IEORE 8100: Advanced Combinatorial Optimization Spring 2025

Syllabus

IEOR, Columbia University

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Course Website. Here.

Content. Combinatorial optimization is a branch of optimization with connections to areas of purely theoretical research such as graph theory, theoretical computer science, and combinatorics, and is used in applications such as market design, logistics, and machine learning, among others.

This course is intended for Ph.D. students who have already gained some basic knowledge in combinatorial optimization (see "Prerequisites" below) and are interested in learning about advanced problems, algorithms, and techniques for the study of combinatorial optimization problems. We will cover a selected list of advanced topics in the area, chosen among those that are at the forefront of current research and/or have most connections with applications.

In particular, the class will be composed of the following 3 sections:

1. **Matching and flows**. Because of their nice mathematical structure and ubiquity in applications, matching and flow problems have been, and still are, a cornerstone of combinatorial optimization. As such, they have been a playground for the development of many algorithmic ideas. We will focus in particular on the following topics:
   (a) Maximum matchings via polynomial identity testing and random walks;
   (b) Extensions of Hall's conditions for matchings;
   (c) Flows: min-cost via capacity scaling, multi-commodity, submodular;
   (d) From polynomial to strongly polynomial: Frank-Tardos' method.
2. **Submodularity**. Submodular functions are set functions that enjoy the diminishing return property. Structural properties of submodular functions and algorithms for optimizing submodular functions have seen tremendous work in the last 30 years, and have been very impactful in applications. We plan to cover the following topics:
   (a) Basics of submodular optimization: base polytope, greedy algorithm;
   (b) Algorithms to minimize submodular functions;
   (c) Algorithms to maximize submodular functions;
   (d) Relationship with discrete convexity and gross substitute, with applications to market design;
3. **Polyhedral combinatorics**. Central to polyhedral combinatorics are the questions of how to represent and solve discrete optimization problems using linear programming. We will see techniques to construct "small" formulations of discrete optimization problems (or proof of the impossibility thereof), and examples of formulations designed at to obtain efficient algorithms. We plan to cover the following topics:
   (a) Basics: polyhedra, faces, facets;
   (b) Iterative methods;

(c) Extended formulations: Yannakakis' factorization theorem, non-existence of compact formulations for the TSP polytope;

(d) Decomposition of TU matrices and their application to solve Integer Programs with subdeterminant at most 2.

Grading. Grading will be based on:

20% Scribe notes. Every student will be asked to scribe one class.

20% 1-2 Assignments on the topics covered in class.

60% Final project and presentation. The final project will be either a presentation of a relevant research paper related to the topics covered in class, or an original research contribution, again related to the topics of the class.

Prerequisites. Linear programming. Basic knowledge in combinatorial optimization (such as shortest paths, bipartite matching, total unimodularity,...) and computational complexity.

Study materials. Research articles and scribe notes.