Redistricting and Representation: The Paradox of Minority Power*

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Abstract

We link partisan and racial gerrymandering to electoral and legislative outcomes. Candidates compete in primaries and general elections, offering ideological and distributive benefits to heterogeneous voters. Redistricting generates systematic trade-offs: concentrated minority districts increase descriptive representation but reduce distributive leverage, while dispersed minorities gain substantive representation as pivotal swing groups but risk losing preferred candidates. Crossover voting by nonminority voters introduces sharp non-linearities; it enhances minority electoral success while increasing majority influence over policy. We document a U-shaped relationship between minority influence and minority concentration. Our framework offers a unified foundation for the paradox of packing, cracking, and minority representation.

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1 Introduction

Redistricting determines not only which candidates are elected, but also how district-level electoral incentives influence the distribution of policy benefits across voter groups. These concerns underpin two central themes in the political economy literature: voter representation and the distribution of policy benefits. The first concerns electoral competition, focusing on who wins office and which constituencies are represented. The second concerns the distribution of policy benefits, focusing on how public resources are allocated and which groups gain. These two dimensions are inherently linked because district boundaries shape candidates' incentives in electoral competition. Yet much of the existing literature treats them separately, overlooking the potential trade-offs between electoral outcomes and policy incentives that arise from redistricting.

We bridge this gap with a model of electoral competition and policy targeting that builds on the Dixit–Londregan framework of distributive politics, extended to a multi-district setting with heterogeneous voters and redistricting. In this environment, redistricting lies at the intersection of electoral competition and public policy. It shapes who gets elected, capturing descriptive representation, and how electoral incentives translate into policy commitments, capturing substantive representation. Although the framework applies broadly, we focus on minority representation to highlight these trade-offs.

Two central questions follow. First, given a fixed demographic distribution and electoral competition over candidates and policies, are minority voters better off when concentrated in a few districts or spread across many? Second, is electing minority legislators essential for advancing minority interests, or can substantive representation be achieved without descriptive representation?

The model yields three main results. First, the extent to which candidates compete for minority voters' support depends nonlinearly on the distribution of those voters across districts. When minority voters have limited influence over policy competition, concentrating them into safe districts maximizes expected utility by increasing the likelihood of electing preferred candidates. When minority voters' support responds more strongly to policy incentives, dispersing them across districts intensifies policy competition and increases distributive benefits. Second, the optimal redistricting strategy depends on how voter responsiveness interacts with primary election rules and district composition. Third, once aggregate legislative influence is taken into account, redistricting involves a trade-off between securing descriptive representation through seat concentration and securing substantive representation through heightened policy competition.

Our contribution is twofold. Substantively, we provide a unified framework for analyzing descriptive and substantive minority representation in electoral competition under redistricting. Methodologically, we generalize the Dixit–Londregan model to a setting with identity, primaries, and endogenous district composition, and derive transparent comparative statics for redistricting choices. The analysis clarifies the conditions under which majority–minority districts enhance minority political power and the conditions under which alternative districting strategies may be more effective.

The remainder of the paper proceeds as follows. The following section situates our analysis within the broader literature on redistricting and representation. Section 3 presents the baseline Dixit–Londregan model and characterizes equilibrium policy competition. Section 4 introduces districts and redistricting. Section 5 incorporates parties and primary elections. Section 6 analyzes optimal redistricting and comparative statics. Section 7 concludes with a discussion of the implications of our analysis for minority representation and redistricting policy.

2 Related Literature: Redistricting and Representation

This review examines empirical and theoretical work on redistricting and representation, highlighting the link between electoral politics and legislative outcomes. Most research on the political and economic impacts of redistricting falls into two strands: studies focusing on partisan politics and studies on racial or ethnic minority groups.

The former research examines how partisan redistricting creates seat–vote curve biases (Tufte, 1973; King, 1989; Gelman and King, 1990; Lublin et al., 2020), incumbent protection (Cox and Katz, 2002; Ansolabehere and Snyder Jr., 2004), and party power consolidation (Butler and Cain, 1991; Issacharoff, 2002; Persily, 2002). The literature on racial redistricting, by contrast, examines Black officeholding in the South (Davidson and Grofman, 1994), the trade-off between descriptive and substantive representation (Cameron et al., 1996; Epstein and O'Halloran, 1999; Lublin, 1997b), its role in partisan shifts in Congress (Lublin and Voss, 2000), and patterns of polarization and segregation (Stephanopoulos, 2016).[1]

Empirical evidence highlights persistent tradeoffs. Jeong and Shenoy (2022) document the "packing-and-cracking" of African American voters, showing how Republican redistricting segre-

[1] Relatedly, Canon (2022) and Ross II (2024) analyze the legal evolution of descriptive and substantive representation from a political and legal perspective, respectively.

gates minority voters into Black-majority districts. Cameron et al. (1996) and Lublin (1997a) argue that such concentration can diminish minorities' substantive policy impact, while Canon (1999) emphasizes descriptive representation and the behind-the-scenes influence minority legislators can wield.[2]

A key point of contention is the willingness of nonminorities to crossover and vote for minority candidates. Ansolabehere et al. (2010) argue it is prevalent, while Lublin et al. (2020) claims it is declining, implying that minority leverage depends more on voter composition than on crossover support. Regardless, the resulting districts are often highly polarized, and the empirical record provides little consensus on optimal districting strategies. This ambiguity has only grown with the Supreme Court's reluctance to review partisan gerrymandering claims, making most challenges legally hinge on race (Tofighbakhsh, 2020).

Formal models of redistricting emphasize partisan rather than racial divisions. Early work analyzes majority-party seat maximization (Musgrove, 1977; Owen and Grofman, 1988). Gul and Pesendorfer (2010) focus on two-party competition and how each party can shape gerrymandering in the districts they control. Coate and Knight (2007) offer a more nuanced framework where partisans and independents evaluate districting schemes that maximize social utility, while Bouton et al. (2023) analyze how turnout differences shape gerrymandering incentives. Kolotilin and Wolitzky (2024) model districting under uncertainty, showing how designers segregate voters into weak and strong districts through partisan packing and cracking. Moscariello (2025) considers the endogenous selection of candidates via primaries and voters' ideological intensities in partisan gerrymandering. These models emphasize partisanship and ideological preferences as the key axis of competition. By contrast, our analysis focuses on ideological and distributive preferences, showing how packing and cracking also emerge from competition over candidate selection and legislative benefits—factors that may diverge from partisanship alone.

Explicit models of racial redistricting are less common. Shotts (2001) studies partisan control of redistricting with observable voter identities, finding that majority-–minority district requirements constrain conservative but not liberal gerrymanders. Friedman and Holden (2008) analyze optimal redistricting with noisy signals of voter preferences, showing that cracking is never optimal. Although they do not consider the Voting Rights Act, their results suggest that spreading strong partisan minority voters across districts is inefficient. Our analysis differs by explicitly

[2] Issacharoff (2002); Grofman et al. (1992); Epstein and O'Halloran (2006) caution, however, that such influence is difficult to measure, suggesting that Section 5 of the Voting Rights Act may have undermined minority power while increasing Republican representation. See Epstein and O'Halloran (1999).

incorporating both voter partisanship and identity, and by focusing on the influence voter groups exert in electoral competition and in shaping distributive outcomes.

Finally, our work connects to the broader literature on distributive politics. Myerson (1993) develops a model of strategic competition in which groups seek rents at the expense of others. Lindbeck and Weibull (1987, 1993) emphasize how altruism, risk aversion, and preferences for redistribution affect policy outcomes. Dixit and Londregan (1996) incorporate income inequality across groups, showing how differences in average income influence the allocation of benefits—a framework we extend to account for partisan and racial identity. By distinguishing between descriptive and substantive representation, our model uncovers new packing-and-cracking patterns that operate along both ideological and distributive dimensions.

These tensions are reflected in redistricting law and civil rights jurisprudence. The Voting Rights Act of 1965 encouraged the creation of majority–minority districts to secure descriptive representation, while subsequent court decisions have grappled with the trade-off between descriptive and substantive representation.[3] Our analysis provides a theoretical benchmark for interpreting these debates, clarifying how districting strategies that concentrate or disperse minority voters affect descriptive and substantive representation through electoral incentives and legislative influence.

This rich literature provides insights into partisan incentives, racial representation, and distributive politics. However, these elements have yet to be integrated into a single framework that links voter allocation by race and party to both electoral outcomes and legislative redistribution. Our analysis fills this gap by providing a unified account of how redistricting strategies shape the trade-offs between descriptive and substantive representation.

3 General Model

We now move to a systematic analysis of redistricting and representation, specifically from the perspective of minority voters' benefits, and adapt the Dixit and Londregan (1996) model of

[3] The tradeoff between descriptive and substantive representation has been central to the evolution of redistricting law. In *Georgia v. Ashcroft* (2003), the Court accepted the logic of "unpacking," reasoning that spreading minority voters could expand overall policy influence even at the expense of electing fewer minority representatives. In *Cooper v. Harris* (2017), the Court struck down racial "packing," emphasizing that concentration does not always enhance minority power, particularly when white crossover voting reduces polarization. Finally, in *Rucho v. Common Cause* (2019), the Court held that partisan gerrymandering was nonjusticiable, effectively separating racial from partisan claims in law even as they remain intertwined in practice. Empirically, Gimpel et al. (2021) show that redistricting litigation has increasingly shifted from federal to state courts, concentrated in regions with large minority populations, underscoring how geography and partisanship structure these trade-offs.

electoral competition.[4] Voters ascribe ideological attachments to different candidates, and these candidates then compete for office by promising group-specific policy benefits. This model is well-suited to our purposes: it captures that voters of one identity may prefer representatives of the same identity and candidate competition over policy outcomes.

To preview the main steps of the model, consider: 1) Democratic primary where minority and nonminority Democrats choose between candidates; 2) a general election between the Democratic primary winner and a Republican candidate; 3) for an election, candidates announce redistributive platforms targeted to each group within the district; 4) voters derive utility from redistributive benefits through the legislature and from ideological alignment with their legislator; and 5) an optimal districting map maximizes minority group utility across all districts.[5] Formally, our model provides a mapping from a districting scheme on a two-dimensional simplex, $\mathbf{D} = \mathbf{D}(S^2)$, to electoral outcomes $\mathbf{L} = \mathbf{L}(\mathbf{D}(S^2))$, and then to legislative outcomes with distributive benefits for districts and voter groups, $\mathbf{P} = \mathbf{P}(\mathbf{L}(\mathbf{D}(S^2)))$.

3.1 Districts

Assume a population of voters, V, divided into a given number of identifiable groups Θ; these may be defined according to voters' ethnicity, language, economic status, religion, political party, etc. Thus, there is a partition from the set of voters V to groups, $\nu : V \rightarrow \Theta$.

For simplicity, we divide a state population along ethnic and partisan lines with voter types $\Theta = i \in \{mD, nD, R\}$, for minority-Democrats, nonminority-Democrats, and Republicans, respectively. Their statewide populations are $\mathbf{N}_{mD}$, $\mathbf{N}_{nD}$, and $\mathbf{N}_R$, with $\sum_i \mathbf{N}_i = \mathbf{N}$, the total state population. Since population proportions must sum to 1, we represent the mix of voter types statewide—or in any given district—as a point in the two-dimensional simplex, S^2, as illustrated in Figure 1.

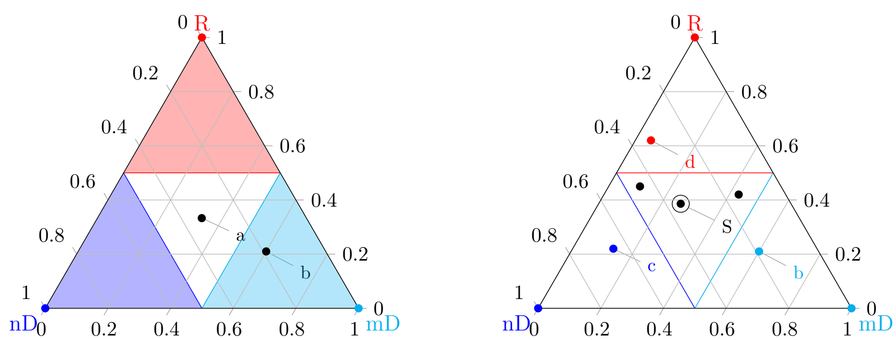
A district is a vector $\mathbf{d} = (N_{mD}, N_{nD}, N_R)$ of voters with $N_i \geq 0$. Let $\mathcal{D}$ be the set of all possible districts, and assume that the state will be divided into K districts with N_{ik} representing the number of voters of type i in district k. We denote the number of all voters in district k with N_k. Then a districting scheme is a function $\mathbf{D} : S^2 \rightarrow \mathcal{D}^K$, yielding a list $(\mathbf{d}_1, \mathbf{d}_2, ..., \mathbf{d}_K)$ of districts. Furthermore, a *valid* districting scheme is one that in any given district, $\sum_i N_{ik} = N_k = \mathbf{N}/K$, and across districts $\sum_k N_{ik} = \mathbf{N}_i$ for all voter types i – i.e., all districts are equally sized, and all

[4] We can also embed the Baron and Ferejohn (1989) model of legislative bargaining as derived in Appendix A.2.

[5] We maximize minority utility to set a benchmark to evaluate alternative voter allocations in the numerical simulations below.

voters are assigned to a district.[6]

Figure 1(a) shows an equilateral triangle representing the two-dimensional simplex S^2, which depicts the possible shares of each group in the electorate. The corners indicate electorates with only one type of voter: nD in the bottom left, mD in the bottom right, and R at the top. Center point (a) is an electorate with an equal division of all three types, each comprising one-third of the district population. The triangle can also be divided into four smaller regions: the bottom left with nD majorities, the bottom right with mD majorities, the top with R majorities, and the center with no single-group majority. Hence, point (b) denotes a majority—minority electorate, while the central region reflects electorates where Democrats hold a majority through the combined share of mD and nD voters. Figure 1(b) illustrates a state with five districts.[7] The statewide distribution of voters is marked by point (S), while the other five points represent the districts, one of which is majority–minority.



(a) Possible Electorates. (b) Sample State with Five Districts.

Figure 1: Possible Electorates and Districts.

3.2 Candidates and Elections

Suppose in each of the K districts, three candidates are competing for a seat in the legislature; these candidates are also of types $\theta = j \in \{mD, nD, R\}$. Candidates maximize their vote share by offering platforms that allocate a proportion T_i of the district's redistributive benefits as transfers to voters of type, capturing *substantative representation*. Let T_{ijk} denote the redistributive transfer from candidate j to group i in district k. Campaign platforms must satisfy $\sum_i T_{ijk} = 1$ for each j and k.[8]

[6]Equivalently, as in the triangle analysis, the average of the percentages of each group in the K districts must equal their statewide population proportion $\mathbf{N_i}/\mathbf{N}$. See the numerical example in Appendix A.1 for illustrations.

[7]Numerical values for Figure 1(b) are provided in Appendix A.1.

[8]We assume both parties and all candidates have equal abilities to distribute benefits, following Dixit and Londregan (1996). Without loss of generality, the model could be extended to allow candidates to vary the marginal

Candidates attain office through a two-stage electoral cycle. First, each district holds a primary election, in which the mD candidate faces an nD opponent. Second, the primary winner competes in the general election against a Republican. We consider closed and open primaries and discuss how candidates' platforms may shift from the primary to the general election stage.[9]

Represent each candidate by a vector $c = (\theta, T_{mD}, T_{nD}, T_R)$, where θ indicates the candidate's type, and $\mathcal{C}$ is the set of all possible candidates. Let $\mathbf{c}_k$ denote three candidates from district k, and $\mathbf{C} = \{\mathbf{c}_1, \mathbf{c}_2, ..., \mathbf{c}_K\}$ be the set of all (3K) candidates across districts. An election is then defined as a mapping from districting schemes and candidate sets to a legislature, $\mathbf{L} : \mathcal{D}^K \times \mathcal{C}^{3K} \rightarrow \mathcal{C}^K$, which assigns to each district a winning representative with a given type and platform.

To smooth out the response functions, we assume probabilistic voting so that the probability a candidate wins a given election rises with the expected proportion of votes she receives. Given expected vote proportion v, let the probability of winning the election be $\Psi(v)$, with $\Psi' > 0$, $\Psi(0) = 0$, $\Psi(1) = 1$, and $\Psi(1 - v) = 1 - \Psi(v)$. We assume here the simplest linear function $\Psi(v) = v$, so that, for instance, a candidate expecting to receive 60% of the vote wins with a 60% probability.[10]

The winners of the K district elections then go to a legislature $\mathbf{L} \in \mathcal{C}^K$. Considering candidates' equilibrium strategies, elections transform a districting scheme into a legislature; that is, $\mathbf{L} = \mathbf{L}(\mathbf{D}(S^2))$.

3.3 Legislative Policies

The legislature then passes a redistributive policy $\mathbf{P}$, dividing K dollars across all districts. Funds allocated to district k are distributed according to the platform of the district's representative. If the type j representative from district k ran on a platform promising T_{ijk} to group i, then members receive $T_{ijk} * B_k$ in total benefits, with individual benefits $b_{ijk} = (T_{ijk} * B_k)/N_{ik}$. Under a Baron and Ferejohn (1989)-closed-rule bargaining process, each legislator (and hence each district) receives one dollar in expected terms, i.e., $B_k = 1$.[11]

value of spending across programs according to voter groups' marginal utilities. We return to this possibility below.

[9] We ignore commitment or credibility problems across election stages and allow for empirically observed strategic platform adjustments by candidates across election rounds (di Tella et al., 2023).

[10] The qualitative results derived below do not depend on our assumption of probabilistic voting.

[11] See Appendix A.2 for the derivation of this assumption.

3.4 Voters

Voters derive distributive benefits from legislative outcomes and ideological benefits from elected candidates. We follow Dixit and Londregan (1995, 1996) in modeling utilities where voters from group i receive utility $U_i(.)$ from consumption and an ideological attachment to winning candidates.[12] In particular, assume that the utility from consumption, b, is given by:

$$U_i(b) = \kappa_i \frac{b^{1-\epsilon}}{1-\epsilon} \tag{3.1}$$

with $\epsilon > 0$ and $\epsilon \neq 1$, where ϵ reflects the degree of diminishing returns in consumption and κ_i the relative weight on consumption relative to ideological benefits. Then the marginal utility of an additional dollar of consumption and the return to consumption are

$$U_i'(b) = \kappa_i b^{-\epsilon} > 0 \text{ and } U_i''(b) = -\epsilon\kappa_i b^{-\epsilon-1} < 0. \tag{3.2}$$

As b increases from 0 to ∞, the marginal utility falls from ∞ to 0, and this assumption avoids corner solutions. A one percent increase in b causes an ϵ percent decrease in marginal utility, so ϵ captures the degree of diminishing returns in consumption.[13] Furthermore, the parameter κ_i captures the relative weight of consumption to ideological benefits for voter group i; higher values of κ_i imply that voters of group i are more responsive to distributive than ideological benefits. Together, ϵ and κ_i characterize the trade-offs between economic and ideological benefits.

Voters' ideological benefits depend on their district's winning candidate and are described by X^j for a candidate of type j, illustrating *descriptive representation*. The overall utility for a voter of type i a representative of type j offering distributive benefits b_{ij} is the sum of their ideological and distributive benefits: $U_i = X_i^j + E[U_i(b_{ij})]$. Thus, for instance, a voter with ideological preference of X^{mD} for minority-Democratic candidates and X^R for Republicans gets extra utility $X^{mD} - X^R \gtreqless 0$ from seeing a minority-Democrat win office instead of a Republican. The voter with a positive ideological gain will, therefore, prefer the minority-Democrat candidate unless the

[12] Lindbeck and Weibull (1987) also consider utility functions with additively separable benefits from consumption and ideological benefits, and with positive but decreasing marginal utility of consumption.

[13] In other words, voters trade off distributive (consumption) benefits against ideological attachment. Sensitivity to distributive transfers depends on ϵ: voters with low ϵ remain highly responsive to transfers, even when they receive many, while voters with high ϵ are less responsive. It is therefore easier and less costly to sway voters with a lower ϵ.

Republican offers her a sufficiently greater consumption value:

$$E[U_i(b_{iR})] - E[U_i(b_{imD})] > X^{mD} - X^R. \tag{3.3}$$

We define the critical value, or "cutpoint" X_i for group i in an election between two candidates labeled 1 and 2 by:

$$X_i^e \equiv U_i(b_{i1}) - U_i(b_{i2}), \tag{3.4}$$

where e indicates the type of election being contested– i.e. a primary election or a general election with either mD vs. R or nD vs. R candidates. Voters are assumed to cast their ballots sincerely for the candidate offering them higher utility. Then, group i voters with values of X_i less than X_i^e will vote for Candidate 1, while the others will vote for Candidate 2. If Candidate 1 offers an additional dollar to each member of the group i, then the critical value will shift in her favor by $U_i'(b_{i1}) = \kappa_i b_{i1}^{-\epsilon}$.[14]

Votes Let Φ_i^e be the concave cumulative distribution of voters of a group i in an election of type e, so that, given the campaign platforms, a proportion $\Phi_i^e(X_i)$ will vote for candidate 1. Given N_i voters of type i, this candidate will receive $N_i\Phi_i^e(X_i)$ votes from group i, with total votes of:

$$V_1^e = \sum_{i\in\Theta} N_i \Phi_i^e(X_i). \tag{3.5}$$

The opposing candidate will then get votes of:

$$V_2^e = \sum_{i\in\Theta} N_i[1 - \Phi_i^e(X_i)] = N - V_1^e. \tag{3.6}$$

Crossover Voting. The distribution functions $\Phi_i^e(X_i)$ play an important role in the following analysis. They indicate the ideological preference of a given voter i for one candidate over another. These preferences could arise partly from a spatial policy model, measuring the degree to which voters agree with the policy choices of their representatives. But they could also arise from group voting preferences: voters might want to support candidates of one type θ over those of another type. In the legal literature, this is what is meant by polarized voting: the willingness, or lack thereof, of voters to cross over and vote for candidates of another race and ethnicity. We assume

[14]We assume throughout that voter groups may differ in their consumption and ideological preferences but neglect within group-differences across districts.

Election	Candidate / Group	mD	nD	R
Primary,	mD	a^1_{mD}	$1 - a^1_{mD}$	
$e = 1$	nD	a^1_{nD}	$1 - a^1_{nD}$	
open primary	R	a^1_R	$1 - a^1_R$	
General mD **vs.** R,	mD	a^2_{mD}		$1 - a^2_{mD}$
$e = 2$	nD	a^2_{nD}		$1 - a^2_{nD}$
	R	a^2_R		$1 - a^2_R$
General nD **vs.** R,	mD		a^3_{mD}	$1 - a^3_{mD}$
$e = 3$	nD		a^3_{nD}	$1 - a^3_{nD}$
	R		a^3_R	$1 - a^3_R$

Table 1: Crossover Rates.

for simplicity that if the distribution of type i voters in the entire population is $\Phi^e_i(.)$, then this is also the distribution of the type i voters in any given district.[15]

Notice that the rates at which different types of voters cast their ballots for various candidates are given by the $\Phi^e_i(0)$ functions for group i in an election of type e, where for convenience we label the primary as election $e = 1$, a general election of mD vs. R as type $e = 2$, and a general election of nD vs. R as $e = 3$. For instance, in an mD vs. nD primary, a proportion $\Phi^1_{mD}(0)$ of minority voters will vote for the minority candidate, and the remaining $1 - \Phi^1_{mD}(0)$ will vote for the nonminority-Democrat candidate.

We redefine these quantities as *crossover rates*, following the usual standard for voting studies, letting a^e_Θ represent the rate at which voters of a group Θ vote for the more liberal candidate in election e.[16] Thus, a proportion a^1_{nD} of nonminority Democrats cross over to vote for the minority candidate in the primary, while $1 - a^1_{nD}$ vote for the nonminority Democratic candidate. Similarly, a proportion a^2_R of Republican voters prefer the minority-Democrat in a general election. For reference, a table of these crossover rates is given in Table 1.

Crossover Voting and Group Power. In the above section, we introduced voters' willingness to trade off ideological and distributive benefits, which determines their electoral power or "swinginess" between candidates. Swinginess is closely related to crossover voting. Specifically, swinginess reflects both (i) trade-offs between redistributive and ideological benefits and (ii) trade-offs across candidates offering different ideological benefits. The second dimension—crossover voting—is di-

[15] We also assume that the number of voters in each district is large enough that we can calculate expected voter utility as the integral of $\Phi^e_i(.)$ for voter types.

[16] Assuming for the purposes of definition that minority-Democrats are more liberal than nonminority-Democrats, who are more liberal than Republicans.

rectly observable as a revealed preference, showing how voters support candidates regardless of partisan or ethnic identity, and has received more attention in both court rulings and the academic literature. For expositional clarity, we treat swinginess and crossover separately when solving for distributive and ideological benefits. In Section 6, we simulate redistricting schemes and discuss how the two can be linked.

Winning Probabilities. Then, for instance, the minority candidate will be expected to win a closed primary, ignoring the notation for district k, if:

$$a^1_{mD}N_{mD} + a^1_{nD}N_{nD} \geq (1 - a^1_{mD})N_{mD} + (1 - a^1_{nD})N_{nD} \;\Rightarrow\; \frac{N_{mD}}{N_{nD}} \geq \frac{1 - 2a^1_{nD}}{2a^1_{mD} - 1}. \tag{3.7}$$

Similarly, we apply a few assumptions on the relative magnitudes of crossover rates: $a^e_{mD} > a^e_{nD} > a^e_R$, reflecting closer ideological alignment among Democrats.[17]

Let Ψ^e_θ represent the probability that a type θ candidate wins election e, and Ψ_θ be the probability that the candidate wins overall. Given that the proportion of votes a candidate receives equals her probability of winning, we have the following for each election type:

$$\text{closed primary} - \Psi^1_{mD} = \frac{a^1_{mD}N_{mD} + a^1_{nD}N_{nD}}{N_{mD} + N_{nD}} \quad \text{and } \Psi^1_{nD} = 1 - \Psi^1_{mD}; \tag{3.8}$$

$$\text{open primary} - \hat{\Psi}^1_{mD} = \frac{a^1_{mD}N_{mD} + a^1_{nD}N_{nD} + a^1_R N_R}{N_{mD} + N_{nD} + N_R} \quad \text{and } \hat{\Psi}^1_{nD} = 1 - \hat{\Psi}^1_{mD}; \tag{3.9}$$

$$\Psi^2_{mD} = \frac{a^2_{mD}N_{mD} + a^2_{nD}N_{nD} + a^2_R N_R}{N_{mD} + N_{nD} + N_R} \quad \text{and } \Psi^2_R = 1 - \Psi^2_{mD}; \tag{3.10}$$

$$\Psi^3_{nD} = \frac{a^3_{mD}N_{mD} + a^3_{nD}N_{nD} + a^3_R N_R}{N_{mD} + N_{nD} + N_R} \quad \text{and } \Psi^3_R = 1 - \Psi^3_{nD}, \tag{3.11}$$

which describes the probabilities of winning the district for each candidate type with

$$\begin{aligned} \Psi_{mD} &= \Psi^1_{mD}\Psi^2_{mD},\ \Psi_{nD} = \Psi^1_{nD}\Psi^3_{nD}, \text{ and } \Psi_R = 1 - \Psi_{mD} - \Psi_{nD} \text{ (closed primary)} \\ \hat{\Psi}_{mD} &= \hat{\Psi}^1_{mD}\Psi^2_{mD},\ \hat{\Psi}_{nD} = \hat{\Psi}^1_{nD}\Psi^3_{nD}, \text{ and } \hat{\Psi}_R = 1 - \hat{\Psi}_{mD} - \hat{\Psi}_{nD} \text{ (open primary)}. \end{aligned} \tag{3.12}$$

These equations define a surface on S^2 with smoothly increasing election probabilities for each candidate type. This set-up simplifies the analysis while capturing voting nuances such as the effects of voter registration and turnout, which may or may not vary across partisanship and

[17] These crossover rates are for the same election e. One can consider different crossover rates across types for each election. For example, Washington (2006) shows that White Democratic and Republican voters are less likely to support a Black candidate of their party – $a^2_{nD} < a^3_{nD}$.

identity, and which we do not model here explicitly.[18]

3.5 Order of Play

To summarize, the order of play is as follows:

1. Given state demographics of $\mathbf{N}_{mD}$, $\mathbf{N}_{nD}$, and $\mathbf{N}_R$ and K districts, a valid districting scheme $\mathbf{D}$ is enacted.
2. Candidates of type j in each district k announce their platforms offering distribute benefits T_{ijk} for $i, j \in \{mD, nD, R\}$.
3. Voters elect candidates in primary and general elections, yielding legislature $\mathbf{L}$ and distributing $\mathbf{P}$.
4. All voters receive their utilities, and the game ends.

