primary surpluses plus interest savings from the issuance of money. By the balanced-budget rule, the primary surplus must equal interest payments on the public debt. Thus, if the nominal interest rate is zero, so is the primary surplus. A zero nominal interest rate also implies that interest savings from the issuance of money are nil. Therefore, when the nominal interest rate is zero, the present discounted value of government revenues vanishes, which is inconsistent with a positive initial level of government liabilities.

This non-existence result is fragile for at least two reasons. First, it follows from our earlier analysis that if the nominal interest rate is positive but arbitrarily close to zero, a rational expectations equilibrium exists and the associated real allocation is arbitrarily close to the one associated with the social optimum. Second, as we show next, an equilibrium consistent with the optimum quantity of money may exist under a balanced-budget rule if the present discounted value of secondary surpluses is expected to be positive.

Consider the case that the government sets an exogenous and deterministic path for either the nominal or the real secondary surplus. When $R = 1$, Eq. (22) simplifies to

$$A_{-1} = \lim_{t\to\infty} E_0 q_t \sum_{j=0}^{t} S_j. \tag{24}$$

We first analyze the case that the real secondary surplus is exogenous and deterministic. From (16) it follows that in any equilibrium $m_t \geq \hat{c} + g$, which implies that $G(m_t) = m_t/\hat{c}$ and, by (17), that $q_t = \beta^t P_0/P_t$, so that $E_0 q_t \sum_{j=0}^{t} P_j s_j = \beta^t P_0 \sum_{j=0}^{t} E_0 P_j s_j / P_t = \beta^t P_0 \sum_{j=0}^{t} \beta^{j-t} s_j$, where the last equality follows from the facts that $P_j/P_t = \beta^{j-t} q_t/q_j$ and $E_0 q_t/q_j = 1$ for any $t \geq j \geq 0$. Thus (24) becomes

$$A_{-1} = P_0 \lim_{t\to\infty} \sum_{j=0}^{t} \beta^j s_j.$$

Suppose further that the limit on the right hand side of this equation, the present discounted value of real secondary surpluses, exists. Since initial nominal government liabilities are assumed to be positive, a rational expectations equilibrium exists and the price level is unique if and only if the present discounted value of real secondary surpluses is positive and finite.

Finally, assume that the nominal secondary surplus is exogenous and deterministic, then (24) becomes

$$A_{-1} = \lim_{t\to\infty} \sum_{j=0}^{t} S_j.$$

This equation will in general not be satisfied for arbitrary paths of the nominal secondary surplus. Thus, in general, a rational expectations equilibrium does not exist. In the unlikely event that the above equation is satisfied, a rational expectations equilibrium exists, but the equilibrium price level is indeterminate.

## 4. Equilibrium under a money growth rate peg

Under the monetary policy regime to be considered in this section, the government pegs the gross growth rate of the money supply at a constant level $\mu > 0$. Thus, $M_t$ is given by

$$M_t = \mu^t M_0. \tag{25}$$

Given this policy rule, Eq. (15) reduces to

$$F(m_t) = \frac{\beta}{\mu} E_t G(m_{t+1}), \tag{26}$$

and the transversality condition (20) becomes

$$\lim_{t \to \infty} \beta^t E_0 \{ G(m_t) \mu^{-t} A_{-1} + [F(m_t) - G(m_t)] M_0 \} = 0. \tag{27}$$

A rational expectations monetary equilibrium is a process $m_t > g$ satisfying Eqs. (26) and (27), given $A_{-1} > 0$ and $M_0 > 0$.

### *4.1. Steady-state equilibria*

We first characterize equilibria in which real balances are constant. Clearly, a steady-state equilibrium does not exist if $\mu \leq \beta$ because in that case no constant value of real balances satisfies (27). In particular, this means that the policymaker cannot bring about the optimal allocation, or optimum quantity of money, by reducing the money supply at the subjective rate of discount. The intuition for the impossibility of implementing the Friedman rule by pegging the money growth rate is essentially the same as in the case of an interest rate peg: under a balanced-budget rule total nominal government liabilities are constant over time and because $\mu = \beta$, the nominal interest rate is equal to zero. Therefore, total nominal government liabilities do not converge to zero in present discounted value.

It follows that any fiscal regime that makes total government liabilities vanish asymptotically will allow for the existence of a steady-state equilibrium consistent with the Friedman rule. An example of such a fiscal policy is one in which the government keeps the stock of public debt equal to zero at all times (Woodford, 1994). In this case, total government liabilities are equal to the money supply, which, by the Friedman rule, shrinks at the rate of discount.[9] Another example is a fiscal policy that specifies an exogenous sequence of nominal secondary surpluses whose sum is equal to the initial stock of total nominal government liabilities.

In what follows, we assume that the growth rate of the money supply exceeds the discount factor ($\mu > \beta$). Given this assumption, any constant value of $m$ satisfies (27). Thus, establishing the existence and uniqueness of a steady-state equilibrium reduces to studying solutions to (26) in which $m_t = m^* > g$ for all $t$,

[9] Clearly, the fiscal policy analyzed by Woodford is not a balanced-budget rule because secondary deficits are financed by seignorage income.

that is, solutions to

$$F(m^*) = \frac{\beta}{\mu} G(m^*). \tag{28}$$

For $m \geq \hat{c} + g$, $F(m) = G(m)$, thus the left side of (28) is greater than the right side (Fig. 1). For $g < m < \hat{c} + g$, $G(m)$ is monotonically decreasing, and $F(m)$ is monotonically increasing. Further, as $m$ approaches $g$ from above, $G(m)$ converges to infinity and $F(m)$ converges to $\theta g/(1-g)$, so the right side of (28) becomes larger than its left side. Therefore, there exists a unique solution $m^*$ to (28), which satisfies $g < m^* < \hat{c} + g$. We have established that if $\mu > \beta$, a steady-state equilibrium exists and is unique. The price level is uniquely determined as $P_t = M_t/m^*$. From (16) it follows that the gross nominal interest rate, $R^*$, is equal to $\mu/\beta$. Because real balances are constant, the rate of inflation must be equal to the growth rate of the money supply. Using the fact that $B_{t+1}/R_t + M_t = A_{-1}$, the real value of public debt can be expressed as

$$\frac{B_{t+1}}{P_t} = \mu^{-t} R^* m^* A_{-1}/M_0 - R^* m^*.$$

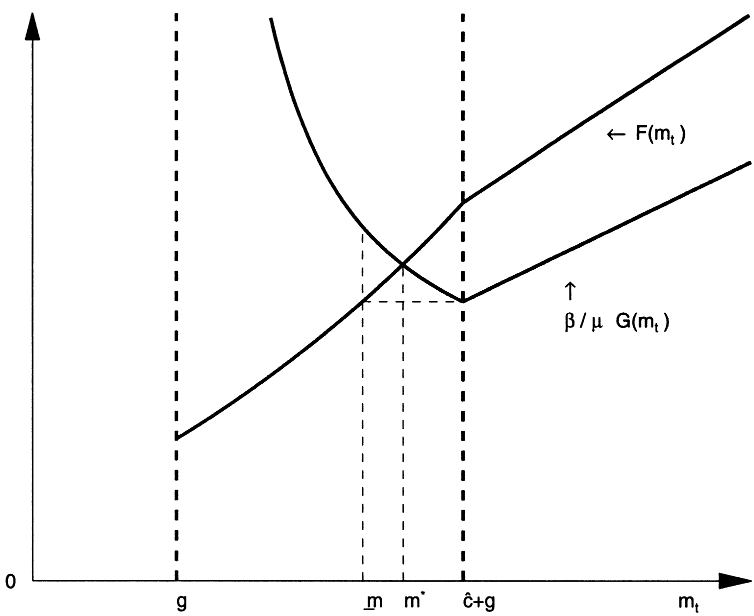


Fig. 1. Money growth rate peg, $\mu > \beta$ and Assumption (A2)

$$G(m_t) = \frac{m_t}{\min(m_t, \hat{c} + g) - g}, \quad F(m_t) = \frac{\theta m_t}{1 - \min(m_t, \hat{c} + g)}.$$

