Tokenization

The idea is to draw connections between RWA tokenization and the "classical" securitization following the work of :

DeMarzo & Duffie, a liquidity-based model of security design

DeMarzo, the pooling and tranching of securities...

Model : consider a private (or real world entity) trying to raise capital :

Capital raise $\xrightarrow{q_1}$ private placement through large instituition

Capital raise $\xrightarrow{q_2}$ RWA tokenization through blockchain (small investors)

Decision variables :

q_1 : amount (in %) raised from private placement (large)

q_2 : amount (in %) raised from RWA tokens (small)

Other quantities :

$P_1(q_1, q_2)$: equilibrium price from private placement

$P_2(q_1)$: " RWA tokenization

(Logic : for large investors, their price depend on both how much is set for larger & small investors.

For small investors (on RWA), they will look at large investors.)

$\tilde{f}$: random variable representing the asset set for capital

Objective :

discounting

$$\underset{q_1, q_2}{\mathrm{Opt}} \; -\delta(q_1+q_2)\tilde{f} + q_1 P_1(q_1, q_2) + q_2 P_2(q_1)$$

$$\text{s.t.} \quad P_1(q_1, q_2) = \tilde{f}(1-q_2)$$

large investors have "access" to the value of asset $\tilde{f}$

(if $q_2 = 0$, then the price $P_1 = \tilde{f}$.

$q_2 = 1$, then $P_1 = 0$ (since nothing from private placement)

Taking derivatives:

$$\begin{cases} -\delta\tilde{f} + P_1(q_1,q_2) + q_1\,\partial_1 P_1 + q_2\,P_2'(q_1) = 0 \\ -\delta\tilde{f} + q_1\,\partial_2 P_1 + P_2(q_1) = 0 \end{cases}$$

Replacing $\tilde{f} = \frac{\delta P_1}{1-q_2}$ (constraint):

$$\begin{cases} \frac{1-q_2-\delta}{1-q_2} P_1 + q_1\,\partial_1 P_1 + q_2\,P_2'(q_1) = 0 & (1) \\ -\frac{\delta}{1-q_2} P_1 + q_1\,\partial_2 P_1 + P_2(q_1) = 0 & (2) \end{cases}$$

$$\begin{array}{l} \partial_2 (1) \\ \partial_1 (2) \end{array} \begin{cases} \frac{\delta}{(1-q_2)^2} P_1 + \frac{1-q_2-\delta}{1-q_2}\,\partial_2 P_1 + q_1\,\partial_1\partial_2 P_1 + P_2'(q_1) = 0 \\ -\frac{\delta}{1-q_2}\,\partial_1 P_1 + \partial_2 P_1 + q_1\,\partial_1\partial_2 P_1 + P_2'(q_1) = 0 \end{cases}$$

$$\Rightarrow \quad \frac{P_1}{1-q_2} = \partial_2 P_1 - \partial_1 P_1$$

(2) - (1): $$-P_1 + q_1(\partial_2 P_1 - \partial_1 P_1) + P_2(q_1) - q_2 P_2'(q_1) = 0$$

$$\Rightarrow \quad \frac{1-q_1-q_2}{1-q_2} P_1 = P_2(q_1) - q_2 P_2'(q_1)$$

$$\begin{cases} \dfrac{1-q_1-q_2}{1-q_2} P_1 = P_2(q_1) - q_2 P_2'(q_1) \\ -\dfrac{\delta}{1-q_2} P_1 + q_1 \partial_2 P_1 + P_2(q_1) = 0 \end{cases}$$

$$\frac{-\delta}{1-q_1-q_2}\left(P_2(q_1) - q_2 P_2'(q_1) \right)$$
$$+ q_1 \left[-\frac{q_1}{(1-q_1-q_2)^2}\left(P_2(q_1) - q_2 P_2'(q_1) \right) - \frac{1-q_2}{1-q_1-q_2} P_2'(q_1) \right]$$
$$+ P_2(q_1) = 0$$

Set $q_2 = 0$:

$$\left(-\frac{\delta}{1-q_1} - \frac{q_1^2}{(1-q_1)^2} + 1 \right) P_2(q_1) - \frac{q_1}{1-q_1} P_2'(q_1) = 0$$

$$q_1 P_2'(q_1) = \left(-\delta + 2 - \frac{1}{1-q_1} \right) P_2(q_1)$$

[Fact: $x f'(x) + \left(\frac{1}{1-x} + a \right) f(x) = 0$
$\Longleftrightarrow f(x) \propto (1-x) x^{-a-1}$]

$$\Rightarrow \quad P_2(q_1) \propto (1-q_1) q_1^{1-\delta}$$

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