The game has perfect information, and we solve it by backward induction, deriving a subgame-perfect Nash equilibrium with specific distributional characteristics. Equilibrium platforms and policies are presented in Section 4, with a formal derivation in Appendix A.3.

3.6 Evaluation of Districting Plans

We evaluate districting plans based on their impact on the overall welfare of minority voters.[19] Let L_k be the legislator elected from district k, and let $\theta(L_k)$ be her type. Then, the plan that maximizes minority groups' utility maximizes the function:

$$\mathbf{D}^* \in \arg\max_{\mathbf{D} \in \mathcal{D}^K} \sum_{i=1}^{\mathbf{N}_{mD}} X_i^{\theta(L_k)} + E[U_i(b_i) | \mathbf{P}(\mathbf{L}(\mathbf{D}))]. \tag{3.13}$$

The optimal districting plan allocates different types of voters across districts, accounting for how the scheme affects minorities' equilibrium distributive and ideological benefits. Concentrating minority voters in a few districts increases the chance of electing minority representatives, but at the cost of electing more Republicans elsewhere. This strategy can yield large distributive benefits within those concentrated districts, while reducing the chances that their representatives will be part of winning legislative coalitions. By contrast, spreading minority voters more evenly

[18] Without loss of generality, uneven turnout and registration levels could be incorporated by scaling results proportionally. For a recent analysis of partisan gerrymandering and voter turnout, see Bouton et al. (2023).

[19] We could apply our framework to evaluate alternative social objectives that might be more egalitarian or utilitarian. Although our focus is on illustrating the tradeoffs between voter preferences and policy outcomes.

allows them to influence outcomes in more districts. But it also raises the risk of marginalization—electing no minorities and securing only limited distributive benefits. The key question is how voters weigh these trade-offs under changing ideological alignments, group population shares, and levels of group power.

4 Equilibrium Platforms and Policy Benefits

First, we describe the equilibrium outcomes of the game above before turning to redistricting considerations and realized benefits. Candidates adopt platforms to maximize their votes in primary and general elections, balancing their offers to various groups. In equilibrium, the candidates adopt identical redistributive platforms: $b_{i1k} = b_{i2k}$ and $T_{i1k} = T_{i2k}$ for each group i in a given district k.[20] See Appendix A.3 for the formal characterization of the equilibrium.

Note that, regardless of whether primaries are open or closed, the equilibrium distributive platforms are determined in the general election, when all voter groups participate. For example, under closed primaries, both Democratic candidates offer identical distributive platforms to minority and nonminority Democrats. Once the winner faces a Republican, competing for Republican voters as well, the Democratic candidate adjusts promises strategically and, in equilibrium, converges to the Republican's distributive platform. Such strategic platform adjustments between primary and general elections are empirically documented by di Tella et al. (2023). When primaries are open, apart from turnout differences, Democrats' distributive promises are identical across primary and general elections, since both stages involve competition for Republican votes. In summary, the structure of primaries and platform shifts across elections do not affect distributive benefits. However, they do affect ideological benefits, as we will document in the next section.

Furthermore, the individual benefits and share of the distributive benefits offered to group Θ by candidate j for district k in equilibrium are

$$b_{ijk} = \frac{\pi_i}{\sum_i \pi_i N_{ik}} B_k \text{ and } T_{ijk} = \frac{\pi_i N_{ik}}{\sum_i \pi_i N_{ik}}, \tag{4.1}$$

[20]See Dixit and Londregan (1996)'s description and existence of an equilibrium in pure strategies of platforms. The existence conditions of Glicksberg's Theorem are fulfilled, and the constrained maximization problem is derived using (3.5) and (3.6) for each candidate's objective function and $\sum_i N_{ik} b_{ijk} = B_k$ as a constraint. The Nash equilibrium follows from a simultaneous solution for all first-order conditions and Lagrange parameters. Consequently, voters cast their ballots for the candidate with whom they have the highest ideological affinity.

where

$$\pi_i = [\kappa_i \phi_i(0)]^{1/\epsilon} \tag{4.2}$$

and $\phi(.) = \Phi'(.)$.[21] The distributive benefits and the groups' distributive shares depend on i) the group's influence to swing election outcomes of π_i, ii) the district's distributive benefits of B_k allocated by the legislature's policy $\mathbf{P}$, and iii) the distribution of voters N_{ik} derived from the districting scheme $\mathbf{D}$.

Voter Group's Power We can see in (4.1) that as group power increases, the greater the value of π_i, the group's share of the legislative pie T_{ijk} increases. The group's ability to affect electoral outcomes increases as

1. Weight on distributive benefits, κ_i, increases;

   Groups with larger values of κ_i care more about distributive rather than ideological issues, and these groups get a bigger share of the legislative pie.

2. Group's candidate indifference, $\phi_i(0)$, increases;

   A group's power also grows with $\phi_i(0)$, which is the density of their distribution function when voters are indifferent between the two candidates running for office. This term captures a group's "swinginess:" the greater the percentage of members indifferent between the candidates or close to it, the more benefits the group and each member receive. The intuition behind this result is straightforward. First, in equilibrium, the candidates offer the same platform to voters, so this will make no difference in voters' decisions. Since the candidates' promises cancel out, those voters who are indifferent between the parties in equilibrium are those for whom $X_i^e = 0$ in the first place. When deciding whether to transfer funds from one group to another, then, it is these marginal voters who will gain or lose; hence, the candidates pay off the groups in ratios proportional to their $\phi_i(0)$ values, and the group's members enjoy greater distributive benefits.

3. Returns in consumption, $1/\epsilon$, are greater;

   The parameter ϵ represents the degree of diminishing returns in consumption; as the additional consumption matters less, the group's power declines, and distributive benefits are more even across groups. For example, as $\epsilon \to \infty$, we get $\pi_i \to 1$ and benefits and shares

[21] The solution follows from applying the utility function of (3.1) and its marginal utility (3.2) to the Nash equilibrium's first-order conditions. More details are in Appendix A.3.

become simple averages: $b_{ijk} \to B_k/N_k$ and $T_{ijk} \to N_{ik}/N_k$.

Clearly, one group's power adversely affects other groups' distributive benefits as there is a fixed legislative pie and the distribution of benefits can be seen as a contest function of (4.1) with a group's total power relative to all groups' powers determining outcomes.

Legislative Policies and Districting An individual's distributive benefits and a group's share of district benefits also depend on the district representative's ability to deliver distributive benefits, B_k, and the district's demographics, N_{ik}. For example, the larger the district-specific transfers, B_k, the greater the individual's gains in consumption from the group's size, which is independent of voters' benefits. On the other hand, the legislator allocates a larger share of district benefits to larger groups, $\partial T_{ijk}/\partial N_{ik} > 0$; though the pie's share, T_{ijk}, is independent of the pie's size, B_k.

5 Distributive and Ideological Benefits

To analyze the equilibrium properties and the trade-offs for redistricting, we divide the analysis into three stages. First, we examine the implications of per-voter distributive benefits b_{ijk}. Ignoring the ideological benefits of electing different types of representatives for the moment, we ask how to allocate minority voters across districts to maximize their total (or average) distributive returns. Second, we analyze the ideological utility associated with electing different types of representatives. Finally, we combine distributive and ideological utilities to characterize the districting schemes that maximize minority voters' overall returns.

5.1 Distributive Benefits and Minority Power

Characterizing districting schemes that provide the most benefits to minorities depends on the behavior of (4.1) on the two-dimensional simplex S^2. We are particularly interested in its behavior on the surface $N_{mDk} + N_{nDk} + N_{Rk} = N_k = \mathbf{N}/K$. We thus rewrite (4.1), minorities' distributive benefits as a group's share in a given district by any candidate,[22] as

$$T_{mDk} = f(N_{mDk}, N_{nDk}) = \frac{\pi_{mD} N_{mDk}}{\pi_{mD} N_{mDk} + \pi_{nD} N_{nDk} + \pi_R N_{Rk}} \quad (5.1)$$

$$= \frac{\pi_{mD} N_{mDk}}{(\pi_{mD} - \pi_R) N_{mDk} + (\pi_{nD} - \pi_R) N_{nDk} + \pi_R N_k} \geq 0. \quad (5.2)$$

[22]We can ignore the candidate subscript as candidates in the same district promise the same benefits.

Note that the denominator is positive throughout. Thus, we can define $\Pi_k \equiv (\pi_{mD} - \pi_R)N_{mDk} + (\pi_{nD} - \pi_R)N_{nDk} + \pi_R N_k$ – i.e., the aggregate group power of district k. We then write the derivatives of the minorities' benefits with respect to the groups' relative powers as

$$\frac{\partial f}{\partial \pi_{mD}} = \frac{N_{mDk}\left(\pi_R(N_k - N_{mDk} - N_{nDk}) + \pi_{nD}N_{nDk}\right)}{\Pi_k^2} \geq 0; \tag{5.3}$$

$$\frac{\partial f}{\partial \pi_{nD}} = -\frac{\pi_{mD}N_{mDk}N_{nDk}}{\Pi_k^2} \leq 0; \tag{5.4}$$

$$\frac{\partial f}{\partial \pi_R} = -\frac{\pi_{mD}N_{mDk}(N_k - N_{mDk} - N_{nDk})}{\Pi_k^2} \leq 0.^{23} \tag{5.5}$$

Assigning signs to the derivatives with $\pi_i > 0$ and $N_{ik} > 0$ shows that increases in the minority group's power is beneficial, while increasing the power of either other group decreases the minority's utility.

5.1.1 Districting Scheme

We now turn to the districting question: how to maximize minority voters' utility by changing the numbers of different types of voters across districts. That is, we seek a valid districting scheme $\tilde{\mathbf{D}}^*$ such that

$$\tilde{\mathbf{D}}^* \in \underset{\mathbf{D}\in\mathcal{D}^K}{\arg\max} \sum_{i=1}^{N_{mD}} E\left[U_i(b_i)|\mathbf{P}(\mathbf{L}(\mathbf{D}))\right]. \tag{5.6}$$

The solution to the utility-maximizing districting scheme may not be unique. Hence, let the set of all possible schemes be $\tilde{\mathcal{D}}^*$ and $\tilde{\mathbf{D}}^*$ a representative element. To determine the characteristics of an optimal districting scheme, we first evaluate the derivatives of minority voters' benefits of (5.2) with respect to the populations of voters:

$$\frac{\partial f}{\partial N_{mDk}} = \frac{\pi_{mD}\left(\pi_{nD}N_{mDk} + \pi_R(N_k - N_{mDk})\right)}{\Pi_k^2} > 0, \tag{5.7}$$

$$\frac{\partial f}{\partial N_{nDk}} = \frac{\pi_{mD}N_{mDk}(\pi_R - \pi_{nD})}{\Pi_k^2} \gtreqless 0. \tag{5.8}$$

As one would expect, the first derivative is always positive; adding more minority voters to a district increases their share of distributive benefits. However, the sign of the first derivative of (5.8) is ambiguous and depends on the other groups' relative power. Minority voters benefit if voters from the more powerful non-minority group are replaced with voters from the less powerful group. Suppose Republicans are politically more powerful than nonminority Democrats, $\pi_R > \pi_{nD}$. In

[23] For $N_k = N_{mDk}$, we have $\frac{\partial f}{\partial \pi_{mD}} = \frac{\partial f}{\partial \pi_{nD}} = \frac{\partial f}{\partial \pi_R} = 0$.

that case, the benefits for minority voters increase as the number of nonminority Democratic voters decreases the number of Republicans in a district, and vice versa. In the districting process, however, changes in voters must be balanced across districts. Hence, minority gains in one district, where less powerful voters increase in number, accompany another district's minority voters' loss as more powerful voters join. We can state

Proposition 1 (Group Power and Districting)**.** *Minority voters' distributive benefits are maximized when they share districts with less powerful nonminority voters, packing powerful nonminority voters into other districts.*

All proofs are in Appendix A. More specifically, we have if nonminority voter groups' power differs, $\pi_{nD} \neq \pi_R$, and for any two districts with different minority voter concentration and aggregate group power, $N_{mDk} \neq N_{mDl}$ and $\Pi_k \neq \Pi_l$ with $k \neq l$, then any districting scheme that maximizes minority distributive benefits concentrates less powerful nonminority voters into minority-populated, less powerful districts and more powerful nonminority voters into nonminority-populated, more powerful districts. Figure 2 and (5.8) illustrate the following intuition.

Consider two districts k_1 and k_2 with $N_{mD1} > N_{mD2}$, $\Pi_1 < \Pi_2$, and $\pi_R > \pi_{nD}$. Focusing on the simplex's interior, the goal is to shift voters of the more powerful nonminority group out of the minority-populated district and into the other district. As shown in Figure 2, accomplishing this requires moving Republicans from k_1 to k_2 and nonminority Democrats from k_2 to k_1, where they exert greater influence. If k_1 is less powerful than k_2 ($\Pi_1 \leq \Pi_2$), then minority benefits rise in k_1 (with more minority voters) by more than they fall in k_2 (with fewer). Hence, average payoffs for minority voters across districts increase, but the district's power across groups decreases. This process continues until one district reaches the simplex's border. It can then be repeated with any other pair of interior districts satisfying $N_{mDk} \neq N_{mDl}$ and $\frac{N_{mDk}^2}{\Pi_k^2} > \frac{N_{mDl}^2}{\Pi_l^2}$ (see districts k_2 and k_3 in Figure 2). In equilibrium, at least one district is on the simplex border. Overall, this pattern reflects a packing-and-cracking dynamic defined by ethnicity and electoral influence rather than by partisanship.[24]

5.1.2 Voter Distribution and Minority Distributive Benefits

All that remains to characterize $\tilde{\mathcal{D}}^*$ completely is to determine the optimal distribution of minority voters across districts. The surfeit of boundary conditions makes the usual maximization solution

[24] Table 2 of the next section provides simulations of optimal districts with varying group power, corroborating our results.

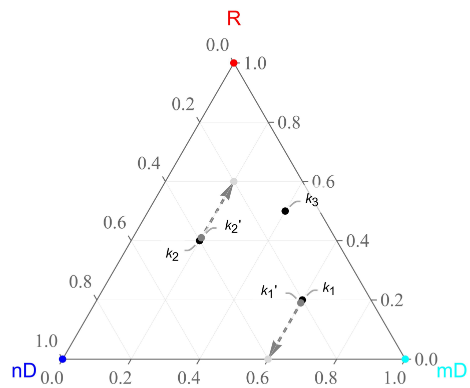


Figure 2: Optimal Districting Process for $\pi_R > \pi_{nD}$ and $\Pi_1 \leq \Pi_2$ with $K = 3$.

via Lagrange multipliers opaque. Still, we can gain insight into the solution by examining the concavity/convexity of the payoff function with respect to the number of minority voters in the district. We thus calculate the determinants of the principal minors of the Hessian matrix:

$$H = \begin{bmatrix} \frac{\partial^2 f}{\partial N_{mDk}^2} & \frac{\partial^2 f}{\partial N_{mDk} N_{nDk}} \\ \frac{\partial^2 f}{\partial N_{nDk} N_{mDk}} & \frac{\partial^2 f}{\partial N_{nDk}^2} \end{bmatrix}. \tag{5.9}$$

with

$$\frac{\partial^2 f}{\partial N_{mDk}^2} = \frac{2\pi_{mD}(\pi_R - \pi_{mD})\left(\pi_R N_k + (\pi_{nD} - \pi_R) N_{nDk}\right)}{\Pi_k^3}, \tag{5.10}$$

$$\frac{\partial^2 f}{\partial N_{mDk} N_{nDk}} = \frac{\pi_{mD}(\pi_{nD} - \pi_R)\left((\pi_{mD} - \pi_R) N_{mDk} + (\pi_R - \pi_{nD}) N_{nDk} - \pi_R N_k\right)}{\Pi_k^3}, \tag{5.11}$$

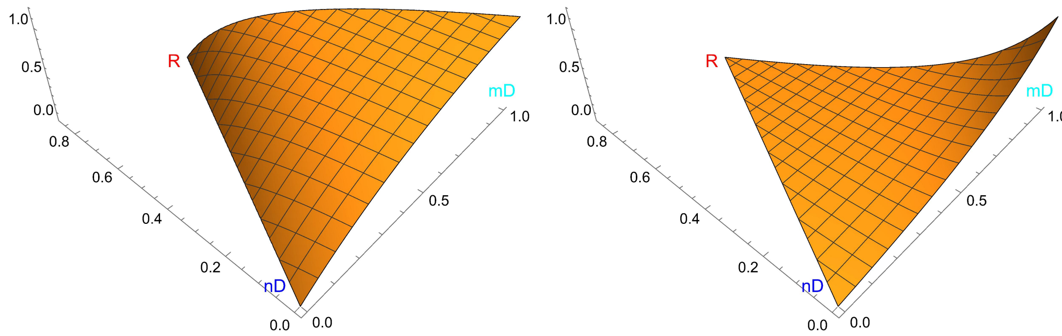
$$\frac{\partial^2 f}{\partial N_{nDk}^2} = \frac{2\pi_{mD}(\pi_{nD} - \pi_R)^2 N_{mDk}}{\Pi_k^3}, \tag{5.12}$$

and

$$det(H) = \frac{\pi_{mD}^2 (\pi_R - \pi_{nD})^2}{\Pi_k^4}. \tag{5.13}$$

The determinant of the entire H matrix is positive for $\pi_{nD} \neq \pi_R$, but the value of $\frac{\partial^2 f}{\partial N_{mDk}^2}$ is indeterminate, indicating that the H matrix can be positive definite, negative definite, or neither, depending on the parameter values. For optimization, the surface could be either concave or convex. Figure 3(a) illustrates a concave function for minority distributive benefits when the minority group's power is larger than that of others. On the other hand, when minority power is lower than the nonminority groups' power, the function is convex, as illustrated in Figure 3(b).[25]

[25] We provide additional examples of nonconcave and nonconvex payoffs in Figure 7 of Appendix A.5 that arise



(a) Concave: $\pi_{mD} = 10$, $\pi_{nD} = 5$, $\pi_R = 2$ (b) Convex: $\pi_{mD} = 2$, $\pi_{nD} = 5$, $\pi_R = 10$

Figure 3: Concave and Convex Minority Distributive Benefits.

The importance of this difference is clear. If we wish to maximize the overall return to minorities, then in the concave case, we would divide minority voters more evenly across districts than with a convex payoff function. Note that the difference between the curvatures of the two surfaces lies in the relative power of minorities compared to other groups: concave for more powerful minorities and convex for less powerful ones. This forms the basis for the following proposition:

Lemma 1. *If $\pi_{mD} = \max_{i\in\Theta}\{\pi_i\}$, then T_{mDk} is concave on S^2; if $\pi_{mD} = \min_{i\in\Theta}\{\pi_i\}$, then T_{mDk} is convex.*

Since optimal values of N_{mD} on a concave surface will be less dispersed than on a convex surface, we have the result that, as minority voters gain power, all else being equal, optimal gerrymanders for distributive benefits divide these voters more equally across districts.[26] Formally, let

$$R(\mathbf{D}) = \max_{d_k, d_l \in \mathbf{D}^*} \left(N_{mDk} - N_{mDl}\right), \tag{5.14}$$

be the range, the maximum difference between the minority population of any two districts in an optimal districting scheme. Then $\frac{\partial R(\mathbf{D})}{\partial \pi_{mD}} \leq 0$, so that minority voters are (weakly) spread out less as their power increases. Combining these results from Lemma 1 with Proposition 1, we can state

Proposition 2 (Distributive Utility and Districting)**.** *Optimal districting schemes will concentrate minority voters in a few districts when their power is low, spread them out when their power is*

when minority group power lies between both nonminority groups' power.

[26] In fact, optimal districts when T_{mDk} is convex concentrate all minority voters into as few districts as possible. Conversely, when T_{mDk} is concave, $N_{mD} > 0$ for all districts.

Group Power			District 1			District 2			District 3			Benefits		
π_{mD}	π_{nD}	π_R	n_{mD1}	n_{nD1}	n_{R1}	n_{mD2}	n_{nD2}	n_{R2}	n_{mD3}	n_{nD3}	n_{R3}	Total	Aver.	$R(\mathbf{D})$
1	3	1	75%	0%	25%	0%	64%	36%	0%	56%	44%	0.750	0.250	75%
2	3	1	44%	0%	56%	0%	100%	0%	31%	20%	49%	0.974	0.325	44%
3	3	1	39%	0%	61%	36%	20%	44%	0%	100%	0%	1.167	0.389	39%
4	3	1	0%	100%	0%	37%	20%	43%	38%	0%	62%	1.300	0.433	38%
5	3	1	30%	35%	35%	30%	0%	70%	15%	85%	0%	1.426	0.475	15%
1	3	3	0%	54%	46%	0%	54%	46%	75%	13%	12%	0.500	0.167	75%
2	3	3	0%	53%	47%	0%	51%	49%	75%	15%	10%	0.667	0.222	75%
3	3	3	25%	40%	35%	25%	40%	35%	25%	40%	35%	0.750	0.250	0%
4	3	3	25%	49%	26%	25%	33%	42%	25%	38%	37%	0.923	0.308	0%
5	3	3	25%	39%	36%	25%	40%	35%	25%	41%	34%	1.071	0.357	0%
1	3	5	0%	93%	7%	0%	2%	98%	75%	25%	0%	0.500	0.167	75%
2	3	5	75%	25%	0%	0%	3%	97%	0%	92%	8%	0.667	0.222	75%
3	3	5	0%	93%	7%	0%	2%	98%	75%	25%	0%	0.750	0.250	75%
4	3	5	40%	60%	0%	35%	60%	5%	0%	0%	100%	0.876	0.292	40%
5	3	5	37%	58%	5%	38%	62%	0%	0%	0%	100%	0.987	0.329	38%

Table 2: Districting Plans Maximizing Minority Distributive Benefits: $N_{mD} = 25\%$, $N_{nD} = 40\%$, and $N_R = 35\%$.

high, and combine them as much as possible with the less powerful of the other two groups.

Table 2 reports optimal districting schemes and distributive benefits for varying levels of group power. The simulations consider a state with three districts, where the population shares are 25% minority Democrats, 40% nonminority Democrats, and 35% Republicans. The power of nonminority Democrats is fixed at $\pi_{nD} = 3$, while the power of the other two groups varies between 1 and 5.

The results confirm the model's predictions. As π_{mD} increases, the range $R(\mathbf{D})$ declines and minority utility rises within each set of observations. Minority voters are allocated to districts with more members of the less powerful group, while more powerful groups are concentrated elsewhere (see the last row for an illustration). When minorities are the least powerful group, they are highly concentrated (rows highlighted in blue). By contrast, when minorities are the most powerful group and the other groups are uniformly weaker, minorities are spread evenly across all districts (rows highlighted in orange).

These four patterns—regarding benefits, the range of outcomes, the power of nonminority voters in minority-concentrated districts, and the equal spread of minority voters when they are stronger than equally powerful nonminority voters—are robust. They hold under variations (i) in group powers (Table 9, Appendix B.1), (ii) in state demographics (Table 10, Appendix B.2), and (iii) in the number of districts (Table 11, Appendix B.3).

5.2 Ideological Benefits

We now turn to the ideological benefits that minority voters derive from their representatives. Our analysis proceeds in two steps: we first examine the probability of minority electoral success, and then evaluate the expected ideological utility of minority voters.

5.2.1 Likelihood of Successful Minority Candidates

In the first step, we evaluate the probability of electing a minority candidate as voter composition changes in a district. We analyze the first and second derivatives of Ψ_{mD} (closed primaries) and $\hat{\Psi}_{mD}$ (open primaries) from equation (3.12), and state the following:

Proposition 3 (Electoral Success and Districting)**.** *The probability of electing a minority candidate:*

1. *increases with the number of minority-Democratic voters;*
2. *is ambiguous with respect to the number of nonminority-Democrat voters when district size N_k is flexible;*
3. *is ambiguous when nonminority-Democrat voters replace Republicans in a closed primary (fixed N_k); and*
4. *increases when nonminority-Democrat voters replace Republicans in an open primary, provided $a^1_{nD} > a^1_R$ (fixed N_k).*

Additionally, $\hat{\Psi}_{mD}$ is convex on S^2, and Ψ_{mD} is convex on S^2 if $a^2_R < (a^2_{mD} - a^2_{nD})\, N_{nDk}/N_k$.

Adding minority voters to a district predictably increases the probability that a minority candidate wins both the primary and the general election, regardless of whether primaries are open or closed. Adding nonminority Democrats has more nuanced effects: they may support minority candidates in the general election but often back nonminority Democrats in the primary. Thus, minority candidates gain only when the general-election effect dominates. Replacing Republicans with nonminority Democrats lowers minority candidates' chances in closed primaries but raises them in general elections, since nonminority Democrats are more likely than Republicans to vote for a minority Democrat. In open primaries, the effect is strictly positive because reducing the number of Republican voters helps in both stages. The logical counterpart is paradoxical: under closed primaries or in states with growing populations, increasing the number of Republican voters can sometimes raise the probability that a minority Democrat is elected.

The fact that $\hat{\Psi}_{mD}(.)$ for open primaries is convex or $\Psi_{mD}(.)$ for closed primaries is convex at low levels of Republican crossover (a_R^2) implies that under these conditions, districting schemes maximizing the number of minority-Democrats elected will concentrate minority voters in as few districts as possible. This finding accords with empirical evidence (see, for instance, Cameron et al. (1996)), although, to our knowledge, it has not previously been established in a general theoretical context.