Hence, if the rate of growth of the money supply is negative ($\mu < 1$), then the real stock of debt associated with the steady-state equilibrium grows without bounds. If, on the other hand, the money growth rate is non-negative ($\mu \geq 1$), then the long-run real stock of debt is finite. If the money growth rate is strictly positive, the real value of the public debt converges to a negative value.[10]

### 4.2. Non-steady-state equilibria

We make the following assumption regarding preferences and the size of government purchases

$$\text{(A1)} \quad F(g) < \frac{\beta}{\mu} \inf_{m > g} G(m).$$

This assumption is satisfied for any value of $\mu$ for which private consumption exceeds government consumption in the steady-state equilibrium. The following proposition shows that if (A1) is satisfied, then real balances held by the private sector are finite and bounded below by a strictly positive value.

**Proposition 1.** *Suppose that preferences satisfy* (A1) *and that* $\mu > \beta$. *Then there exist bounds* $\underline{m} \in (g, \hat{c} + g)$ *and* $\bar{m} < \infty$ *such that in any monetary equilibrium* $\underline{m} \leq m_t \leq \bar{m}$. *Thus, self-fulfilling inflations as well as self-fulfilling deflations are impossible.*

**Proof.** The proof of this proposition, which draws heavily on the proof of Propositions 4 and 5 in Woodford (1994), is presented in Appendix A. □

Proposition 1 establishes that in any equilibrium there are at most bounded fluctuations. The existence of an upper bound and hence the impossibility of self-fulfilling deflations depends crucially on the assumed balanced-budget requirement. For example, under a fiscal policy in which the stock of public debt is zero at all times speculative deflations are possible (Woodford, 1994). The intuition for the impossibility of speculative deflations under a balanced-budget rule is simple. In a speculative deflation prices grow at a rate smaller than the rate of monetary expansion. Thus real balances become arbitrarily large, exceeding $\hat{c} + g$ after a finite number of periods, at which point the cash-in-advance constraint ceases to bind and the nominal interest rate becomes zero. In any

[10] It is straightforward to show that the results of this subsection also hold when preferences are of the more general type described in footnote 4 or when government purchases are not subject to a cash-in-advance constraint or both.

rational expectations equilibrium, the present discounted value of total nominal government liabilities net of interest savings from the issuance of money has to converge to zero. But as discussed earlier, this can never be the case if the nominal interest rate is zero because under a balanced-budget rule total nominal government liabilities are constant.

On the other hand, the balanced-budget requirement does not rule out the existence of speculative inflations. The balanced-budget rule ensures that in any speculative inflation the transversality condition will necessarily be satisfied, that is, the present discounted value of total nominal government liabilities net of interest savings from the issuance of money necessarily converges to zero. The intuition for this is as follows. In a hyperinflation real balances converge to zero and the nominal interest rate increases over time. Since under a balanced-budget rule total nominal government liabilities are constant, the fact that the nominal interest rate is positive ensures that their present discounted value converges to zero. At the same time, as real balances are shrinking, interest savings from the issuance of money disappear in present-discounted-value terms.

The existence of a lower bound on real balances is guaranteed individually by both the assumption that government purchases are subject to a cash-in-advance constraint and the assumed log-linear preference specification. More generally, any preference specification with a single-period utility function of the form $U(c,1-h)$ satisfying $\lim_{c\to 0} cU_1(c,1-c) > 0$ rules out self-fulfilling inflations in the absence of a cash-in-advance constraint on government purchases (see also Woodford, 1994).

To establish the uniqueness of the rational expectations equilibrium, we make the following assumption:

$$\text{(A2)} \quad g < \frac{\beta}{\mu\theta}.$$

Like assumption (A1), assumption (A2) is satisfied whenever private consumption exceeds government consumption in the steady-state equilibrium. Assumption (A2) is stronger than assumption (A1). To see this, note that (A1) is satisfied whenever $g < \beta/(\mu\theta)/(1-\beta/\mu)$, and that $1/(1-\beta/\mu) > 1$. The following proposition completes the characterization of equilibrium under a money growth rate peg.

**Proposition 2.** *If* $\mu > \beta$ *and assumption* (A2) *is satisfied, then the steady-state equilibrium is the unique rational expectations monetary equilibrium. Thus, the equilibrium price level is unique.*

**Proof.** See Appendix A.

The main implication of this proposition is that under a balanced-budget requirement, the monetary authority can control the price level by controlling the rate of expansion of the monetary aggregate. This result stands in stark contrast with the one obtained under an interest rate peg. In that case, a balanced-budget rule necessarily renders the price level indeterminate.

## 5. Equilibrium under an interest rate feedback rule

Since Taylor (1993), a growing empirical literature has documented that post-war US monetary policy can be characterized as following an interest-rate feedback rule whereby the nominal interest rate is set as an increasing function of inflation and output.[11] Particular attention has been placed on the implications of the magnitude of the inflation elasticity of such a rule for macroeconomic stability. In this section, we consider the consequences of interest rate feedback rules for the determinacy of equilibrium in the presence of a balanced-budget requirement. Specifically, we assume that the nominal interest rate is set according the following non-linear rule

$$R_t = \max[1, R + \alpha(\pi_t - \beta R)], \quad \alpha, R > 0, \tag{29}$$

where $\pi_t \equiv P_t/P_{t-1}$ denotes the gross rate of inflation. This specification of monetary policy ensures the non-negativity of the nominal interest rate for all possible paths of the inflation rate. We will refer to monetary policy as active if in response to an increase in the inflation rate, the monetary authority raises the nominal interest rate by more than one-for-one and as passive otherwise. One implication of the zero bound requirement on nominal interest rates is that if monetary policy is globally non-decreasing and active for some values of the inflation rate, then it must also be passive for some other values. As will become clear below, this property of the feedback rule gives rise to multiple steady-state rates of inflation and has important consequences for the global stability of equilibrium.

Under the monetary policy regime described by Eq. (29), the supply of both money and bonds is endogenous, as is the case under the pure interest rate peg discussed in Section 3. Throughout this section, we restrict the analysis to perfect-foresight equilibria. Using (16) to replace $R_t$, the feedback rule can be written as

$$\frac{G(m_t)}{F(m_t)} = \max\left[1, R(1 - \alpha\beta) + \alpha \frac{m_{t-1}}{m_t} \frac{M_t}{M_{t-1}}\right]. \tag{30}$$

Using (15) to eliminate $M_{t+1}/M_t$ yields

$$\frac{G(m_{t+1})}{F(m_{t+1})} = \max\left[1, R(1 - \alpha\beta) + \alpha\beta \frac{m_t}{m_{t+1}} \frac{G(m_{t+1})}{F(m_t)}\right], \quad t \geq 0 \tag{31}$$

[11] See for example, Orphanides (1998) and Clarida et al. (1997).

and

$$\frac{G(m_0)}{F(m_0)} = \max\left[1, R(1-\alpha\beta) + \alpha\frac{M_0}{m_0 P_{-1}}\right]. \tag{32}$$

A perfect-foresight equilibrium consists of a pair of sequences $\{m_t, M_t\}$ satisfying $m_t > g$, $M_t > 0$, Eqs. (15), (20), (31), and (32), given $A_{-1}$ and $P_{-1}$.