Two points follow from the analysis. First, the relationship between electing minority-Democrats and concentrating minority voters depends on crossover rates. Under closed primaries with low a_R^2, concentration is optimal, but when a_R^2 is higher, optimal schemes for descriptive representation spread minority voters more evenly across districts. Second, the convexity of $\Psi_{mD}(.)$ derives from the two-step primary-general election process. Adding minority voters to a district increases the chances that a minority candidate wins both stages. And because $\hat{\Psi}_{mD}(.)$ or $\Psi_{mD}(.)$ is the product of these two probabilities, adding minority voters at the margin has a quadratic effect on the overall probability of electing minority candidates to office.[27]

5.2.2 Expected Minority Ideological Benefits

With these expected electoral outcomes, we can examine the ideological benefits that minority voters anticipate from candidates joining the legislature as their representatives. We define the average utility per voter of a given type i for a j type representative:

$$\bar{X}_i^j = \int_{-\infty}^{\infty} X_i^j d[\Phi(X_i)]. \tag{5.15}$$

Then the total utility to voters electing a type j representative is $N_{ij}\bar{X}_i^j$. For convenience, recalibrate utilities so that $\bar{X}_{mD}^{mD} = 1$ and $\bar{X}_{mD}^{R} = 0$, and define $\beta \equiv \bar{X}_{mD}^{nD}$, with $0 \leq \beta \leq 1$. Overall expected utility for minority voters includes both the type elected and their average attachment to representatives of that type:

$$\begin{aligned} E[X] &= \Psi_{mD}\bar{X}_{mD}^{mD} + \Psi_{nD}\bar{X}_{mD}^{nD} + \Psi_R\bar{X}_{mD}^{R} \\ &= \Psi_{mD} + \Psi_{nD}\beta - \text{closed primaries} \end{aligned}$$

[27]Individually, the election function is concave in N_{mDk} for the primary and linear in N_{mDk} for the general, making the combined convexity all the more striking.

$$= \quad \hat{\Psi}_{mD} + \hat{\Psi}_{nD}\beta - \text{open primaries.} \quad (5.16)$$

It is natural to ask whether the districting schemes that maximize minority voters' overall expected ideological utility are the same as those that elect minority representatives.

Proposition 4 (Ideological Utility and Districting)**.** *Districting schemes that maximize minority voters' ideological utility coincide with those that maximize the number of minority Democrats elected when the additional utility of descriptive representation is sufficiently large. Formally, there exists a $\tilde{\beta} > 0$ such that for $\beta < \tilde{\beta}$, $E(X)$ is convex on S^2.*

When the extra utility of electing a minority-Democrat is high enough (β is close to 0), the $E(X)$ function is convex, independent of whether primaries are closed or open. Districting schemes that maximize overall utility coincide with those that elect as many minority-Democrats as possible to office. Conversely, when it is more important to avoid electing Republicans (β is close to 1), the function becomes concave, and optimal schemes spread minority voters more across districts. As partisan concerns rise, then, minority voters prefer to work more through electoral coalitions, joining with nonminority-Democratic voters to minimize the number of Republicans elected to office.[28,29]

6 Optimal Districts

We identify districting schemes that maximize minority voters' overall utility by combining the distributive and ideological benefits derived above. On the one hand, these benefits are additive; seemingly, the task is to add up the above-mentioned effects. On the other hand, this rosy scenario is complicated by the two effects being inextricably linked: groups receive greater distributive benefits with increasing "swinginess," their density at $\phi_i(0)$ rises, but this quantity also indicates the amount of crossover voting by that group.

The observation cuts two ways. First, as nonminority voters are increasingly willing to cross over and vote for minority candidates, the chances of electing minorities to office rise, which raises the average ideological utility of minority voters. However, this greater willingness to crossover

[28] As a special case for elections with closed primaries, we note that the $E(X)$ function is also convex when minority voters are more likely to support a minority candidate in the general election ($a^2_{mD} > a^2_{nD}$ by assumption), Republican voters more likely support a nonminority-Democrat than a minority candidate ($a^3_R\beta > a^2_R$), with Democrats similarly voting against a Republican candidate ($a^3_{nD} \geq a^3_{mD}$).

[29] As a special case for elections with open primaries, we note that the $E(X)$ function is also convex when minority voters are more likely to support a minority candidate in the general election ($a^2_{mD} > a^2_R$ by assumption) and Republican voters are more likely to support a nonminority-Democrat candidate than minority voters ($a^3_{mD} \leq a^3_R$).

means that nonminority voters are now more swingy and decisive, so they will receive larger shares of distributive benefits B_k in equilibrium. From minorities' point of view, then, the price for greater electoral support from other groups is a loss of distributive benefits.

Second, the more politically cohesive the minority-Democrats are–that is, the more consistently they vote only for minority-Democrats–the less influential they become relative to other groups, and the few distributive benefits they receive. In this sense, the model captures the idea that the most loyal democratic supporters are also the most easily "taken for granted." Declining racial or ethnic polarization in voting patterns is therefore a mixed blessing for minorities, creating a tradeoff between ideological and distributive benefits.

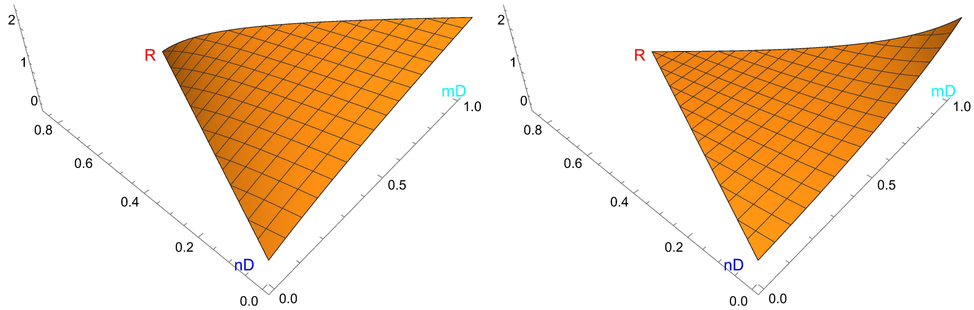
How do these considerations affect the nature of optimal districting schemes as minorities gain power? We know that the distributive payoff function T_{ijk} becomes concave as π_{mD} rises; how does this interact with ideological utility, given that $E(X)$ is convex under certain circumstances? We can state the following:

Proposition 5 (Overall Utility and Districting)**.** *Districting schemes that maximize minorities' total utility concentrate minority voters less as their group power, π_{mD}, increases.*

If minority voters are motivated more by distributional than ideological benefits (κ_i is increasing π_i) and their voting rates are decreasing in each election round, making them more influential ($\phi_i(0)$ is increasing π_i), the concavity of minority distributive benefits will eventually outweigh any convexity in minority ideological benefits. Hence, more powerful minorities are sufficiently motivated by distributive benefits, resulting in a greater spread of minority voters across districts and a greater realization of distributive benefits. Less powerful minority voters are more concentrated and gain ideological benefits in those districts, but on average, they receive fewer distributive benefits. Overall, then, if minority voters prioritize tangible benefits over political beliefs, the spread of benefits among them will outweigh the concentration of benefits. Powerful minorities, therefore, benefit more from a dispersion of minority voters, while less powerful ones gain more from concentration. Minority voters become more influential as their voting rates decrease.

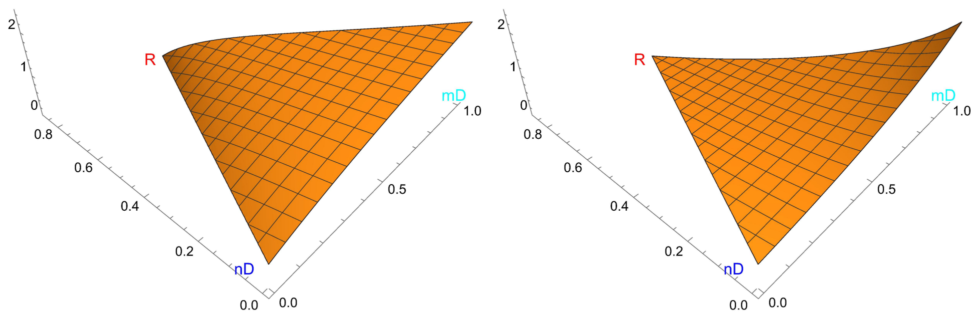
6.1 Numerical Illustrations and Patterns

Our numerical simulations illustrate how the payoff function of different redistricting schemes can be convex or concave with respect to the concentration of minority voters across districts. They also reveal additional patterns concerning: (i) changes in minority benefits as their electoral influence increases, (ii) the degree of minority voter concentration, (iii) the types and concentration of nonminority voters across districts, and (iv) the potential number of majority–minority districts.



(a) $\pi_{mD} = 10$, $\pi_{nD} = 5$, $\pi_R = 2$, closed primary

(b) $\pi_{mD} = 2$, $\pi_{nD} = 5$, $\pi_R = 10$, closed primary



(c) $\pi_{mD} = 10$, $\pi_{nD} = 5$, $\pi_R = 2$, open primary

(d) $\pi_{mD} = 2$, $\pi_{nD} = 5$, $\pi_R = 10$, open primary

Figure 4: Minority Total Benefits – $a^1_{nD} = .3$, $a^2_{nD} = .7$, $\beta = .5$.

For example, Figure 4 shows the overall utility of minority voters, which combines distributive benefits from (5.2)—concave or convex depending on group power—with the expected (convex) ideological gains from (5.16). The resulting total utility is given by

$$
\begin{aligned}
EU_{mD} &= \frac{\pi_{mD} N_{mDk}}{\sum_i \pi_i N_{ik}} + \Psi_{mD} + \Psi_{nD}\beta \text{ – closed primary,} \\
&= \frac{\pi_{mD} N_{mDk}}{\sum_i \pi_i N_{ik}} + \hat{\Psi}_{mD} + \hat{\Psi}_{nD}\beta \text{ – open primary.} \qquad (6.1)
\end{aligned}
$$

Our illustrations follow the same patterns as Figure 3, but with payoffs bounded between 0 and 2 rather than 0 and 1 for distributive shares. The graphs show smooth payoff functions, since both distributive and ideological benefits are determined by probabilistic voting and the

Group Power			*District 1*			*District 2*			*District 3*			**Benefits**		
π_{mD}	π_{nD}	π_R	n_{mD1}	n_{nD1}	n_{R1}	n_{mD2}	n_{nD2}	n_{R2}	n_{mD3}	n_{nD3}	n_{R3}	Total	Aver.	$R(\mathbf{D})$
1	3	1	75%	0%	25%	0%	70%	30%	0%	50%	50%	2.172	0.724	75%
2	3	1	46%	0%	54%	0%	100%	0%	29%	20%	51%	2.370	0.790	46%
3	3	1	35%	20%	45%	0%	100%	0%	40%	0%	60%	2.562	0.854	40%
4	3	1	37%	20%	43%	0%	100%	0%	38%	0%	62%	2.695	0.898	38%
5	3	1	35%	0%	65%	6%	94%	0%	34%	26%	40%	2.795	0.932	28%
6	3	1	29%	36%	35%	16%	84%	0%	30%	0%	70%	2.905	0.968	14%
7	3	1	28%	39%	33%	19%	81%	0%	28%	0%	72%	3.007	1.002	8%
8	3	1	27%	41%	32%	21%	79%	0%	27%	0%	73%	3.097	1.032	6%
9	3	1	27%	43%	31%	23%	77%	0%	26%	0%	74%	3.177	1.059	4%
10	3	1	26%	44%	30%	24%	76%	0%	25%	0%	75%	3.248	1.083	3%
1	3	3	0%	100%	0%	0%	20%	80%	75%	0%	25%	1.922	0.641	75%
2	3	3	0%	98%	2%	0%	22%	78%	75%	0%	25%	2.089	0.696	75%
3	3	3	0%	100%	0%	0%	20%	80%	75%	0%	25%	2.172	0.724	75%
4	3	3	40%	0%	60%	0%	100%	0%	35%	20%	45%	2.284	0.761	40%
5	3	3	19%	81%	0%	25%	39%	36%	31%	0%	69%	2.424	0.808	11%
1	3	5	0%	61%	39%	0%	34%	66%	75%	25%	0%	1.883	0.628	75%
2	3	5	75%	25%	0%	0%	32%	68%	0%	63%	37%	2.049	0.683	75%
3	3	5	0%	90%	10%	0%	5%	95%	75%	25%	0%	2.133	0.711	75%
4	3	5	0%	0%	100%	32%	63%	5%	43%	57%	0%	2.202	0.734	43%
5	3	5	38%	62%	0%	37%	58%	5%	0%	0%	100%	2.312	0.771	38%

Table 3: Districting Plans Maximizing Minority Total Benefits: $N_{mD} = 25\%$, $N_{nD} = 40\%$, $N_R = 35\%$, $a^1_{nD} = .3$, $a^2_{nD} = .7$, $\beta = .5$, closed primaries.

resulting contest functions. Although the structure of primaries affects the numerical values of ideological benefits, the figures reveal little visual difference between closed and open primaries when compared vertically.

To show the trade-offs identified in Proposition 5, we calculate optimal districting schemes using the same values of π_{mD}, π_{nD}, and π_R as in Table 2, holding overall population proportions constant. We assume the extra utility of electing a nonminority Democrat is $\beta = 0.5$, and 1 for electing a minority candidate, relative to a baseline of 0 for a Republican. The nonminority-Democrat crossover rate is set at 30% in primaries and 70% in general elections.[30]

The results, reported in Tables 3 and 4, show higher ranges compared to Table 2, reflecting a stronger incentive to concentrate minority voters to ensure minority candidates can win in at least some districts. The earlier rule that $R(D)$ weakly decreases within each subgroup of five simulations continues to hold. Finally, the same pattern emerges at the extremes: when minorities are the least powerful group, at least one district contains no minority voters, while when they are the most powerful, $N_{mDk} > 0$ for all districts k.

Comparing simulation results for states with closed versus open primaries, we see similar patterns of rising minority benefits, declining concentration, and sorting by group power. However,

[30]Formally, $a^e_{mD} = 1$, $a^e_R = 0$, and $a^e_{nD} \in 0.3, 0.7, 1$.

Group Power			District 1			District 2			District 3			Benefits		
π_{mD}	π_{nD}	π_R	n_{mD1}	n_{nD1}	n_{R1}	n_{mD2}	n_{nD2}	n_{R2}	n_{mD3}	n_{nD3}	n_{R3}	Total	Aver.	$R(\mathbf{D})$
1	3	1	0%	100%	0%	0%	20%	80%	75%	0%	25%	2.069	0.69	75%
2	3	1	59%	0%	41%	0%	100%	0%	16%	20%	64%	2.183	0.73	59%
3	3	1	38%	0%	62%	0%	100%	0%	37%	20%	43%	2.370	0.79	38%
4	3	1	35%	0%	65%	0%	100%	0%	40%	20%	40%	2.505	0.83	40%
5	3	1	30%	38%	32%	18%	82%	0%	27%	0%	73%	2.637	0.88	11%
6	3	1	28%	43%	29%	23%	77%	0%	24%	0%	76%	2.764	0.92	5%
7	3	1	27%	45%	28%	25%	75%	0%	23%	0%	77%	2.874	0.96	5%
8	3	1	27%	46%	27%	26%	74%	0%	22%	0%	78%	2.969	0.99	6%
9	3	1	27%	47%	26%	27%	73%	0%	21%	0%	79%	3.052	1.02	6%
10	3	1	27%	48%	25%	28%	72%	0%	20%	0%	80%	3.125	1.04	7%
1	3	3	0%	95%	5%	0%	0%	100%	75%	25%	0%	1.880	0.63	75%
2	3	3	0%	95%	5%	0%	0%	100%	75%	25%	0%	2.046	0.68	75%
3	3	3	0%	95%	5%	0%	0%	100%	75%	25%	0%	2.130	0.71	75%
4	3	3	35%	60%	5%	0%	0%	100%	40%	60%	0%	2.205	0.74	40%
5	3	3	38%	62%	0%	36%	58%	7%	2%	0%	98%	2.317	0.77	36%
1	3	5	0%	0%	100%	0%	95%	5%	75%	25%	0%	1.880	0.63	75%
2	3	5	0%	95%	5%	0%	0%	100%	75%	25%	0%	2.046	0.68	75%
3	3	5	0%	95%	5%	0%	0%	100%	75%	25%	0%	2.130	0.71	75%
4	3	5	27%	68%	5%	0%	0%	100%	48%	52%	0%	2.194	0.73	48%
5	3	5	40%	60%	0%	0%	0%	100%	35%	60%	5%	2.304	0.77	40%

Table 4: Districting Plans Maximizing Minority Total Benefits: $N_{mD} = 25\%$, $N_{nD} = 40\%$, $N_R = 35\%$, $a^1_{nD} = .3$, $a^2_{nD} = .7$, $\beta = .5$, open primaries.

the concentration of minority voters, $R(D)$, differs numerically across primary structures. Tables 3 and 4 show that these differences are inconsistent in magnitude across the two electoral procedures. We return to this discrepancy in the next section, focusing on the light-gray highlighted rows in Table 4.

The four main patterns—regarding (i) benefits, (ii) range, (iii) the role of nonminority power within minority-concentrated districts, and (iv) the equal spread of minority voters when they are stronger than equally powerful nonminority voters— are robust under:

1. variations in group powers (Tables 12-13 in Appendix B.4);
2. variations in state demographics (Tables 14-15 in Appendix B.5);
3. variations in minority ideological benefits (Tables 16-17 for closed primaries and Tables 18-19 for open primaries in Appendix B.6); and
4. variations in primary and general crossover rates (Tables 20-21 for closed primaries and Tables 22-23 for open primaries in Appendix B.7).

Group Power – **Minority Power**					
π_{mD}	π_{nD}	π_R	Total	Average	$R(\mathbf{D})$
1	3	1	2.172	0.72	75%
2	3	1	2.370	0.79	46%
3	3	1	2.562	0.85	40%
4	3	1	2.695	0.90	38%
5	3	1	2.795	0.93	28%
6	3	1	2.905	0.97	14%
7	3	1	3.007	1.00	8%
8	3	1	3.097	1.03	6%
9	3	1	3.177	1.06	4%
10	3	1	3.248	1.08	3%
15	3	1	3.503	1.17	3%
20	3	1	3.662	1.22	4%
50	3	1	4.025	1.34	5%
100	3	1	4.176	1.39	3%
150	3	1	4.231	1.41	0%
200	3	1	4.259	1.42	3%
250	3	1	4.277	1.43	7%
300	3	1	4.290	1.43	11%
400	3	1	4.307	1.44	18%
500	3	1	4.318	1.44	23%

Table 5: Closed Primaries.

Group Power – **Minority Power**					
π_{mD}	π_{nD}	π_R	Total	Average	$R(\mathbf{D})$
1	3	1	2.069	0.69	75%
2	3	1	2.183	0.73	59%
3	3	1	2.370	0.79	38%
4	3	1	2.505	0.83	40%
5	3	1	2.637	0.88	11%
6	3	1	2.764	0.92	5%
7	3	1	2.874	0.96	5%
8	3	1	2.969	0.99	6%
9	3	1	3.052	1.02	6%
10	3	1	3.125	1.04	7%
15	3	1	3.388	1.13	10%
20	3	1	3.550	1.18	12%
50	3	1	3.921	1.31	17%
100	3	1	4.078	1.36	22%
150	3	1	4.137	1.38	26%
200	3	1	4.169	1.39	30%
250	3	1	4.189	1.40	34%
300	3	1	4.204	1.40	37%
400	3	1	4.225	1.41	43%
500	3	1	4.240	1.41	47%

Table 6: Open Primaries.

Table 7: Minority Voter Concentration and Benefits – $N_{mD} = 25\%$, $N_{nD} = 40\%$, and $N_R = 35\%$.

6.2 Minority Power and U-Shaped Concentration

Our simulation results in Table 4 indicate another paradox. As minority power increases, the concentration of minority voters generally decreases, consistent with Proposition 5. Yet the light-gray rows reveal a non-monotonic relationship between group power π_{mD} and $R(D)$, where concentration rises again at higher levels of minority power. Extending the simulations by further increasing π_{mD} while holding other groups' power constant, we find a U-shaped relationship for both closed and open primaries (Table 7).

Mathematically, this result follows from the concavity of distributive benefits, the convexity of ideological benefits, and the bounded payoff between 0 and 2. The U-shape is especially pronounced under open primaries because ideological benefits are more strongly convex (Propositions 3 and 4). Graphically, the U-shaped minority power paradox depends on the curvature of the surface S^2, as shown in Figures 8 and 9, where most of the surface is flat at high values of π_{mD} and the concavity appears mainly along the nD–R diagonal as $N_{mDk}/N_k \to 0$.

Intuitively, and focusing on the interaction of $\pi_{mD} * N_{mDk}$ in (6.1), when minority power is low, the optimal strategy concentrates minority voters in a few districts as they do not receive much distributive benefits in electoral competition but can influence the election of the candidate's type. Hence, for low minority power, the optimization focuses on descriptive representation. As

Group Power			Crossover							Closed Primaries		Open Primaries	
π_{mD}	π_{nD}	π_R	a^1_{mD}	a^2_{mD}	a^3_{mD}	a^1_{nD}	a^2_{nD}	a^3_{nD}	a^e_R	Total	$R(\mathbf{D})$	Total	$R(\mathbf{D})$
1	3	1	1	1	1	0.3	0.7	0.7	0	2.046	75%	1.980	75%
2	3	1	1	0.8	0.8	0.3	0.7	0.7	0	2.115	45%	2.006	46%
3	3	1	1	0.7	0.7	0.3	0.7	0.7	0	2.238	39%	2.141	38%
4	3	1	1	0.6	0.6	0.3	0.7	0.7	0	2.301	34%	2.223	22%
5	3	1	1	0.5	0.5	0.3	0.7	0.7	0	2.368	12%	2.312	8%
1	3	1	1	1	1	0.3	0.7	0.7	0	2.046	75%	1.980	75%
2	3	1	1	0.8	0.8	0.3	0.7	0.7	0	2.115	45%	2.006	46%
3	3	1	1	0.7	0.7	0.3	0.7	0.7	0	2.238	39%	2.141	38%
10	3	1	1	0.6	0.6	0.3	0.7	0.7	0	2.908	3%	2.854	6%
20	3	1	1	0.5	0.5	0.3	0.7	0.7	0	3.264	7%	3.222	10%
1	3	1	1	1	1	0.3	0.7	0.7	0	2.046	75%	1.980	75%
2	3	1	1	0.8	0.8	0.3	0.7	0.7	0	2.115	45%	2.006	46%
3	3	1	1	0.7	0.7	0.3	0.7	0.7	0	2.238	39%	2.141	38%
25	3	1	1	0.6	0.6	0.3	0.7	0.7	0	3.435	8%	3.387	11%
50	3	1	1	0.5	0.5	0.3	0.7	0.7	0	3.629	10%	3.589	14%

Table 8: Districting Plans Maximizing Minority Total Benefits: Minority Power and Crossover.

minority power increases, and candidates compete for their votes, distributive benefits rise. It is then optimal to spread minority voters across districts to maximize total benefits across districts. However, at higher levels of π_{mD}, the gradient of T_{mDk} with respect to minority power flattens due to concavity. These diminishing marginal returns allow minority voters to be reallocated across districts—keeping distributive benefits roughly equal across k, while raising ideological benefits in at least one district.

Group Power and Crossover In the simulations above we increased π_{mD} while holding $a^e_{mD} = 1$, $a^e_R = 0$, and $a^e_{nD} \in \{0.3, 0.7, 1\}$. Of course, electoral crossover a^e_i and group power π^e_i are not independent, since a group's swinginess is defined by its willingness to cross over along partisan or identity lines. The precise functional form linking the two is an empirical question. To test robustness, we impose a negative relationship between π_i and a_i: as minority group power increases, its crossover value converges to one-half, reflecting the order of $\pi_{nD}, \pi_R, a^e_{nD}$, and a^e_R but varying the mapping between π_{mD} and a^e_{mD}. The simulations in Tables 8 confirm that as minority power rises and crossover converges to 0.5, the concentration of minority voters again follows a U-shaped pattern.

We recognize that this result is primarily theoretical, since the values of π_{mD} would need to be extremely large relative to those of other groups. Nevertheless, it highlights the trade-offs in redistricting once we account for the electoral structure, the influence of voter groups on elections and platforms, and the resulting patterns of representation.

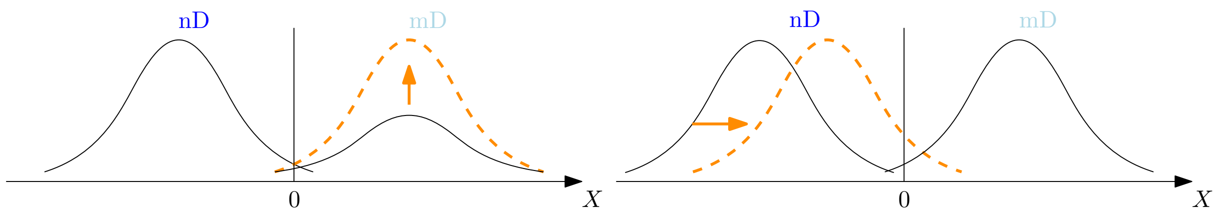
7 Discussion and Conclusion

This paper offers a unified approach to minority representation under electoral competition and redistricting, explicitly incorporating identity and partisan differences. We analyze how voters' ideological and policy preferences interact with candidate identity and party affiliation to influence minority political power (swinginess) and the distributive benefits they receive in a majoritarian system. Optimal redistricting emerges as a trade-off between ideological and distributive gains.

First, grouping less influential minority voters with less influential nonminority voters is more effective in promoting minority interests than grouping influential minority voters with influential nonminority voters, regardless of their political party. Second, concentrated minority districts are effective when minority voters are less swingy and prioritize ideological benefits, thereby emphasizing coalition formation at the electoral stage. By contrast, spreading minority voters across multiple districts is more effective when they are swingy and focused on distributive benefits, shifting coalition building to the legislature. Third, the extent to which districting benefits minorities depends largely on the likelihood of Democrats or Republicans winning in both primary and general elections.

Lastly, our simulations reveal a U-shaped relationship between minority power and preferred concentration: low power favors majority–minority districts, moderate power favors dispersion to maximize distributive benefits, and high power favors renewed concentration that enhances both substantive and descriptive representation. To conclude, we apply the framework developed in the previous sections to examine the impact of various changes in the political landscape–such as increased Black voter registration, the defection of white Democrats to the Republican Party, and decreased racism–on minority electoral success, policy benefits, and redistricting plans.

Increasing Minority Registration and Voting Turnout. Before the passage of the 1965 Voting Rights Act (VRA), many Southern states enacted laws to de facto disenfranchise Blacks. Such devices as the grandfather clause, poll taxes, and white-only primaries, not to mention direct intimidation, minimized Blacks' participation in politics. When one form of discrimination was outlawed, the states would switch to another. This macabre game of wack-a-mole continued until the VRA swept away all such "tests and devices," and its Section 5 preclearance provisions required covered states, those with historical patterns of discrimination, to obtain the permission of the federal government before adopting any new law that might impact minorities' ability to



(a) Minority Voter Registration. (b) Nonminority-Democratic Crossover.

Figure 5: Relative Power of nonminority and minority-Democratic Voters.

vote. The most direct result of passing the VRA was thus to greatly increase Blacks' participation to the point where now, in most areas of the South, minorities register and vote at rates at or above those of white voters.[31]

From the model above, the impact of an increase in statewide minority-Democrats is (usually) unambiguous: it acts just like an increase in their share of legislative benefits, $T_{mDk} = \pi_{mD} N_{mDk}/\Pi_k$, and so both increase the flow of benefits to minority constituents and make it easier to elect minorities to office, thereby increasing their ideological benefits. The shift in legislative benefits is also illustrated in Figure 5(a), where the horizontal axis shows the ideological distributions of the minority and nonminority Democrat voters, with the 0, or indifference, point in the middle. The increase in the size of the minority electorate increases their power π_{mD} by raising $\phi_{mD}(0)$ while also increasing the number of districts that could elect a minority candidate. Cascio and Washington (2014) document how the reforms following the VRA increased voter turnout and public spending for counties with a higher Black population.

Furthermore, according to the model, as the number of Black registered voters increases from low numbers, the first response of state district drawers should be to create concentrated minority districts. Indeed, this happened in the 1970s and 1980s, with one rule of thumb stating that districts had to be at least 65% Black to be "effectively" majority-minority. As minority participation continues to increase, the response should be to concentrate minority voters less, spreading them out more evenly across districts. Some worry that reducing the majority in these districts will dilute their influence over policy and reduce the number of minorities in office, thereby giving back some of the hard-won gains of the civil rights movement. Others view it as a natural progression of minorities into mainstream politics and a means to expand their political influence.

Economic Inequality. Robust empirical evidence shows that minority and nonminority voter groups differ in income and wealth levels in the United States, with some minority groups exhibit-

[31] By 2020, Black registration and electoral turnout in Southern states only differed by 5 percentage points.

ing lower economic means (Black, Hispanic, and Native American households) and other minority groups with greater economic means (Asian households). As pointed out by Dixit and Londregan (1996), voters' degree of diminishing returns to consumption affects their sensitivity in trading off distributive for ideological benefits. If one would focus on diminishing returns to consumption, all else being equal, less affluent voters are easier to sway by candidates offering distributive benefits, making their support less costly but also more competitive. In our analysis, this would imply that one would spread minority voters with less economic means across more districts to benefit them, while concentrating those minority voters who are more affluent and less responsive to distributive benefits. However, one must also consider the degree of diminishing returns, as captured by the parameter ϵ. Even minority groups with low economic means may be less sensitive to distributive benefits in their vote choice, large ϵ, and would prefer to be concentrated in few districts; while minority voter groups, even affluent ones, with high sensitivity, low ϵ, would prefer to a representation across many districts. These theoretical nuances of our model highlight the need for more empirical investigations and inform the legal challenges around descriptive and substantive representation.

Hispanic and Latino Vote. Recent trends in the Hispanic/Latino vote in the United States are complex and multifaceted. Historically, Hispanics have aligned with the Democratic party and tend to be concentrated in minority districts largely overlooked by the Democratic party. However, as Hispanics increasingly become swing voters, shifting between the Republican and Democratic parties, their ability to gain distributive gains will inevitably increase.

One factor that appears to be driving this shift is the changing nature of working-class labor trends, with some Hispanic/Latino voters moving towards the Republican Party in response to economic concerns and a desire for greater job security. Evidence suggests that conservative social issues, such as abortion and same-sex marriage, also play a role in this shift. Another factor is the diversity within the Hispanic/Latino community, which includes individuals with a wide range of cultural, linguistic, and socioeconomic backgrounds. Consequently, there is significant variation in voting patterns based on individual and community-level preferences.

Increasing Crossover. Finally, we come to the increased willingness of nonminority voters of all stripes to vote for minority candidates due to steadily decreasing racism. Decreasing racism does help minorities win office, and indeed, the number of elected minorities in the South has skyrocketed since adopting the VRA.[32] Nevertheless, as mentioned above and illustrated in Figure 5(b), the impact on distributive benefits is complex. For nonminority voters to be less racist, they must be less ideologically averse to minorities' holding office, represented by a right-hand shift in the distribution of X-values as in Figure 5(b). This shift increases the density of nonminority Democrat voters at $X = 0$; as minorities become more influential in elections, they will enjoy greater legislative benefits. These gains continue until the central hump of the distribution passes the 0 threshold, beyond which decreased racism (increased crossover) also leads to a smaller share of the legislative pie.

Since less than 50% of nonminority voters reliably support minority candidates, though, we may assume that we still reside on the upward slope of the distribution function. Hence, nonminority voters may be gaining greater benefits from candidates' platforms at the expense of minority voters. Of course, the tradeoff regarding increased descriptive representation may be worthwhile. However, it is still interesting that decreased racism is not an unalloyed good for minorities. There have long been rumblings that Democrats in office, white and Black alike, take their minority constituents for granted and give them less than their fair share of benefits. While this may be true, and if so, the model here provides a plausible rationale for why it would happen.

Future Research We end by pointing to several possible extensions to our model. First, our current legislative model is simple to focus on the logic of changing preferences and group powers without building an incentive to form party-based coalitions in the legislature. Hence, there are no real legislative parties or permanent coalitions. Nevertheless, if one wanted to investigate the impact of changing legislative rules, committee powers, or party leadership on districting, these elements could be incorporated into the legislative model.

Voter Demographics and Partisanship. Our analysis centers around the districting, electoral, and legislative characteristics and demographics in the United States. We consider two parties and divide the population into a minority and a nonminority. Both demographics and partisanship describe the groups and their political motivations. However, our analysis can be generalized to majoritarian systems in which parties compete in single-representative districts, holding some form of primaries to choose a candidate from within the party who faces other parties' candidates, a legislature awarding distributive benefits across districts and voting groups, and voting groups

[32]See the essays Davidson and Grofman (1994) for detailed state-by-state analyses attributing the rise in Black office holding directly to the VRA. Indeed, 2020 voter data shows that whites register at a 75% rate, Blacks 68%, Asians 64%, and Hispanics 59%—-similarly, Whites turnout at a 69% rate, Blacks 61%, Asians 59%, and Hispanics 52%.

identified by partisanship and identity. The two voters' identities can follow ethnic, racial, religious, economic, or gender attributes that may define a group and describe the group's voting behavior. For more than two voter identities, groups may still be similar regarding their willingness to cross partisan and identity lines. For example, many minority voters, such as Black, Afro-American, Hispanic, Asian-American, or LGBTQIA+, tend to support in various degrees the Democratic party in the United States, and their willingness or reluctance to vote for Republican candidates may unify them. The key tensions in our analysis arise from i) a group's willingness to trade off distributive benefits from a legislature for ideological benefits from a candidate's identity, ii) a group's willingness to cross partisan and identity lines in elections (e.g., minority voters' attachment to a party, nonminority voters' tradeoff between minority candidate from the same party or nonminority candidate from the other party), and iii) the institutional principles of districting.

Electoral Platforms and Fiscal Policies. Our results follow Dixit and Londregan (1996)'s characterization of electoral platforms in which candidates make identical promises regarding redistribution of consumption benefits and taxes for each voter group in a district, all under the assumption that both parties have identical abilities to distribute benefits and raise taxes. Our analysis streamlines the budget process and neglects the collection of taxes, assuming that there is sufficient fiscal capacity and discretion in the budget. Instead, we focus on distributive benefits only, fiscal transfers from the legislature's budget to the district(s) that come as subsidies, tax credits, tax deductions, or welfare payments. If candidates were to run on in-kind transfers or public goods, where voter groups may have different preferences across programs, each candidate would need to evaluate the marginal value of a dollar spent for any program on the voter group's marginal utility from the program, affecting the change in the voter group's vote.

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A Appendix: Derivations and Proofs

A.1 Sample State and Five Districts

The example of Figure 1(b) is created with $S = (.36, 0.26, .38)$ and valid districting matrix for five districts with

$$\begin{pmatrix} 0.19 & 0.6 & 0.21 \\ 0.33 & 0.05 & 0.62 \\ 0.45 & 0.1 & 0.45 \\ 0.14 & 0.43 & 0.42 \\ 0.65 & 0.13 & 0.22 \end{pmatrix}.$$

A.2 A Solution to Legislative Policies and Bargaining

The derivation and solution of a possible subgame of legislative bargaining follow Baron and Ferejohn (1989).

Close Rule Legislative Bargaining Suppose the legislature passes a redistributive policy, dividing K dollars across all districts. They do so via a closed rule bargaining process: a legislator is selected randomly to offer a proposed budget division. The entire legislature then votes on the proposal (under a closed rule); if it is adopted, the game ends. If a majority vote rejects it, discounting occurs (all payoffs are lowered by a factor of δ, $0 < \delta \leq 1$), and the legislative subgame starts again with another member chosen randomly to make an offer. In this game, members try to maximize the benefits directed toward their district.[33]

The outcome of this legislative process will be a vector $(B_1, B_2, ..., B_K)$ of district-specific benefits, with $B_k \geq 0$ and $\sum_k B_k = K$, allocating a given budget,[34] across K odd districts.[35].So

[33] Our analysis streamlines the fiscal budgetary process with a focus on a single legislature, discretionary spending, and a given budget amount. The analysis ignores the complexities of redistribution that arise when considering federal vs. state legislatures in a multi-level system of redistribution, legislative budgeting vs. executive budget implementation in a system with separate powers, discretionary vs. mandatory spending in an environment of multi-means redistribution, or balanced budget vs. flexible budget in a fiscal setting with tax collection, government borrowing or lending, and endogenous budget amounts. Here, we focus on who gets elected and the policy outcome enacted by the legislature. For a budget process with separation of powers, see Grossman and Helpman (2008); for redistribution with the same different abilities of parties to collect taxes and distribute benefits, see Dixit and Londregan (1996).

[34] We ignore considerations of balanced budgets and taxation and instead assume that a given budget is divided across districts as the federal government can borrow or has revenues from various other tax sources unrelated to the distributive policies of interest here. For balanced budget implications and redistributive policies, see Cox and McCubbins (1986) and Lindbeck and Weibull (1987).

[35] We will assume that each district elects one legislator and the odd number of legislators avoids legislative ties.

the legislative policy function is $\mathbf{P} : \mathcal{C}^K \rightarrow \mathbb{R}_+^K$. This follows from the results of the elections, which in turn depend on the districting scheme, so $\mathbf{P} = \mathbf{P}(\mathbf{L}(\mathbf{D}(S^2)))$.

Any funds allocated to district k in the legislative process are divided according to the platform adopted by that district's representative. So if the type j representative from district k ran on a platform promising T_{ijk} to members of a group i, then voters in this group will receive $T_{ijk} * B_k$ in total benefits, with individual benefits $b_{ijk} = (T_{ijk} * B_k)/N_{ik}$.

Legislative Outcomes The legislative game is elementary; in equilibrium, the legislator who makes the first offer constructs a random minimum-winning coalition of $(K-1)/2$ other legislators and keeps the remainder for herself.[36] Let l be the legislator who makes the offer, C be the legislators selected to be in the coalition, and O be the remaining legislators. Then, the equilibrium offers to share the K being distributed are:

$$B_k = \begin{cases} \frac{(2-\delta)K+\delta}{2} & \text{if } k = l; \\ \delta & \text{if } k \in C; \\ 0 & \text{if } k \in O. \end{cases} \tag{A.1}$$

Since the game is symmetric, each legislator has an expected return of 1 from the legislative bargaining session. If a candidate promises a group T_{ijk} in transfers during the election, this is also their expected total legislative payout if that candidate wins office.

In this setup, the chosen legislator constructs a minimum-winning coalition at the lowest possible cost for herself, which describes its Shapley value. Hence, the cost of swaying other legislators may or may not be related to partisan control of the legislators, but it is less if legislators are driven by securing transfers for their districts, and competition for joining the minimum-winning coalition dominates.

A.3 Derivation of Solution to Candidate Platforms

The derivation and solution of the subgame follow Dixit and Londregan (1996) and incorporate the districting plans and characterization of minority benefits from our model.

[36]See Baron and Ferejohn (1989)'s Proposition 3 for a stationary subgame-perfect equilibrium with infinite sessions, majority and closed rule, and n-(odd)-legislators as well as $n = K$ districts/legislators and a total distributive benefit of K instead of 1.

Elections and Voter Groups First, we characterize the candidates' platforms. Candidates adopt platforms to maximize their votes, subject to an allocation of district benefits across voter groups. The voter groups in a closed primary are $\Theta \in \{mD, nD\}$, in an open primary $\Theta \in \{mD, nD, R\}$, and in a general election $\Theta \in \{mD, nD, R\}$.

Candidate Platforms Two candidates announce simultaneously in each election. Candidate 1's problem is

$$\max V_1^e = \sum_{i \in \Theta} N_i \Phi_i^e(X_i) \text{ s.t. } \sum_{i \in \Theta} T_{i1k} B_k = \sum_{i \in \Theta} N_{ik} b_{i1k} \leq B_k \tag{A.2}$$

and candidate 2's is equivalently

$$\max V_2^e = \sum_{i \in \Theta} N_i [1 - \Phi_i^e(X_i)] \text{ s.t. } \sum_{i \in \Theta} T_{i2k} B_k = \sum_{i \in \Theta} N_{ik} b_{i2k} \leq B_k. \tag{A.3}$$

For the existence of a Nash equilibrium, Glicksberg's Theorem requires that each candidate's payoffs are a quasi-concave function of their strategy and a continuous function of other players' strategies. First, the distributive benefits for voters, b_{ij}, are an increasing linear function of the candidates' platforms, T_{ijk}. Second, the voters' cutoff for differences in candidates' promised benefits, $X_i^e = U_i(b_{i1}) - U_i(b_{i2})$, is increasing and concave in b_{ij}. Finally, the expected candidate's number of vote, $V_1^e = \sum_{i \in \Theta} N_i \Phi_i^e(X_i)$ and $V_2^e = \sum_{i \in \Theta} N_i(1 - \Phi_i^e(X_i))$, is increasing in the cutoff X_i and concave due to the concavity of $\Phi_i(.)$. Hence, existence holds.

For the solution of the Nash equilibrium, we use Lagrange parameters λ_1 and λ_2 for each respective candidate and solve all first-order conditions simultaneously. Consider candidate 1 first:

$$L = \sum_{i \in \Theta} N_i \Phi_i^e(X_i) + \lambda_1 \left(B_k - \sum_{i \in \Theta} N_{ik} b_{i1k} \right) \tag{A.4}$$

with

$$\frac{\partial L}{\partial T_{i1k}} = N_{ik} \phi_i^e(X_i) U_i'(b_{i1k}) \frac{\partial b_{i1k}}{\partial T_{i1k}} - \lambda_1 N_{ik} \frac{\partial b_{i1k}}{\partial T_{i1k}} = 0, \tag{A.5}$$

which can be written as

$$\lambda_1 = \phi_i^e(X_i) U_i'(b_{i1k}) \;\Leftrightarrow\; b_{i1k} = H_i \left(\frac{\lambda_1}{\phi_i^e(X_i)} \right), \tag{A.6}$$

where $H_i(.)$ is the inverse of the marginal utility function. Since $U_i(.)$ is a decreasing function, $H_i(.)$ is decreasing as well, and there is a unique solution for λ_1.

Candidate 2's problem is symmetric, and we have

$$L = \sum_{i \in \Theta} N_i(1 - \Phi_i^e(X_i)) + \lambda_2 \left(B_k - \sum_{i \in \Theta} N_{ik} b_{i2k} \right) \tag{A.7}$$

with

$$\frac{\partial L}{\partial T_{i2k}} = -N_{ik}\phi_i^e(X_i)(-U_i'(b_{i2k}))\frac{\partial b_{i2k}}{\partial T_{i2k}} - \lambda_2 N_{ik}\frac{\partial b_{i2k}}{\partial T_{i2k}} = 0, \tag{A.8}$$

which can be written as

$$\lambda_2 = \phi_i^e(X_i)U_i'(b_{i2k}) \;\Leftrightarrow\; b_{i2k} = H_i\left(\frac{\lambda_2}{\phi_i^e(X_i)}\right), \tag{A.9}$$

which provides a unique solution for λ_2.

The Lagrange parameters are independent of candidates' characteristics, which implies both candidates face the same shadow value in equilibrium, $\lambda_1 = \lambda_2$. As a result, a voter's marginal utility in distributive benefits is equal across both candidates:

$$\lambda_1 = \lambda_2 \;\Leftrightarrow\; U_i'(b_{i1k}) = U_i'(b_{i2k}). \tag{A.10}$$

Due to $U_i(.)$ being a continuous, increasing function, we have that the distributive benefits are identical across both candidates, $b_{i1k} = b_{i2k}$, which implies that both candidates choose identical platforms, $T_{i1k} = T_{i2k}$, and distributive promises cancel each other out such that voters choose based on ideological alignments.

Group and Member Benefits Now, we describe the distribution of district benefits across groups and their members. We use the first-order conditions above with the voters' utility function described by (3.1) and (3.2):

$$\lambda_j = \phi_i^e(0)U_i'(b_{ijk}) \;\Rightarrow\; b_{ijk} = \left(\frac{\phi_i(0)\kappa_i}{\lambda_j}\right)^{\frac{1}{\epsilon}} = \frac{\pi_i}{\lambda_j^{1/\epsilon}} \;\Leftrightarrow\; \lambda_j^{1/\epsilon} = \frac{\pi_i}{b_{ijk}}. \tag{A.11}$$

Applying $\lambda_1 = \lambda_2$ and $b_{i1k} = b_{i2k}$, we get for group i compared to group $h \neq i$ that

$$b_{ijk} = \frac{\pi_i b_{hjk}}{\pi_h}. \tag{A.12}$$

Using the budget constraint of $\sum_i N_{ik} b_{ijk} = B_k$ with (A.12), we get

$$b_{hjk} = \frac{\pi_h}{\sum_i N_{ik}\pi_i} B_k, \tag{A.13}$$

which provides the individual benefits for a member of group Θ in (4.1). The group shares follow from rearranging $b_{ijk} = (T_{ijk} * B_k)/N_{ik}$ and applying (A.13):

$$T_{hjk} = \frac{b_{hjk} N_{ik}}{B_k} = \frac{\pi_h N_{hk}}{\sum_i N_{ik}\pi_i}, \tag{A.14}$$

which completes (4.1).

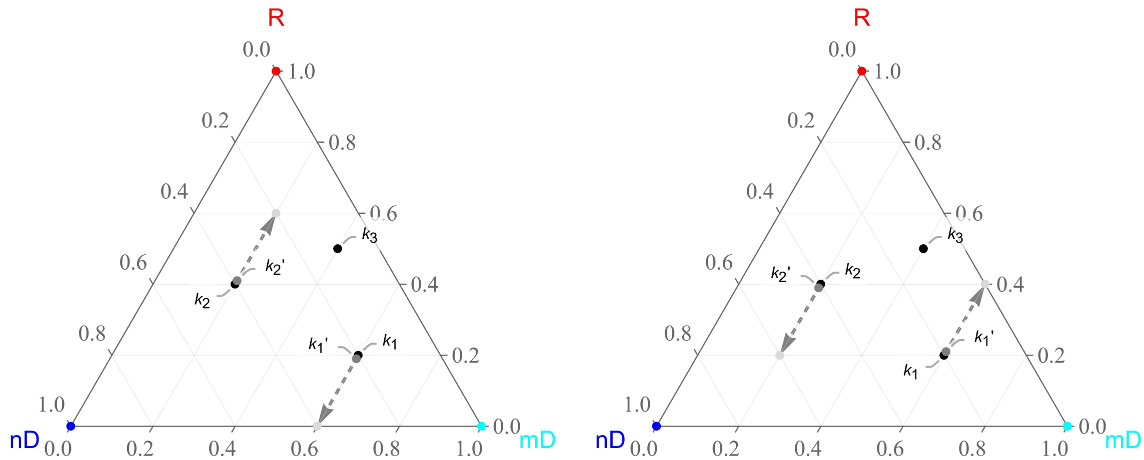
Primaries and General Election Note that candidates will offer identical platforms, but the platforms themselves depend on the voter groups and their characteristics. Hence, both Democrats offer identical platforms in a closed primary to minority and non-minority Democrat voters. Still, their platforms differ (equally) in an open primary when they also make promises to potential Republican voters, reducing promises to Democrat voters. In the general election, ignoring any credibility cost or commitments, the primary winner focuses on winning and follows a flexible platform strategy. The platform would change if the primary election were closed and the Democratic candidate competed for Republican voters, or if the primary were open but Republican turnout across the election differed.

A.4 Proof of Proposition 1

Consider a valid districting scheme $\mathbf{D}$, and assume to the contrary that $N_{ik} > 0$ for all $i \in \Theta$ and k. First, assume that Republicans are more powerful than nonminority Democrats, $\pi_{nD} < \pi_R$. Consider two districts k_1 and k_2, with N_{mD1} minority voters in k_1 and N_{mD2} in k_2, and assume that $N_{mD1} > N_{mD2}$. This is illustrated in Figure 6(a).

We now move one Republican voter from k_1 to k_2 and one nonminority-Democrat voter from k_2 and k_1, while holding minority voters constant in each district. Such a change preserves the validity of the districting scheme, and the arrows in Figure 6(a) illustrate the direction of changes. For optimality, it has to increase minority voters' distributive benefits. Hence, we compare any gains and losses across the two districts.

Considering the changes in minority benefits from (5.8), minority voters' distributive benefits in k_1 are increasing when a Republican is replaced with a less powerful nonminority-Democratic



(a) $\pi_{nD} < \pi_R$ and $\Pi_1 \leq \Pi_2$.

(b) $\pi_{nD} > \pi_R$ and $\Pi_1 \leq \Pi_2$.

Figure 6: Optimal Districting Process with $K = 3$.

voter, by

$$N_{mD1}\frac{\partial f}{\partial N_{nD1}} = N_{mD1}^2\frac{\pi_{mD}(\pi_R - \pi_{nD})}{\Pi_1^2}; \tag{A.15}$$

while minority voters' distributive benefits in k_2 are decreasing when a more powerful Republican replaces the nonminority-Democrat voter, by

$$N_{mD2}\frac{\partial f}{\partial N_{nD2}} = N_{mD2}^2\frac{\pi_{mD}(\pi_R - \pi_{nD})}{\Pi_2^2}. \tag{A.16}$$

Comparing A.15 with A.16, we get

$$\frac{N_{mD1}^2}{\Pi_1^2} \gtreqless \frac{N_{mD2}^2}{\Pi_2^2}. \tag{A.17}$$

Given the assumption of $N_{mD1} > N_{mD2}$, the minority gains in k_1 outweigh the minority losses in k_2 if $\Pi_1^2 \leq \Pi_2^2$, and such redistricting would be optimal as it increases net gains for minority voters.

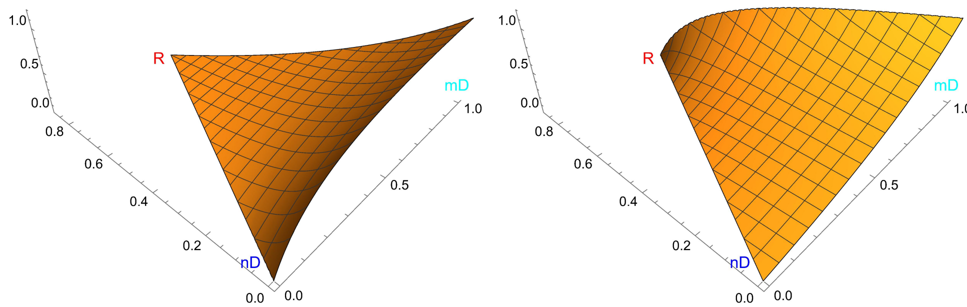
Second, assume that nonminority-Democrat voters are more powerful than Republican voters, $\pi_{nD} > \pi_R$, and the number of minorities differs, $N_{mD1} > N_{mD2}$. Then it would be beneficial to move a nonminority Democrat from k_1 to k_2, and a Republican vice versa, if the minority gains in k_1 are greater than the minority losses in k_2. This comparison follows again

$$\frac{N_{mD1}^2}{\Pi_1^2} \gtreqless \frac{N_{mD2}^2}{\Pi_2^2} \text{ with } \pi_{nD} > \pi_R. \tag{A.18}$$

If the minority-concentrated district is less powerful, then this would be beneficial. Figure 6(b) illustrates this process.

Hence, the proposed districting scheme **D** cannot be optimal. Through re-iteration of the process – the district's power decreases when less powerful nonminority voters replace more powerful nonminority voters, $\frac{\partial \Pi_k}{\partial N_{nD}} = \pi_{nD} - \pi_R$ – minority populated districts with low district power or nonminority populated districts with high district power will not lie in the interior of S^2.

A.5 Nonconcave and Nonconvex Minority Distributive Benefits



(a) $\pi_{mD} = 7$, $\pi_{nD} = 1$, $\pi_R = 15$

(b) $\pi_{mD} = 7$, $\pi_{nD} = 15$, $\pi_R = 1$

Figure 7: Nonconcave and Nonconvex Minority Distributive Benefits.

A.6 Proof of Lemma 1

Consider a valid districting scheme **D** and a district $\mathbf{d}^* = (N_{mD}, N_{nD}, N_R)$. Define $t = N_{mD}/N_k$ and $\alpha = N_{nD}/(N_{nD} + N_R)$, and let $l = \{\mathbf{d} \in \mathcal{D} | \alpha = N_{nD}/(N_{nD} + N_R)\}$. Thus l is a line running through $\mathbf{d}^*$, connecting it to $(1, 0, 0)$, which is the corner of S^2 where the district composes of minority voters only ("mD" in triangles) while keeping the ratio of nonminority voters constant throughout. Applying (5.1) divided by N_k with $t = N_{mDk}/N_k$, $(1-t)*\alpha = \frac{N_{nDk}+N_{Rk}}{N_k} * \frac{N_{nDk}}{N_{nDk}+N_R} = N_{nDk}/N_k$, and $(1-t)*(1-\alpha) = \frac{N_{nDk}+N_{Rk}}{N_k} * \frac{N_{Rk}}{N_{nDk}+N_R} = N_{Rk}/N_k$, the parameterized path for the minority payoff function is

$$g(t) = \frac{\pi_{mD} t}{\pi_{mD} t + \pi_{nD}(1-t)\alpha + \pi_R(1-t)(1-\alpha)} \tag{A.19}$$

Note that the denominator is positive, and we can evaluate the curvature of the path with its second derivative with respect to t:

$$g''(t) = -\frac{2\pi_{mD} \overbrace{(\alpha\pi_{nD} + (1-\alpha)\pi_R)}^{(+)} \overbrace{(\pi_{mD} - \alpha\pi_{nD} - (1-\alpha)\pi_R)}^{(?)}}{\underbrace{(\pi_{mD} t + \pi_{nD}(1-t)\alpha + \pi_R(1-t)(1-\alpha))^3}_{(+)}}. \tag{A.20}$$

The second derivative is negative if $\pi_{mD} > \alpha\pi_{nD} + (1-\alpha)\pi_R$ – i.e., when minorities' power is greater than the weighted average of the other groups' powers, based on district population. Hence, $\pi_{mD} = \max_{i\in\Theta}\{\pi_\Theta\}$ implies that $g''(t) < 0$ for all t, indicating that the entire surface is concave. Conversely, the second derivative is positive if $\pi_{mD} < \alpha\pi_{nD} - (1-\alpha)\pi_R$, which implies that for $\pi_{mD} = \min_{i\in\Theta}\{\pi_\Theta\}$ we get $g''(t) > 0$ for all t, and the surface is convex.