### 5.1. Steady-state equilibria

Consider first steady-state equilibria, that is, perfect-foresight equilibria in which real balances are constant from period 0 onward. In such an equilibrium, real balances, $m^*$, must satisfy $m^* < \hat{c} + g$. To see why, suppose that $m^* \geq \hat{c} + g$. This implies that $G(m^*) = F(m^*)$ and, by (15), that $M_t = \beta^t M_0$. Using these two relations, the left side of (20) can be written as $G(m^*)A_{-1}/M_0 > 0$, so that the transversality condition is violated. For $g < m^* < \hat{c} + g$, $G(m^*)/F(m^*) > 1$, and hence $m^*$ must satisfy

$$\frac{G(m^*)}{F(m^*)} = R(1-\alpha\beta) + \alpha\beta\frac{G(m^*)}{F(m^*)}.$$

Suppose that $\alpha\beta \neq 1$. Clearly, if $R \leq 1$, the above equation is inconsistent, and thus no steady-state equilibrium exists. If $R > 1$, then $m^*$ solves

$$\frac{G(m^*)}{F(m^*)} = R. \tag{33}$$

As discussed in Section 3, this equation has a unique solution. Evaluating (15) and (32) at $m_t = m^*$ yields $M_t = (\beta R)^t M_0$ and $M_0 = R\beta m^* P_{-1} > 0$. As is readily verified, this sequence of nominal balances and $m_t = m^*$ for all $t$ satisfy the transversality condition (20). Since both nominal and real balances are unique, so is the price level, $P_t = M_t/m^*$. Comparing Eqs. (16) and (33), it follows that the nominal interest rate is equal to $R$.[12]

Note that in the steady-state equilibrium real balances are independent of $\alpha$ and identical to the level of real balances that would obtain if the nominal interest rate were pegged at $R$. However, unlike the interest rate peg, the feedback rule induces a unique price level. This key difference is accounted for by the link between the nominal interest rate and the contemporaneous inflation rate introduced by the feedback rule.

[12] If $\alpha\beta = 1$, there exists a continuum of steady-state equilibria. Specifically, any level of real balances $m^* \in (g, \hat{c} + g)$ and a sequence of nominal balances $M_t = (\beta G(m^*)/F(m^*))^t M_0$ with $M_0 = (G(m^*)/F(m^*))\beta m^* P_{-1} > 0$ constitute a steady-state equilibrium.

To highlight the role of a balanced-budget rule for price level determination under an interest rate feedback rule, consider an alternative fiscal policy whereby the government specifies an exogenous path for the real primary surplus. In this case, unlike in the case of a balanced-budget rule, there exists no steady-state equilibrium satisfying $g < m^* < \hat{c} + g$. To see this, note that Eqs. (32) and (33) imply that $P_0 = \beta R P_{-1}$. At the same time, the transversality condition (18) implies that $P_0 = (M_{-1} + B_0)/(\sum_{t=0}^{\infty}\beta^t[T_t/P_t - g + (1 - R^{-1})m^*])$. These two expressions are inconsistent because their right sides are exogenous and in general different from each other.[13]

### 5.2. Non-steady-state equilibria

For the analysis of non-steady-state equilibria, we assume that $R > 1$ and $\alpha\beta \neq 1$. Two elements of the model play a key role for price level determination in addition to the assumed fiscal regime: the elasticity of the feedback rule with respect to inflation, $\alpha\beta$, and the steady-state leisure-to-consumption ratio, which, by Eq. (16), is equal to $\theta R$. Our main finding is that if the leisure-to-consumption ratio is greater than one, – which as we will argue below is the case of greatest empirical relevance – then the equilibrium price level and real allocation are indeterminate for both relatively low and relatively high values of the inflation elasticity of the feedback rule. Specifically, the equilibrium is indeterminate for $\alpha\beta < 1$ and for $\alpha\beta > (\theta R + 1)/(\theta R - 1)$. For intermediate values of the inflation elasticity of the feedback rule, $1 < \alpha\beta < (\theta R + 1)/(\theta R - 1)$, the perfect-foresight equilibrium is unique. The following proposition formalizes this result.

**Proposition 3.** *Suppose that the steady-state leisure-to-consumption ratio, $\theta R$, is greater than one. If the inflation elasticity of the feedback rule, $\alpha\beta$, satisfies*

$$0 < \alpha\beta < 1 \quad \textit{or} \quad \alpha\beta > \frac{\theta R + 1}{\theta R - 1},$$

*then there exists a continuum of perfect-foresight equilibria in each of which the sequence of real balances is different. In each of these equilibria, the initial price level can be taken to be different. In addition, if $\alpha\beta > \max[1 + \theta^{-1}, (\theta R + 1)/(\theta R - 1)]$, there exists an infinite number of perfect-foresight equilibria in which the real allocation is equal to the optimal allocation for arbitrarily many periods.*

[13] Another difference between this fiscal regime and a balanced-budget rule is that under this fiscal regime a steady-state equilibrium in which $m^* \geq \hat{c} + g$ may exist. Specifically, this is the case if we impose the following restrictions on the parameters of the model: $(\alpha\beta - 1)(R - 1) \geq 0$ and $(1 + (\alpha\beta - 1)R)/\alpha \geq (M_{-1} + B_0)/(P_{-1}\sum_{t=0}^{\infty}\beta^t(T_t/P_t - g)) > 0$.

*On the other hand, if*

$$1 < \alpha\beta < \frac{\theta R + 1}{\theta R - 1},$$

*then the only perfect-foresight equilibrium is the steady-state equilibrium. Thus, the equilibrium price level is unique.*

**Proof.** See Appendix A.

Three implications of this proposition are noteworthy. First, unlike the case of a pure interest rate peg, if under an interest rate feedback rule the price level is indeterminate, so is the real allocation. Second, in the presence of a balanced-budget requirement the monetary authority can control the price level without directly controlling a monetary aggregate by following an interest rate feedback rule with an appropriately chosen inflation elasticity. Third, our result differs from that of existing studies on price-level determinacy under interest rate feedback rules in that indeterminacy arises not only under passive monetary policy ($\alpha\beta < 1$) but also under active monetary policy ($\alpha\beta > 1$). For example, Leeper (1991) finds that under active monetary policy, if an equilibrium exists, then it is unique.

In order to understand the difference between our result and Leeper's, it is important to take into account the behavior of the real interest rate in each of the two models. Leeper studies an endowment economy with separable preferences over consumption and real balances. In such a model, the equilibrium real interest rate is exogenous. To see this, note that the real interest rate is equal to the intertemporal marginal rate of substitution, which in equilibrium depends only on present and future endowments. Thus, given a constant endowment stream, the equilibrium nominal interest rate moves one-for-one with expected inflation. Suppose now that the public expects an increase in next period's inflation. Then the current nominal interest rate increases by the same amount. If the inflation elasticity of the feedback rule is greater than one, the current rate of inflation increases by less than the increase in the nominal interest rate and hence by less than next period's expected inflation. As a result, an increase in today's inflation is accompanied by an even higher increase in expected future inflation. Such an explosive path is inconsistent with a stationary equilibrium and therefore expectations-driven fluctuations are impossible.