A.7 Proof of Proposition 3

We separate the proof into an analysis for a state with closed primaries and then repeat the steps for a state with open primaries.

Closed Primaries For the first part, we substitute the closed primary and general election probabilities of a minority candidate winning at each stage, (3.8) and (3.10), take the respective derivative from (3.12) with respect to the number of minority voters, and get:

$$\frac{\partial\Psi_{mDk}}{\partial N_{mDk}} = \overbrace{\frac{\partial\Psi^1_{mDk}}{\partial N_{mDk}}\Psi^2_{mDk}}^{primary} + \overbrace{\Psi^1_{mDk}\frac{\partial\Psi^2_{mDk}}{\partial N_{mDk}}}^{general} \tag{A.21}$$

$$= \underbrace{\frac{(a^1_{mD} - a^1_{nD})N_{nDk}}{(N_{mDk}+N_{nDk})^2}\underbrace{\Psi^2_{mDk}}_{(\geq 0)}}_{primary:(+)} + \underbrace{\underbrace{\Psi^1_{mDk}}_{(\geq 0)}\frac{(a^2_{mDk} - a^2_{nD})N_{nDk} + (a^2_{mD} - a^2_R)N_{Rk}}{(N_{mDk}+N_{nDk}+N_{Rk})^2}}_{general:(+)} \geq 0. \tag{A.22}$$

Given our assumption that minority voters are more likely to vote for a minority candidate than nonminority voters, $a^e_{mD} > a^e_{nD} > a^e_R$, we see immediately that the first term is positive, $a^1_{mD} > a^1_{nD}$, and the two subsequent terms are positive or zero. The last term is positive or zero as well as for $a^2_{mD} \geq a^2_{nD}$ and $a^2_{mD} \geq a^2_R$. Note that our statement is independent of whether $a^2_{nD} \gtreqless a^2_R$. The same holds for an evaluation on S^2 with $N_{Rk} = N_k - N_{mDk} - N_{nDk}$.

For the second part, we repeat the substitution but take the respective derivative from (3.12) with respect to the number of nonminority-Democrat voters and get:

$$\frac{\partial \Psi_{mDk}}{\partial N_{nDk}} = \overbrace{\frac{\partial \Psi^1_{mDk}}{\partial N_{nDk}} \Psi^2_{mDk}}^{primary} + \overbrace{\Psi^1_{mDk} \frac{\partial \Psi^2_{mDk}}{\partial N_{nDk}}}^{general} \tag{A.23}$$

$$= \underbrace{\frac{(a^1_{nD} - a^1_{mD}) N_{mDk}}{(N_{mDk} + N_{nDk})^2} \underbrace{\Psi^2_{mDk}}_{(\geq 0)}}_{primary:(-)} + \underbrace{\underbrace{\Psi^1_{mDk}}_{(\geq 0)} \frac{(a^2_{nD} - a^2_{mD}) N_{nDk} + (a^2_{nD} - a^2_{R}) N_{Rk}}{(N_{mDk} + N_{nDk} + N_{Rk})^2}}_{general:(+/-)} \gtreqless 0. \tag{A.24}$$

The first term is negative due to $a^1_{mD} > a^1_{nD}$, the two subsequent terms are positive or zero, and the last term is ambiguous. We have that $a^2_{mD} > a^2_{nD}$, illustrating the negative effect in the primary election for minority candidates. The derivative may be negative overall, but that may be offset by $a^2_{mD} > a^2_{nD}$ and $a^2_{nD} > a^2_{R}$, which illustrates the positive effect in the general election for minority candidates – nonminority-Democratic voters being more likely to support a minority-Democratic candidate than Republican voters ($a^2_{nD} > a^2_{R}$). Here, the result depends on the relationship between differences in crossover voting of nonminority voters.

For the third part, regarding replacing Republican voters with nonminority-Democrats, we rewrite the probability of a minority candidate winning the election (3.12) as

$$\tilde{\Psi}_{mDk} = \left(\frac{a^1_{mD} N_{mDk} + a^1_{nD} N_{nDk}}{N_{mDk} + N_{mDk}} \right) \left(\frac{a^2_{mD} N_{mDk} + a^2_{nD} N_{nDk} + a^2_{R} (N_k - N_{mDk} - N_{nDk})}{N_k} \right), \tag{A.25}$$

where we employ the district's population for substitution, $N_{Rk} = N_k - N_{mD} - N_{nD}$. To illustrate the replacement effect – increasing nonminority-Democratic voters and decreasing Republican voters in a district – we take the derivative with respect to the number of nonminority-Democratic voters and get:

$$\frac{\partial \tilde{\Psi}_{mDk}}{\partial N_{nDk}} = \overbrace{\frac{\partial \Psi^1_{mDk}}{\partial N_{nDk}} \tilde{\Psi}^2_{mDk}}^{primary} + \overbrace{\Psi^1_{mDk} \frac{\partial \tilde{\Psi}^2_{mDk}}{\partial N_{nDk}}}^{general} \tag{A.26}$$

$$= \underbrace{\frac{(a^1_{nD} - a^1_{mD}) N_{mDk}}{(N_{mDk} + N_{nDk})^2} \underbrace{\tilde{\Psi}^2_{mDk}}_{(\geq 0)}}_{primary:(-)} + \underbrace{\underbrace{\Psi^1_{mDk}}_{(\geq 0)} \frac{a^2_{nD} - a^2_{R}}{N_k}}_{general:(+)} \gtreqless 0, \tag{A.27}$$

where the minority candidate's chances decrease in the primary but increase in the general election.

For the last part, we employ again (A.25) and take the second derivative with respect to the number of minority voters:

$$\frac{\partial^2 \tilde{\Psi}_{mDk}}{\partial N_{mDk}^2} = \frac{2(a_{mD}^1 - a_{nD}^1)N_{nDk}\left((a_{mD}^2 - a_{nD}^2)N_{nDk} - a_R^2 N_k\right)}{N_k(N_{mDk} + N_{nDk})^3}, \tag{A.28}$$

which is positive when

$$(a_{mD}^2 - a_{nD}^2)N_{nDk} - a_R^2 N_k > 0. \tag{A.29}$$

Hence, for $a_R^2 < (a_{mD}^2 - a_{nD}^2)N_{nDk}/N_k$, Ψ_{mD} is convex on S^2.

Open Primaries For the first part, we substitute the open primary and general election probabilities of a minority candidate winning at each stage, (3.9) and (3.10), take the respective derivative from (3.12) with respect to the number of minority voters, and get:

$$\begin{aligned}\frac{\partial \hat{\Psi}_{mDk}}{\partial N_{mDk}} &= \overbrace{\frac{\partial \hat{\Psi}_{mDk}^1}{\partial N_{mDk}} \Psi_{mDk}^2}^{primary} + \overbrace{\hat{\Psi}_{mDk}^1 \frac{\partial \Psi_{mDk}^2}{\partial N_{mDk}}}^{general} \\ &= \underbrace{\frac{(a_{mD}^1 - a_{nD}^1)N_{nDk} + (a_{mD}^1 - a_R^1)N_{Rk}}{(N_{mDk} + N_{nDk} + N_{Rk})^2} \underbrace{\Psi_{mDk}^2}_{(\geq 0)}}_{primary:(+)} \end{aligned} \tag{A.30}$$

$$+ \underbrace{\underbrace{\hat{\Psi}_{mDk}^1}_{(\geq 0)} \frac{(a_{mDk}^2 - a_{nD}^2)N_{nDk} + (a_{mD}^2 - a_R^2)N_{Rk}}{(N_{mDk} + N_{nDk} + N_{Rk})^2}}_{general:(+)} \geq 0. \tag{A.31}$$

Given our assumption that minority voters are more likely to vote for a minority candidate than nonminority voters, $a_{mD}^e > a_{mD}^e > a_R^e$, we see immediately that the first term is positive, $a_{mD}^1 > a_{nD}^1$, and the two subsequent terms are positive or zero. The last term is positive or zero as well as for $a_{mD}^2 \geq a_{nD}^2$ and $a_{mD}^2 \geq a_R^2$. Note that our statement is independent of whether $a_{nD}^2 \gtreqless a_R^2$. The same holds for an evaluation on S^2 with $N_{Rk} = N_k - N_{mDk} - N_{nDk}$.

For the second part, we take the respective derivative from (3.12) with respect to the number of nonminority-Democrat voters and get:

$$\begin{aligned}\frac{\partial \hat{\Psi}_{mDk}}{\partial N_{nDk}} &= \overbrace{\frac{\partial \hat{\Psi}_{mDk}^1}{\partial N_{nDk}} \Psi_{mDk}^2}^{primary} + \overbrace{\hat{\Psi}_{mDk}^1 \frac{\partial \Psi_{mDk}^2}{\partial N_{nDk}}}^{general} \\ &= \underbrace{\frac{(a_{nD}^1 - a_{mD}^1)N_{mDk} + (a_{nD}^1 - a_R^1)N_{Rk}}{(N_{mDk} + N_{nDk} + N_{Rk})^2} \underbrace{\Psi_{mDk}^2}_{(\geq 0)}}_{primary:(+/-)} \end{aligned} \tag{A.32}$$

$$+\underbrace{\underbrace{\hat{\Psi}^1_{mDk}}_{(\geq 0)} \frac{(a^2_{nD}-a^2_{mD})N_{nDk}+(a^2_{nD}-a^2_{R})N_{Rk}}{(N_{mDk}+N_{nDk}+N_{Rk})^2}}_{general:(+/-)} \gtreqless 0. \tag{A.33}$$

The first term is this time ambiguous due to $a^1_{nD} < a^1_{mD}$ and $a^1_{nD} > a^1_R$, the two subsequent terms are positive or zero, and the last term is ambiguous, too. With an open primary, the negative effect in the closed primary election for minority candidates no longer holds. Here again, the result depends on the relationship between differences in crossover voting of nonminority voters, both in the primary and general election, where it was only dependent on the general election with closed primaries.

For the third part, regarding nonminority-Democrats replacing Republican voters, we rewrite the probability of a minority candidate winning the election (3.12) as

$$\begin{aligned}\hat{\tilde{\Psi}}_{mDk} &= \left(\frac{a^1_{mD}N_{mDk}+a^1_{nD}N_{nDk}+a^1_R(N_k-N_{mDk}-N_{nDk})}{N_k}\right) \\ &\quad\times\left(\frac{a^2_{mD}N_{mDk}+a^2_{nD}N_{nDk}+a^2_R(N_k-N_{mDk}-N_{nDk})}{N_k}\right),\end{aligned} \tag{A.34}$$

where we employ the district's population for substitution, $N_{Rk} = N_k - N_{mD} - N_{nD}$. To illustrate the replacement effect – increasing nonminority-Democratic voters and decreasing Republican voters in a district – we take the derivative with respect to the number of nonminority-Democratic voters and get:

$$\frac{\partial\hat{\tilde{\Psi}}_{mDk}}{\partial N_{nDk}} = \overbrace{\frac{\partial\hat{\tilde{\Psi}}^1_{mDk}}{\partial N_{nDk}}\tilde{\Psi}^2_{mDk}}^{primary} + \overbrace{\hat{\tilde{\Psi}}^1_{mDk}\frac{\partial\tilde{\Psi}^2_{mDk}}{\partial N_{nDk}}}^{general} \tag{A.35}$$

$$= \underbrace{\frac{a^1_{nD}-a^1_R}{N_k}\underbrace{\tilde{\Psi}^2_{mDk}}_{(\geq 0)}}_{primary:(+)} + \underbrace{\underbrace{\hat{\tilde{\Psi}}^1_{mDk}}_{(\geq 0)}\frac{a^2_{nD}-a^2_R}{N_k}}_{general:(+)} \geq 0, \tag{A.36}$$

where the minority candidate's chances increase in the primary and general elections when nD voters are crossing more over than R voters.

For the last part, we employ again (A.34) and take the second derivative with respect to the number of minority voters:

$$\frac{\partial^2\hat{\tilde{\Psi}}_{mDk}}{\partial N^2_{mDk}} = \frac{2(a^1_{mD}-a^1_R)(a^2_{mD}-a^2_R)}{N^2_k} > 0, \tag{A.37}$$

which is positive and hence, $\hat{\tilde{\Psi}}_{mDk}$ is convex on S^2.

A.8 Proof of Proposition 4

We separate the proof into an analysis for a state with closed primaries and then repeat the steps for a state with open primaries.

Closed Primaries The expected utility for minority voters from (5.16) can be rewritten with (3.8) to (3.12) and $N_R = N_k - N_{mD} - N_{nD}$ as

$$\begin{aligned} E(X) &= \Psi^1_{mDk}\Psi^2_{mDk} + \Psi^1_{nDk}\Psi^3_{nDk}\beta = \Psi^1_{mDk}\Psi^2_{mDk} + (1-\Psi^1_{mDk})\Psi^3_{nDk}\beta \\ &= \left(\frac{a^1_{mD}N_{mDk} + a^1_{nD}N_{nDk}}{N_{mDk}+N_{nDk}}\right)\left(\frac{a^2_{mD}N_{mDk} + a^2_{nD}N_{nDk} + a^2_R(N_k - N_{mDk} - N_{nDk})}{N_k}\right) \\ &\quad + \left(1 - \frac{a^1_{mD}N_{mDk} + a^1_{nD}N_{nDk}}{N_{mDk}+N_{nDk}}\right)\left(\frac{a^3_{mD}N_{mDk} + a^3_{nD}N_{nDk} + a^3_R(N_k - N_{mDk} - N_{nDk})}{N_k}\right) \end{aligned} \tag{A.38}$$

The second derivative with respect to the number of minority voters in a district is

$$\frac{\partial^2 E(X)}{\partial N^2_{mDk}} = \frac{2(a^1_{mD} - a^1_{nD})N_{nDk}\left((a^2_{mD} - a^2_{nD})N_{nDk} - (a^2_R - a^3_R\beta)N_k - (a^3_{mD} - a^3_{nD})\beta N_{nDk}\right)}{N_k(N_{mDk} + N_{nDk})^3}, \tag{A.39}$$

which is positive if

$$\gamma \equiv (a^2_{mD} - a^2_{nD})N_{nDk} - (a^2_R - a^3_R\beta)N_k - (a^3_{mD} - a^3_{nD})\beta N_{nDk} > 0. \tag{A.40}$$

As $a^2_R - a^3_R\beta \to 0$ or $a^2_R \to 0$ and $a^3_R \to 0$, we can state

$$\beta < \frac{a^2_{mD} - a^2_{nD}}{a^3_{mD} - a^3_{nD}}. \tag{A.41}$$

Note that (A.39) is also positive if $a^2_{mD} > a^2_{nD}$ (by assumption), $a^3_R\beta > a^2_R$ (Republican voters much more likely to support a nonminority-Democrat than a minority candidate), and $a^3_{nD} \geq a^3_{mD}$ (similar support among Democrats against a Republican candidate).

Open Primaries The expected utility for minority voters from (5.16) can be rewritten with (3.8) to (3.12) and $N_R = N_k - N_{mD} - N_{nD}$ as

$$E(X) = \hat{\Psi}^1_{mDk}\Psi^2_{mDk} + \hat{\Psi}^1_{nDk}\Psi^3_{nDk}\beta = \hat{\Psi}^1_{mDk}\Psi^2_{mDk} + (1-\hat{\Psi}^1_{mDk})\Psi^3_{nDk}\beta$$

$$= \left(\frac{a^1_{mD}N_{mDk} + a^1_{nD}N_{nDk} + a^1_R N_{Rk}}{N_k}\right)\left(\frac{a^2_{mD}N_{mDk} + a^2_{nD}N_{nDk} + a^2_R(N_k - N_{mDk} - N_{nDk})}{N_k}\right)$$
$$+ \left(1 - \frac{a^1_{mD}N_{mDk} + a^1_{nD}N_{nDk} + a^1_R N_{Rk}}{N_k}\right)\left(\frac{a^3_{mD}N_{mDk} + a^3_{nD}N_{nDk} + a^3_R(N_k - N_{mDk} - N_{nDk})}{N_k}\right) \tag{A.42}$$

The second derivative with respect to the number of minority voters in a district is

$$\frac{\partial^2 E(X)}{\partial N^2_{mDk}} = \frac{2(a^1_{mD} - a^1_R)\left(a^2_{mD} - a^2_R - \beta(a^3_{mD} - a^3_R)\right)}{N^2_k}, \tag{A.43}$$

which is positive if

$$\delta \equiv a^2_{mD} - a^2_R - \beta(a^3_{mD} - a^3_R) > 0. \tag{A.44}$$

We can state

$$\beta < \frac{a^2_{mD} - a^2_R}{a^3_{mD} - a^3_R}. \tag{A.45}$$

A.9 Proof of Proposition 5

We know from Proposition 2 that T_{ijk} becomes concave as π_{mD} rises; we wish to determine the conditions under which overall utility $\mathcal{U}_{mD} = U_{mD}(T_{ijk}) + E(X)$ is concave on S^2 with respect to π_{mD}. Recall from (4.2) that

$$\pi_i = [\kappa_i \phi_i(0)]^{1/\epsilon} \tag{A.46}$$

so that π_{mD} can increase either through a rise in κ_{mD} or $\phi_{mD}(0)$.

Taking the former, a rise in κ_{mD} indicates that minority voters prefer more distributive to ideological benefits at the margin. Since voters' overall utility is given by

$$X + \kappa_{mD}\frac{b^{1-\epsilon}}{1-\epsilon}, \tag{A.47}$$

an increase in κ_{mD} indicates that the weight placed on distributive returns increases relative to ideology. This means that the concavity of T_{ijk} will eventually dominate the sum, even if $E(X)$ is convex, making $\mathcal{U}_{mD}$ concave in π_{mD}.

Closed Primaries Taking the latter, an increase in $\phi_{mD}(0)$ indicates that minority voters are becoming more decisive; meaning that their voting rates a^e_{mD} decline for each election type e.

Taking the total derivative of (A.39) with respect to a^e_{mD} yields

$$\frac{\partial\left(\frac{\partial^2 E(X)}{\partial N^2_{mDk}}\right)}{\partial a^e_{mD}} = \frac{2N_{mD}\gamma + \overbrace{2(1-\beta)(a^1_{mD} - a^1_{nD})}^{(+)}}{N_k(N_{mDk} + N_{nDk})^3}, \tag{A.48}$$

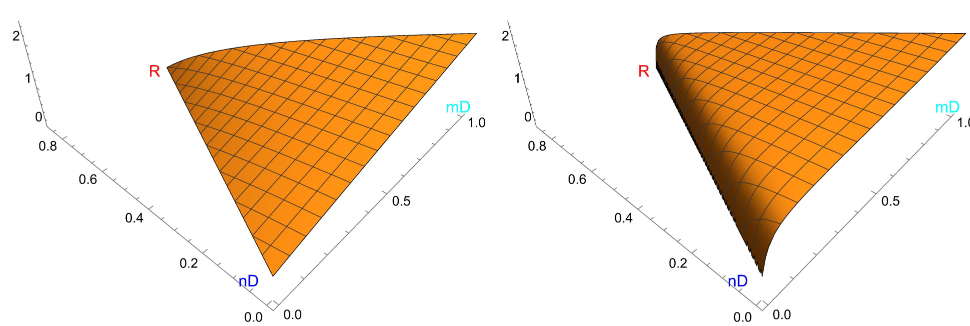
where γ follows from (A.40). If $\gamma > 0$, then $\frac{\partial^2 E(X)}{\partial N^2_{mDk}}$ is positive due to $a^1_{mD} > a^1_{nD}$, $E(X)$ is convex, and (A.48) is positive. So lower values of a^e_{mD} will make the surface of $E(X)$ more concave, again implying that $\mathcal{U}_{mD}$ becomes concave on S^2. If $\gamma < 0$, then $\frac{\partial^2 E(X)}{\partial N^2_{mDk}}$ is negative, $E(X)$ is concave, and (A.48) is ambiguous. However, low values of a^e_{mD} will not alter much the concavity of $E(X)$, and the concavity of T_{ijk} will dominate.

Open Primaries Taking the total derivative of (A.43) with respect to a^e_{mD} yields

$$\frac{\partial\left(\frac{\partial^2 E(X)}{\partial N^2_{mDk}}\right)}{\partial a^e_{mD}} = \frac{2\left(\delta + \overbrace{(a^1_{mD} - a^1_{R})(1-\beta)}^{(+)}\right)}{N_k^2}, \tag{A.49}$$

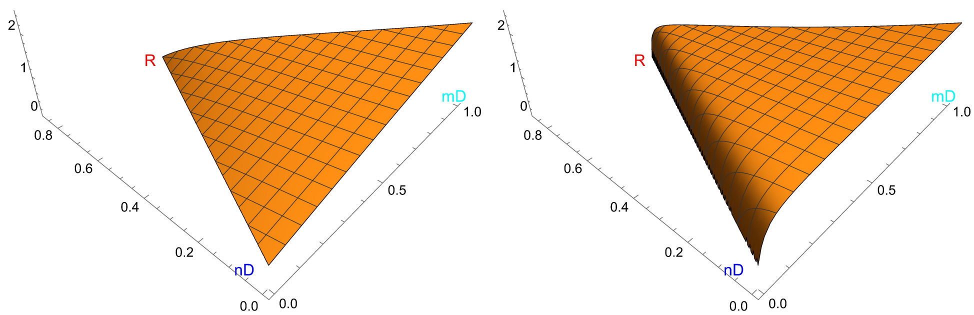
where δ follows from (A.44). If $\delta > 0$, then $\frac{\partial^2 E(X)}{\partial N^2_{mDk}}$ is positive due to $a^1_{mD} > a^1_{R}$, $E(X)$ is convex, and (A.49) is positive. So lower values of a^e_{mD} will make the surface of $E(X)$ more concave, again implying that $\mathcal{U}_{mD}$ becomes concave on S^2. If $\delta < 0$, then $\frac{\partial^2 E(X)}{\partial N^2_{mDk}}$ is negative, $E(X)$ is concave, and (A.49) is ambiguous. However, low values of a^e_{mD} will not alter much the concavity of $E(X)$, and the concavity of T_{ijk} will dominate.

A.10 Minority Power and Total Benefits



(a) $\pi_{mD} = 4$, $\pi_{nD} = 3$, $\pi_R = 1$ (b) $\pi_{mD} = 150$, $\pi_{nD} = 3$, $\pi_R = 1$

Figure 8: Total Minority Benefits – Minority Power, closed primaries.



(a) $\pi_{mD} = 4$, $\pi_{nD} = 3$, $\pi_R = 1$

(b) $\pi_{mD} = 150$, $\pi_{nD} = 3$, $\pi_R = 1$

Figure 9: Total Minority Benefits – Minority Power, open primaries.

B Online Appendix: Simulations

B.1 Minority Distributive Payoffs and Voter Distribution – Group Power

Group Power – **Minority Power**					
π_{mD}	π_{nD}	π_R	Total	Average	$R(\mathbf{D})$
1	3	1	0.750	0.25	75%
3	3	1	1.167	0.39	39%
5	3	1	1.426	0.48	15%
7	3	1	1.654	0.55	5%
10	3	1	1.899	0.63	3%
1	3	2	0.600	0.20	75%
3	3	2	0.912	0.30	42%
5	3	2	1.208	0.40	11%
7	3	2	1.450	0.48	4%
10	3	2	1.713	0.57	1%
1	3	3	0.500	0.17	75%
3	3	3	0.750	0.25	1%
5	3	3	1.071	0.36	0%
7	3	3	1.313	0.44	0%
10	3	3	1.579	0.53	0%
1	3	4	0.500	0.17	75%
3	3	4	0.750	0.25	75%
5	3	4	0.995	0.33	28%
7	3	4	1.217	0.41	9%
10	3	4	1.477	0.49	3%
1	3	5	0.500	0.17	75%
3	3	5	0.750	0.25	75%
5	3	5	0.987	0.33	38%
7	3	5	1.159	0.39	26%
10	3	5	1.399	0.47	9%

Group Power – **Large Differences**					
π_{mD}	π_{nD}	π_R	Total	Average	$R(\mathbf{D})$
1	5	1	0.750	0.25	75%
3	5	1	1.087	0.36	42%
5	5	1	1.318	0.44	38%
7	5	1	1.458	0.49	35%
10	5	1	1.669	0.56	9%
1	5	3	0.500	0.17	75%
3	5	3	0.750	0.25	75%
5	5	3	0.953	0.32	41%
7	5	3	1.129	0.38	23%
10	5	3	1.373	0.46	8%
1	5	5	0.375	0.13	75%
3	5	5	0.643	0.21	75%
5	5	5	0.750	0.25	0%
7	5	5	0.955	0.32	0%
10	5	7	1.098	0.37	18%
1	5	7	0.375	0.13	75%
3	5	7	0.643	0.21	75%
5	5	7	0.750	0.25	75%
7	5	7	0.905	0.30	39%
10	5	7	1.098	0.37	18%
1	5	10	0.375	0.13	75%
3	5	10	0.643	0.21	75%
5	5	10	0.750	0.25	75%
7	5	10	0.895	0.30	40%
10	5	10	1.072	0.36	38%

Group Power – **Homogeneous Nonminority Power**					
π_{mD}	π_{nD}	π_R	Total	Average	$R(\mathbf{D})$
1	1	1	0.750	0.25	0%
2	1	1	1.200	0.40	0%
3	1	1	1.500	0.50	0%
4	1	1	1.714	0.57	0%
5	1	1	1.875	0.63	0%
1	3	3	0.500	0.17	75%
2	3	3	0.667	0.22	75%
3	3	3	0.750	0.25	1%
4	3	3	0.923	0.31	0%
5	3	3	1.071	0.36	0%
1	5	5	0.375	0.13	75%
2	5	5	0.545	0.18	75%
3	5	5	0.643	0.21	75%
4	5	5	0.706	0.24	75%
5	5	5	0.750	0.25	0%
1	7	7	0.300	0.10	75%
2	7	7	0.462	0.15	75%
3	7	7	0.563	0.19	75%
4	7	7	0.632	0.21	75%
5	7	7	0.682	0.23	75%
1	10	10	0.231	0.08	75%
2	10	10	0.375	0.13	75%
3	10	10	0.474	0.16	75%
4	10	10	0.545	0.18	75%
5	10	10	0.600	0.20	75%

Group Power – **Heterogeneous Nonminority Power**					
π_{mD}	π_{nD}	π_R	Total	Average	$R(\mathbf{D})$
1	5	1	0.750	0.25	75%
2	5	1	0.911	0.30	52%
3	5	1	1.087	0.36	42%
4	5	1	1.219	0.41	39%
5	5	1	1.318	0.44	38%
1	5	3	0.500	0.17	75%
2	5	3	0.667	0.22	75%
3	5	3	0.750	0.25	75%
4	5	3	0.846	0.28	46%
5	5	3	0.953	0.32	41%
1	5	5	0.375	0.13	75%
2	5	5	0.545	0.18	75%
3	5	5	0.643	0.21	75%
4	5	5	0.706	0.24	75%
5	5	7	0.750	0.25	75%
1	5	7	0.375	0.13	75%
2	5	7	0.545	0.18	75%
3	5	7	0.643	0.21	75%
4	5	7	0.706	0.24	75%
5	5	7	0.750	0.25	75%
1	5	10	0.375	0.13	75%
2	5	10	0.545	0.18	75%
3	5	10	0.643	0.21	75%
4	5	10	0.706	0.24	75%
5	5	10	0.750	0.25	75%

Table 9: Districting Plans Maximizing Minority Distributive Benefits – Group Power, $N_{mD} = 25\%$, $N_{nD} = 40\%$, and $N_R = 35\%$.

B.2 Minority Distributive Payoffs and Voter Distribution – Demographics

Group Power			District 1			District 2			District 3			Benefits		
π_{mD}	π_{nD}	π_R	n_{mD1}	n_{nD1}	n_{R1}	n_{mD2}	n_{nD2}	n_{R2}	n_{mD3}	n_{nD3}	n_{R3}	Total	Aver.	$R(\mathbf{D})$
1	3	1	0%	75%	25%	4%	0%	96%	86%	0%	14%	0.900	0.300	86%
2	3	1	0%	75%	25%	45%	0%	55%	45%	0%	55%	1.241	0.414	45%
3	3	1	38%	0%	62%	38%	0%	62%	14%	75%	11%	1.447	0.482	24%
4	3	1	34%	0%	66%	34%	0%	66%	23%	75%	2%	1.625	0.542	11%
5	3	1	33%	0%	67%	32%	0%	68%	25%	75%	0%	1.770	0.590	8%
1	3	2	0%	4%	96%	0%	71%	29%	90%	0%	10%	0.818	0.273	90%
2	3	2	0%	75%	25%	8%	0%	92%	82%	0%	18%	0.900	0.300	82%
3	3	2	41%	0%	59%	41%	0%	59%	8%	75%	17%	1.106	0.369	33%
4	3	2	35%	0%	65%	35%	0%	65%	21%	75%	4%	1.291	0.430	14%
5	3	2	33%	0%	67%	33%	0%	67%	25%	75%	0%	1.450	0.483	8%
1	3	3	0%	35%	65%	0%	35%	65%	90%	4%	6%	0.750	0.250	90%
2	3	3	0%	35%	65%	0%	35%	65%	90%	5%	5%	0.857	0.286	90%
3	3	3	30%	25%	45%	30%	25%	45%	30%	25%	45%	0.900	0.300	0%
4	3	3	30%	27%	43%	30%	25%	45%	30%	23%	47%	1.091	0.364	0%
5	3	3	30%	11%	59%	30%	11%	59%	30%	54%	16%	1.250	0.417	0%
1	3	4	0%	33%	67%	0%	32%	68%	90%	10%	0%	0.750	0.250	90%
2	3	4	0%	62%	38%	0%	3%	97%	90%	10%	0%	0.857	0.286	90%
3	3	4	0%	63%	37%	0%	2%	98%	90%	10%	0%	0.900	0.300	90%
4	3	4	53%	47%	0%	37%	28%	35%	0%	0%	100%	0.998	0.333	53%
5	3	4	40%	60%	0%	29%	15%	57%	22%	0%	78%	1.126	0.375	18%
1	3	5	0%	32%	68%	0%	33%	67%	90%	10%	0%	0.750	0.250	90%
2	3	5	0%	43%	57%	0%	22%	78%	90%	10%	0%	0.857	0.286	90%
3	3	5	0%	35%	65%	0%	30%	70%	90%	10%	0%	0.900	0.300	90%
4	3	5	60%	40%	0%	30%	35%	35%	0%	0%	100%	0.967	0.322	60%
5	3	5	51%	49%	0%	39%	26%	35%	0%	0%	100%	1.070	0.357	51%

Intermediate Minority Population: $N_{mD} = 0.3,\ N_{nD} = 0.25,\ N_R = 0.45.$

Group Power			District 1			District 2			District 3			Benefits		
π_{mD}	π_{nD}	π_R	n_{mD1}	n_{nD1}	n_{R1}	n_{mD2}	n_{nD2}	n_{R2}	n_{mD3}	n_{nD3}	n_{R3}	Total	Aver.	$R(\mathbf{D})$
1	3	1	20%	0%	80%	100%	0%	0%	0%	75%	25%	1.200	0.400	100%
2	3	1	59%	0%	41%	59%	0%	41%	2%	75%	23%	1.500	0.500	58%
3	3	1	48%	0%	53%	48%	0%	53%	25%	75%	0%	1.712	0.571	23%
4	3	1	47%	0%	53%	48%	0%	53%	25%	75%	0%	1.875	0.625	23%
5	3	1	45%	0%	55%	29%	71%	0%	46%	4%	50%	1.996	0.665	17%
1	3	2	20%	0%	80%	100%	0%	0%	0%	75%	25%	1.111	0.370	100%
2	3	2	0%	75%	25%	79%	0%	21%	41%	0%	59%	1.200	0.400	79%
3	3	2	51%	0%	49%	51%	0%	49%	19%	75%	6%	1.403	0.468	32%
4	3	2	25%	75%	0%	47%	0%	53%	47%	0%	53%	1.596	0.532	22%
5	3	2	43%	9%	48%	43%	0%	57%	34%	66%	0%	1.752	0.584	9%
1	3	3	100%	0%	0%	0%	42%	58%	20%	33%	47%	1.077	0.359	100%
2	3	3	20%	38%	42%	100%	0%	0%	0%	37%	63%	1.143	0.381	100%
3	3	3	43%	24%	34%	39%	26%	36%	39%	26%	36%	1.200	0.400	4%
4	3	3	40%	23%	37%	40%	25%	35%	40%	27%	33%	1.412	0.471	0%
5	3	3	40%	23%	37%	40%	21%	39%	40%	30%	30%	1.579	0.526	0%
1	3	4	100%	0%	0%	20%	75%	5%	0%	0%	100%	1.075	0.358	100%
2	3	4	0%	0%	100%	20%	75%	5%	100%	0%	0%	1.140	0.380	100%
3	3	4	100%	0%	0%	0%	0%	100%	20%	75%	5%	1.197	0.399	100%
4	3	4	59%	36%	5%	61%	39%	0%	0%	0%	100%	1.324	0.441	61%
5	3	4	31%	0%	69%	41%	23%	36%	48%	52%	0%	1.456	0.485	17%
1	3	5	100%	0%	0%	20%	75%	5%	0%	0%	100%	1.074	0.358	100%
2	3	5	20%	75%	5%	100%	0%	0%	0%	0%	100%	1.138	0.379	100%
3	3	5	100%	0%	0%	20%	75%	5%	0%	0%	100%	1.194	0.398	100%
4	3	5	62%	38%	0%	0%	0%	100%	58%	37%	5%	1.316	0.439	62%
5	3	5	61%	39%	0%	59%	36%	5%	0%	0%	100%	1.412	0.471	61%

Large Minority Population: $N_{mD} = 0.4,\ N_{nD} = 0.25,\ N_R = 0.35.$

Table 10: Districting Plans Maximizing Minority Distributive Benefits – State Demographics.

B.3 Minority Distributive Payoffs and Voter Distribution – Number of Districts

5 Districts – $N_{mD} = 0.25,\ N_{nD} = .4,\ N_R = 0.35$					
π_{mD}	π_{nD}	π_R	Total	Average	$R(\mathbf{D})$
1	3	1	1.250	0.25	88%
2	3	1	1.765	0.35	42%
3	3	1	2.045	0.41	42%
4	3	1	2.222	0.44	42%
5	3	1	2.404	0.48	15%
1	3	2	1.143	0.23	100%
2	3	2	1.250	0.25	63%
3	3	2	1.552	0.31	42%
4	3	2	1.778	0.36	25%
5	3	2	2.020	0.40	11%
1	3	3	1.100	0.22	100%
2	3	3	1.182	0.24	100%
3	3	3	1.250	0.25	0%
4	3	3	1.538	0.31	0%
5	3	3	1.786	0.36	0%
1	3	4	1.100	0.22	100%
2	3	4	1.182	0.24	100%
3	3	4	1.250	0.25	63%
4	3	4	1.464	0.29	40%
5	3	4	1.667	0.33	21%
1	3	5	1.100	0.22	100%
2	3	5	1.182	0.24	100%
3	3	5	1.250	0.25	63%
4	3	5	1.463	0.29	42%
5	3	5	1.634	0.38	39%

12 Districts – $N_{mD} = 0.25,\ N_{nD} = .4,\ N_R = 0.35$					
π_{mD}	π_{nD}	π_R	Total	Average	$R(\mathbf{D})$
1	3	1	3.000	0.25	94%
2	3	1	4.200	0.35	43%
3	3	1	4.868	0.41	41%
4	3	1	5.354	0.45	36%
5	3	1	5.856	0.49	10%
1	3	2	3.000	0.25	100%
2	3	2	3.000	0.25	100%
3	3	2	3.708	0.31	42%
4	3	2	4.281	0.36	21%
5	3	2	4.868	0.41	10%
1	3	3	3.000	0.25	100%
2	3	3	3.000	0.25	100%
3	3	3	3.000	0.25	42%
4	3	3	3.692	0.31	0%
5	3	3	4.286	0.36	0%
1	3	4	2.470	0.21	100%
2	3	4	3.000	0.25	100%
3	3	4	3.000	0.25	78%
4	3	4	3.533	0.29	39%
5	3	4	3.995	0.33	20%
1	3	5	3.000	0.25	100%
2	3	5	3.000	0.25	100%
3	3	5	3.000	0.25	94%
4	3	5	3.516	0.29	40%
5	3	5	3.955	0.33	38%

Table 11: Districting Plans Maximizing Minority Distributive Benefits – Number of Districts.

B.4 Minority Total Payoffs and Voter Distribution – Group Power

Group Power – **Minority Power**

π_{mD}	π_{nD}	π_R	Total	Average	$R(\mathbf{D})$
1	3	1	2.172	0.72	75%
3	3	1	2.562	0.85	40%
5	3	1	2.795	0.93	28%
7	3	1	3.007	1.00	8%
10	3	1	3.248	1.08	3%
1	3	2	2.022	0.67	75%
3	3	2	2.308	0.77	44%
5	3	2	2.571	0.86	23%
7	3	2	2.804	0.93	9%
10	3	2	3.064	1.02	4%
1	3	3	1.922	0.64	75%
3	3	3	2.172	0.72	75%
5	3	3	2.424	0.81	11%
7	3	3	2.663	0.89	6%
10	3	3	2.929	0.98	4%
1	3	4	1.883	0.63	75%
3	3	4	2.133	0.71	75%
5	3	4	2.331	0.78	12%
7	3	4	2.562	0.85	2%
10	3	4	2.825	0.94	1%
1	3	5	1.883	0.63	75%
3	3	5	2.128	0.71	75%
5	3	5	2.312	0.77	38%
7	3	5	2.493	0.83	17%
10	3	5	2.743	0.91	4%

Group Power – **Large Differences**

π_{mD}	π_{nD}	π_R	Total	Average	$R(\mathbf{D})$
1	5	1	2.172	0.72	75%
3	5	1	2.483	0.83	43%
5	5	1	2.713	0.90	38%
7	5	1	2.851	0.95	39%
10	5	1	3.023	1.01	14%
1	5	3	1.922	0.64	75%
3	5	3	2.172	0.72	75%
5	5	3	2.349	0.78	43%
7	5	3	2.514	0.84	40%
10	5	3	2.729	0.91	14%
1	5	5	1.797	0.60	75%
3	5	5	2.065	0.69	75%
5	5	5	2.172	0.72	75%
7	5	5	2.311	0.77	25%
10	5	7	2.438	0.81	8%
1	5	7	1.758	0.59	75%
3	5	7	2.025	0.68	75%
5	5	7	2.133	0.71	75%
7	5	7	2.230	0.74	38%
10	5	7	2.438	0.81	8%
1	5	10	1.758	0.59	75%
3	5	10	2.025	0.68	75%
5	5	10	2.133	0.71	75%
7	5	10	2.190	0.73	75%
10	5	10	2.397	0.80	38%

Group Power – **Homogeneous Nonminority Power**

π_{mD}	π_{nD}	π_R	Total	Average	$R(\mathbf{D})$
1	1	1	2.172	0.72	75%
2	1	1	2.551	0.85	7%
3	1	1	2.850	0.95	4%
4	1	1	3.064	1.02	3%
5	1	1	3.225	1.07	3%
1	3	3	1.922	0.64	75%
2	3	3	2.089	0.70	75%
3	3	3	2.172	0.72	75%
4	3	3	2.284	0.76	40%
5	3	3	2.424	0.81	11%
1	5	5	1.797	0.60	75%
2	5	5	1.967	0.66	75%
3	5	5	2.065	0.69	75%
4	5	5	2.128	0.71	75%
5	5	5	2.172	0.72	75%
1	7	7	1.722	0.57	75%
2	7	7	1.884	0.63	75%
3	7	7	1.984	0.66	75%
4	7	7	2.054	0.68	75%
5	7	7	2.104	0.70	75%
1	10	10	1.653	0.55	75%
2	10	10	1.797	0.60	75%
3	10	10	1.896	0.63	75%
4	10	10	1.967	0.66	75%
5	10	10	2.022	0.67	75%

Group Power – **Heterogeneous Nonminority Power**

π_{mD}	π_{nD}	π_R	Total	Average	$R(\mathbf{D})$
1	5	1	2.172	0.72	75%
2	5	1	2.311	0.77	54%
3	5	1	2.483	0.83	43%
4	5	1	2.614	0.87	39%
5	5	1	2.713	0.90	38%
1	5	3	1.922	0.64	75%
2	5	3	2.089	0.70	75%
3	5	3	2.172	0.72	75%
4	5	3	2.244	0.75	50%
5	5	3	2.349	0.78	43%
1	5	5	1.797	0.60	75%
2	5	5	1.967	0.66	75%
3	5	5	2.065	0.69	75%
4	5	5	2.128	0.71	75%
5	5	7	2.133	0.71	75%
1	5	7	1.758	0.59	75%
2	5	7	1.928	0.64	75%
3	5	7	2.025	0.68	75%
4	5	7	2.089	0.70	75%
5	5	7	2.133	0.71	75%
1	5	10	1.758	0.59	75%
2	5	10	1.928	0.64	75%
3	5	10	2.025	0.68	75%
4	5	10	2.089	0.70	75%
5	5	10	2.133	0.71	75%

Table 12: Districting Plans Maximizing Minority Total Benefits – Group Power, $N_{mD} = 25\%$, $N_{nD} = 40\%$, $N_R = 35\%$, $K = 3$, closed primaries.

Group Power – **Minority Power**					
π_{mD}	π_{nD}	π_R	Total	Average	$R(\mathbf{D})$
1	3	1	2.069	0.69	75%
3	3	1	2.370	0.79	38%
5	3	1	2.637	0.88	11%
7	3	1	2.874	0.96	5%
10	3	1	3.125	1.04	7%
1	3	2	1.919	0.64	75%
3	3	2	2.137	0.71	75%
5	3	2	2.422	0.81	2%
7	3	2	2.669	0.89	3%
10	3	2	2.935	0.98	5%
1	3	3	1.880	0.63	75%
3	3	3	2.130	0.71	75%
5	3	3	2.317	0.77	36%
7	3	3	2.540	0.85	12%
10	3	3	2.803	0.93	7%
1	3	4	1.880	0.63	75%
3	3	4	2.130	0.71	75%
5	3	4	2.310	0.77	39%
7	3	4	2.477	0.83	37%
10	3	4	2.708	0.90	14%
1	3	5	1.880	0.63	75%
3	3	5	2.130	0.71	75%
5	3	5	2.304	0.77	40%
7	3	5	2.470	0.82	38%
10	3	5	2.648	0.88	26%

Group Power – **Large Differences**					
π_{mD}	π_{nD}	π_R	Total	Average	$R(\mathbf{D})$
1	5	1	2.069	0.69	75%
3	5	1	2.290	0.76	43%
5	5	1	2.524	0.84	41%
7	5	1	2.666	0.89	38%
10	5	1	2.890	0.96	8%
1	5	3	1.819	0.61	75%
3	5	3	2.069	0.69	75%
5	5	3	2.157	0.72	50%
7	5	3	2.335	0.78	13%
10	5	3	2.589	0.86	2%
1	5	5	1.755	0.58	75%
3	5	5	2.023	0.67	75%
5	5	5	2.130	0.71	75%
7	5	5	2.230	0.74	39%
10	5	7	2.400	0.80	38%
1	5	7	1.755	0.58	75%
3	5	7	2.023	0.67	75%
5	5	7	2.130	0.71	75%
7	5	7	2.222	0.74	42%
10	5	7	2.400	0.80	38%
1	5	10	1.755	0.58	75%
3	5	10	2.023	0.67	75%
5	5	10	2.130	0.71	75%
7	5	10	2.213	0.74	46%
10	5	10	2.388	0.80	39%

Group Power – **Homogeneous Nonminority Power**					
π_{mD}	π_{nD}	π_R	Total	Average	$R(\mathbf{D})$
1	1	1	2.130	0.71	75%
2	1	1	2.431	0.81	18%
3	1	1	2.725	0.91	8%
4	1	1	2.938	0.98	6%
5	1	1	3.098	1.03	5%
1	3	3	1.880	0.63	75%
2	3	3	2.046	0.68	75%
3	3	3	2.130	0.71	75%
4	3	3	2.205	0.74	40%
5	3	3	2.317	0.77	36%
1	5	5	1.755	0.58	75%
2	5	5	1.925	0.64	75%
3	5	5	2.023	0.67	75%
4	5	5	2.086	0.70	75%
5	5	5	2.130	0.71	75%
1	7	7	1.680	0.56	75%
2	7	7	1.841	0.61	75%
3	7	7	1.942	0.65	75%
4	7	7	2.011	0.67	75%
5	7	7	2.062	0.69	75%
1	10	10	1.611	0.54	75%
2	10	10	1.755	0.58	75%
3	10	10	1.853	0.62	75%
4	10	10	1.925	0.64	75%
5	10	10	1.980	0.66	75%

Group Power – **Heterogeneous Nonminority Power**					
π_{mD}	π_{nD}	π_R	Total	Average	$R(\mathbf{D})$
1	5	1	2.069	0.69	75%
2	5	1	2.176	0.73	75%
3	5	1	2.290	0.76	43%
4	5	1	2.423	0.81	39%
5	5	1	2.524	0.84	41%
1	5	3	1.819	0.61	75%
2	5	3	1.985	0.66	75%
3	5	3	2.069	0.69	75%
4	5	3	2.119	0.71	75%
5	5	3	2.157	0.72	50%
1	5	5	1.755	0.58	75%
2	5	5	1.925	0.64	75%
3	5	5	2.023	0.67	75%
4	5	5	2.086	0.70	75%
5	5	7	2.130	0.71	75%
1	5	7	1.755	0.58	75%
2	5	7	1.925	0.64	75%
3	5	7	2.023	0.67	75%
4	5	7	2.086	0.70	75%
5	5	7	2.130	0.71	75%
1	5	10	1.755	0.58	75%
2	5	10	1.925	0.64	75%
3	5	10	2.023	0.67	75%
4	5	10	2.086	0.70	75%
5	5	10	2.130	0.71	75%

Table 13: Districting Plans Maximizing Minority Total Benefits – Group Power, $N_{mD} = 25\%$, $N_{nD} = 40\%$, $N_R = 35\%$, $K = 3$, open primaries.

B.5 Minority Total Payoffs and Voter Distribution – State Demographics

Group Power			District 1			District 2			District 3			Benefits		
π_{mD}	π_{nD}	π_R	n_{mD1}	n_{nD1}	n_{R1}	n_{mD2}	n_{nD2}	n_{R2}	n_{mD3}	n_{nD3}	n_{R3}	Total	Aver.	$R(\mathbf{D})$
1	3	1	0%	8%	92%	0%	67%	33%	90%	0%	10%	2.220	0.74	90%
2	3	1	45%	0%	55%	0%	75%	25%	45%	0%	55%	2.561	0.85	45%
3	3	1	41%	0%	59%	7%	75%	18%	41%	0%	59%	2.747	0.92	34%
4	3	1	36%	0%	64%	19%	75%	6%	36%	0%	64%	2.911	0.97	17%
5	3	1	33%	0%	67%	24%	75%	1%	33%	0%	67%	3.051	1.02	9%
1	3	2	0%	38%	62%	0%	37%	63%	90%	0%	10%	2.138	0.71	90%
2	3	2	0%	0%	100%	0%	75%	25%	90%	0%	10%	2.220	0.74	90%
3	3	2	45%	0%	55%	0%	75%	25%	45%	0%	55%	2.422	0.81	45%
4	3	2	38%	0%	62%	14%	75%	11%	38%	0%	62%	2.581	0.86	24%
5	3	2	35%	0%	65%	21%	75%	4%	35%	0%	65%	2.733	0.91	14%
1	3	3	0%	0%	100%	0%	75%	25%	90%	0%	10%	2.070	0.69	90%
2	3	3	0%	0%	100%	0%	75%	25%	90%	0%	10%	2.177	0.73	90%
3	3	3	45%	0%	55%	0%	75%	25%	45%	0%	55%	2.220	0.74	45%
4	3	3	37%	0%	63%	16%	75%	9%	37%	0%	63%	2.373	0.79	21%
5	3	3	24%	75%	1%	33%	0%	67%	33%	0%	67%	2.528	0.84	9%
1	3	4	0%	58%	42%	0%	7%	93%	90%	10%	0%	2.051	0.68	90%
2	3	4	0%	50%	50%	0%	15%	85%	90%	10%	0%	2.158	0.72	90%
3	3	4	0%	36%	64%	0%	29%	71%	90%	10%	0%	2.201	0.73	90%
4	3	4	25%	75%	0%	4%	0%	96%	61%	0%	39%	2.238	0.75	57%
5	3	4	25%	75%	0%	32%	0%	68%	33%	0%	67%	2.389	0.80	8%
1	3	5	0%	59%	41%	0%	6%	94%	90%	10%	0%	2.051	0.68	90%
2	3	5	0%	57%	43%	0%	8%	92%	90%	10%	0%	2.158	0.72	90%
3	3	5	90%	10%	0%	0%	0%	100%	0%	65%	35%	2.201	0.73	90%
4	3	5	90%	10%	0%	0%	0%	100%	0%	65%	35%	2.224	0.74	90%
5	3	5	46%	54%	0%	0%	0%	100%	44%	21%	35%	2.305	0.77	46%

Intermediate Minority Population - D Majority: $N_{mD} = 0.3,\ N_{nD} = 0.25,\ N_R = 0.45.$

Group Power			District 1			District 2			District 3			Benefits		
π_{mD}	π_{nD}	π_R	n_{mD1}	n_{nD1}	n_{R1}	n_{mD2}	n_{nD2}	n_{R2}	n_{mD3}	n_{nD3}	n_{R3}	Total	Aver.	$R(\mathbf{D})$
1	3	1	47%	0%	53%	0%	45%	55%	43%	0%	57%	2.052	0.68	47%
2	3	1	45%	0%	55%	45%	0%	55%	0%	45%	55%	2.393	0.80	45%
3	3	1	36%	0%	64%	36%	0%	64%	18%	45%	37%	2.620	0.87	18%
4	3	1	33%	0%	67%	33%	0%	67%	24%	45%	31%	2.812	0.94	9%
5	3	1	31%	0%	69%	31%	0%	69%	27%	45%	28%	2.963	0.99	4%
1	3	2	0%	39%	61%	90%	0%	10%	0%	6%	94%	1.970	0.66	90%
2	3	2	24%	0%	76%	66%	0%	34%	0%	45%	55%	2.052	0.68	66%
3	3	2	44%	0%	56%	44%	0%	56%	3%	45%	52%	2.254	0.75	41%
4	3	2	35%	0%	65%	35%	0%	65%	20%	45%	35%	2.441	0.81	14%
5	3	2	33%	0%	67%	33%	0%	67%	24%	45%	31%	2.601	0.87	8%
1	3	3	0%	25%	75%	90%	0%	10%	0%	20%	80%	1.902	0.63	90%
2	3	3	0%	0%	100%	90%	0%	10%	0%	45%	55%	2.009	0.67	90%
3	3	3	45%	0%	55%	45%	0%	55%	0%	45%	55%	2.052	0.68	45%
4	3	3	35%	0%	65%	35%	0%	65%	20%	45%	35%	2.209	0.74	15%
5	3	3	32%	0%	68%	32%	0%	68%	26%	45%	29%	2.366	0.79	6%
1	3	4	0%	18%	82%	0%	17%	83%	90%	10%	0%	1.883	0.63	90%
2	3	4	0%	0%	100%	90%	10%	0%	0%	35%	65%	1.990	0.66	90%
3	3	4	0%	22%	78%	90%	10%	0%	0%	13%	87%	2.033	0.68	90%
4	3	4	0%	0%	100%	35%	0%	65%	55%	45%	0%	2.070	0.69	55%
5	3	4	27%	0%	73%	27%	0%	73%	36%	45%	19%	2.203	0.73	9%
1	3	5	0%	0%	100%	0%	35%	65%	90%	10%	0%	1.883	0.63	90%
2	3	5	0%	0%	100%	90%	10%	0%	0%	35%	65%	1.990	0.66	90%
3	3	5	0%	25%	75%	90%	10%	0%	0%	10%	90%	2.033	0.68	90%
4	3	5	0%	27%	73%	90%	10%	0%	0%	8%	92%	2.056	0.69	90%
5	3	5	0%	0%	100%	35%	0%	65%	55%	45%	0%	2.121	0.71	55%

Intermediate Minority Population - R Majority: $N_{mD} = 0.3,\ N_{nD} = 0.15,\ N_R = 0.55.$

Table 14: Districting Plans Maximizing Minority Total Benefits – State Demographics, closed primaries.