By contrast, in our model the real interest rate is endogenous because the intertemporal marginal rate of substitution is a function of current and future consumption, which, being a cash good, depends on the nominal interest rate. To see why expectations of higher future rates of inflation can be self-fulfilling under active monetary policy, suppose that next period's inflation is expected to be above its steady-state level. By the feedback rule, this produces an increase in next period's nominal interest rate, which in turn depresses future consumption. Given current consumption, the decline in future consumption implies a lower current real interest rate. For some parameter configurations, the decline in the real interest rate outweighs the increase in expected inflation leading to a decline in this period's nominal interest rate. By the feedback rule, the decline in the current nominal interest rate must be associated with lower current inflation. If current inflation decreases by more than future inflation is expected to increase, the resulting path converges to the steady-state inflation rate – in an oscillating fashion – and can therefore be supported as an equilibrium outcome.

It is clear from this explanation that what is necessary for the change in the nominal interest rate to be dominated by movements in the real interest rate rather than in expected inflation is a high elasticity of consumption with respect to the nominal interest rate. It can be shown that the magnitude of this elasticity only depends on and is increasing in the leisure-to-consumption ratio $\theta R$. The following proposition, together with Proposition 3, shows that a leisure-to-consumption ratio greater than one is a necessary condition for indeterminacy to arise under active monetary policy.

**Proposition 4.** *Suppose that the steady-state leisure-to-consumption ratio*, $\theta R$, *is less than one. If the inflation elasticity of the interest rate feedback rule*, $\alpha\beta$, *satisfies*

$$0 < \alpha\beta < 1,$$

*then there exists a continuum of perfect-foresight equilibria in each of which the sequence of real balances is different. In each of these equilibria the initial price level can be taken to be different.*

*If*

$$\alpha\beta > 1,$$

*the only perfect-foresight equilibrium is the steady-state equilibrium. Thus, the equilibrium price level is unique.*

**Proof.** See Appendix A.

Fig. 2 summarizes the relation between price level determinacy, the elasticity of the interest rate feedback rule with respect to inflation, $\alpha\beta$, and the steady-state leisure-to-consumption ratio, $\theta R$. The case of greatest empirical interest is the one in which the leisure-to-consumption ratio exceeds one. In the context of our model, the leisure-to-consumption ratio equals the ratio of leisure to labor divided by the share of private consumption in GDP. In the real-business-cycle

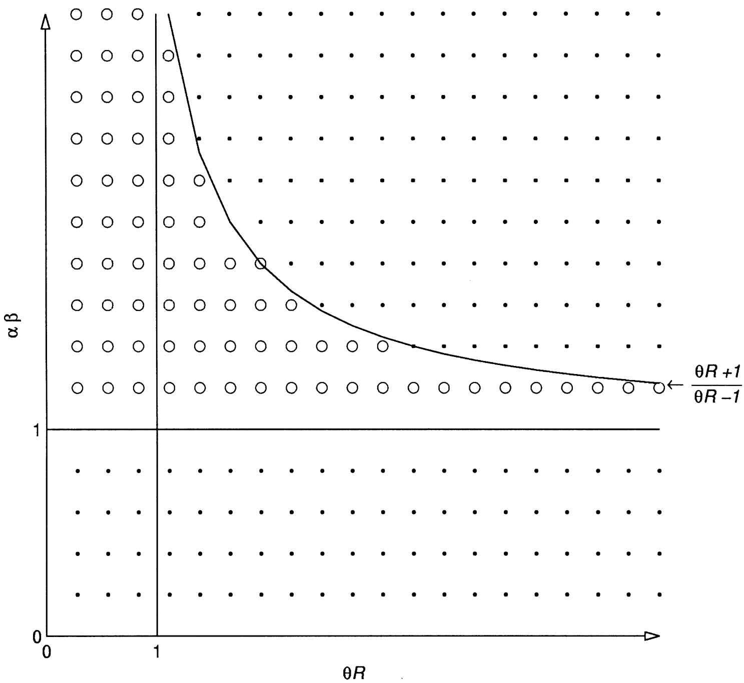


Fig. 2. Relationship between price level determinacy, the elasticity of the feedback rule with respect to inflation, $\alpha\beta$, and the leisure-to-consumption ratio, $\theta R$. ($\circ$) price level is determinate, ($\cdot$) price level is indeterminate.

literature, the value assigned to the leisure-to-labor ratio ranges from 2 and 4.[14] Thus, assuming a ratio of private consumption to GDP greater than 2/3 implies a range for $\theta R$ between 2 and 6. Consequently, the implied threshold of $\alpha\beta$ beyond which the equilibrium becomes indeterminate under active monetary policy, $(\theta R + 1)/(\theta R - 1)$, lies between 1.3 and 3. At the same time, empirical estimates of the inflation elasticity of the feedback rule take values between 1 and 2.[15] We interpret these back-of-the-envelope calculations as suggesting

[14] Based on the microeconomic evidence from time allocation studies reported in Ghez and Becker (1975) and Juster and Stafford (1991), Cooley and Prescott (1995) assign a value of 2 to the leisure-to-labor ratio. King et al. (1988) assume a value of 4, which is based on the empirical evidence that about 20 percent of total weekly hours are devoted to market work.

[15] For example, Orphanides (1998) reports values for the sensitivity of the feedback rule with respect to current inflation between 0.8 and 1.6 in a sample of US data beginning in 1987 and ending in 1993. Rotemberg and Woodford (1997) estimate a three-variable VAR using quarterly US data from 1980:1 to 1995:2 and find that the long-run sensitivity of the nominal interest rate with respect to the inflation rate is 2.13.

that in the US economy the adoption of a balanced-budget rule, if not accompanied by a change in the stance of monetary policy, may lead to the loss of the nominal anchor.

However, this conclusion should be interpreted with caution. In the first place, empirical estimates of interest rate feedback rules typically allow for partial adjustment, that is, the monetary authority is assumed to implement its target interest rate only slowly over time. This is important because if we were to allow for partial adjustment in our model, the threshold beyond which the equilibrium is indeterminate under active monetary policy would change. Second, a growing body of empirical work argues that US monetary policy may be better described by a forward-looking interest rate feedback rule, that is, a rule in which the current nominal interest rate is set as a function of expected future inflation. One can show that, in the context of our model economy, a forward-looking rule implies that the price level is indeterminate regardless of the inflation elasticity of the feedback rule. However, the resource allocation is determinate under passive monetary policy and indeterminate under active monetary policy provided the leisure-to-consumption ratio exceeds one, which is, as we argue, the case of greatest interest. Interestingly, empirical estimates of forward-looking feedback rules suggest that at least since the early 1980s US monetary policy has been active (e.g., Clarida et al., 1997, 1998; Orphanides, 1998). Thus, our model predicts that even under a forward-looking feedback rule, the adoption of a balance budget requirement, if not accompanied by a change in monetary policy, could entail both real and nominal indeterminacy.[16]

Before closing this section, we briefly discuss how the results change under alternative fiscal regimes. Consider first a specification of the balanced-budget requirement that allows for bounded secondary surpluses.[17] If the policy stipulates an exogenous path for nominal secondary surpluses, it can be shown that the results are unchanged. However, if the policy stipulates an exogenous path for real secondary surpluses that are sufficiently large in present discounted value, then under active monetary policy ($\alpha\beta > 1$) the steady-state equilibrium is no longer unique. Specifically, in addition to the steady-state equilibrium in

[16] In the context of a sticky-price model, Clarida et al. (1998) obtain the opposite result, namely, that the resource allocation is indeterminate under passive forward-looking feedback rules and determinate under active ones. The difference in results can be explained by the implication of their assumption of nominal rigidities for the interaction between output and inflation. Specifically, their model includes a forward-looking Phillips curve linking current output to current and expected future inflation. By contrast, our model implies a forward-looking Phillips curve linking current output only to expected future inflation. See Woodford (1996), Bernanke and Woodford (1997), and Benhabib et al. (1998a) for further analysis of the determinacy of equilibrium under interest-rate feedback rules in models with nominal rigidities.