Group Power			District 1			District 2			District 3			Benefits		
π_{mD}	π_{nD}	π_R	n_{mD1}	n_{nD1}	n_{R1}	n_{mD2}	n_{nD2}	n_{R2}	n_{mD3}	n_{nD3}	n_{R3}	Total	Aver.	$R(\mathbf{D})$
1	3	1	90%	0%	10%	0%	75%	25%	0%	0%	100%	2.164	0.72	90%
2	3	1	45%	0%	55%	0%	75%	25%	45%	0%	55%	2.303	0.77	45%
3	3	1	37%	0%	63%	16%	75%	9%	37%	0%	63%	2.497	0.83	20%
4	3	1	33%	0%	67%	25%	75%	0%	32%	0%	68%	2.685	0.90	8%
5	3	1	25%	75%	0%	32%	0%	68%	33%	0%	67%	2.831	0.94	8%
1	3	2	0%	75%	25%	0%	0%	100%	90%	0%	10%	2.082	0.69	90%
2	3	2	0%	0%	100%	0%	75%	25%	90%	0%	10%	2.164	0.72	90%
3	3	2	0%	0%	100%	0%	75%	25%	90%	0%	10%	2.195	0.73	90%
4	3	2	25%	75%	0%	32%	0%	68%	33%	0%	67%	2.350	0.78	8%
5	3	2	27%	73%	0%	31%	2%	66%	31%	0%	69%	2.511	0.84	4%
1	3	3	0%	65%	35%	0%	0%	100%	90%	10%	0%	2.037	0.68	90%
2	3	3	0%	0%	100%	0%	65%	35%	90%	10%	0%	2.145	0.71	90%
3	3	3	0%	65%	35%	0%	0%	100%	90%	10%	0%	2.187	0.73	90%
4	3	3	69%	31%	0%	0%	0%	100%	21%	44%	35%	2.219	0.74	69%
5	3	3	46%	54%	0%	15%	0%	85%	30%	21%	50%	2.332	0.78	31%
1	3	4	0%	0%	100%	0%	65%	35%	90%	10%	0%	2.037	0.68	90%
2	3	4	0%	65%	35%	0%	0%	100%	90%	10%	0%	2.145	0.71	90%
3	3	4	90%	10%	0%	0%	0%	100%	0%	65%	35%	2.187	0.73	90%
4	3	4	90%	10%	0%	0%	0%	100%	0%	65%	35%	2.211	0.74	90%
5	3	4	58%	42%	0%	0%	0%	100%	32%	33%	35%	2.282	0.76	58%
1	3	5	0%	65%	35%	0%	0%	100%	90%	10%	0%	2.037	0.68	90%
2	3	5	90%	10%	0%	0%	0%	100%	0%	65%	35%	2.145	0.71	90%
3	3	5	90%	10%	0%	0%	0%	100%	0%	65%	35%	2.187	0.73	90%
4	3	5	90%	10%	0%	0%	0%	100%	0%	65%	35%	2.211	0.74	90%
5	3	5	66%	34%	0%	0%	0%	100%	24%	41%	35%	2.253	0.75	66%

Intermediate Minority Population - D Majority: $N_{mD} = 0.3,\ N_{nD} = 0.25,\ N_R = 0.45.$

Group Power			District 1			District 2			District 3			Benefits		
π_{mD}	π_{nD}	π_R	n_{mD1}	n_{nD1}	n_{R1}	n_{mD2}	n_{nD2}	n_{R2}	n_{mD3}	n_{nD3}	n_{R3}	Total	Aver.	$R(\mathbf{D})$
1	3	1	0%	0%	100%	90%	0%	10%	0%	45%	55%	1.992	0.66	90%
2	3	1	45%	0%	55%	45%	0%	55%	0%	45%	55%	2.131	0.71	45%
3	3	1	33%	0%	67%	33%	0%	67%	23%	45%	32%	2.358	0.79	10%
4	3	1	30%	0%	70%	30%	0%	70%	31%	45%	24%	2.561	0.85	1%
5	3	1	28%	0%	72%	28%	0%	72%	34%	45%	21%	2.719	0.91	6%
1	3	2	0%	0%	100%	90%	0%	10%	0%	45%	55%	1.910	0.64	90%
2	3	2	0%	0%	100%	90%	0%	10%	0%	45%	55%	1.992	0.66	90%
3	3	2	0%	0%	100%	90%	0%	10%	0%	45%	55%	2.023	0.67	90%
4	3	2	28%	0%	72%	28%	0%	72%	34%	45%	21%	2.186	0.73	5%
5	3	2	28%	0%	72%	28%	0%	72%	34%	45%	21%	2.353	0.78	6%
1	3	3	0%	0%	100%	90%	10%	0%	0%	35%	65%	1.869	0.62	90%
2	3	3	0%	0%	100%	90%	10%	0%	0%	35%	65%	1.977	0.66	90%
3	3	3	0%	0%	100%	90%	10%	0%	0%	35%	65%	2.019	0.67	90%
4	3	3	0%	0%	100%	86%	14%	0%	4%	31%	65%	2.043	0.68	86%
5	3	3	18%	0%	82%	17%	0%	83%	55%	45%	0%	2.149	0.72	38%
1	3	4	0%	0%	100%	90%	10%	0%	0%	35%	65%	1.869	0.62	90%
2	3	4	0%	0%	100%	90%	10%	0%	0%	35%	65%	1.977	0.66	90%
3	3	4	0%	0%	100%	90%	10%	0%	0%	35%	65%	2.019	0.67	90%
4	3	4	0%	0%	100%	90%	10%	0%	0%	35%	65%	2.043	0.68	90%
5	3	4	0%	0%	100%	72%	28%	0%	18%	17%	65%	2.073	0.69	72%
1	3	5	0%	0%	100%	90%	10%	0%	0%	35%	65%	1.869	0.62	90%
2	3	5	0%	35%	65%	90%	10%	0%	0%	0%	100%	1.977	0.66	90%
3	3	5	0%	35%	65%	90%	10%	0%	0%	0%	100%	2.019	0.67	90%
4	3	5	0%	0%	100%	90%	10%	0%	0%	35%	65%	2.043	0.68	90%
5	3	5	0%	0%	100%	90%	10%	0%	0%	35%	65%	2.057	0.69	90%

Intermediate Minority Population - R Majority: $N_{mD} = 0.3,\ N_{nD} = 0.15,\ N_R = 0.55.$

Table 15: Districting Plans Maximizing Minority Total Benefits – State Demographics, open primaries.

B.6 Minority Total Payoffs and Voter Distribution – Ideological Benefits

Group Power			District 1			District 2			District 3			Benefits		
π_{mD}	π_{nD}	π_R	n_{mD1}	n_{nD1}	n_{R1}	n_{mD2}	n_{nD2}	n_{R2}	n_{mD3}	n_{nD3}	n_{R3}	Total	Aver.	$R(\mathbf{D})$
1	3	1	0%	39%	61%	0%	81%	19%	75%	0%	25%	1.752	0.58	75%
2	3	1	46%	0%	54%	0%	100%	0%	29%	20%	51%	1.950	0.65	46%
3	3	1	40%	0%	60%	0%	100%	0%	35%	20%	45%	2.142	0.71	40%
4	3	1	37%	20%	43%	0%	100%	0%	38%	0%	62%	2.275	0.76	38%
5	3	1	35%	0%	65%	6%	94%	0%	34%	26%	40%	2.375	0.79	28%
1	3	2	0%	60%	40%	0%	60%	40%	75%	0%	25%	1.602	0.53	75%
2	3	2	75%	0%	25%	0%	60%	40%	0%	60%	40%	1.752	0.58	75%
3	3	2	31%	20%	49%	0%	100%	0%	44%	0%	56%	1.888	0.63	44%
4	3	2	40%	0%	60%	0%	100%	0%	35%	20%	45%	2.029	0.68	40%
5	3	2	35%	0%	65%	12%	88%	0%	29%	32%	40%	2.151	0.72	23%
1	3	3	0%	100%	0%	0%	20%	80%	75%	0%	25%	1.502	0.50	75%
2	3	3	0%	98%	2%	0%	22%	78%	75%	0%	25%	1.669	0.56	75%
3	3	3	0%	54%	46%	0%	66%	34%	75%	0%	25%	1.752	0.58	75%
4	3	3	40%	0%	60%	0%	100%	0%	35%	20%	45%	1.864	0.62	40%
5	3	3	25%	39%	36%	19%	81%	0%	31%	0%	69%	2.004	0.67	11%
1	3	4	0%	48%	52%	0%	47%	53%	75%	25%	0%	1.463	0.49	75%
2	3	4	0%	95%	5%	0%	0%	100%	75%	25%	0%	2.696	0.90	75%
3	3	4	0%	81%	19%	0%	14%	86%	75%	25%	0%	1.713	0.57	75%
4	3	4	0%	0%	100%	36%	59%	5%	39%	61%	0%	1.788	0.60	39%
5	3	4	30%	70%	0%	27%	50%	23%	18%	0%	82%	1.911	0.64	12%
1	3	5	0%	11%	89%	0%	84%	16%	75%	25%	0%	1.463	0.49	75%
2	3	5	0%	53%	47%	0%	42%	58%	75%	25%	0%	1.629	0.54	75%
3	3	5	0%	17%	83%	0%	78%	22%	75%	25%	0%	1.713	0.57	75%
4	3	5	0%	48%	52%	0%	47%	53%	75%	25%	0%	1.763	0.59	75%
5	3	5	34%	66%	0%	28%	54%	18%	13%	0%	87%	1.865	0.62	20%

Low Minority Benefit from Majority Democrat Candidate: $\beta = 0$.

Group Power			District 1			District 2			District 3			Benefits		
π_{mD}	π_{nD}	π_R	n_{mD1}	n_{nD1}	n_{R1}	n_{mD2}	n_{nD2}	n_{R2}	n_{mD3}	n_{nD3}	n_{R3}	Total	Aver.	$R(\mathbf{D})$
1	3	1	75%	0%	25%	0%	60%	40%	0%	60%	40%	1.962	0.65	75%
2	3	1	29%	20%	51%	0%	100%	0%	46%	0%	54%	2.160	0.72	46%
3	3	1	35%	20%	45%	0%	100%	0%	40%	0%	60%	2.352	0.78	40%
4	3	1	38%	0%	62%	0%	100%	0%	37%	20%	43%	2.485	0.83	38%
5	3	1	35%	0%	65%	6%	94%	0%	34%	26%	40%	2.585	0.86	28%
1	3	2	0%	60%	40%	0%	60%	40%	75%	0%	25%	1.812	0.60	75%
2	3	2	75%	0%	25%	0%	60%	40%	0%	60%	40%	1.962	0.65	75%
3	3	2	31%	20%	49%	0%	100%	0%	44%	0%	56%	2.098	0.70	44%
4	3	2	40%	0%	60%	0%	100%	0%	35%	20%	45%	2.239	0.75	40%
5	3	2	29%	32%	40%	12%	88%	0%	35%	0%	65%	2.361	0.79	23%
1	3	3	0%	100%	0%	0%	20%	80%	75%	0%	25%	1.712	0.57	75%
2	3	3	0%	100%	0%	0%	20%	80%	75%	0%	25%	1.879	0.63	75%
3	3	3	0%	43%	57%	0%	77%	23%	75%	0%	25%	1.962	0.65	75%
4	3	3	40%	0%	60%	0%	100%	0%	35%	20%	45%	2.074	0.69	40%
5	3	3	25%	39%	36%	19%	81%	0%	31%	0%	69%	2.214	0.74	11%
1	3	4	0%	0%	100%	0%	95%	5%	75%	25%	0%	1.673	0.56	75%
2	3	4	0%	95%	5%	0%	0%	100%	75%	25%	0%	1.839	0.61	75%
3	3	4	0%	95%	5%	0%	0%	100%	75%	25%	0%	1.923	0.64	75%
4	3	4	0%	58%	42%	38%	62%	0%	37%	0%	63%	1.982	0.66	38%
5	3	4	30%	70%	0%	27%	50%	23%	18%	0%	82%	2.121	0.71	12%
1	3	5	0%	0%	100%	0%	95%	5%	75%	25%	0%	1.673	0.56	75%
2	3	5	0%	0%	100%	0%	95%	5%	75%	25%	0%	1.839	0.61	75%
3	3	5	0%	65%	35%	0%	30%	70%	75%	25%	0%	1.923	0.64	75%
4	3	5	0%	0%	100%	8%	87%	5%	67%	33%	0%	1.981	0.66	67%
5	3	5	37%	58%	5%	38%	62%	0%	0%	0%	100%	2.102	0.70	38%

Low Minority Benefit from Majority Democrat Candidate: $\beta = 0.25$.

Table 16: Districting Plans Maximizing Minority Total Benefits – Low nD Benefit, closed primaries.

Group Power			District 1			District 2			District 3			Benefits		
π_{mD}	π_{nD}	π_R	n_{mD1}	n_{nD1}	n_{R1}	n_{mD2}	n_{nD2}	n_{R2}	n_{mD3}	n_{nD3}	n_{R3}	Total	Aver.	$R(\mathbf{D})$
1	3	1	0%	60%	40%	75%	0%	25%	0%	60%	40%	2.382	0.79	75%
2	3	1	0%	100%	0%	46%	0%	54%	29%	20%	51%	2.580	0.86	46%
3	3	1	0%	100%	0%	40%	0%	60%	35%	20%	45%	2.772	0.92	40%
4	3	1	0%	100%	0%	38%	0%	62%	37%	20%	43%	2.905	0.97	38%
5	3	1	6%	94%	0%	35%	0%	65%	34%	26%	40%	3.005	1.00	28%
1	3	2	0%	84%	16%	75%	0%	25%	0%	36%	64%	2.232	0.74	75%
2	3	2	0%	60%	40%	75%	0%	25%	0%	60%	40%	2.382	0.79	75%
3	3	2	0%	100%	0%	44%	0%	56%	31%	20%	49%	2.518	0.84	44%
4	3	2	0%	100%	0%	40%	0%	60%	35%	20%	45%	2.659	0.89	40%
5	3	2	12%	88%	0%	35%	0%	65%	29%	32%	40%	2.781	0.93	23%
1	3	3	0%	100%	0%	75%	0%	25%	0%	20%	80%	2.132	0.71	75%
2	3	3	0%	100%	0%	75%	0%	25%	0%	20%	80%	2.299	0.77	75%
3	3	3	0%	20%	80%	75%	0%	25%	0%	100%	0%	2.382	0.79	75%
4	3	3	0%	100%	0%	40%	0%	60%	35%	20%	45%	2.494	0.83	40%
5	3	3	19%	81%	0%	31%	0%	69%	25%	39%	36%	2.634	0.88	11%
1	3	4	0%	40%	60%	75%	25%	0%	0%	55%	45%	2.093	0.70	75%
2	3	4	0%	0%	100%	75%	25%	0%	0%	95%	5%	2.259	0.75	75%
3	3	4	0%	0%	100%	75%	25%	0%	0%	95%	5%	2.343	0.78	75%
4	3	4	0%	0%	100%	39%	61%	0%	36%	59%	5%	2.418	0.81	39%
5	3	4	30%	70%	0%	18%	0%	82%	27%	50%	23%	2.541	0.85	12%
1	3	5	0%	0%	100%	75%	25%	0%	0%	95%	5%	2.093	0.70	75%
2	3	5	0%	0%	100%	75%	25%	0%	0%	95%	5%	2.259	0.75	75%
3	3	5	0%	0%	100%	75%	25%	0%	0%	95%	5%	2.343	0.78	75%
4	3	5	43%	57%	0%	32%	63%	5%	0%	0%	100%	2.412	0.80	43%
5	3	5	0%	0%	100%	37%	58%	5%	38%	62%	0%	2.522	0.84	38%

Low Minority Benefit from Majority Democrat Candidate: $\beta = 0.75$.

Group Power			District 1			District 2			District 3			Benefits		
π_{mD}	π_{nD}	π_R	n_{mD1}	n_{nD1}	n_{R1}	n_{mD2}	n_{nD2}	n_{R2}	n_{mD3}	n_{nD3}	n_{R3}	Total	Aver.	$R(\mathbf{D})$
1	3	1	75%	0%	25%	0%	60%	40%	0%	60%	40%	2.592	0.86	75%
2	3	1	29%	20%	51%	0%	100%	0%	46%	0%	54%	2.790	0.93	46%
3	3	1	40%	0%	60%	0%	100%	0%	35%	20%	45%	2.982	0.99	40%
4	3	1	37%	20%	43%	0%	100%	0%	38%	0%	62%	3.115	1.04	38%
5	3	1	35%	0%	65%	6%	94%	0%	34%	26%	40%	3.215	1.07	28%
1	3	2	0%	100%	0%	75%	0%	25%	0%	20%	80%	2.442	0.81	75%
2	3	2	0%	60%	40%	0%	60%	40%	75%	0%	25%	2.592	0.86	75%
3	3	2	31%	20%	49%	0%	100%	0%	44%	0%	56%	2.728	0.91	44%
4	3	2	35%	20%	45%	0%	100%	0%	40%	0%	60%	2.869	0.96	40%
5	3	2	29%	32%	40%	12%	88%	0%	35%	0%	65%	2.991	1.00	23%
1	3	3	0%	53%	47%	0%	67%	33%	75%	0%	25%	2.342	0.78	75%
2	3	3	0%	100%	0%	0%	20%	80%	75%	0%	25%	2.509	0.84	75%
3	3	3	0%	57%	43%	0%	63%	37%	75%	0%	25%	2.592	0.86	75%
4	3	3	40%	0%	60%	0%	100%	0%	35%	20%	45%	2.704	0.90	40%
5	3	3	25%	39%	36%	19%	81%	0%	31%	0%	69%	2.844	0.95	11%
1	3	4	0%	95%	5%	0%	0%	100%	75%	25%	0%	2.303	0.77	75%
2	3	4	0%	0%	100%	0%	95%	5%	75%	25%	0%	2.469	0.82	75%
3	3	4	0%	95%	5%	0%	0%	100%	75%	25%	0%	2.638	0.88	75%
4	3	4	0%	0%	100%	39%	61%	0%	36%	59%	5%	2.628	0.88	39%
5	3	4	30%	70%	0%	27%	50%	23%	18%	0%	82%	2.751	0.92	12%
1	3	5	0%	0%	100%	0%	95%	5%	75%	25%	0%	2.303	0.77	75%
2	3	5	0%	95%	5%	0%	0%	100%	75%	25%	0%	2.469	0.82	75%
3	3	5	0%	73%	27%	0%	22%	78%	75%	25%	0%	2.553	0.85	75%
4	3	5	32%	63%	5%	0%	0%	100%	43%	57%	0%	2.622	0.87	43%
5	3	5	38%	62%	0%	37%	58%	5%	0%	0%	100%	2.732	0.91	38%

Low Minority Benefit from Majority Democrat Candidate: $\beta = 1$.

Table 17: Districting Plans Maximizing Minority Total Benefits – High nD Benefit, closed primaries.

Group Power			District 1			District 2			District 3			Benefits		
π_{mD}	π_{nD}	π_R	n_{mD1}	n_{nD1}	n_{R1}	n_{mD2}	n_{nD2}	n_{R2}	n_{mD3}	n_{nD3}	n_{R3}	Total	Aver.	$R(\mathbf{D})$
1	3	1	0%	100%	0%	75%	0%	25%	0%	20%	80%	1.531	0.51	75%
2	3	1	0%	100%	0%	75%	0%	25%	0%	20%	80%	1.638	0.55	75%
3	3	1	18%	0%	82%	57%	20%	23%	0%	100%	0%	1.760	0.59	57%
4	3	1	24%	76%	0%	19%	0%	81%	32%	44%	24%	1.912	0.64	13%
5	3	1	29%	71%	0%	18%	0%	82%	29%	49%	23%	2.074	0.69	11%
1	3	2	0%	0%	100%	75%	25%	0%	0%	95%	5%	1.453	0.48	75%
2	3	2	0%	0%	100%	75%	25%	0%	0%	95%	5%	1.619	0.54	75%
3	3	2	0%	0%	100%	75%	25%	0%	0%	95%	5%	1.703	0.57	75%
4	3	2	0%	0%	100%	34%	61%	5%	41%	59%	0%	1.779	0.59	41%
5	3	2	37%	63%	0%	6%	0%	94%	32%	57%	11%	1.896	0.63	31%
1	3	3	0%	0%	100%	75%	25%	0%	0%	95%	5%	1.453	0.48	75%
2	3	3	0%	0%	100%	75%	25%	0%	0%	95%	5%	1.619	0.54	75%
3	3	3	0%	0%	100%	75%	25%	0%	0%	95%	5%	1.703	0.57	75%
4	3	3	0%	0%	100%	30%	65%	5%	45%	55%	0%	1.773	0.59	45%
5	3	3	35%	60%	5%	0%	0%	100%	40%	60%	0%	1.883	0.63	40%
1	3	4	0%	0%	100%	75%	25%	0%	0%	95%	5%	1.453	0.48	75%
2	3	4	0%	0%	100%	75%	25%	0%	0%	95%	5%	1.619	0.54	75%
3	3	4	0%	0%	100%	75%	25%	0%	0%	95%	5%	1.703	0.57	75%
4	3	4	0%	0%	100%	27%	68%	5%	48%	52%	0%	1.767	0.59	48%
5	3	4	0%	0%	100%	35%	60%	5%	40%	60%	0%	1.877	0.63	40%
1	3	5	0%	0%	100%	75%	25%	0%	0%	95%	5%	1.453	0.48	75%
2	3	5	0%	0%	100%	75%	25%	0%	0%	95%	5%	1.619	0.54	75%
3	3	5	0%	0%	100%	75%	25%	0%	0%	95%	5%	1.703	0.57	75%
4	3	5	0%	0%	100%	23%	72%	5%	52%	48%	0%	1.763	0.59	52%
5	3	5	0%	0%	100%	41%	59%	0%	34%	61%	5%	1.871	0.62	41%

Low Minority Benefit from Majority Democrat Candidate: $\beta = 0$.

Group Power			District 1			District 2			District 3			Benefits		
π_{mD}	π_{nD}	π_R	n_{mD1}	n_{nD1}	n_{R1}	n_{mD2}	n_{nD2}	n_{R2}	n_{mD3}	n_{nD3}	n_{R3}	Total	Aver.	$R(\mathbf{D})$
1	3	1	0%	100%	0%	0%	20%	80%	75%	0%	25%	1.800	0.60	75%
2	3	1	0%	20%	80%	0%	100%	0%	75%	0%	25%	1.907	0.64	75%
3	3	1	44%	20%	36%	0%	100%	0%	31%	0%	69%	2.058	0.69	44%
4	3	1	38%	30%	32%	10%	90%	0%	27%	0%	73%	2.198	0.73	27%
5	3	1	29%	44%	27%	24%	76%	0%	22%	0%	78%	2.350	0.78	7%
1	3	2	0%	100%	0%	0%	0%	100%	75%	20%	5%	1.668	0.56	75%
2	3	2	0%	95%	5%	0%	0%	100%	75%	25%	0%	1.833	0.61	75%
3	3	2	0%	0%	100%	0%	95%	5%	75%	25%	0%	1.916	0.64	75%
4	3	2	37%	63%	0%	32%	57%	11%	6%	0%	94%	1.999	0.67	30%
5	3	2	31%	69%	0%	28%	51%	20%	15%	0%	85%	2.146	0.72	16%
1	3	3	0%	95%	5%	0%	0%	100%	75%	25%	0%	1.666	0.56	75%
2	3	3	0%	95%	5%	0%	0%	100%	75%	25%	0%	1.833	0.61	75%
3	3	3	0%	95%	5%	0%	0%	100%	75%	25%	0%	1.916	0.64	75%
4	3	3	33%	62%	5%	0%	0%	100%	42%	58%	0%	1.989	0.66	42%
5	3	3	39%	61%	0%	36%	59%	5%	0%	0%	100%	2.100	0.70	39%
1	3	4	0%	95%	5%	0%	0%	100%	75%	25%	0%	1.666	0.56	75%
2	3	4	0%	95%	5%	0%	0%	100%	75%	25%	0%	1.833	0.61	75%
3	3	4	0%	95%	5%	0%	0%	100%	75%	25%	0%	1.916	0.64	75%
4	3	4	29%	66%	5%	0%	0%	100%	46%	54%	0%	1.983	0.66	46%
5	3	4	40%	60%	0%	0%	0%	100%	35%	60%	5%	2.093	0.70	40%
1	3	5	0%	0%	100%	0%	95%	5%	75%	25%	0%	1.666	0.56	75%
2	3	5	0%	95%	5%	0%	0%	100%	75%	25%	0%	1.833	0.61	75%
3	3	5	0%	95%	5%	0%	0%	100%	75%	25%	0%	1.916	0.64	75%
4	3	5	0%	0%	100%	25%	70%	5%	50%	50%	0%	1.979	0.66	50%
5	3	5	35%	60%	5%	0%	0%	100%	40%	60%	0%	2.087	0.70	40%

Low Minority Benefit from Majority Democrat Candidate: $\beta = 0.25$.

Table 18: Districting Plans Maximizing Minority Total Benefits – Low nD Benefit, open primaries.

Group Power			District 1			District 2			District 3			Benefits		
π_{mD}	π_{nD}	π_R	n_{mD1}	n_{nD1}	n_{R1}	n_{mD2}	n_{nD2}	n_{R2}	n_{mD3}	n_{nD3}	n_{R3}	Total	Aver.	$R(\mathbf{D})$
1	3	1	0%	60%	40%	75%	0%	25%	0%	60%	40%	2.342	0.78	75%
2	3	1	0%	100%	0%	49%	0%	51%	26%	20%	54%	2.496	0.83	49%
3	3	1	0%	100%	0%	40%	0%	60%	35%	20%	45%	2.686	0.90	40%
4	3	1	0%	100%	0%	38%	0%	62%	37%	20%	43%	2.819	0.94	38%
5	3	1	31%	33%	36%	31%	0%	69%	13%	87%	0%	2.933	0.98	18%
1	3	2	0%	60%	40%	75%	0%	25%	0%	60%	40%	2.192	0.73	75%
2	3	2	0%	60%	40%	75%	0%	25%	0%	60%	40%	2.342	0.78	75%
3	3	2	0%	100%	0%	47%	0%	53%	28%	20%	52%	2.433	0.81	47%
4	3	2	0%	100%	0%	40%	0%	60%	35%	20%	45%	2.573	0.86	40%
5	3	2	31%	0%	69%	18%	82%	0%	26%	38%	36%	2.714	0.90	13%
1	3	3	0%	48%	52%	75%	25%	0%	0%	47%	53%	2.100	0.70	75%
2	3	3	0%	48%	52%	75%	25%	0%	0%	47%	53%	2.267	0.76	75%
3	3	3	0%	47%	53%	75%	25%	0%	0%	48%	52%	2.350	0.78	75%
4	3	3	25%	40%	35%	25%	40%	35%	25%	40%	35%	2.433	0.81	0%
5	3	3	25%	40%	35%	25%	40%	35%	25%	40%	35%	2.581	0.86	0%
1	3	4	0%	48%	52%	75%	25%	0%	0%	47%	53%	2.100	0.70	75%
2	3	4	0%	48%	52%	75%	25%	0%	0%	47%	53%	2.267	0.76	75%
3	3	4	0%	48%	52%	75%	25%	0%	0%	47%	53%	2.350	0.78	75%
4	3	4	33%	62%	5%	42%	58%	0%	0%	0%	100%	2.416	0.81	42%
5	3	4	0%	0%	100%	38%	62%	0%	37%	58%	5%	2.527	0.84	38%
1	3	5	0%	48%	52%	75%	25%	0%	0%	47%	53%	2.100	0.70	75%
2	3	5	0%	48%	52%	75%	25%	0%	0%	47%	53%	2.267	0.76	75%
3	3	5	0%	48%	52%	75%	25%	0%	0%	47%	53%	2.350	0.78	75%
4	3	5	0%	0%	100%	45%	55%	0%	30%	65%	5%	2.410	0.80	45%
5	3	5	0%	0%	100%	39%	61%	0%	36%	59%	5%	2.520	0.84	39%

Low Minority Benefit from Majority Democrat Candidate: $\beta = 0.75$.