[17] Secondary surpluses are not restricted to be positive.

which the nominal interest rate is positive ($R_t = R > 1$), there exist an infinite number of steady-state equilibria in which the nominal interest rate is zero ($R_t = 1$). The equilibrium price level is identical across all steady-state equilibria in which the nominal interest rate is zero, but different from the one associated with the steady-state equilibrium in which the nominal interest rate is positive.

A more complex picture arises when one considers non-steady-state equilibria. It can be shown that if the fiscal policy specifies a non-zero exogenous path for real secondary surpluses, then under active monetary policy the equilibrium is always indeterminate. In particular, there exist an infinite number of equilibrium sequences of nominal interest rates originating arbitrarily close to the steady state in which the nominal interest rate is positive that converge either to that steady state or to the steady state in which the nominal interest rate is zero. In addition, there may also exist equilibria in which the nominal interest rate fluctuates forever around $R$. Clearly, the existence of equilibria in which the interest rate converges to zero hinges critically on the assumed non-linear form of the interest rate feedback rule. Under the alternative assumption that the interest rate feedback rule (29) takes the form $R_t = R + \alpha(\pi_t - \beta R)$, equilibria in which the nominal interest rate converges to zero are impossible.[18] We conclude that, unlike the case in which the fiscal authority specifies a zero secondary surplus or an exogenous path for the nominal secondary surplus, when the fiscal policy consists of a non-zero exogenous sequence for the real secondary surplus, the price level is indeterminate regardless of the magnitude of the inflation elasticity of the feedback rule.

Finally, consider a fiscal regime in which the path of the real primary surplus is exogenous. The set of perfect-foresight equilibria that arises in this case is fundamentally different from that obtained under a balanced-budget rule. On the one hand, if monetary policy is active and in the intermediate range, $1 < \alpha\beta < (\theta R + 1)/(\theta R - 1)$, then as shown above under a balanced-budget rule the only perfect-foresight equilibrium is the steady-state equilibrium with $m^* < \hat{c} + g$, a positive nominal interest rate, and a suboptimal real allocation. By contrast, under a fiscal regime in which real primary surpluses are exogenous, such a perfect-foresight equilibrium does not exist. Instead the set of perfect-foresight equilibria includes – but may not be limited to – equilibria in which $m_t \geq \hat{c} + g$ for all $t$, the nominal interest rate is equal to zero, and the real allocation is optimal. On the other hand, if monetary policy is passive, $0 < \alpha\beta < 1$, then under a balanced-budget rule the perfect-foresight equilibrium is indeterminate, while under a fiscal policy in which real primary surpluses are exogenous, if an equilibrium exists, it is unique.[19]

[18] Similar implications of a zero lower bound on nominal interest rates for global stability apply to models with nominal rigidities (see Benhabib et al., 1998b).

[19] It is possible to show that in this case a perfect-foresight equilibrium always exists provided that $R(1 - \alpha\beta) > 1$.

## 6. Conclusion

In this paper, we argue that an important but largely ignored aspect of balanced-budget fiscal policy rules is their implication for nominal stability. For example, we show cases in which given the monetary policy regime the mere implementation of a balanced-budget rule may lead to the loss of the nominal anchor. Our findings complement our earlier work on the real consequences of balanced-budget rules, Schmitt-Grohé and Uribe (1997), where we show that in a standard neoclassical growth model without money a balanced-budget policy may lead to real instability by allowing for equilibria in which expectations of future income tax increases can be self-fulfilling. The study of the macroeconomic consequences of balanced-budget rules could be extended in several directions. First, it would be worth studying how the results of this paper are modified in a model augmented with nominal frictions such as sticky prices. Second, expanding the set of monetary policies might provide additional insights into the restrictions that the particular fiscal policy studied in this paper imposes on the conduct of monetary policy. The family of Taylor rules, that is, feedback rules whereby the nominal interest rate depends not only on current inflation but also on the output gap, are especially interesting from an empirical point of view.

## Appendix A

**Proof of Proposition 1.** We first prove the existence of the lower bound $\underline{m}$. Define

$$\underline{m} \equiv \inf\left\{ m \geq g \,\middle|\, F(m) \geq \beta/\mu \inf_{m' \geq g} G(m') \right\}.$$

By (A1), $\underline{m} > g$. Since $m \geq \hat{c} + g$ implies $F(m) = G(m) > \beta/\mu G(m)$, it follows that $\underline{m} < \hat{c} + g$. Suppose $m_t < \underline{m}$, then

$$F(m_t) < \beta/\mu \inf_{m' > g} G(m') \leq \beta/\mu E_t[G(m_{t+1})]$$

contradicting (26). Therefore, $m_t \geq \underline{m}$ in any equilibrium. Next we show the existence of the upper bound $\bar{m}$. Define

$$\bar{m} \equiv \max\left\{ \beta/\mu\hat{c} \sup_{\underline{m} \leq m \leq \hat{c}+g} G(m),\ \hat{c} + g \right\}.$$

The fact that $G(m)$ is continuous on the compact interval $[\underline{m},\hat{c}+g]$ implies that $\bar{m}<\infty$. The definition of $\bar{m}$ furthermore implies that for all $m$ in the interval $[\underline{m},\hat{c}+g]$,

$$G(m) \le \mu/\beta \frac{\bar{m}}{\hat{c}}. \tag{A.1}$$

Observe also that for all $\hat{c}+g<m\le\bar{m}$,

$$G(m) = \frac{m}{\hat{c}} \le \frac{\bar{m}}{\hat{c}} \le \mu/\beta \frac{\bar{m}}{\hat{c}}.$$

Thus (A.1) holds for all $m$ in the interval $\underline{m}\le m\le\bar{m}$. Suppose that at some date $m_t\ge\bar{m}$. Then, letting $P_t(x)$ denote the probability of the event $x$ conditional upon information available at time $t$, it follows that

$$\begin{aligned}
\frac{m_t}{\hat{c}} &= F(m_t) = \frac{\beta}{\mu}E_t[G(m_{t+1})] \\
&= \frac{\beta}{\mu}P_t(m_{t+1}\le\bar{m})E_t[G(m_{t+1})|m_{t+1}\le\bar{m}] \\
&\quad + \frac{\beta}{\mu}P_t(m_{t+1}>\bar{m})E_t[G(m_{t+1})|m_{t+1}>\bar{m}] \\
&\le P_t(m_{t+1}\le\bar{m})\frac{\bar{m}}{\hat{c}} + \frac{\beta}{\mu}P_t(m_{t+1}>\bar{m})E_t[m_{t+1}|m_{t+1}>\bar{m}]\frac{1}{\hat{c}} \\
&= \frac{\bar{m}}{\hat{c}} - P_t(m_{t+1}>\bar{m})\frac{\bar{m}}{\hat{c}} + \frac{\beta}{\mu}P_t(m_{t+1}>\bar{m})E_t[m_{t+1}|m_{t+1}>\bar{m}]\frac{1}{\hat{c}} \\
&\le \frac{\bar{m}}{\hat{c}} + \frac{\beta}{\mu}P_t(m_{t+1}>\bar{m})\frac{1}{\hat{c}}[E_t(m_{t+1}|m_{t+1}>\bar{m})-\bar{m}] \\
&= \frac{\bar{m}}{\hat{c}} + \frac{\beta}{\mu}E_t[\max(m_{t+1}-\bar{m},0)]\frac{1}{\hat{c}}.
\end{aligned}$$