Group Power			District 1			District 2			District 3			Benefits		
π_{mD}	π_{nD}	π_R	n_{mD1}	n_{nD1}	n_{R1}	n_{mD2}	n_{nD2}	n_{R2}	n_{mD3}	n_{nD3}	n_{R3}	Total	Aver.	$R(\mathbf{D})$
1	3	1	0%	60%	40%	0%	60%	40%	75%	0%	25%	2.635	0.88	75%
2	3	1	28%	20%	52%	0%	100%	0%	47%	0%	53%	2.812	0.94	47%
3	3	1	34%	20%	46%	0%	100%	0%	41%	0%	59%	3.002	1.00	41%
4	3	1	39%	0%	61%	0%	100%	0%	36%	20%	44%	3.134	1.04	39%
5	3	1	35%	0%	65%	8%	92%	0%	32%	28%	40%	3.237	1.08	27%
1	3	2	0%	60%	40%	0%	60%	40%	75%	0%	25%	2.485	0.83	75%
2	3	2	0%	60%	40%	0%	60%	40%	75%	0%	25%	2.635	0.88	75%
3	3	2	45%	0%	55%	0%	100%	0%	30%	20%	50%	2.749	0.92	45%
4	3	2	34%	20%	46%	0%	100%	0%	41%	0%	59%	2.890	0.96	41%
5	3	2	28%	32%	40%	12%	88%	0%	35%	0%	65%	3.015	1.00	23%
1	3	3	0%	60%	40%	0%	60%	40%	75%	0%	25%	2.385	0.80	75%
2	3	3	0%	60%	40%	0%	60%	40%	75%	0%	25%	2.552	0.85	75%
3	3	3	0%	60%	40%	0%	60%	40%	75%	0%	25%	2.635	0.88	75%
4	3	3	25%	40%	35%	25%	40%	35%	25%	40%	35%	2.740	0.91	0%
5	3	3	25%	40%	35%	25%	40%	35%	25%	40%	35%	2.888	0.96	0%
1	3	4	0%	48%	52%	0%	47%	53%	75%	25%	0%	2.348	0.78	75%
2	3	4	0%	47%	53%	0%	48%	52%	75%	25%	0%	2.514	0.84	75%
3	3	4	0%	47%	53%	0%	48%	52%	75%	25%	0%	2.598	0.87	75%
4	3	4	17%	39%	43%	0%	38%	62%	58%	42%	0%	2.650	0.88	58%
5	3	4	25%	40%	35%	25%	40%	35%	25%	40%	35%	2.791	0.93	0%
1	3	5	0%	47%	53%	0%	48%	52%	75%	25%	0%	2.348	0.78	75%
2	3	5	0%	47%	53%	0%	48%	52%	75%	25%	0%	2.514	0.84	75%
3	3	5	0%	48%	52%	0%	47%	53%	75%	25%	0%	2.598	0.87	75%
4	3	5	0%	48%	52%	0%	47%	53%	75%	25%	0%	2.648	0.88	75%
5	3	5	36%	59%	5%	39%	61%	0%	0%	0%	100%	2.737	0.91	39%

Low Minority Benefit from Majority Democrat Candidate: $\beta = 1$.

Table 19: Districting Plans Maximizing Minority Total Benefits – High nD Benefit, open primaries.

B.7 Minority Total Payoffs and Voter Distribution – Crossover Rates

Group Power			District 1			District 2			District 3			Benefits		
π_{mD}	π_{MD}	π_R	n_{mD1}	n_{MD1}	n_{R1}	n_{mD2}	n_{MD2}	n_{R2}	n_{mD3}	n_{MD3}	n_{R3}	Total	Aver.	$R(\mathbf{D})$
1	3	1	0%	60%	40%	75%	0%	25%	0%	60%	40%	2.124	0.71	75%
2	3	1	0%	100%	0%	46%	0%	54%	29%	20%	51%	2.315	0.77	46%
3	3	1	0%	100%	0%	40%	0%	60%	35%	20%	45%	2.506	0.84	40%
4	3	1	0%	100%	0%	38%	0%	62%	37%	20%	43%	2.639	0.88	38%
5	3	1	2%	98%	0%	37%	0%	63%	36%	22%	42%	2.736	0.91	34%
1	3	2	0%	23%	77%	75%	0%	25%	0%	97%	3%	1.974	0.66	75%
2	3	2	0%	60%	40%	75%	0%	25%	0%	60%	40%	2.124	0.71	75%
3	3	2	0%	100%	0%	44%	0%	56%	31%	20%	49%	2.253	0.75	44%
4	3	2	0%	100%	0%	40%	0%	60%	35%	20%	45%	2.394	0.80	40%
5	3	2	8%	92%	0%	36%	0%	64%	31%	28%	41%	2.508	0.84	28%
1	3	3	0%	100%	0%	75%	0%	25%	0%	20%	80%	1.874	0.62	75%
2	3	3	0%	100%	0%	75%	0%	25%	0%	20%	80%	2.041	0.68	75%
3	3	3	0%	20%	80%	75%	0%	25%	0%	100%	0%	2.124	0.71	75%
4	3	3	0%	100%	0%	40%	0%	60%	35%	20%	45%	2.228	0.74	40%
5	3	3	17%	83%	0%	33%	0%	67%	25%	37%	38%	2.358	0.79	16%
1	3	4	0%	0%	100%	75%	25%	0%	0%	95%	5%	1.823	0.61	75%
2	3	4	0%	0%	100%	75%	25%	0%	0%	95%	5%	1.990	0.66	75%
3	3	4	0%	0%	100%	75%	25%	0%	0%	95%	5%	2.073	0.69	75%
4	3	4	0%	46%	54%	26%	74%	0%	49%	0%	51%	2.131	0.71	49%
5	3	4	25%	48%	27%	22%	0%	78%	28%	72%	0%	2.261	0.75	6%
1	3	5	0%	0%	100%	75%	25%	0%	0%	95%	5%	1.823	0.61	75%
2	3	5	0%	45%	55%	75%	25%	0%	0%	50%	50%	1.990	0.66	75%
3	3	5	0%	43%	57%	75%	25%	0%	0%	52%	48%	2.073	0.69	75%
4	3	5	0%	0%	100%	52%	48%	0%	23%	72%	5%	2.127	0.71	52%
5	3	5	0%	0%	100%	37%	58%	5%	38%	62%	0%	2.236	0.75	38%

Low Majority Democrat Primary Crossover $a^1_{MD} = 0.1$.

Group Power			District 1			District 2			District 3			Benefits		
π_{mD}	π_{nD}	π_R	n_{mD1}	n_{nD1}	n_{R1}	n_{mD2}	n_{nD2}	n_{R2}	n_{mD3}	n_{nD3}	n_{R3}	Total	Aver.	$R(\mathbf{D})$
1	3	1	0%	46%	54%	75%	0%	25%	0%	74%	26%	2.220	0.74	75%
2	3	1	0%	100%	0%	45%	0%	55%	30%	20%	50%	2.426	0.81	45%
3	3	1	0%	100%	0%	40%	0%	60%	35%	20%	45%	2.618	0.87	40%
4	3	1	0%	100%	0%	38%	0%	62%	37%	20%	43%	2.750	0.92	38%
5	3	1	9%	91%	0%	33%	0%	67%	32%	29%	38%	2.856	0.95	24%
1	3	2	0%	60%	40%	75%	0%	25%	0%	60%	40%	2.070	0.69	75%
2	3	2	0%	100%	0%	75%	0%	25%	0%	20%	80%	2.220	0.74	75%
3	3	2	0%	100%	0%	43%	0%	57%	32%	20%	48%	2.363	0.79	43%
4	3	2	0%	100%	0%	40%	0%	60%	35%	20%	45%	2.505	0.84	40%
5	3	2	33%	0%	67%	14%	86%	0%	28%	34%	38%	2.634	0.88	19%
1	3	3	0%	20%	80%	75%	0%	25%	0%	100%	0%	1.970	0.66	75%
2	3	3	0%	100%	0%	75%	0%	25%	0%	20%	80%	2.137	0.71	75%
3	3	3	0%	20%	80%	75%	0%	25%	0%	100%	0%	2.220	0.74	75%
4	3	3	26%	35%	39%	34%	0%	66%	15%	85%	0%	2.345	0.78	19%
5	3	3	25%	41%	34%	29%	0%	71%	21%	79%	0%	2.491	0.83	8%
1	3	4	0%	17%	83%	75%	25%	0%	0%	78%	22%	1.942	0.65	75%
2	3	4	0%	0%	100%	75%	25%	0%	0%	95%	5%	2.109	0.70	75%
3	3	4	0%	27%	73%	75%	25%	0%	0%	68%	32%	2.192	0.73	75%
4	3	4	0%	0%	100%	39%	61%	0%	36%	59%	5%	2.283	0.76	39%
5	3	4	32%	68%	0%	15%	0%	85%	29%	52%	20%	2.403	0.80	17%
1	3	5	0%	48%	52%	75%	25%	0%	0%	47%	53%	1.942	0.65	75%
2	3	5	0%	0%	100%	75%	25%	0%	0%	95%	5%	2.109	0.70	75%
3	3	5	0%	67%	33%	75%	25%	0%	0%	28%	72%	2.192	0.73	75%
4	3	5	0%	0%	100%	41%	59%	0%	34%	61%	5%	2.277	0.76	41%
5	3	5	0%	0%	100%	38%	62%	0%	37%	58%	5%	2.388	0.80	38%

High Majority Democrat Primary Crossover $a^1_{nD} = 0.5$.

Table 20: Districting Plans Maximizing Minority Total Benefits – Primary Crossover, closed primaries.

Group Power			District 1			District 2			District 3			Benefits		
π_{mD}	π_{MD}	π_R	n_{mD1}	n_{MD1}	n_{R1}	n_{mD2}	n_{MD2}	n_{R2}	n_{mD3}	n_{MD3}	n_{R3}	Total	Aver.	$R(\mathbf{D})$
1	3	1	0%	60%	40%	75%	0%	25%	0%	60%	40%	1.956	0.65	75%
2	3	1	0%	100%	0%	49%	0%	51%	26%	20%	54%	2.106	0.70	49%
3	3	1	0%	100%	0%	42%	0%	58%	33%	20%	47%	2.293	0.76	42%
4	3	1	0%	100%	0%	39%	0%	61%	36%	20%	44%	2.425	0.81	39%
5	3	1	0%	100%	0%	38%	0%	62%	37%	20%	43%	2.521	0.84	38%
1	3	2	0%	60%	40%	75%	0%	25%	0%	60%	40%	1.806	0.60	75%
2	3	2	0%	89%	11%	75%	0%	25%	0%	31%	69%	1.956	0.65	75%
3	3	2	0%	100%	0%	49%	0%	51%	26%	20%	54%	2.042	0.68	49%
4	3	2	0%	100%	0%	42%	0%	58%	33%	20%	47%	2.181	0.73	42%
5	3	2	0%	100%	0%	41%	0%	59%	34%	20%	46%	2.289	0.76	41%
1	3	3	0%	31%	69%	75%	0%	25%	0%	89%	11%	1.706	0.57	75%
2	3	3	0%	85%	15%	75%	0%	25%	0%	35%	65%	1.873	0.62	75%
3	3	3	0%	97%	3%	75%	0%	25%	0%	23%	77%	1.956	0.65	75%
4	3	3	0%	100%	0%	46%	0%	54%	29%	20%	51%	2.016	0.67	46%
5	3	3	0%	100%	0%	41%	0%	59%	34%	20%	46%	2.125	0.71	41%
1	3	4	0%	60%	40%	75%	0%	25%	0%	60%	40%	1.635	0.54	75%
2	3	4	0%	76%	24%	75%	0%	25%	0%	44%	56%	1.806	0.60	75%
3	3	4	0%	60%	40%	75%	0%	25%	0%	60%	40%	1.898	0.63	75%
4	3	4	0%	25%	75%	75%	0%	25%	0%	95%	5%	1.956	0.65	75%
5	3	4	14%	86%	0%	61%	0%	39%	0%	34%	66%	2.005	0.67	61%
1	3	5	0%	45%	55%	75%	25%	0%	0%	50%	50%	1.588	0.53	75%
2	3	5	0%	69%	31%	75%	25%	0%	0%	26%	74%	1.755	0.58	75%
3	3	5	0%	60%	40%	75%	0%	25%	0%	60%	40%	1.849	0.62	75%
4	3	5	0%	60%	40%	75%	0%	25%	0%	60%	40%	1.912	0.64	75%
5	3	5	4%	96%	0%	71%	0%	29%	0%	24%	76%	1.957	0.65	71%

Low Majority Democrat General Crossover $a^2_{MD} = 0.1$.

Group Power			District 1			District 2			District 3			Benefits		
π_{mD}	π_{nD}	π_R	n_{mD1}	n_{nD1}	n_{R1}	n_{mD2}	n_{nD2}	n_{R2}	n_{mD3}	n_{nD3}	n_{R3}	Total	Aver.	$R(\mathbf{D})$
1	3	1	0%	60%	40%	75%	0%	25%	0%	60%	40%	2.100	0.70	75%
2	3	1	0%	100%	0%	47%	0%	53%	28%	20%	52%	2.282	0.76	47%
3	3	1	0%	100%	0%	41%	0%	59%	34%	20%	46%	2.472	0.82	41%
4	3	1	0%	100%	0%	39%	0%	61%	36%	20%	44%	2.605	0.87	39%
5	3	1	0%	100%	0%	38%	0%	62%	37%	20%	43%	2.701	0.90	38%
1	3	2	0%	60%	40%	75%	0%	25%	0%	60%	40%	1.950	0.65	75%
2	3	2	0%	60%	40%	75%	0%	25%	0%	60%	40%	2.100	0.70	75%
3	3	2	0%	100%	0%	45%	0%	55%	30%	20%	50%	2.219	0.74	45%
4	3	2	0%	100%	0%	41%	0%	59%	34%	20%	46%	2.360	0.79	41%
5	3	2	2%	98%	0%	39%	0%	61%	34%	22%	44%	2.469	0.82	37%
1	3	3	0%	92%	8%	75%	0%	25%	0%	28%	72%	1.850	0.62	75%
2	3	3	0%	63%	37%	75%	0%	25%	0%	57%	43%	2.017	0.67	75%
3	3	3	0%	71%	29%	75%	0%	25%	0%	49%	51%	2.100	0.70	75%
4	3	3	0%	100%	0%	41%	0%	59%	34%	20%	46%	2.194	0.73	41%
5	3	3	12%	88%	0%	36%	0%	64%	28%	32%	41%	2.312	0.77	24%
1	3	4	0%	0%	100%	75%	25%	0%	0%	95%	5%	1.784	0.59	75%
2	3	4	0%	75%	25%	75%	0%	25%	0%	45%	55%	1.950	0.65	75%
3	3	4	0%	60%	40%	75%	0%	25%	0%	60%	40%	2.042	0.68	75%
4	3	4	0%	100%	0%	75%	0%	25%	0%	20%	80%	2.100	0.70	75%
5	3	4	25%	75%	0%	28%	0%	72%	22%	45%	33%	2.209	0.74	6%
1	3	5	0%	60%	40%	75%	25%	0%	0%	35%	65%	1.784	0.59	75%
2	3	5	0%	92%	8%	75%	25%	0%	0%	3%	97%	1.951	0.65	75%
3	3	5	0%	0%	100%	75%	25%	0%	0%	95%	5%	2.034	0.68	75%
4	3	5	0%	0%	100%	75%	25%	0%	0%	95%	5%	2.084	0.69	75%
5	3	5	0%	0%	100%	38%	62%	0%	37%	58%	5%	2.176	0.73	38%

High Majority Democrat General Crossover $a^2_{nD} = 0.5$.

Table 21: Districting Plans Maximizing Minority Total Benefits – General Crossover, closed primaries.

Group Power			District 1			District 2			District 3			Benefits		
π_{mD}	π_{nD}	π_R	n_{mD1}	n_{nD1}	n_{R1}	n_{mD2}	n_{nD2}	n_{R2}	n_{mD3}	n_{nD3}	n_{R3}	Total	Aver.	$R(\mathbf{D})$
1	3	1	0%	100%	0%	75%	0%	25%	0%	20%	80%	2.027	0.68	75%
2	3	1	0%	100%	0%	63%	0%	37%	12%	20%	68%	2.138	0.71	63%
3	3	1	0%	100%	0%	39%	0%	61%	36%	20%	44%	2.321	0.77	39%
4	3	1	0%	100%	0%	36%	0%	64%	39%	20%	41%	2.455	0.82	39%
5	3	1	17%	83%	0%	28%	0%	72%	30%	37%	33%	2.578	0.86	14%
1	3	2	0%	100%	0%	75%	0%	25%	0%	20%	80%	1.877	0.63	75%
2	3	2	0%	100%	0%	75%	0%	25%	0%	20%	80%	2.027	0.68	75%
3	3	2	0%	100%	0%	75%	0%	25%	0%	20%	80%	2.095	0.70	75%
4	3	2	9%	91%	0%	33%	0%	67%	32%	29%	38%	2.211	0.74	24%
5	3	2	23%	77%	0%	26%	0%	74%	26%	43%	31%	2.362	0.79	4%
1	3	3	0%	0%	100%	75%	25%	0%	0%	95%	5%	1.822	0.61	75%
2	3	3	0%	0%	100%	75%	25%	0%	0%	95%	5%	1.989	0.66	75%
3	3	3	0%	0%	100%	75%	25%	0%	0%	95%	5%	2.072	0.69	75%
4	3	3	0%	0%	100%	31%	64%	5%	44%	56%	0%	2.132	0.71	44%
5	3	3	32%	56%	12%	7%	0%	93%	36%	64%	0%	2.245	0.75	29%
1	3	4	0%	0%	100%	75%	25%	0%	0%	95%	5%	1.822	0.61	75%
2	3	4	0%	0%	100%	75%	25%	0%	0%	95%	5%	1.989	0.66	75%
3	3	4	0%	0%	100%	75%	25%	0%	0%	95%	5%	2.072	0.69	75%
4	3	4	0%	0%	100%	53%	47%	0%	22%	73%	5%	2.127	0.71	53%
5	3	4	0%	0%	100%	36%	59%	5%	39%	61%	0%	2.236	0.75	39%
1	3	5	0%	0%	100%	75%	25%	0%	0%	95%	5%	1.822	0.61	75%
2	3	5	0%	0%	100%	75%	25%	0%	0%	95%	5%	1.989	0.66	75%
3	3	5	0%	0%	100%	75%	25%	0%	0%	95%	5%	2.072	0.69	75%
4	3	5	0%	0%	100%	63%	37%	0%	12%	83%	5%	2.124	0.71	63%
5	3	5	0%	0%	100%	40%	60%	0%	35%	60%	5%	2.230	0.74	40%

Low Majority Democrat Primary Crossover $a^1_{nD} = 0.1$.

Group Power			District 1			District 2			District 3			Benefits		
π_{mD}	π_{nD}	π_R	n_{mD1}	n_{nD1}	n_{R1}	n_{mD2}	n_{nD2}	n_{R2}	n_{mD3}	n_{nD3}	n_{R3}	Total	Aver.	$R(\mathbf{D})$
1	3	1	0%	100%	0%	75%	0%	25%	0%	20%	80%	2.110	0.70	75%
2	3	1	0%	100%	0%	55%	0%	45%	20%	20%	60%	2.228	0.74	55%
3	3	1	0%	100%	0%	37%	0%	63%	38%	20%	42%	2.420	0.81	38%
4	3	1	33%	0%	67%	39%	23%	38%	3%	97%	0%	2.555	0.85	36%
5	3	1	25%	0%	75%	20%	80%	0%	30%	40%	30%	2.696	0.90	9%
1	3	2	0%	100%	0%	75%	0%	25%	0%	20%	80%	1.960	0.65	75%
2	3	2	0%	100%	0%	75%	0%	25%	0%	20%	80%	2.110	0.70	75%
3	3	2	0%	100%	0%	75%	20%	5%	0%	0%	100%	2.190	0.73	75%
4	3	2	23%	0%	77%	29%	43%	28%	23%	77%	0%	2.328	0.78	6%
5	3	2	22%	0%	78%	26%	74%	0%	27%	46%	27%	2.484	0.83	5%
1	3	3	0%	0%	100%	75%	25%	0%	0%	95%	5%	1.937	0.65	75%
2	3	3	0%	0%	100%	75%	25%	0%	0%	95%	5%	2.104	0.70	75%
3	3	3	0%	0%	100%	75%	25%	0%	0%	95%	5%	2.187	0.73	75%
4	3	3	0%	0%	100%	36%	59%	5%	39%	61%	0%	2.279	0.76	39%
5	3	3	38%	62%	0%	0%	0%	100%	37%	58%	5%	2.390	0.80	38%
1	3	4	0%	0%	100%	75%	25%	0%	0%	95%	5%	1.937	0.65	75%
2	3	4	0%	0%	100%	75%	25%	0%	0%	95%	5%	2.104	0.70	75%
3	3	4	0%	0%	100%	75%	25%	0%	0%	95%	5%	2.187	0.73	75%
4	3	4	0%	0%	100%	33%	62%	5%	42%	58%	0%	2.273	0.76	42%
5	3	4	0%	0%	100%	36%	59%	5%	39%	61%	0%	2.384	0.79	39%
1	3	5	0%	0%	100%	75%	25%	0%	0%	95%	5%	1.937	0.65	75%
2	3	5	0%	0%	100%	75%	25%	0%	0%	95%	5%	2.104	0.70	75%
3	3	5	0%	0%	100%	75%	25%	0%	0%	95%	5%	2.187	0.73	75%
4	3	5	0%	0%	100%	44%	56%	0%	31%	64%	5%	2.268	0.76	44%
5	3	5	0%	0%	100%	40%	60%	0%	35%	60%	5%	2.378	0.79	40%

High Majority Democrat Primary Crossover $a^1_{nD} = 0.5$.

Table 22: Districting Plans Maximizing Minority Total Benefits – Primary Crossover, open primaries.

Group Power			District 1			District 2			District 3			Benefits		
π_{mD}	π_{nD}	π_R	n_{mD1}	n_{nD1}	n_{R1}	n_{mD2}	n_{nD2}	n_{R2}	n_{mD3}	n_{nD3}	n_{R3}	Total	Aver.	$R(\mathbf{D})$
1	3	1	0%	60%	40%	75%	0%	25%	0%	60%	40%	1.920	0.64	75%
2	3	1	0%	60%	40%	75%	0%	25%	0%	60%	40%	2.027	0.68	75%
3	3	1	0%	100%	0%	44%	0%	56%	31%	20%	49%	2.142	0.71	44%
4	3	1	0%	100%	0%	40%	0%	60%	35%	20%	45%	2.273	0.76	40%
5	3	1	0%	100%	0%	38%	0%	62%	37%	20%	43%	2.369	0.79	38%
1	3	2	0%	60%	40%	75%	0%	25%	0%	60%	40%	1.770	0.59	75%
2	3	2	0%	60%	40%	75%	0%	25%	0%	60%	40%	1.920	0.64	75%
3	3	2	0%	60%	40%	75%	0%	25%	0%	60%	40%	1.988	0.66	75%
4	3	2	0%	100%	0%	48%	0%	52%	27%	20%	53%	2.031	0.68	48%
5	3	2	0%	100%	0%	42%	0%	58%	33%	20%	47%	2.138	0.71	42%
1	3	3	0%	60%	40%	75%	0%	25%	0%	60%	40%	1.670	0.56	75%
2	3	3	0%	60%	40%	75%	0%	25%	0%	60%	40%	1.837	0.61	75%
3	3	3	0%	60%	40%	75%	0%	25%	0%	60%	40%	1.920	0.64	75%
4	3	3	0%	60%	40%	75%	0%	25%	0%	60%	40%	1.970	0.66	75%
5	3	3	25%	40%	35%	25%	40%	35%	25%	40%	35%	2.008	0.67	0%
1	3	4	0%	48%	52%	75%	25%	0%	0%	48%	52%	1.648	0.55	75%
2	3	4	0%	48%	52%	75%	25%	0%	0%	47%	53%	1.814	0.60	75%
3	3	4	0%	48%	52%	75%	25%	0%	0%	47%	53%	1.898	0.63	75%
4	3	4	0%	48%	52%	75%	25%	0%	0%	47%	53%	1.948	0.65	75%
5	3	4	0%	47%	53%	75%	25%	0%	0%	48%	52%	1.981	0.66	75%
1	3	5	0%	48%	52%	75%	25%	0%	0%	47%	53%	1.648	0.55	75%
2	3	5	0%	47%	53%	75%	25%	0%	0%	48%	52%	1.814	0.60	75%
3	3	5	0%	48%	52%	75%	25%	0%	0%	47%	53%	1.898	0.63	75%
4	3	5	0%	48%	52%	75%	25%	0%	0%	47%	53%	1.948	0.65	75%
5	3	5	0%	48%	52%	75%	25%	0%	0%	47%	53%	1.981	0.66	75%

Low Majority Democrat General Crossover $a_{nD}^2 = 0.1$.

Group Power			District 1			District 2			District 3			Benefits		
π_{mD}	π_{nD}	π_R	n_{mD1}	n_{nD1}	n_{R1}	n_{mD2}	n_{nD2}	n_{R2}	n_{mD3}	n_{nD3}	n_{R3}	Total	Aver.	$R(\mathbf{D})$
1	3	1	0%	60%	40%	75%	0%	25%	0%	60%	40%	2.006	0.67	75%
2	3	1	0%	100%	0%	67%	0%	33%	8%	20%	72%	2.116	0.71	67%
3	3	1	0%	100%	0%	40%	0%	60%	35%	20%	45%	2.293	0.76	40%
4	3	1	0%	100%	0%	37%	0%	63%	38%	20%	42%	2.427	0.81	38%
5	3	1	30%	0%	70%	32%	33%	35%	13%	87%	0%	2.538	0.85	19%
1	3	2	0%	60%	40%	75%	0%	25%	0%	60%	40%	1.856	0.62	75%
2	3	2	0%	60%	40%	75%	0%	25%	0%	60%	40%	2.006	0.67	75%
3	3	2	0%	60%	40%	75%	0%	25%	0%	60%	40%	2.074	0.69	75%
4	3	2	0%	100%	0%	39%	0%	61%	36%	20%	44%	2.181	0.73	39%
5	3	2	29%	0%	71%	26%	39%	34%	19%	81%	0%	2.319	0.77	10%
1	3	3	0%	14%	86%	75%	25%	0%	0%	81%	19%	1.784	0.59	75%
2	3	3	0%	57%	43%	75%	25%	0%	0%	38%	62%	1.951	0.65	75%
3	3	3	0%	0%	100%	75%	25%	0%	0%	95%	5%	2.034	0.68	75%
4	3	3	0%	95%	5%	75%	25%	0%	0%	0%	100%	2.084	0.69	75%
5	3	3	33%	67%	0%	13%	0%	87%	29%	53%	18%	2.191	0.73	20%
1	3	4	0%	50%	50%	75%	25%	0%	0%	45%	55%	1.784	0.59	75%
2	3	4	0%	48%	52%	75%	25%	0%	0%	47%	53%	1.951	0.65	75%
3	3	4	0%	0%	100%	75%	25%	0%	0%	95%	5%	2.034	0.68	75%
4	3	4	0%	13%	87%	75%	25%	0%	0%	82%	18%	2.084	0.69	75%
5	3	4	0%	0%	100%	35%	60%	5%	40%	60%	0%	2.177	0.73	40%
1	3	5	0%	73%	27%	75%	25%	0%	0%	22%	78%	1.784	0.59	75%
2	3	5	75%	25%	0%	0%	48%	52%	0%	47%	53%	1.951	0.65	75%
3	3	5	0%	48%	52%	75%	25%	0%	0%	47%	53%	2.034	0.68	75%
4	3	5	0%	73%	27%	75%	25%	0%	0%	22%	78%	2.084	0.69	75%
5	3	5	0%	0%	100%	34%	61%	5%	41%	59%	0%	2.171	0.72	41%

High Majority Democrat General Crossover $a_{nD}^2 = 0.5$.

Table 23: Districting Plans Maximizing Minority Total Benefits – General Crossover, open primaries.