Then for any $m_t\ge\underline{m}$, it follows that

$$E_t[\max(m_{t+1}-\bar{m},0)] \ge \mu/\beta \max(m_t-\bar{m},0).$$

Since $G(m_t)\ge(1/\hat{c})\max(m_t-\bar{m},0)$, it follows that $E_0G(m_t)\ge(1/\hat{c})(\mu/\beta)^t\max[m_0-\bar{m},0]$. Consider now the transversality condition (27). Since $0\le G(m)-F(m)\le G(\underline{m})-F(\underline{m})$ for all $m\ge\underline{m}$, it follows that

$\lim_{t\to\infty} \beta^t E_0[F(m_t) - G(m_t)] = 0$. Thus,

$$\lim_{t\to\infty} \beta^t E_0\left[ G(m_t)\frac{A_{-1}}{\mu^t} + [F(m_t) - G(m_t)]M_0 \right] \geq \max[m_0 - \bar{m}, 0]\frac{A_{-1}}{\hat{c}},$$

so that the transversality condition is violated if $m_0 > \bar{m}$. Therefore, in any equilibrium $m_0 \leq \bar{m}$. Since a transversality condition like (27) must hold not only in period 0 but also for all $t > 0$, it follows that $m_t \leq \bar{m}$ for all $t \geq 0$. □

The following lemma establishes the uniqueness of the perfect-foresight equilibrium and serves as the basis for the proof of Proposition 2.

**Lemma 1.** *If* $\mu > \beta$ *and assumption* (A2) *is satisfied, then the steady-state equilibrium is the unique perfect-foresight monetary equilibrium.*

**Proof.** If assumption (A2) holds, then $\bar{m} = \hat{c} + g$. Since in any equilibrium $m_t \leq \bar{m}$, (26) can be solved for $m_{t+1}$ as a function of $m_t$ to get

$$m_{t+1} = \frac{gm_t}{(1+\gamma)m_t - \gamma} \equiv f(m_t), \tag{A.2}$$

where $\gamma \equiv \beta/(\theta\mu)$. Since $\mu > \beta$ and $m_t$ is bounded, the transversality condition (27) is satisfied. Since assumption (A1) is satisfied whenever assumption (A2) holds, it follows that a steady-state equilibrium, $m_t = m^*$ for all $t$, exists, is unique, and satisfies $g < m^* < \hat{c} + g$. Suppose there exists a perfect-foresight equilibrium $\{m_t\}$ with $m_0 < m^*$. Let $\tilde{m}_t = m_{2t}$, $t \geq 0$; From (A.2), it follows that $\tilde{m}_{t+1} = (g^2\tilde{m}_t)/[(1+\gamma)(g-\gamma)\tilde{m}_t + \gamma^2]$. Assumption (A2) and the fact that $m_0 < m^*$ imply that $\{\tilde{m}_t\}$ is a monotonically decreasing sequence, and Proposition 1 implies that it must be bounded below by $\underline{m}$. Therefore, the sequence $\{\tilde{m}_t\}$ must converge to some $\tilde{m} \in [\underline{m}, m^*)$ satisfying $\tilde{m} = (g^2\tilde{m})/[(1+\gamma)(g-\gamma)\tilde{m} + \gamma^2]$. But the only non-zero solution to this equation is $\tilde{m} = (\gamma + g)/(1+\gamma) = m^*$, which is a contradiction. By a similar argument, one can show that there exists no perfect-foresight equilibrium with $m_0 > m^*$. Hence, the unique perfect-foresight equilibrium is $m_t = m^*$ for all $t$. □

**Proof of Proposition 2.** Suppose there exists a rational expectations equilibrium with $\underline{m} \leq m_t < m^*$ for some $t$. Let $m^P_{t+j} \equiv f^j(m_t)$, where $f(\cdot)$ is defined by (A.2). It follows that there exists an even integer $J$ such that for all $s \leq J-1$, $m^P_{t+s} \in [\underline{m}, \bar{m}]$ and $m^P_{t+J} < \underline{m}$. Note that $m^P_{t+1} > m^*$ and that $P(m_{t+1} \geq m^P_{t+1}) > 0$ because $E_t G(m_{t+1}) = G(m^P_{t+1})$ and $G(\cdot)$ is strictly decreasing on $[\underline{m}, \bar{m}]$. Let $\varepsilon_{t+s} \equiv P(m_{t+s} \geq m^P_{t+s} | m_{t+s-1} \leq m^P_{t+s-1})$, if $s$ is odd, and let $\varepsilon_{t+s} \equiv P(m_{t+s} \leq m^P_{t+s} | m_{t+s-1} \geq m^P_{t+s-1})$, if $s$ is even. The probabilities $\varepsilon_{t+s}$ are strictly positive for all $s \leq J$. To see this, assume first that $m_{t+s-1} \leq m^P_{t+s-1}$. Then, since

$F(m)$ is strictly increasing, $F(m_{t+s-1}) \leq F(m^P_{t+s-1})$. Using (26), this implies that $E_{t+s-1}G(m_{t+s}) \leq G(m^P_{t+s})$ and thus, since $G(\cdot)$ is strictly decreasing, $\varepsilon_{t+s} > 0$. Similarly, if $m_{t+s-1} \geq m^P_{t+s-1}$, then $E_{t+s-1}G(m_{t+s}) \geq G(m^P_{t+s})$ and hence $\varepsilon_{t+s} > 0$. Next, note that

$$\begin{aligned} P(m_{t+J} < \underline{m}) &\geq P(m_{t+J} \leq m^P_{t+J}) \\ &= \varepsilon_{t+J} P(m_{t+J-1} \geq m^P_{t+J-1}) \\ &\quad + P(m_{t+J} \leq m^P_{t+J} | m_{t+J-1} < m^P_{t+J-1}) P(m_{t+J-1} < m^P_{t+J-1}) \\ &\geq \varepsilon_{t+J} P(m_{t+J-1} \geq m^P_{t+J-1}) \\ &\geq \cdots \\ &\geq \prod_{s=2}^{J} \varepsilon_{t+s} P(m_{t+1} \geq m^P_{t+1}) \\ &> 0. \end{aligned}$$

But this is impossible according to Proposition 1. Similarly, one can show that $m_t > m^*$ leads to the contradiction $P(m_{t+J} < \underline{m}) > 0$ for some $J$. Hence, in any rational expectations equilibrium $m_t = m^*$ at all times. □

**Proof of Propositions 3 and 4.** Using the definitions of $F(\cdot)$ and $G(\cdot)$ Eq. (31) can be expressed as

$$\tilde{m}_{t+1} = \begin{cases} \max[a + b\tilde{m}_t, c + d\tilde{m}_{t+1}] & \text{if } 1 + \theta R(1 - \alpha\beta) < 0 \\ \min[a + b\tilde{m}_t, c + d\tilde{m}_{t+1}] & \text{if } 1 + \theta R(1 - \alpha\beta) > 0 \end{cases} \tag{A.3}$$

where $\tilde{m}_t \equiv \min[m_t, \hat{c} + g]$ and

$$a = \frac{(1 - \alpha\beta)(1 + \theta R g)}{1 + \theta R(1 - \alpha\beta)},$$

$$b = \frac{\alpha\beta}{1 + \theta R(1 - \alpha\beta)},$$

$$c = \frac{1 + \theta g}{1 + \theta R(1 - \alpha\beta)},$$

and

$$d = \frac{-\theta + \theta R(1 - \alpha\beta)}{1 + \theta R(1 - \alpha\beta)}.$$

Suppose $1 < \alpha\beta < (1 + \theta R)/(\theta R)$. Then $b > 1$ and (A.3) takes the form

$$\tilde{m}_{t+1} = \min[a + b\tilde{m}_t, c + d\tilde{m}_{t+1}]. \quad \text{(A.4)}$$

The upper left panel of Fig. 3 shows with solid lines the functions $a + b\tilde{m}_t$ and $\tilde{m}_t$. It indicates with a broken line the value of $\tilde{m}_{t+1}$ that satisfies (A.4) as a function of $m_t$. Since for $\tilde{m} < \hat{c} + g$, $c + d\tilde{m} > \tilde{m}$ and for $\tilde{m} = \hat{c} + g$, $c + d\tilde{m} = \tilde{m}$, $\tilde{m}_{t+1} = \hat{c} + g$ is a solution to (A.4) only if $a + b\tilde{m}_t \geq \hat{c} + g$. Suppose $m_0 < m^*$ and construct a sequence for $m_t$ that satisfies (A.4). After a finite number of periods, $a + b\tilde{m}_t < g$, so that no $\tilde{m}_{t+1} > g$ satisfying (A.4) exists. Alternatively, consider $m_0 > m^*$ and construct a sequence for $m_t$ that satisfies (A.4). After a finite number of periods $t'$, $\tilde{m}_t = \hat{c} + g$ for all $t > t'$. Such a path violates the transversality condition (20). To see this, note that for any $t > t'$, $F(m_t) = G(m_t)$ and $G(m_t)/M_t = \beta^{t'-t}G(m_{t'})/M_{t'}$; so that $\beta^t[G(m_t)A_{-1}/M_t + F(m_t) - G(m_t)] = \beta^{t'}A_{-1}G(m_{t'})/M_{t'} > 0$ for any $t > t'$. Therefore, if $b > 1$, the only perfect-foresight equilibrium is the steady-state equilibrium.

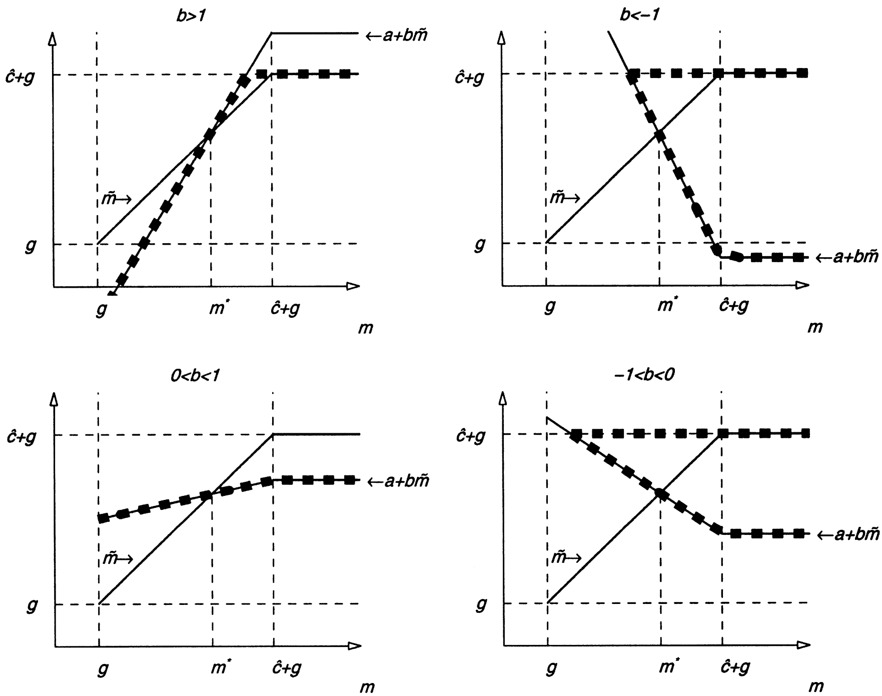


Fig. 3. Feedback rule: $R_t = \max[1, R + \alpha(\pi_t - \beta R)]$ $\tilde{m}_t \equiv \min[m_t, \hat{c} + g]$,

$$\blacksquare\,\blacksquare\,\blacksquare\,\blacksquare\,\blacksquare: \left\{\tilde{m}_{t+1} | \tilde{m}_{t+1} = \begin{cases} \max[a + b\tilde{m}_t, c + d\tilde{m}_{t+1}] & \text{if } b < 0 \\ \min[a + b\tilde{m}_t, c + d\tilde{m}_{t+1}] & \text{if } b > 0 \end{cases}\right\}.$$

Suppose now that

$$\frac{1+\theta R}{\theta R} < \alpha\beta < \begin{cases} \frac{\theta R+1}{\theta R-1}, & \text{if } \theta R > 1, \\ \infty, & \text{otherwise,} \end{cases}$$

so that $b < -1$. Then (A.3) takes the form

$$\tilde{m}_{t+1} = \max[a + b\tilde{m}_t, c + d\tilde{m}_{t+1}]. \tag{A.5}$$

The upper right panel of Fig. 3 illustrates this case. Note that for $\tilde{m} < \hat{c} + g$, $c + d\tilde{m} < \tilde{m}$ and that for $\tilde{m} = \hat{c} + g$, $c + d\tilde{m} = \tilde{m}$, so that $\tilde{m}_{t+1} = \hat{c} + g$ is a solution to (A.5) only if $a + b\tilde{m}_t \leq \hat{c} + g$. In any equilibrium it must be the case that $m_t > \underline{m} \equiv \max[g,(\hat{c} + g - a)/b]$ because for $m_t < \underline{m}$, $a + b\tilde{m}_t > \hat{c} + g$ and thus no $\tilde{m}_{t+1}$ that satisfies Eq. (A.5) exists. Consider $m_0 > \underline{m}$ different from $m^*$; the corresponding sequences of $\tilde{m}_t$ constructed from (A.5) must converge to $\hat{c} + g$. However, as just shown, such a sequence cannot be an equilibrium because it violates the transversality condition, (20). Therefore, if $b < -1$, the only perfect-foresight equilibrium is again the steady-state equilibrium.

If $\alpha\beta > (1 + \theta R)/(\theta R - 1) > 1$, then $-1 < b < 0$ and $1 + \theta R(1 - \alpha\beta) < 0$. Thus, (A.3) takes the form

$$\tilde{m}_{t+1} = \max[c + d\tilde{m}_{t+1}, a + b\tilde{m}_t]. \tag{A.6}$$

This case is shown in the lower right panel of Fig. 3. Choose $m_0 \in (\underline{m},\hat{c} + g)$, where $\underline{m} \equiv \max[g,(\hat{c} + g - a)/b] < \hat{c} + g$, and construct a sequence for $m_t$ from $m_{t+1} = a + bm_t$. Since $|b| < 1$ the so constructed sequence of real balances converges to $m^*$. It is straightforward to show that this sequence satisfies (20). This sequence is also a solution to (A.6): note first that for any $t$, $m_t < \hat{c} + g$; further, $0 > b > -1$ implies that $c + dm < m$ for $m < \hat{c} + g$. From (32) one can find the level of nominal balances, $M_0$, associated with a particular choice of $m_0$. Since $R(1 - \alpha\beta) < 0$ whenever $b < 0$, and $G(m_0)/F(m_0) > 1$ for $m_0 \in (\underline{m},\hat{c} + g)$, there exists a unique positive $M_0$ that satisfies (32) and a unique $P_0$ $(= [G(m_0)/F(m_0) - R(1 - \alpha\beta)]P_{-1}/\alpha)$. Therefore, there exists a continuum of perfect-foresight equilibria indexed by $m_0$. Since $G(m_0)/F(m_0$ is monotonically decreasing for $m_0 \in (g,\hat{c} + g)$, in each of these the price level can be taken to be different. If in addition $\alpha\beta > 1 + \theta^{-1}$, then for any $m_0 > \underline{m}$ there exist multiple solutions to (A.6), that is, $m_{t+1} \geq \hat{c} + g$ as well as $m_{t+1} = a + b\tilde{m}_t < \hat{c} + g$ satisfy (A.6). Consider the following sequence: $m_t \geq \hat{c} + g$ for $t$ even and $m_t = a + b\tilde{m}_{t-1}$ for $t$ odd. For this sequence to be an equilibrium it must satisfy the transversality condition. To see that it does, note that (15) and (16) imply that $M_t/G(m_t) = \beta^t M_0/F(m_0)\prod_{s=1}^{t-1} R_s$. Also, note that $F(m) - G(m)$ is bounded for $m > \underline{m}$. It follows that

$$\lim_{t\to\infty} \beta^t[G(m_t)A_{-1}/M_t + F(m_t) - G(m_t)] = \lim_{t\to\infty} \frac{\beta^t A_{-1}F(m_0)}{\beta^t M_0 \prod_{s=1}^{t-1} R_s}$$

$$= \lim_{t\to\infty} \frac{A_{-1}F(m_0)}{M_0} \prod_{s=1}^{t-1} R_s^{-1}$$

$$= 0.$$

The last equality follows from the fact that for any $t \exists t'$ such that $m_{t'} < \hat{c} + g$, so that $R_{t'} > 1$. This implies that $1/\prod_{s=1}^{t-1} R_s$ is decreasing over time and converges to zero. Therefore the transversality condition is satisfied. Finally, as shown above, the fact that $b < 0$ ensures that the nominal money supply associated with any $m_t > \underline{m}$ is positive.

If $0 < \alpha\beta < 1$, then $0 < b < 1$ and $1 + \theta R(1 - \alpha\beta) > 0$. Thus, (A.3) takes the form

$$\tilde{m}_{t+1} = \min[a + b\tilde{m}_t, c + d\tilde{m}_{t+1}].$$

Further, for any $m > g$, $a + b\tilde{m} < \hat{c} + g$ and $c + d\tilde{m} \geq \hat{c} + g$; so sequences of real balances that satisfy (A.3) are the same as those that satisfy

$$\tilde{m}_{t+1} = a + b\tilde{m}_t. \tag{A.7}$$

The lower left panel of Fig. 3 plots this case. Eq. (A.7) implies a sequence for $m_t$ that converges monotonically to $m^*$ given any $m_0 > g$. It is straightforward to show that any such sequence satisfies the transversality condition. From (32) one can find the level of nominal balances, $M_0$, associated with a particular choice of $m_0$. A requirement of equilibrium is that $M_0 > 0$. If $R(1 - \alpha\beta) < 1$, $M_0$ is positive for any $m_0 > g$. If, on the other hand, $R(1 - \alpha\beta) \geq 1$, the initial money supply, $M_0$, associated with a particular choice for $m_0$ is positive only if $m_0 < \bar{m} \equiv \frac{1 + g\theta R(1-\alpha\beta)}{1 + \theta R(1-\alpha\beta)} \in (m^*, \hat{c} + g)$. Note that for any $m_0 < \min[\bar{m}, \hat{c} + g]$, the initial price level $P_0$ is given by $[G(m_0)/F(m_0) - R(1 - \alpha\beta)]P_{-1}/\alpha$ and that $G(m)/F(m)$ is monotonically decreasing in $m$, which implies that the initial price level is a monotonically decreasing function of $m_0$. Since $\min[\bar{m}, \hat{c} + g] > m^* > g$, there exists a continuum of perfect-foresight equilibria in each of which the sequence of real balances and prices can be taken to be different.

Finally, note that if $1 + \theta R(1 - \alpha\beta) = 0$, then (31) becomes $0 = \max[(1 + \theta)\tilde{m}_{t+1} - (1 + \theta g), (1 + g\theta R)/(\theta R) - (1 + \theta R)/(\theta R)\tilde{m}_t]$. The only solution to this expression that does not violate the transversality condition is $m_t = m^*$ for all $t$. □

## References


Auernheimer, L., Contreras, B., 1990. Control of the interest rate with a government budget constraint: determinacy of the price level and other results. Texas A&M University, unpublished manuscript.

Benhabib, J., Schmitt-Grohé, S., Uribe, M., 1998a. Monetary policy and multiple equilibria. New York University, unpublished manuscript.

Benhabib, J., Schmitt-Grohé, S., Uribe, M., 1998b. The perils of Taylor rules. New York University, forthcoming, Journal of Economic Theory.

Bernanke, B., Woodford, M., 1997. Inflation forecasts and monetary policy. Journal of Money, Credit, and Banking 29, 653–684.

Clarida, R., Galí, J., Gertler, M., 1997. Monetary policy rules and macroeconomic stability: evidence and some theory. New York University, unpublished manuscript.

Clarida, R., Galí, J., Gertler, M., 1998. Monetary policy rules in practice: some international evidence. European Economic Review 42, 1033–1067.

Cooley, T.F., Prescott, E.C., 1995. Economic growth and business cycles. In: Cooley, T.F. (Ed.), Frontiers of Business Cycle Research, Princeton University Press, Princeton, pp. 1–38.

Ghez, G., Becker, G.S., 1975. The allocation of time and goods over the life cycle. National Bureau of Economic Research, New York.

Juster, F.T., Stafford, F.P., 1991. The allocation of time: empirical findings, behavior models, and problems of measurement. Journal of Economic Literature 29, 471–522.

King, R.G., Plosser, C., Rebelo, S., 1988. Production, growth and business cycles: I. The basic neoclassical model. Journal of Monetary Economics 21, 195–232.

Leeper, E., 1991. Equilibria under 'active' and 'passive' monetary and fiscal policies. Journal of Monetary Economics 27, 129–147.

Lucas, R.E. Jr., Stokey, N., 1987. Money and interest in a cash-in-advance economy. Econometrica 55, 491–514.

Orphanides, A., 1998. Monetary policy rules bases on real-time data. Finance and Economics Discussion Series No. 1998-03, Board of Governors of the Federal Reserve System, Washington, DC.

Rotemberg, J.J., Woodford, M., 1997. An optimization-based econometric framework for the evaluation of monetary policy. In: Bernanke, B.S., Rotemberg, J.J. (Eds.), NBER Macroeconomics Annual 1997, MIT Press, Cambridge, pp. 297–346.

Sargent, T.J., Wallace, N., 1975. 'Rational' expectations, the optimal monetary instrument, and the optimal money supply rule. Journal of Political Economy 83, 241–254.

Schmitt-Grohé, S., Uribe, M., 1997. Balanced-budget rules, distortionary taxes, and aggregate instability. Journal of Political Economy 105, 976–1000.

Sims, C., 1994. A simple model for the study of the determination of the price level and the interaction of monetary and fiscal policy. Economic Theory 4, 381–399.

Taylor, J.B., 1993. Discretion versus policy rules in practice. Carnegie-Rochester Conference Series on Public Policy 39, 195–214.

Woodford, M., 1994. Monetary policy and price level determinacy in a cash-in-advance economy. Economic Theory 4, 345–380.

Woodford, M., 1995. Price-level determinacy without control of the monetary aggregate. Carnegie-Rochester Conference Series on Public Policy 43, 1–46.

Woodford, M., 1996. Control of the public debt: A requirement for price stability? NBER working paper 5684.